Mode-Decomposed Equation Diagnosis for Atmospheric Blocking Development

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ABSTRACT

This paper proposes a new method to identify atmospheric blocking development without the time filtering used in previous studies. A mode-decomposed vorticity equation is formulated from the principal components (PCs) of 500-hPa geopotential height by applying a new idea; the orthonormality of PCs allows any variable to be decomposed into a projection corresponding to the PCs. To test this, sectorial blocking episodes in Northern Hemisphere winter were identified by Barriopedro’s method. A blocking index was defined for each longitudinal range as the linear combination of the 10 largest PCs by means of the composite for the blocking episodes. Blocking development was diagnosed, in terms of the low modes of PC1–PC10 and the high modes of PC11–PC50. The results suggest that the intensification of blocking over the North Pacific and Eurasia is associated with nonlinear interaction among high modes, whereas the intensification (decay) of North Atlantic blocks is related mainly to enhanced nonlinear interaction among low-frequency (high-frequency) eddies. This main result is insensitive to the choice of definition for blocks and the choice of the mode separation boundary.

1. Introduction

Atmospheric blocks are largely anomalous anticyclones that frequently emerge in the Pacific, in the Atlantic, and over Europe (Tibaldi and Molteni 1990) and persist for a week or longer. They often bring prolonged abnormal weather in the extratropics. Many papers have been devoted to the physical mechanism of blocks. One impact of a Rossby wave breaking with a daily time scale typically causes blocking onset (Nakamura and Wallace 1993; Nakamura et al. 1997). Ji and Tibaldi (1983) suggest that synoptic-scale eddies play a more important role in blocking formation than large-scale topography does. Tsou and Smith (1990), Lupo and Smith (1995), and Colucci (2001) further note that self-interaction of synoptic-scale eddies and synoptic-to-planetary interaction contribute to blocking development. The barotropic vorticity equation has been widely used to diagnose the eddy feedback that acts as forcing against dissipation and mean-flow advection (Mullen 1986; Holopainen and Fortelius 1987). However, the diagnosis based on the balance equation has not identified causality in numerical and observational studies (Shutts 1983; Illari 1984; Haines and Marshall 1987). Although the eddy straining mechanism coincides with blocking development (Shutts 1983), it might not be an essential cause of blocking intensification or maintenance (Maeda et al. 2000; Arai and Mukougawa 2002). Yamazaki and Itoh (2013) found eddy–eddy merging during blocking episodes and attributed blocking maintenance to this
merging. Luo (2000, 2005) and Luo et al. (2014) clarified a causal link between synoptic-scale eddies and blocking intensification. Specially, many studies have shown that the beta term counteracts vorticity advection by low-frequency eddies (Holopainen and Fortelius 1987), and high-frequency eddy-induced vorticity flux convergence (VFC) directly produces a large-scale vorticity tendency. Preexisting meridianlly elongated transient eddies lead to subsequent blocking intensification by projecting strongly onto a large-scale dipole VFC in the downstream of the preexisting synoptic-scale eddies.

In most of the studies mentioned above, temporal filtering was used to separate blocking events and to distinguish between low-frequency and high-frequency eddies that triggered blocks. However, temporal filtering prevents analysis of the contribution of synoptic-scale eddies from a sequence of snapshots used in real-time analysis. Decomposing the equation into spherical harmonic function bases (Metz 1986) could resolve this problem, but the bases do not necessarily include the most notable variability such as blocks. A similar mode decomposition method has been applied to blocking establishment (Colucci 1987; Tsou and Smith 1990; Lupo and Smith 1995). A brand-new idea presented in this paper is an application of the equation decomposed into empirical orthogonal function (EOF) bases (Selten 1997) to analyze blocking development. This is possible because large-scale atmospheric blockings project mostly to a few of gravest EOFs. Metz (1986) and Egger et al. (1986) originally discussed the role of stochastic forcing from synoptic-scale eddies in blocking maintenance, in which the temporal evolution substantially follows the stochastic dynamics of low-order EOFs. Using a rough truncation of the quasigeostrophic equations (Marshall and Molteni 1993) and a parameterization strategy that includes the unresolved high-frequency modes as stochastic processes in the resolved-mode dynamic system (D’Andrea and Vautard 2001; D’Andrea 2002; Majda et al. 2001), Franzke et al. (2005) constructed a prototype general circulation model that can represent dominant weather patterns including atmospheric blocking. They demonstrated that a model with at most 10 modes could recreate the basic statistics of the weather patterns, such as the variances, temporal correlations, and pattern correlations of the transient eddy fluxes (Franzke and Majda 2006). This kind of model requires a closure procedure and often introduces climate drift. A low-order system of orthonormal EOF bases has also been constructed by an empirical fitting procedure. The linear inverse model consists of a linear equation with an empirically determined operator forced by a random process (Penland and Ghil 1993; Winkler et al. 2001; Newman et al. 1997; Zhang and Held 1999). Kravtsov et al. (2005) relaxed the limitation of linearity in the linear inverse model by including quadratic terms. More empirical fitting provided a stochastic differential equation with multiplicative noise (Sura et al. 2005; Berner 2005; Inatsu et al. 2013), but the complicated models found in this way require very long-term data to estimate many parameters or rough truncation. The equations physically or empirically formulated in the previous studies mentioned above have largely avoided analysis of specific phenomena due to possibly significant errors stemming from the assumption of closure or from statistical instability.

