Precipitation Susceptibility and Aerosol Buffering of Warm- and Mixed-Phase Orographic Clouds in Idealized Simulations

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ABSTRACT

The sensitivity of warm- and mixed-phase orographic precipitation to the aerosol background with simultaneous changes in the abundance of cloud condensation nuclei and ice nucleating particles is explored in an idealized, two-dimensional modeling study. The concept of precipitation susceptibility \( \frac{d\ln P}{d\ln N} \), where \( P \) is the precipitation mixing ratio and \( N \) the cloud droplet number, is adapted for orographic clouds. Precipitation susceptibility is found to be low because perturbations to different precipitation formation pathways compensate each other. For mixed-phase conditions, this in particular means a redistribution between warm and cold pathways. The compensating behavior is described as a consequence of a balance equation for the cloud water along parcel trajectories that constrains the total precipitation formation to match the drying from condensation and vapor deposition on ice-phase hydrometeors caused by the mountain flow. For an aerosol-independent condensation rate (saturation adjustment), this balance requirement limits aerosol impacts on orographic precipitation (i) to the evaporation of hydrometeors and (ii) to the glaciation state of the cloud, which controls the contribution of vapor deposition to drying. The redistribution of precipitation formation pathways is coupled to a redistribution of the total hydrometeor mass between hydrometeor categories. Aerosol effects on the glaciation state of the cloud enhance this redistribution effect such that liquid and ice adjustments do not compensate. For the externally constrained, fully adjusted steady-state situation studied, precipitation susceptibility quantifies the redistribution effect rather than changes in precipitation production as in previous studies.

1. Introduction

Orographic clouds are especially prone to exhibit aerosol–cloud–precipitation interactions because the mountain flow limits the time available for precipitation formation. By controlling the time scale at which small cloud hydrometeors are converted into precipitation hydrometeors, aerosol perturbations can shift the horizontal locations where hydrometeor types occur. Shifting the location of precipitation controls the leeward-precipitation fraction (spillover factor); shifting the location of small hydrometeors affects the extent of reevaporation (drying ratio). Aerosol effects on orographic precipitation are typically studied either in the context of deliberate cloud seeding, where the abundance of ice-formation aerosol [ice nucleating particles (INPs)] is increased (e.g., Xue et al. 2013; Geresdi et al. 2017), or in the context of anthropogenic pollution, which is usually assumed to be represented by an increase in cloud droplet–forming aerosol [cloud condensation nuclei (CCN); e.g., Saleeby et al. 2009, 2011]. For warm orographic clouds, increases in CCN lead to more but smaller cloud droplets, which decrease the autoconversion efficiency and thus lead to a delay in precipitation formation, equivalent to the mechanism discussed by Albrecht (1989) for shallow maritime clouds. Miltenberger et al. (2015) showed that the warm orographic precipitation efficiency, defined as the fraction of cloud water that is converted into precipitation, scales with the ratio of an advective time scale and a time scale of microphysical conversion, which is influenced by the

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abundance of CCN. According to this scaling relationship, precipitation efficiency increases for decreasing droplet, or CCN, numbers for low and intermediate precipitation efficiencies. At high precipitation efficiencies, where the time for precipitation formation is always sufficiently long, the aerosol effect levels off.

In polluted mixed-phase clouds, the slowing effect of decreased droplet size on autoconversion is replaced by a slowing effect on riming (Borys et al. 2003). The corresponding delay in precipitation formation and spillover effect is found to be enhanced by the slower fall speeds and longer vertical trajectories of lightly rimed ice (Saleeby et al. 2009, 2011). Although it is statistically difficult to assess the success of deliberate cloud seeding of orographic clouds with INPs in the field (Chu et al. 2017), modeling studies generally find precipitation enhancement for stratiform clouds. Increased depletion of the water vapor (increased drying) by diffusional growth of ice-phase hydrometeors has been identified as the most important contribution to precipitation enhancement (Xue et al. 2013; Geresdi et al. 2017). Similar to the warm case, seeding is found to be less efficient for higher precipitation efficiencies in these studies.

In general, perturbations to the liquid- and ice-phase pathways to precipitation formation cannot be discussed separately. Atmospheric aerosol usually provides CCN and INPs simultaneously, and the interaction of liquid and ice microphysics is the very essence of mixed-phase clouds. As discussed, INPs and the ice-phase pathway of vapor deposition tend to accelerate and increase precipitation formation, while CCN and droplet-collection processes like autoconversion and riming are associated with fewer and later precipitation formation. In accordance with this complementary behavior, the response of mixed-phase clouds to simultaneous CCN and INP perturbations has been found of inconclusive sign and is in general small (Mühlbauer et al. 2010). Next to the opposing effects of CCN and INP perturbations, the complexity of mixed-phase cloud microphysics itself predisposes compensating responses of different processes to aerosol perturbations and buffering behavior (Glassmeier and Lohmann 2016).

This study aims to clarify how different pathways to precipitation formation interact in their response to aerosol perturbations and identify orographic controls of this interplay. In our approach, we adapt the concept of precipitation susceptibility to orographic clouds. In its original form (Feingold and Siebert 2009), precipitation susceptibility $s$ is a vertically averaged concept and quantifies the relative change $d\ln R = dR/R \approx \Delta R/R$ in warm rain rate $R$ that follows from a relative change in cloud droplet number concentration $N_{cl}$ (Feingold and Siebert 2009),

$$ s = \frac{d\ln R}{d\ln N_{cl}} = \frac{\partial \ln R}{\partial \ln N_{cl}} \bigg|_{\text{LWP}} + \frac{\partial \ln R}{\partial \ln \text{LWP}} \bigg|_{N_{cl}} \frac{d\ln \text{LWP}}{d\ln N_{cl}}, \quad (1) $$

such that the numerical value of $s$ indicates the percentage change in $R$ that follows from a 1% change in $N_{cl}$. The right-hand side of Eq. (1) separates the total precipitation susceptibility into the partial precipitation susceptibility when fixing the liquid water path (LWP; indicated by |LWP) and the secondary effect, or adjustment, of changes in LWP that are typically triggered by a perturbation in $N_{cl}$. Precipitation susceptibility is usually interpreted in relation to the sensitivities of microphysical conversion rates. For global models with diagnostic precipitation, precipitation susceptibility corresponds to the autoconversion exponent (Stevens and Feingold 2009). For prognostic precipitation, precipitation susceptibility decreases as warm rain formation transitions from droplet-number-dependent autoconversion to droplet-number-independent accretion (at fixed liquid water content; Sorooshian et al. 2009; Wood et al. 2009; Glassmeier and Lohmann 2016). If aerosol perturbations do not trigger adjustments in the amount of cloud ice (i.e., the glaciation state of the cloud), mixed-phase clouds are expected to feature lower precipitation susceptibilities than warm clouds. This is a consequence of the compensations discussed above and of riming not being as sensitive to changes in droplet number as autoconversion (Glassmeier and Lohmann 2016). Strong precipitation susceptibilities in mixed-phase clouds can be observed when the aerosol perturbation triggers glaciation adjustments (Glassmeier and Lohmann 2016). Precipitation susceptibility can thus qualitatively capture the sensitivities of precipitation efficiency to aerosol perturbations.

The intuitive arguments above are based on the assumption that precipitation susceptibility $s$ is dominated by the first term on the right-hand side of Eq. (1). As adjustments require time to respond to the perturbation, this assumption is expected to be justified on a short time scale after a perturbation. In contrast, orographic clouds can be idealized as steady-state systems that have fully adjusted to applied perturbations. These adjustments can lead to counterintuitive sensitivities. For an illustration, consider a box model of a cloud in which cloud water $C$ is created by condensation (cond) and converted with a parameter $\alpha$ into precipitation hydrometeors $P$, which are depleted by sedimentation with a sedimentation velocity $\sigma$:

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1 We have omitted the conventional minus sign in our definition of precipitation susceptibility because this minus was motivated by the use for warm clouds.
\[
\dot{C} = \text{cond} - \alpha CP, \tag{2}
\]
\[
\dot{P} = -\sigma P + \alpha CP. \tag{3}
\]

The steady-state solution \( \dot{C} = \dot{P} = 0 \) of this system is \( C = \frac{\alpha}{\alpha} P = \text{cond}/\alpha \). A decrease in the background aerosol corresponds to an increase of \( \alpha \). On the short time scale, Eq. (3) indicates an increase in \( P \) for increasing \( \alpha \). When the system has readjusted to a steady state, \( P \) will have the same value that it had before the perturbation. More realistic versions of such box models take into account nonlinear conversion rates from \( C \) to \( P \). The paper is organized as follows: In section 2, we describe the numerical model and its setup for this study. The discussion of our results in section 3 starts with a qualitative discussion of aerosol effects on orographic clouds from the trajectory-averaged perspective, continues with implications of the cloud water budget along trajectories, and concludes with the discussion of orographic precipitation susceptibilities. Section 4 provides a discussion and summary of the results. The appendix provides a list of symbols and acronyms.

2. Model and simulations

We use the nonhydrostatic limited-area atmospheric model of the Consortium for Small Scale Modelling (COSMO; Baldauf et al. 2011) in its 2D setup and make use of the Aerosols and Reactive Trace Gases (ART) extension of the model (COSMO-ART; Vogel et al. 2009), which comprises aerosol–cloud interactions based on chemical and physical properties of aerosol populations (Bangert et al. 2011, 2012) with the M7 aerosol scheme (COSMO-ART-M7; Glassmeier et al. 2017). As the present study focuses on the aerosol effect on precipitation formation rather than on activation and freezing, we prescribe measurement-based aerosol fields (section 2b) rather than dynamically calculating its evolution from emissions. Therefore, we disable aerosol microphysical evolution (i.e., condensation, coagulation, sedimentation, and washout). Aerosol activation follows Fountoukis and Nenes (2005) and is based on a CCN spectrum derived from Köhler theory (Köhler 1936) in combination with a population-splitting approach to solve the supersaturation budget (Nenes and Seinfeld 2003). Improved in-cloud activation is included according to Barahona et al. (2010) by treating existing droplets as giant CCN. Following Glassmeier et al. (2017), we assume immersion freezing to be the sole ice formation process in mixed-phase clouds. We consider dust as immersing by using the parameterization of Phillips et al. (2008). Immersion freezing is considered the most important ice formation process (Kanji et al. 2017) for mixed-phase clouds. Deposition nucleation is only important in the absence of liquid water (i.e., for cirrus clouds). Contact nucleation is poorly understood and could potentially be important for mixed-phase clouds under specific conditions (Hande et al. 2017). For the simulated case, the exclusion of contact freezing is no limitation because the case does not feature uncoated INPs, which are required to initiate contact freezing in existing parameterizations. We do not consider ice multiplication. Aerosol is not scavenged by droplet activation. Instead, CCN and INP depletion is implemented as a number adjustment. We employ the two-moment cloud microphysics scheme of Seifert and Beheng (2006, hereafter SB) in the updated version of Noppel et al. (2010) with five hydrometeor classes (cloud water, ice, rain, snow, and graupel). While vapor deposition on ice-phase hydrometeors is explicitly described in the scheme, condensation is implemented by means of a saturation adjustment. This has implications for the Wegener–Bergeron–Findeisen (WBF) process (Wegener 1911; Bergeron 1935; Findeisen 1938), which describes the growth of ice-phase hydrometeors at the expense of cloud droplets because saturation vapor pressure with respect to water is higher than that with respect to ice. The WBF process is technically separated into two steps: First, vapor deposition on ice is calculated, and the corresponding depletion of water vapor leads to subsaturation with respect to water. Later in the time step, the saturation adjustment transfers cloud water to the vapor phase to reach water saturation. Radiation is not considered.

a. Setup

Following Miltenberger et al. (2015), we use a setup with 150 vertical and 750 horizontal grid points at a domain height of 22 km and a horizontal resolution of 1 km. The time step is 6 s. A bell-shaped hill with a maximum height of \( h_{\text{max}} = 0.8 \) km and a half-width \( a = 20 \) km is placed in the domain center (cf. Fig. 3). The corresponding orography profile is given by
\[ h(d) = \frac{h_{\text{max}}}{1 + (d/a)^2}, \]

where \( d \) denotes the horizontal distance from the center.

Our initial and boundary conditions are motivated by Muhlbauer et al. (2010) and are illustrated in Fig. 1. The temperature profile corresponds to a dry Brunt–Väisälä frequency of \( N_d = 0.018 \text{s}^{-1} \) with a surface temperature of \( T_s = 296.15 \text{K} \) for the warm case and \( N_d = 0.013 \text{s}^{-1} \) with \( T_s = 270.15 \text{K} \) for the mixed-phase cloud. The moisture profile is in both cases given by the following expression for the saturation ratio:

\[ S(z) = a + \frac{b - a}{1 - \exp[-c(z - z_0)]}, \]

where \( z \) denotes the metric height and the parameters take the values \( a = 0.95, \ b = 0.03, \ c = 0.0015 \text{m}^{-1} \), and \( z_0 = 6000 \text{m} \). The wind profile has no updraft component and is vertically constant with a value of \( 20 \text{m s}^{-1} \) in the analysis region. Above, it increases linearly toward a value of \( 40 \text{m s}^{-1} \) at the model top.

We run this setup for 50 h. We allow the system to equilibrate for 30 h and base our analysis on 15-min output from the remaining 20 h. We average temporally to obtain steady-state fields.

In the steady-state situation the boundary conditions are chosen to obtain a linear mountain wave [see the updraft profile (black contours) in Fig. 3] and a stratification that is stable enough to prevent convection. The profiles of equivalent potential temperature \( \theta_e \) indicate stability (Cotton and Anthes 1992):

\[ \theta_e = T \left( \frac{p_0}{p_d} \right) R_d (c_{pd} + c_{qi}) \exp \left[ \frac{L_v q_v - L_f q_i}{(c_{pd} + c_{qi}) T} \right], \]

where \( T \) denotes the temperature; \( p_0 = 1000 \text{hPa} \) the reference pressure; \( p_d \) the pressure of dry air; \( R_d \), the gas constants of dry and moist air; \( c_{pd} \) the heat capacities at constant pressure of dry air and of water; \( q_{vi} \), the mixing ratios of water vapor, ice, and total water; and \( L_{vi} \), the latent heat of vaporization and fusion. While the thermodynamic initial and boundary profiles result in a stable stratification for the mixed-phase cloud, the high surface temperatures of the warm profile result in a slightly unstable stratification, which does not affect the subsequent analysis. The high surface temperatures are necessary to ensure that the cloud top lies at temperatures warmer than the freezing level.

b. Aerosol perturbations

In addition to the dynamic and thermodynamic profiles, an aerosol composition is prescribed at the left boundary of the domain. The aerosol composition is fixed but transported with the flow. Following Muhlbauer et al. (2010), we apply a vertically constant aerosol profile based on point measurements from the high-alpine station Jungfraujoch (JFJ) in Switzerland for clean and polluted conditions as summarized in Table 1.