Moreover, identifying blocks from model outputs or analysis data is another problem. Among many algorithms proposed for objective blocking identification (Barriopedro et al. 2010), the methods can be classified into the following two. Tibaldi and Molteni (1990) detect the longitudes of blocks by flow reversal at absolute geopotential heights, and this has been extended to detection of two-dimensional blocking episodes (Pelly and Hoskins 2003; Schwierz et al. 2004; Barriopedro et al. 2006; Diao et al. 2006). In contrast, Dole and Gordon (1983) defined two-dimensional blocking episodes as persistent, positive anomalies in geopotential height (Davini et al. 2012). Although a recent sophisticated algorithm combined these methods to discover blocking episodes mostly consistent with the synopticians’ intuition (Barriopedro et al. 2010; Dunn-Sigouin and Son 2013), there is not yet consensus on a universal method of identifying blocks. This lack of agreement limits understanding of the blocking maintenance mechanism, because a catalogue of the episodes depends on the choice of algorithm.

The purpose of this study is to propose a new extended method of blocking diagnosis without time filtering, on the basis of a mode-decomposed vorticity equation and to apply this method to the eddy interactions that contribute to blocking development from an analysis dataset. By expanding the so-called EOF’s dual formalism (Hirose and Kutzbach 1969), an application of an orthogonal EOF set of geopotential height patterns to other variables enables us to decompose the vorticity equation, not in the quasigeostrophic framework, into each mode contribution of linear advection, the beta term, vortex-tube stretching, nonlinear mode-to-mode interaction, and dissipation. The equation used here is only for the diagnosis, not conducting time integrations, so that neither a closure technique for higher-mode interactions nor a correction for climate drift are necessary. The rest of this paper is organized as follows: section 2 introduces the analysis data used here; section 3 formulates the mode-decomposed vorticity equation; section 4 describes the results from analysis of blocking
FIG. 1. The spatial pattern of the (a) first to (l) twelfth empirical orthogonal functions (EOFs) for 500-hPa geopotential height in the domain north of 20°N for 150 winter days from 1 Nov nearly to the end of March, examining the years from 1960/61 to 2016/17. The contour interval is 10 m with negative contours dashed and the zero contour omitted. The explained variance of the first to twelfth modes is 8.2%, 7.2%, 6.2%, 5.8%, 5.2%, 4.9%, 4.4%, 3.7%, 3.3%, 3.2%, 3.0%, and 2.3%, respectively.
maintenance with the mode-decomposed equation; section 5 discusses the robustness of the result; and section 6 concludes the paper.

2. Data and method

The datasets used here are the 6-h JRA-55 dataset archived by the Japan Meteorological Agency (Kobayashi et al. 2015), which provide horizontal wind vector, air temperature, and geopotential height at several isobaric surfaces with a resolution of 1.25° × 1.25° (longitude × latitude). The analysis was limited to the domain north of 20°N and a period of 150 winter days, from 1 November nearly to the end of March, examining the years from 1960/61 to 2016/17. Reference principal components (PCs) were obtained by EOF decomposition of 500-hPa geopotential height north of 20°N during the whole period (Fig. 1). The first mode (Fig. 1a) had projected strongly onto the North Atlantic Oscillation and the second mode (Fig. 1b) resembled the Pacific–North American pattern (cf. Kimoto and Ohiil 1993). A partial similarity was found between the western Atlantic pattern and the third mode (Fig. 1c), the western Pacific pattern and the fourth mode (Fig. 1d), the Eurasian pattern and the fifth mode (Fig. 1e), and the eastern Atlantic pattern and the sixth mode (Fig. 1f; Wallace and Gutzler 1981; Barnston and Livezey 1987). We tested 50 modes of 500-hPa geopotential height; together these explained about 90% of the total variance. The modes 10 and less with much low-frequency power (hereinafter, the low modes; Fig. 2) were separated from the remaining modes (the high modes) in this paper. It is noteworthy that the diurnal peaks found in most of the modes did not essentially affect the following results.