Total aerosol mass and number concentrations have been derived by Muhlbauer et al. (2010) based on measurements from Weingartner et al. (1999). We assume that all aerosol has been coated during transport to the remote location JFJ. Since no significant contribution of the coarse mode to the climatological values was observed and nucleation mode aerosol neither directly nor indirectly affects cloud microphysics in our setup, the relevant aerosol composition is thus described by a coated Aitken and a coated accumulation mode. Similar to Muhlbauer et al. (2010), we base the chemical composition of these modes on Cozic et al. (2008). We assume the following mass ratios of black carbon (BC) to organic carbon (OC) to sulfate (SU) aerosol:
In the accumulation mode, we distinguish natural and anthropogenically influenced aerosol composition by replacing a part of the sulfate mass $M_{SU}$ by dust mass $M_{DU}$ such that the fraction

$$f_{SU} = \frac{M_{SU}}{M_{SU} + M_{DU}}$$

(8)
takes values in between $f_{SU} = 0.6$ (natural) and $f_{SU} = 0.9$ (anthropogenic) and Eq. (7) corresponds to $f_{SU} = 1$.

Our investigations are based on 29 simulations with different aerosol conditions. These are constructed by linear interpolation between the clean–natural and the polluted–anthropogenic aerosol conditions with respect to mass concentration, number concentration, and chemical composition: Each aerosol condition is described by a vector $(f_{SU}, f_N, f_M)$. The chemical dimension is described by $f_{SU}$ and varied between the natural and anthropogenic composition, $0.6 \leq f_{SU} < 0.9$. The weighting of clean $C = (N_c, M_c)$ and polluted $P = (N_p, M_p)$ mass and number concentrations at fixed composition is described by $f = (f_N, f_M)$ according to $(N, M) = fP + (1 - f)C$, where $f_M$ and $f_N$ vary between 0 and 1.

Figure 2 illustrates the resulting three-dimensional parameter space. The choice of the parameter space is, on the one hand, motivated by capturing the variability in the two prognostic moments of the aerosol distribution. By varying the composition in terms of SU and DU, we, on the other hand, try to change the effective ratio of CCN and INPs and thus the relative importance of the ice- and warm-phase pathways to precipitation formation. Table 2 provides a key to some specific aerosol conditions.

### Table 1. Measurement-based aerosol conditions for clean and polluted situations based on Muhlbauer et al. (2010) with natural ($f_{SU} = M_{SU}/(M_{SU} + M_{DU}) = 0.6$) and anthropogenic ($f_{SU} = 0.9$) partitioning of accumulation-mode sulfate mass $M_{SU}$ and dust mass $M_{DU}$ [cf. Eq. (8)].

<table>
<thead>
<tr>
<th>Mode</th>
<th>Situation</th>
<th>Number ($10^8$ m$^{-3}$)</th>
<th>Total</th>
<th>SU</th>
<th>OC</th>
<th>BC</th>
<th>DU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aitken</td>
<td>Clean–natural</td>
<td>3.1</td>
<td>0.07</td>
<td>0.0287</td>
<td>0.0385</td>
<td>0.0028</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>Polluted–anthropogenic</td>
<td>5.3</td>
<td>0.26</td>
<td>0.1066</td>
<td>0.1430</td>
<td>0.0104</td>
<td>—</td>
</tr>
<tr>
<td>Accumulation</td>
<td>Clean–natural</td>
<td>0.4</td>
<td>0.44</td>
<td>0.1082</td>
<td>0.2420</td>
<td>0.0176</td>
<td>0.0722</td>
</tr>
<tr>
<td></td>
<td>Polluted–anthropogenic</td>
<td>2.6</td>
<td>1.74</td>
<td>0.6421</td>
<td>0.9570</td>
<td>0.0696</td>
<td>0.0713</td>
</tr>
</tbody>
</table>

3. Results

Figure 3 shows the warm- and mixed-phase cloud that form by orographic lifting at the upwind slope of the mountain. Temperatures $T$ in the warm cloud are everywhere above the freezing level (Fig. 3a), while temperatures in the mixed-phase cloud take values in between the subfreezing surface temperature and the onset temperature of homogeneous freezing (Fig. 3b). The microphysical cloud processes are shown in detail in Figs. 4a and 5a.

The clouds form by activation (act) of aerosol particles to cloud droplets at the windward rim, especially in the lower half with stronger updrafts. Note that activation regions with apparently vanishing cloud mixing ratio occur because microphysical rates are diagnosed before and cloud variables after tracer transport in the time step. In-cloud activation creates additional droplets at a horizontal distance of $\approx 350$ km from the domain boundary where the mountain slope is steepest and causes high supersaturations.

Some of the droplets activated above a height of 2 km in the mixed-phase cloud are converted into ice crystals by immersion freezing (freez). In contrast to cloud
droplets, ice crystals have a nonnegligible sedimentation velocity. Sedimenting ice crystals form the lower-left part of the cloud in Fig. 5a that occurs downwind of activation and freezing. Sedimentation does not proceed vertically but at an angle because of the horizontal flow, as can also be observed for hydrometeors sedimenting downwind of the mountain (cf. Fig. 3).

Droplets and crystals grow by cond and vapor deposition on ice-phase hydrometeors (diff), respectively. Both processes together deplete the water vapor mixing ratio $M_{\text{v}}$, and their combined rate will be denoted as $M_{\text{v}}$ sink (Figs. 4a and 5a). Coagulation (coag) converts small hydrometeors (i.e., cloud droplets and ice crystals) into large hydrometeors (i.e., raindrops, snowflakes, and graupel particles). Rain formation in the warm cloud proceeds via accretion (acc) and autoconversion (aut). The former is shifted downwind as compared to the latter (Fig. 4a) because accretion requires raindrops, which are initially produced by autoconversion. The glaciation of the mixed-phase cloud proceeds via the WBF process (Fig. 5a) and riming. WBF regions correspond to regions where $M_{\text{v}}$ sink $<$ diff.

The liquid saturation adjustment applied in the model ensures saturation ratios corresponding to water saturation, $S = 1$, in the warm cloud (not shown). This condition is also fulfilled in the major part of the mixed-phase cloud as shown in Fig. 5b. Only the outermost rim of the downwind side of the cloud at 380–400 km is completely glaciated.

To summarize, the mixing ratio of cloud water in the droplet category of the microphysics scheme $M_{\text{d}}$ is gradually transferred into the rain category $M_{\text{prl}}$ in the warm cloud while the flow passes the mountain. The degree of this conversion can be characterized by the precipitation fraction $R_{\text{prl}} = \frac{M_{\text{prl}}}{M_{\text{prl}} + M_{\text{cl}}}$.

$$R_{\text{glac}} = \frac{M_{\text{ci}} + M_{\text{prs}} + M_{\text{prg}}}{M_{\text{cl}} + M_{\text{ci}} + M_{\text{pr}}},$$

which is illustrated in Fig. 4b (shaded regions). For the mixed-phase cloud, cloud water $M_{\text{cl}}$ is converted into ice $M_{\text{ci}}$, rain $M_{\text{prl}}$, snow $M_{\text{prs}}$, and graupel $M_{\text{prg}}$. This conversion is described by the glaciation fraction $R_{\text{glac}}$.

The extent of the rain-dominated region defined by $R_{\text{pr}} > 0.5$ (cf. Fig. 4b) and of mixed-phase regions with $0.1 < R_{\text{glac}} < 0.9$ (cf. Fig. 5b) change with the aerosol background: Precipitation formation is delayed and glaciation sets in earlier for polluted conditions, as is expected from increased abundance of CCN and INPs. For the mixed-phase case, the location and extent of the cloud vary with the aerosol condition, while a corresponding effect is absent for the warm cloud (cf. red and blue contours in Figs. 4b and 5b). Changes in the extent of the mixed-phase cloud are caused by corresponding changes in the flow field (not shown). We find that the variability in the cloud extent and flow field is well correlated with the abundance of dust aerosol ($r^2 \approx 0.8$ in a bilinear fit of cloud extent as a function of total dust mass concentration and the number concentration of dust containing particles; not shown). As dust is a proxy for the abundance of INPs in different aerosol conditions, this correlation supports the concept that the changes in the flow field can be attributed to increased latent heat
release for more glaciated conditions and are not the result of nonsystematic variability in the quasi-steady state. We thus find a thermodynamic effect of INPs on the orographic mountain flow. This change on the flow does not affect our analysis.

a. Trajectory averages

Performing a susceptibility analysis on high-resolution simulation output bears two problems. First, if a variable is more strongly influenced by advection than by microphysics, susceptibilities would be artificially small because the former is largely independent of the details of microphysics. Second, temporal delays (e.g., between the influence of aerosol on activation and precipitation formation) are not taken into account. Both issues are alleviated by choosing a Lagrangian framework for the analysis. Besides the use of cloud parcel models (Sorooshian et al. 2009; Feingold et al. 2013), the use of vertical averages or vertically integrated quantities like LWP (Jiang et al. 2010; Sorooshian et al. 2010; Duong et al. 2011; Terai et al. 2012; Jung et al. 2016; Dadashazar et al. 2017) roughly corresponds to averaging over the adiabatic trajectories of rising parcels in vertically developing clouds. For the horizontal development of orographic clouds, vertical averages would not approximate averages along parcel trajectories. Trajectories of orographic cloud parcels approximately follow lines of constant equivalent potential temperature $u_e$. Averaging along the moist isentropes of parcel ascent can thus be replaced by averaging over the moist isentropes of mountain flow. The parcel approach cannot account for vertical mixing and the sedimentation of precipitation hydrometeors. Our analysis assumes that both effects can be neglected in comparison to the transport along the trajectories. Vertical mixing is expected to be small because of the quasi-stable stratification of our setup. The dominance of horizontal as compared to vertical transport of precipitation hydrometeors is illustrated by the flat slopes of falling precipitation in Fig. 3.

Our analysis is based on normalized sums of cloud properties $X^a(\theta_e, l)$,

$$\langle X^a \rangle(\theta_e) = \frac{1}{L(\theta_e)} \sum_l X^a(\theta_e, l), \quad (11)$$

where $l$ denotes the position along a trajectory and the superscript $a$ indicates the different aerosol conditions. We normalize the trajectory sum with the aerosol-independent length $L(\theta_e)$ that counts the number of grid boxes that the $\theta_e$ trajectory of the reference cloud (obtained as average over all aerosol conditions) runs within the cloud:

$$L(\theta_e) = \sum_l C(\theta_e, l), \quad \text{where}$$

$$C(\theta_e, l) = \begin{cases} 1 & \text{for } M_{\text{ci}}^{\text{ref}} > 10^{-10} \text{kg kg}^{-1} \\ 0 & \text{otherwise} \end{cases} \quad (12)$$

We choose to normalize the trajectory sum to obtain meaningful values for the cloud properties. We use an aerosol-independent normalization, rather than calculating a conventional average, which amounts to normalizing with the aerosol-dependent number of cloudy grid points for a given aerosol condition, so that we do not mask the aerosol-dependent variability in trajectory...
lengths. As illustrated in Fig. 3, trajectories, or lines of constant \( u_e \), respectively, cross the mountain horizontally. The variables \( l \) and \( u_e \) can thus be considered a horizontal and vertical coordinate, respectively, such that the normalized sum \( h/C_1 i \) can be considered a horizontal average that results in profiles of cloud properties.

Figures 6–8 show such trajectory profiles of cloud characteristics for selected aerosol conditions obtained in this way. A comparison of the curves for the “base” and “M poll” cases, which lead to comparable cloud conditions, illustrates that aerosol mass and number perturbation have comparable effects on the profiles. Comparing the polluted–anthropogenic to polluted aerosol conditions shows that aerosol composition mainly matters for the glaciated part of the cloud, where it controls the amount of INP-active dust.

For the warm cloud, the average condensation rate, which corresponds to the sink of water vapor \( M_y \) in the absence of ice-phase diffusional growth, decreases as the height of the trajectories increases but is independent of the aerosol condition (Fig. 6b). As a result, the mixing ratios of cloud water \( M_{cl} \) and rainwater \( M_{prl} \) likewise decrease with height (Figs. 6c,d). As expected, droplet number \( N_{cl} \) increases with increasing pollution (Fig. 6e). The size of droplets as measured by their average mass \( M_{cl}/N_{cl} \) decreases (Fig. 6f) although cloud water \( M_{cl} \) increases. For polluted aerosol conditions, \( M_{pl} \) is slightly decreased (Fig. 6c).

Despite the aerosol-induced differences in the flow pattern and extent of the mixed-phase cloud (Fig. 5b), the trajectory-averaged updraft \( W \) turns out to be independent of the aerosol condition, as is also the case in the warm cloud (Figs. 6a and 7a). Nevertheless, the vapor depletion rate, \( M_y \), sink, is slightly decreased for clean as compared to polluted conditions in the upper, glaciated part of the cloud (Fig. 7b). Here, an increase in aerosol means an increase in glaciation, which means an increase in diffusional vapor deposition and thus an increase in overall vapor depletion.

For the mixed-phase cloud, we distinguish cloud water from sedimenting hydrometeor classes, \( M_{ci}+pr = M_{ci} + M_{prl} + M_{prs} + M_{prg} \), with the mixing ratios of ice crystals \( M_{ci} \), rain \( M_{prl} \), snow \( M_{prs} \), and graupel \( M_{prg} \). We may interpret \( M_{ci+pr} \) as precipitation: even if small ice crystals constitute cloud rather than precipitation, they rapidly grow by vapor deposition and coagulation with larger hydrometeors after sedimenting into the lower part of the cloud. In the lower part of the cloud, differences in \( M_{ci+pr} \) are dominated by rain with a compensation from graupel (Figs. 8d,e). The upper part is
dominated by $M_{ci}$ (Fig. 8a). Snow qualitatively follows the behavior of $M_{ci}$ but is quantitatively less relevant (Figs. 8a,f).

The lower part of the mixed-phase cloud depends on aerosol conditions in a way qualitatively similar to the warm cloud when considering $M_{ci}$ as precipitation mixing ratio (Figs. 6c and 7c). Quantitatively, the precipitation response is more pronounced in the mixed-phase case than in the warm case. As glaciation increases with increasing height and decreasing temperature, the signals in $M_{ci}$ and $M_{ci+pr}$ to increasing pollution change their sign: pollution increases the number concentration of INPs and leads to an increased ice crystal number concentration $N_{ci}$ and mixing ratio $M_{ci}$ (Figs. 8a,b). As cold microphysics increases in importance, cloud liquid is depleted (Fig. 7d).

The signal in droplet size remains decreasing for increasing pollution (Fig. 7f), while ice crystal size features three regimes (Fig. 8c): Crystal size decreases with increasing pollution in the lower and upper parts of the cloud. In between, a sudden increase in ice mixing ratio with height and pollution, which coincides with the shifting location and extension of the mixed-phase region (Fig. 5), corresponds to increasing crystal sizes.