The blocking episodes were determined by following the method of Barriopedro et al. (2010), based on 500-hPa geopotential height anomaly $z_a$, defined as the deviation from the 6-h climatology. This method consisted of three steps: first, the identification of meridional reversals at each longitude and each time frame as local, instantaneous blocks (e.g., Tibaldi and Molteni 1990); second, the detection of two-dimensional images with geopotential height anomaly exceeding a criterion at each time frame (e.g., Dole and Gordon 1983); and third, the tracking of the blocked images over time (cf. Inatsu 2009). In the first step, we prepared the reference latitude $\phi_j$ as a function of the longitude and the month, in which the variance of 500-hPa geopotential height deviation from its 5-day running average attained the maximum. The longitude with the index

$$
\frac{1}{2\pi} \sum_{j_{b}+6}^{j_{b}+12} \frac{1}{\max_{\lambda_{i}, \phi_{j}} \left[ z_{a}(\lambda_{i}, \phi_{j}) - \sum_{j=b-1}^{j_{b}+12} z_{a}(\lambda_{i}, \phi_{j}) \right]}, \quad (1)
$$

being positive was identified as the local instantaneous block longitude. Here $\lambda_i$ and $\phi_j$ are longitude and latitude, respectively, for the cell at longitude index $i$ and latitude index $j$, and $i_c$ is the number of longitude grid in this paper. It is noteworthy that the diurnal peaks found in most of the modes did not essentially affect the following results.

Fig. 2. Power spectrum of each EOF mode. The contours are plotted at 0.5, 1, 2, …, 200, 500, and 1000 m with 1, 10, 100 and 1000 m thickened. The value exceeding 100 m is shaded.
than about 260 km in 6 h (a speed of about 12 m s\(^{-1}\)), a temporal connection was accepted. The blocking episode was finally defined as the time sequence of temporally connected two-dimensional images having a lifetime longer than 96 h. The peak of a blocking episode was defined as the time at which the maximum geopotential height anomaly was largest.

3. Mode decomposed equation

We start with the orthogonality in the dimensional EOF spatial patterns of 500-hPa geopotential height denoted as \(\{Z_1, Z_2, \ldots, Z_N\}\),

\[
(Z_m, Z_n) = \int_\Omega Z_m(x) Z_n(x) \, dx = \|Z_n\|^2 \delta_{mn}, \tag{2}
\]

and in the normalized PC time series \(\{A_1, A_2, \ldots, A_N\}\),

\[
\frac{1}{T} \int_0^T A_m(t) A_n(t) \, dt = \delta_{mn}, \tag{3}
\]

where \(\Omega\) is the two-dimensional domain north of 20\(^\circ\)N and \(T\) is the total period of 57 winter segments. A variable \(Y\) can be expanded to a set of orthonormal time series as

\[
Y(x, t) \approx Y_0(x, t) + \sum_{n=1}^{N} Y_n(x) A_n(t), \tag{4}
\]

where \(Y_0(x, t)\) is the 6-h climatology of \(Y\), which is defined as the 31-day running mean of the average for each time frame over 1960 to 2017. The \(Y_n\) exactly equals to the \(Y\) regression on the \(n\)-mode PC, and \(\{Y_1, Y_2, \ldots, Y_N\}\) are not generally pairwise orthogonal. We next introduce the vorticity equation for an isobaric surface, here at 500 hPa,
where \( \mathbf{u} \) is the horizontal wind vector, \( \zeta \) is relative vorticity, \( D \) is horizontal divergence, \( f \) and \( \beta \) are, respectively, the Coriolis force and its meridional derivative, and \( F \) is the residual. Separating the anomalies from the climatology, the anomalous relative vorticity has the form