To summarize, the trajectory profiles show that the response of the precipitation mixing ratio to aerosol perturbations in the warm as well as in the mixed-phase orographic cloud is buffered as compared to the signals in cloud water and cloud ice.

b. Cloud water budgets

The buffered precipitation response described in the previous section can be understood as a consequence of balancing the budget of cloud water $M_{ci}$ in the cloud (i.e., the need of sources and sinks of cloud water to compensate for all aerosol conditions). The main source of $M_{ci}$ is cond. In mixed-phase clouds, the melting (melt) of small ice crystals provides an additional, smaller source. Possible sink processes are aut, acc, ice–droplet riming (ic-rim), snow–droplet riming (sc-rim), graupel–droplet riming (gc-rim), and evaporation (evp), which includes the WBF process, and the freezing of cloud
droplets. As sedimentation is negligible for cloud droplets and entrainment of cloud water is not parameterized in the model used for this study, these processes do not affect the cloud water budget. For normalized sums [Eq. (11)] along trajectories, we can thus study the following balanced cloud water budget $B_{cl}$:

$$0 = \langle B_{cl} \rangle,$$

$$= \langle \text{cond} \rangle + \langle \text{melt} \rangle - \langle \text{evp} \rangle - \langle \text{aut} \rangle - \langle \text{acc} \rangle - \langle \text{ic-rim} \rangle - \langle \text{sc-rim} \rangle - \langle \text{gc-rim} \rangle - \langle \text{freez} \rangle$$

$$= \langle M_{v} \text{ sink} \rangle - \langle (i\text{-diff}) + (s\text{-diff}) + (g\text{-diff}) + (\text{aut}) + (\text{acc}) + (\text{ic-rim}) + (\text{sc-rim}) + (\text{gc-rim}) \rangle,$$

$$= \langle M_{v} \text{ prod} \rangle.$$
where we have substituted \( \langle \text{cond} \rangle = \langle M_c \text{ sink} \rangle - \langle i\text{-diff} \rangle - \langle g\text{-diff} \rangle \) and neglected freezing, melting, and evaporation to obtain the approximate expression in the third row of Eq. (13). Freezing can be neglected because the size of freezing cloud droplets is very small compared to the typical sizes that ice crystals quickly reach by vapor deposition. Melting and evaporation can be neglected in our case because our setup assumes subfreezing surface temperatures and because it features a complete conversion of cloud water. All the sink terms are sources of the precipitation mixing ratio \( M_{ci+pr} \) such that they constitute the total precipitation production (precip prod). This balance is approximately met by our simulations (cf. graphs of \( M_c \) sink and precip prod in Figs. 9a and 10a).

As discussed in the context of Figs. 6 and 7, the source term \( \langle M_c \text{ sink} \rangle \) in Eq. (13) is independent of the aerosol background for the warm cloud and in the lower part of the mixed-phase cloud. When comparing two simulations with different aerosol conditions, an aerosol-induced change to one of the sink processes in Eq. (13) thus has to be compensated by an opposite change in another sink process to ensure that their sum still matches the constant source term. In the upper part of the mixed-phase cloud, \( \langle M_c \text{ sink} \rangle \) increases with increasing glaciation, and aerosol-induced changes in

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**Fig. 7.** As in Fig. 6, but for the mixed-phase cloud and with the precipitation mixing ratio \( M_{ci+pr} \) instead of rain.
precipitation formation pathways are constrained to be consistent with this increase.

The compensating behavior of the different processes contributing to precipitation production is illustrated in Figs. 9b and 10b: For the warm cloud, a decrease in autoconversion with increasing pollution is compensated by a matching increase in accretion (Fig. 9b). Comparing the vertical difference profiles of microphysical rates in the mixed-phase cloud illustrates that changes in autoconversion and cloud–droplet riming compensate each other (Fig. 10b). This shift is driven by an increased abundance of ice. Pollution-induced shifts from graupel to snow lead to compensating changes in snow–droplet and graupel–droplet riming. Changes in accretion seem to correspond to a combination of changes in graupel–cloud riming and diffusional growth. In the upper part of the cloud, a partial compensation between changes in ice–droplet riming and diffusion may be identified. The compensating tendencies between microphysical rates are not restricted to the relationship between the two aerosol conditions compared in Figs. 9 and 10. For most trajectories, we find strong negative correlations (coefficient of determination $r^2 > 0.8$) between aerosol-induced changes in the rates of
The droplet water budget of the warm cloud consists of the vapor depletion rate ($M_v$ sink) and the rates of autoconversion (aut) and accretion (acc), whose sum amounts to the total rain production $M_{ci+pr}$ for trajectory profiles of (a) these rates for the “base” aerosol condition (Table 2) and (b) their differences between polluted ($P$) and clean ($C$) aerosol conditions ($P - C$).
FIG. 10. As in Fig. 9, but for the mixed-phase cloud and with additional rates of ice–cloud riming (ic-rim), snow–cloud riming (sc-rim), graupel–cloud riming (gc-rim), diffusional growth of ice (i-diff), snow (s-diff) and graupel (g-diff) and the summed diffusional growth (diff).
FIG. 11. Total precipitation susceptibilities of rainwater mixing ratio $M_{pr}$ to (a) droplet number $N_{cl}$ for the warm cloud and of the precipitation mixing ratio $M_{ci+pr}$ to (c) droplet number $N_{cl}$ and (d) crystal number $N_{ci}$. Also shown are (b),(e) the cloud liquid adjustment to droplet number, $d\ln M_{cl}/d\ln N_{cl}$ in the (b) warm- and (e) mixed-phase cloud, and (f) the ice crystal number adjustment to droplet number $d\ln N_{ci}/d\ln N_{cl}$ in the mixed-phase cloud. Data points represent fit coefficients according to Eq. (14) as a function of equivalent potential temperature $\theta_e$. The color scale encodes the value of the coefficient of determination $r^2$ of the corresponding regression. Error bars show the 95% confidence interval of the fitted coefficients/susceptibilities. Only fits significant at the 95% level based on a two-sided $t$ test have been considered.
autoconversion and accretion in the warm cloud and the sum of autoconversion and accretion and riming in the mixed-phase cloud (not shown).

In summary, the constraint of retaining a balanced budget of cloud water $M_{cl}$ means that different precipitation production processes need to adapt in compensating ways to an aerosol perturbation. While an aerosol perturbation can thus redistribute the pathways of precipitation formation, the total precipitation production in nonglaciated cloud parts remains largely constant and is thus almost completely buffered. The only control aerosols have on precipitation production is via variability in INPs that change the glaciation state and thus control the depletion of water vapor by deposition on ice-phase hydrometeors.

c. Precipitation susceptibility

The aerosol dependence of the precipitation mixing ratio $M_{cl+pr}$ in the upper glaciated part of the mixed-phase cloud from Fig. 7 can be explained by the glaciation-mediated aerosol dependence of the sink of $M_{v}$. For the lower part of the mixed-phase cloud and the warm cloud, however, $M_{cl+pr}$ changes with the aerosol condition although the sink of $M_{v}$ is aerosol independent. Here, the redistribution of precipitation formation pathways corresponds to a redistribution of the total hydrometeor mixing ratio $M_{cl+ci+pr}$ between the different categories. The precipitation mixing ratio thus changes although its source rate does not. This is a situation typically observed in fully equilibrated systems. Shortly after a perturbation, in contrast, the source rate and mixing ratio would be correlated [recall the discussion of Eqs. (2) and (3)]. To quantify the redistribution effects and discuss their relationship to the budget constraint, we apply the concept of precipitation susceptibility [Eq. (1)].

We determine total precipitation susceptibilities $d\ln M_{cl+pr}/d\ln N_{e}$ to cloud droplet and/or ice crystal number concentration, $N_{e} = N_{cl}, N_{ci}$, individually for each trajectory, labeled by its value of $\theta_{e}$. The dependence of the susceptibility on the trajectory corresponds to regime dependence because the height of a trajectory determines its temperature and condensation rate. Different trajectories show a strong meteorological covariability between cloud variables. This
variability is stronger than the variability from different aerosol conditions and would mask the aerosol effect when disregarding the trajectory dependence of susceptibilities. Based on the variability arising for the 29 different aerosol conditions simulated, we thus fit for each trajectory the log-linear model

$$\ln \langle M_{ci+prl} \rangle (\theta_e) = s_{N_{ci}} \ln \langle N_{ci} \rangle (\theta_e) + s_{M_{cl}} \ln \langle M_{cl} \rangle (\theta_e) + c(\theta_e),$$  \hspace{1cm} (14)

where \( c \) is the axis intercept of the fit and coefficients \( s_{N_{ci}}, s_{M_{cl}} \) correspond to total susceptibilities. The validity of this linear model is quantified by the correlation coefficient \( r \) or the coefficient of determination \( r^2 \).