\[
\frac{\partial \zeta^I}{\partial t} + \nabla \cdot (\mathbf{u} \zeta) + \beta \mathbf{u} + fD = F',
\]

where the prime symbol is for the anomaly. By substituting the PC time series decomposition [Eq. (4)] of horizontal wind vector and relative vorticity into Eq. (6), we obtain

\[
\sum_n \zeta_n \frac{dA_n}{dt} = -\sum_n (\nabla \cdot (\mathbf{u}_n \zeta_n + \mathbf{u}_n \zeta_0) + \beta \mathbf{u}_n + fD_n) A_n \\
- \sum_m \sum_n \nabla \cdot (\mathbf{u}_m \zeta_n) A_n A_m + F'.
\]

Taking the inner product with \( \ell \)-mode relative vorticity pattern \( \zeta_\ell \),

\[
\sum_n \mathbf{C}_{n \ell} \frac{dA_n}{dt} = \sum_n (\mathbf{L}_{A_n}^\ell + \mathbf{L}_{C_n}^\ell + \mathbf{L}_{B_n}^\ell + \mathbf{L}_{D_n}^\ell) A_n \\
- \sum_m \mathbf{N}_{m \ell m} A_m A_n + F',
\]

where \( \mathbf{C}_{n \ell} = (\zeta_\ell, \zeta_\ell), \mathbf{L}_{A_n}^\ell = (\zeta_\ell, \nabla \cdot [\mathbf{u}_n \zeta_\ell]), \mathbf{L}_{C_n}^\ell = (\zeta_\ell, \beta \mathbf{u}_n), \mathbf{L}_{B_n}^\ell = (\zeta_\ell, \nabla \cdot [\mathbf{u}_n \zeta_0]), \mathbf{L}_{D_n}^\ell = (\zeta_\ell, fD_n), \mathbf{N}_{m \ell m} = (\zeta_\ell, \nabla \cdot [\mathbf{u}_m \zeta_\ell]), \) and \( F_\ell = (\zeta_\ell, F') \). Note that the linear terms vary slowly because they depend on climatological variables. If the matrix \( \mathbf{C}_{n \ell} \) were diagonal, Eq. (8) would be just reduced to the mode equation. Geostrophy relates zonal and meridional wind and air temperature to the spatial derivative of geopotential height, so that each set of these spatial patterns is almost orthogonal. The matrix \( \mathbf{C}_{n \ell} \) is regular because its diagonal components are almost one order larger than nondiagonal components (Fig. 3a). Operating the inverse matrix of \( \mathbf{C}_{n \ell} \) from the left of Eq. (8) and noting that

\[
\sum_{\ell, n} \sum_{m \ell m} \mathbf{C}_{n \ell}^\ell \frac{dA_n}{dt} = \sum_{\ell, n} \sum_{m \ell m} \frac{dA_n}{dt} = \frac{dA_\ell}{dt},
\]

the mode equation can be simplified to

\[
\frac{dA_\ell}{dt} = -\sum_{\ell, n} \sum_{m \ell m} \mathbf{C}_{n \ell}^\ell \mathbf{L}_{A_n}^\ell + \mathbf{L}_{C_n}^\ell + \mathbf{L}_{B_n}^\ell + \mathbf{L}_{D_n}^\ell A_n \\
- \sum_{\ell, m, n} \mathbf{C}_{n \ell}^\ell \mathbf{N}_{m \ell m} A_m A_n + \sum_{\ell, f} \mathbf{C}_{n \ell}^\ell F_\ell.
\]