Total precipitation susceptibilities to droplet number are negative for the warm as well as for the lower part of the mixed-phase cloud (Figs. 11a,c), as is expected for a classical lifetime effect [recall that our definition of precipitation susceptibility [Eq. (1)] omits the conventional minus sign]. The mixed-phase susceptibility to droplet number with a value of about \(-0.5\) is stronger than the warm value of about \(-0.25\). The precipitation mixing ratio in the upper part of the mixed-phase cloud, which is dominated by ice crystals, is not correlated to cloud droplet number (Fig. 11e). Instead, it is well predicted by the number of ice crystals, which is in turn not correlated to \( M_{ci+prl} \) in the lower part (Fig. 11d).

Total precipitation susceptibilities comprise the effects of aerosol-induced changes in cloud droplet and ice crystal number and adjustments to these changes. For the warm cloud, a decomposition into number effect and adjustments reveals how the budget constraint controls the partitioning of cloud and rainwater and thus precipitation susceptibility: Eq. (1) corresponds to a bilinear regression

$$\ln \langle M_{prl} \rangle (\theta_e) = s_{N_{cl}} \ln \langle N_{cl} \rangle (\theta_e) + s_{M_{cl}} \ln \langle M_{cl} \rangle (\theta_e) + c(\theta_e),$$  \hspace{1cm} (15)

with coefficients \( s_{N_{cl}} = \partial \ln \langle M_{prl} \rangle / \partial \ln \langle N_{cl} \rangle \) and \( s_{M_{cl}} = \partial \ln \langle M_{prl} \rangle / \partial \ln \langle M_{cl} \rangle \) corresponding to partial susceptibilities. Fitted results for partial susceptibilities are shown in Fig. 12. The partial precipitation susceptibility with respect to cloud droplet number \( s_{N_{cl}} \) tends to be positive when only considering data points with \( r^2 > 0.95 \). This result seems counterintuitive and in contrast to the classical lifetime effect. It arises from higher-order adjustments in the cloud to reequilibrate the budget: An increase in cloud droplet number reduces autoconversion and thus requires an increase in accretion to keep precipitation production constant. At fixed \( M_{cl} \), accretion can only be increased by increasing \( M_{prl} \). Along similar lines, the budget constraint explains a negative partial susceptibility \( s_{M_{prl}} \): At fixed \( N_{cl} \), an increase in \( M_{cl} \) increases autoconversion and accretion, which is then compensated by a decrease in accretion mediated by a decrease in \( M_{prl} \). In other words, the negative values of \( s_{M_{prl}} \) describe a redistribution of rainwater, which decreases, to cloud water, which increases, and corresponds to a decreasing precipitation fraction.

The budget constraint can be formalized by expressing partial susceptibilities as an implicit derivative of the balance equation [Eq. (13)]:

$$\frac{\partial \ln \langle M_{prl} \rangle}{\partial \ln \langle N_{cl} \rangle} = -\frac{\partial B_{prl}[N, M(N)]}{\partial \ln \langle N_{cl} \rangle} + \frac{\partial \langle N_{cl} \rangle}{\partial \ln \langle M_{cl} \rangle} \frac{\partial \ln \langle M_{prl} \rangle}{\partial \ln \langle M_{cl} \rangle} - \frac{\partial \langle N_{cl} \rangle}{\partial \ln \langle M_{cl} \rangle},$$

where we have omitted indices and average brackets for brevity and sums run over \( r \in \{ \text{ -aut, sink, acc, ic-rim, is-rim, ig-rim, i-diff, } \text{s-diff, g-diff} \} \). The rearrangement on the right-hand side shows the relationship between susceptibilities of conversion rates \( r \) and state variables \( M \) and \( N \). Assuming that rate susceptibilities take values of magnitude 1, the rearrangement also illustrates that conversion processes with absolutely small rates like evaporation, melting, and freezing in our case can be neglected when estimating the sign of a partial susceptibility.

From Eq. (16), the sign of the partial precipitation susceptibility to droplet number in the warm cloud follows to be positive,

$$\text{sgn} \left( \frac{\partial \ln \langle M_{prl} \rangle}{\partial \ln \langle N_{cl} \rangle} \right) = \text{sgn} \left[ -\frac{\partial (\text{aut + acc} - M_{\text{sink}})}{\partial \ln \langle N_{cl} \rangle} / \frac{\partial \langle N_{cl} \rangle}{\partial \ln \langle M_{prl} \rangle} \right] = \frac{(-1) + (0) - (0)}{(0) + (+1) - (0)} = +1, \hspace{1cm} (17)$$

where the evaluation is based on assuming that (i) autoconversion decreases with increasing \( N_{cl} \) and is independent of \( M_{prl} \), (ii) accretion increases with increasing \( M_{prl} \) and is independent of \( N_{cl} \), and (iii) condensation (which is identical to \( M_{\text{sink}} \) for the warm cloud) is independent of both variables and all rates are trajectory-averaged rates as functions of trajectory-averaged cloud variables. Assuming that autoconversion and accretion
both increase with increasing \( M_d \) results in \( s_{M_d} < 0 \), as observed. It needs to be stressed that trajectory-averaged rates as functions of trajectory-averaged variables do not follow the functional forms implemented in the cloud microphysics scheme. This difference explains that fitted values of partial susceptibilities differ from the values derived based on the microphysical equations in Glassmeier and Lohmann (2016). This is also the reason that we do not include the rain dependence of SB autoconversion (Glassmeier and Lohmann 2016) in assumption (i). Based on Eq. (1) in combination with the signs of the partial susceptibilities (Fig. 12a) and a positive adjustment \( dN_{ci} / dN_{cl} \) (Fig. 11b), we conclude that the negative value of the total precipitation susceptibility to droplet number is the result of the negative adjustment term overcompensating the positive partial susceptibility. This is not surprising for a fully equilibrated system.

By considering all sedimenting hydrometeors as precipitation, the equivalent of Eq. (1) for adjustments in the lower part of the mixed-phase cloud reads

\[
\frac{dM_{ci+pr}}{dN_{cl}} = \frac{\partial \ln M_{ci+pr}}{\partial N_{cl}} + \frac{\partial \ln M_{ci+pr}}{\partial N_{cl}} \frac{dM_{ci}}{dN_{cl}} + \frac{\partial \ln M_{ci+pr}}{\partial N_{cl}} \frac{dN_{ci}}{dN_{cl}} \frac{dM_{ci}}{dN_{cl}} \tag{18}
\]

Given our limited dataset of 29 aerosol conditions and cross correlations between the variables, we refrain from performing the three-dimensional regression corresponding to Eq. (15). Instead, we limit our analysis to an analytical estimation of partial susceptibilities similar to Eq. (17):

\[
\text{sgn} \left( \frac{\partial \ln M_{ci+pr}}{\partial N_{cl}} \right) = \text{sgn} \left[ -\frac{\partial \text{aut} / \partial N_{cl}}{\partial \text{acc} + \text{rim} + \text{diff} / \partial M_{ci+pr}} \right] = +1 \tag{19}
\]

\[
\text{sgn} \left( \frac{\partial \ln M_{ci}}{\partial M_{cl}} \right) = \text{sgn} \left[ -\frac{\partial \text{aut} + \text{acc} + \text{rim} / \partial N_{cl}}{\partial \text{acc} + \text{rim} + \text{diff} / \partial M_{ci+pr}} \right] = -1 \tag{20}
\]

\[
\text{sgn} \left( \frac{\partial \ln M_{ci}}{\partial N_{cl}} \right) = \text{sgn} \left[ -\frac{\partial \text{rim} + \text{diff} / \partial N_{cl}}{\partial \text{acc} + \text{rim} + \text{diff} / \partial M_{ci+pr}} \right] = -1 \tag{21}
\]

where we have assumed that all partial derivatives with the exception of \( \partial \text{aut} / \partial N_{cl} \) are positive and omitted nonsusceptible rates. The signs of \( s_{M_i} \) and \( s_{M_d} \) are the same as in the warm case and can be interpreted in the same way. As the ice-phase variables in the mixed-phase cloud feature strong covariability, \( N_{ci} \) can be interpreted as a proxy for the importance of ice- and mixed-phase pathways. The positive partial susceptibility \( s_{N_{ci}} = \partial \ln M_{ci+pr} / \partial N_{ci} \) means that at fixed \( N_{cl} \) and fixed \( M_{ci} \), a shift from warm to cold pathways requires a reduction in \( M_{ci+pr} \) to reequilibrate the budget. Such a reduction is needed because ice-phase hydrometeors contribute to precipitation production not only by coalescence like rain but also by diffusion. Consequently, ice-phase hydrometeors produce precipitation more efficiently such that only a smaller amount of frozen hydrometeors as compared to rain is needed to cause the same total precipitation production rate. With positive adjustments \( dN_{ci} / dN_{cl} \) and \( dN_{ci} / dN_{cl} \) (Figs. 11e,f), the sign of mixed-phase precipitation susceptibility to droplet number is dominated by adjustments in the same way as warm precipitation susceptibility. The liquid and ice adjustments enhance each other because both terms in Eq. (18) are positive.

4. Conclusions

In an idealized, two-dimensional modeling study, we have explored the sensitivity of warm- and mixed-phase orographic precipitation to aerosol backgrounds that simultaneously vary in their abundance of cloud condensation nuclei (CCN) and ice nucleating particles (INPs). For quantification, we adapt the concept of precipitation susceptibility [Eq. (1)] to orographic clouds. To account for the horizontal rather than vertical development of orographic clouds, our analysis is based on averages of variables along moist isentropes [Eq. (11)], which trace parcel trajectories, rather than vertical averages. For warm, mixed-phase, and glaciated trajectories, we generally find low precipitation susceptibilities, which means that the precipitation response to aerosol perturbations is buffered as compared to the response of cloud variables like droplet number concentration and ice water path (Figs. 6–8).

The Lagrangian perspective of the trajectory approach allows us to formulate a budget equation for cloud water \( M_d \). Under steady-state conditions, this budget is balanced, which constrains the total precipitation formation to match the depletion of water vapor caused by the mountain flow [Eq. (13)]. In our simulations, the evaporation of cloud droplets is negligible such that vapor depletion corresponds to condensation in the warm cloud and to condensation and vapor deposition on ice-phase hydrometeors in the mixed-phase cloud. Condensation is independent of the aerosol background in our model because of the applied saturation adjustment. For the warm cloud, the application of a condensation scheme that allows
for supersaturations with respect to water could introduce an aerosol dependence of condensation if changes in (the location of) latent heat release changed the flow field or if supersaturation was sustained until a trajectory leaves the cloud. Both scenarios are probably not very important in our simulations. For the mixed-phase cloud, the saturation adjustment might limit the aerosol effect on vapor depletion: With saturation adjustment, WBF-active cloud regions feature water saturation, although a lower saturation ratio in between water and ice saturation is expected (Korolev 2007). Increased glaciation shifts saturation closer to ice saturation and thus increases diffusional vapor deposition. When applying a saturation adjustment, this effect is limited to shifting mixed-phase regions to full glaciation and does not allow for increased vapor deposition in mixed-phase regions.

The balance constraint on the precipitation production rate explains the buffered precipitation susceptibility. A change in the aerosol background leads to a redistribution among the different pathways of precipitation formation, but the total amount of precipitation formation can only increase because of increased glaciation and a shift from condensation to vapor deposition as discussed in the previous paragraph. In the warm cloud, aerosol-induced changes in autoconversion and accretion compensate each other, polluted conditions favoring accretion because autoconversion efficiency is reduced by smaller droplet sizes, which leads to an accumulation of cloud water. In the mixed-phase cloud, precipitation production via collision–coalescence is replaced by riming for more polluted aerosol conditions, which correspond to increased glaciation and vapor deposition. Under polluted conditions, glaciation proceeds by snow–cloud riming at the expense of graupel–cloud riming and by vapor deposition on ice crystals at the expense of ice–cloud riming.

In accordance with Glassmeier and Lohmann (2016), Saleeby and Cotton (2013), and Muhlbauer et al. (2010), we thus observe a compensation between the liquid- and mixed-phase as well as between the mixed- and ice-phase pathways to precipitation formation for aerosol-induced increases in glaciation. The “externally constrained” buffering observed here needs to be distinguished from the “statistical” buffering discussed by Glassmeier and Lohmann (2016). In the latter case, buffering is not required to meet an external constraint but occurs on a statistical basis because compensating responses to aerosol perturbations are likely to be found when a multitude of processes are affected. In both cases, buffering is implemented by compensating responses to an aerosol perturbation, but the underlying causes are different.

In view of our budget analysis, the decreasing sensitivity to aerosols with increasing precipitation efficiency discussed by Miltenberger et al. (2015), Xue et al. (2013), and Geresdi et al. (2017) can be explained as follows: Disregarding effects on glaciation and given an aerosol-independent condensation rate, aerosols can only affect precipitation production by changing the degree of hydrometeor evaporation. Aerosol can affect evaporation by redistributing the total hydrometeor mixing ratio \( M_{ci+pr} \) between the different hydrometeor categories because different hydrometeor types feature different fall velocities. The partitioning of the total hydrometeor mixing ratio between nonsedimenting cloud water \( M_{cl} \) and the sedimenting precipitation hydrometeors \( M_{ci+pr} \) in particular controls evaporation and total precipitation as well as the spillover. The importance of evaporation decreases with increasingly complete conversion of cloud water into precipitation hydrometeors (i.e., precipitation efficiency).

In terms of the interpretation of the precipitation susceptibility concept, the constrained total precipitation production excludes the traditional view that precipitation susceptibility quantifies the strength of precipitation production that changes with changes in \( N_{cl} \) along the lines of Albrecht (1989). Our analysis finds that the total precipitation susceptibility is dominated by adjustments, which is consistent given that the steady-state condition corresponds to a maximally adjusted state. This indicates that precipitation susceptibility \( d\ln M_{ci+pr}/d\ln N_{cl} \) can be interpreted as a quantification of the redistribution between \( M_{ci+pr} \) and \( M_{cl} \) that corresponds to a cloud water adjustment \( d\ln M_{cl}/d\ln N_{cl} \).

It is interesting to discuss the applicability of the traditional as opposed to the fully adjusted, or steady-state, perspective on precipitation susceptibility. Although the atmosphere is constantly changing, approximate steady-state situations are possible when the time scale at which the atmospheric boundary conditions change is slow as compared to the thermodynamic and microphysical adjustment time scale. Orographic clouds as discussed in this study are one such example because their lifetime can be much longer than the time that individual air parcels spent in the cloud. Stratocumulus clouds are another example (Bretherton et al. 2010). The crucial difference between these two examples is that updraft and condensation in stratocumulus clouds are not aerosol independent. Precipitation formation in stratocumulus thus lacks the balance constraint discussed for orographic clouds. As discussed in the context of Eqs. (3) and (2), the traditional, process-focused perspective on susceptibilities applies on short time scales after a perturbation or change in the atmospheric boundary conditions.

In a fully adjusted situation, precipitation susceptibility can nevertheless be related to the process rates.
In a steady-state cloud, the distribution of total hydrometeor mixing ratio $M_{\text{cl+ci+pr}}$ into the different hydrometeor categories does not change with time. For each category, sources and sink thus need to compensate, such that an aerosol-induced redistribution of precipitation formation pathways corresponds to a redistribution of $M_{\text{cl+ci+pr}}$ between categories. We formalize this relationship between the redistributions of $M_{\text{cl+ci+pr}}$ (i.e., susceptibilities) and the redistribution of precipitation formation pathways (i.e., microphysical rates) by implicit derivation [Eq. (16)]. This relationship implies signs for partial precipitation susceptibilities that are counterintuitive based on the traditional interpretation (e.g., $\frac{\partial \ln M_{\text{ci+pr}}}{\partial \ln N_{\text{cl}}} > 0$). Their explanation instead requires taking into account feedbacks that lead to the redistribution of the total hydrometeor mixing ratio: A droplet-number-induced decrease of the autoconversion rate requires accretion to increase for a reequilibration of the budget. For a fixed cloud water mixing ratio, the precipitation mixing ratio has to increase to achieve an increase as accretion is largely independent of droplet number.

In summary, we find the following picture of aerosol–cloud–precipitation interactions in completely adjusted, externally constrained systems: An increase in $N_{\text{cl}}$ reduces autoconversion. This implies that a larger fraction of the total hydrometeor mixing ratio $M_{\text{cl+ci+pr}}$ resides in the droplet instead of the precipitation category and leads to such an increase in accretion that compensates for the decrease in autoconversion. For the warm cloud, we find a precipitation susceptibility of approximately $-0.25$ for this process. In the weakly glaciated part of the mixed-phase cloud, the redistribution leads to an absolutely larger precipitation susceptibility of approximately $-0.5$ because ice-phase hydrometeors are more effective than rain in producing $M_{\text{cl+pr}}$. This additional ice-phase adjustment enhances the liquid adjustment, while aerosol effects on the warm and cold precipitation processes compensate each other. The strongly glaciated part of the mixed-phase cloud is controlled by aerosol-induced changes in total diffusional vapor deposition and is thus not susceptible to changes in $N_{\text{cl}}$. Our discussion illustrates that complex cloud systems governed by feedbacks cannot be explained from intuitive arguments based on individual processes alone. Instead, systemwide arguments, for example based on external constraints as shown here, are required.

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## APPENDIX

### List of Symbols and Acronyms

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle \cdot \rangle$</td>
<td>Spatial average or normalized sum along a trajectory</td>
</tr>
<tr>
<td><code>ci</code></td>
<td>Index denoting cloud ice category in SB</td>
</tr>
<tr>
<td><code>cl</code></td>
<td>Index denoting cloud droplet category in SB</td>
</tr>
<tr>
<td><code>pr</code></td>
<td>Index denoting precipitation-size hydrometeor categories in SB ($=\text{prl} + \text{prs} + \text{prg}$)</td>
</tr>
<tr>
<td><code>prl</code></td>
<td>Index denoting rain category in SB</td>
</tr>
<tr>
<td><code>prs</code></td>
<td>Index denoting snow category in SB</td>
</tr>
<tr>
<td><code>prg</code></td>
<td>Index denoting water vapor</td>
</tr>
<tr>
<td>$\theta_c$</td>
<td>Equivalent potential temperature</td>
</tr>
<tr>
<td><code>acc</code></td>
<td>Mass accretion rate</td>
</tr>
<tr>
<td><code>act</code></td>
<td>Mass rate of cloud droplet activation</td>
</tr>
<tr>
<td><code>aut</code></td>
<td>Mass autoconversion rate</td>
</tr>
<tr>
<td>$B_{\text{cl}}$</td>
<td>Budget of $M_{\text{cl}}$</td>
</tr>
<tr>
<td><code>BC</code></td>
<td>Black carbon aerosol</td>
</tr>
<tr>
<td><code>CCN</code></td>
<td>Cloud condensation nuclei</td>
</tr>
<tr>
<td><code>coag</code></td>
<td>Total mass rate of coagulation ($=\text{aut} + \text{acc} + \text{rim}$)</td>
</tr>
<tr>
<td><code>cond</code></td>
<td>Mass condensation rate</td>
</tr>
<tr>
<td><code>diff</code></td>
<td>Total mass rate of vapor deposition in ice-phase hydrometeors ($=i\text{-diff} + g\text{-diff} + s\text{-diff}$)</td>
</tr>
<tr>
<td><code>DU</code></td>
<td>Dust aerosol</td>
</tr>
<tr>
<td><code>evp</code></td>
<td>Mass evaporation rate</td>
</tr>
<tr>
<td>$f_N$</td>
<td>Fractional contribution of polluted number concentration to aerosol condition</td>
</tr>
<tr>
<td>$f_M$</td>
<td>Fractional contribution of polluted mass concentration to aerosol condition</td>
</tr>
<tr>
<td>$f_{\text{SU}}$</td>
<td>Fractional contribution of sulfate to sulfate-coated dust aerosol</td>
</tr>
<tr>
<td><code>freez</code></td>
<td>Mass rate of droplet freezing</td>
</tr>
<tr>
<td><code>gc-rim</code></td>
<td>Mass rate of graupel–droplet riming</td>
</tr>
<tr>
<td><code>g-diff</code></td>
<td>Mass rate of vapor deposition on graupel</td>
</tr>
<tr>
<td><code>ic-rim</code></td>
<td>Mass rate of ice–droplet riming</td>
</tr>
<tr>
<td><code>i-diff</code></td>
<td>Mass rate of vapor deposition on ice</td>
</tr>
<tr>
<td><code>INP</code></td>
<td>Ice nucleation particle</td>
</tr>
<tr>
<td><code>LWP</code></td>
<td>Liquid water path</td>
</tr>
<tr>
<td>$M_{\text{ci+pr}}$</td>
<td>Precipitation mixing ratio (i.e., mixing ratio in sedimenting hydrometeors categories)</td>
</tr>
</tbody>
</table>
\( M_{cl} \) Cloud water mixing ratio (i.e., mixing ratio in cloud droplet category)

\( M_{cl+ci+pr} \) Total hydrometeor mixing ratio

\( M_{ci+pr} \) Prodution rate of \( M_{ci+pr} \)

\( M_{sink} \) Depletion rate of \( M_x \)

\( M_x \) Mixing ratio of hydrometeors in category \( x \)

\( \text{melt} \) Mass rate of droplet melting

\( N_x \) Number of hydrometeors in category \( x \)

\( \text{OC} \) Organic aerosol

\( r \) Mass rate of change of a microphysical process

\( R \) Rain rate

\( R_{\text{glac}} \) Fraction of total hydrometeor mixing ratio \( M_{cl+ci+pr} \) in ice, snow, or graupel category of SB

\( R_{\text{pr}} \) Fraction of total hydrometeor mixing ratio \( M_{cl+ci+pr} \) in rain, snow, or graupel category of SB

\( \text{rim} \) Total mass rate of \( M_{cl} \) riming (=ic-rim + ge-rim + sc-rim)

\( s \) Total precipitation susceptibility

\( S \) Saturation ratio

\( s_x \) Partial precipitation susceptibility with respect to \( x \)

\( \text{sc-rim} \) Mass rate of snow–droplet riming

\( s\text{-diff} \) Mass rate of vapor deposition on snow

\( \text{SO}_4 \) Sulfate aerosol

\( w \) Vertical velocity

\( \text{WBF} \) Wegener–Bergeron–Findeisen

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