The \( k \)-mode PC tendency then equals the linear effects by following. The first term of the right-hand side of Eq. (10) consists of the \( n \)-mode VFC by climatological wind vector, \(-\sum_{\ell, n} \sum_{m \ell m} \mathbf{C}_{n \ell}^\ell \mathbf{L}_{A_n}^\ell \) (the \( A \) term), the climatological VFC by \( n \)-mode wind vector, \(-\sum_{\ell, n} \sum_{m \ell m} \mathbf{C}_{n \ell}^\ell \mathbf{L}_{C_n}^\ell A_n \) (the \( C \) term), beta term with \( n \)-mode meridional wind, \(-\sum_{\ell, n} \sum_{m \ell m} \mathbf{C}_{n \ell}^\ell \mathbf{L}_{B_n}^\ell A_n \) (the \( B \) term), and vortex stretching term with \( n \)-mode horizontal wind divergence, \(-\sum_{\ell, n} \sum_{m \ell m} \mathbf{C}_{n \ell}^\ell \mathbf{L}_{D_n}^\ell A_n \) (the \( D \) term). The nonlinear effects of the \( n \)-mode VFC by \( m \)-mode wind are \(-\sum_{\ell, n} \sum_{m \ell m} \mathbf{C}_{n \ell}^\ell \mathbf{N}_{m \ell m} A_m A_n \), as in the second term of the right-hand side of Eq. (10). Linear operators, \(-\sum_{\ell, n} \sum_{m \ell m} \mathbf{C}_{n \ell}^\ell \mathbf{L}_{A_n}^\ell, -\sum_{\ell, n} \sum_{m \ell m} \mathbf{C}_{n \ell}^\ell \mathbf{L}_{C_n}^\ell, -\sum_{\ell, n} \sum_{m \ell m} \mathbf{C}_{n \ell}^\ell \mathbf{L}_{B_n}^\ell, \) and \(-\sum_{\ell, n} \sum_{m \ell m} \mathbf{C}_{n \ell}^\ell \mathbf{L}_{D_n}^\ell \) are displayed in Fig. 3b. The \( A \)- and \( B \)-term operators had large magnitudes in some projections and they were often opposite in sign. The \( C \)- and \( D \)-term operators were small relative to the \( A \)- and \( B \)-term operators. Most projections of the total linear operator had a comparable magnitude to the nonlinear operators (see Fig. 3c for the operator with climatological state at 0000 UTC 1 January). Among the projection, the total linear term
for the first-mode projection had a maximum at the second mode and a minimum at the twentieth mode, and that for the second-mode projection had a maximum at the fifteenth mode and a minimum at the sixth mode. The nonlinear operators $W_{kmn} = -\sum C_{kl}^2 N_{kmn}$ were functions of $(m, n)$; Figs. 3d and 3e display the operators for the first- and second-mode equations (i.e., $k = 1, 2$). The nonlinear operator is largely antisymmetric. For example, for the first-mode projection, the top five values were found with $(m, n) = (5, 6), (10, 12), (6, 5), (12, 13), (3, 12)$ and the bottom with $(m, n) = (3, 6), (6, 5), (12, 10), (8, 12), (13, 12)$. We here evaluated the magnitude of the nonlinear operators from $|W_{kmn}|^2$ for $n > m$ and $|W_{kmn}|^2$ for $n = m$. We then summed this magnitude for low modes (both $m$ and $n \leq 10$) and for high modes (all others) for each projection mode $k$. From this evaluation, we found no clear distinction between first-mode and second-mode projections (not shown). Note that the dependence on the scale-separation mode will be discussed after the results (section 5).

As will be stated later, the composite analysis allows us to measure a sectorial blocking by linear combination of EOF modes. A sectorial blocking index for a sector $s$ was defined as

$$I(s) = \sum_k P_k(s)A_k,$$

where $P_k$ is the $k$th coefficient of the linear combination. From this, the following equation is readily derived from Eq. (10):

$$\frac{dI(s)}{dt} = \sum_n \mathcal{E}_n(s)A_n + \sum_{m,n} \mathcal{M}_{mn}(s)A_mA_n + \mathcal{F}(s),$$

where
characterizes the linear-term contribution of the \( n \)th mode,

\[
\mathcal{L}_n = \mathcal{L}_n^A + \mathcal{L}_n^C + \mathcal{L}_n^B + \mathcal{L}_n^D \\
= -\sum_{k,l} P_k c_{kl}^{-1} (L_{ln}^A + L_{ln}^C + L_{ln}^B + L_{ln}^D) 
\]

(13)

characterizes the nonlinear interaction between \( m \)th and \( n \)th modes, and

\[
\mathcal{M}_{mn} = -\sum_{k,l} P_k c_{kl}^{-1} N_{pmn} 
\]

(14)

is the nonconserved term. In the diagnosis, we separated low modes from high modes. There were eight components of the linear term: from \( A \) to \( D \) terms, the low-mode contribution to the linear term \( \sum_{n=1}^{10} L_{n} A_{n} \) (lin-L term) and the linear high-mode contribution \( \sum_{n=11}^{50} L_{n} A_{n} \) (lin-H term). Three nonlinear categories were used: nonlinear terms among low modes \( \sum_{n=1}^{10} \sum_{m=1}^{10} M_{mn} A_{m} A_{n} \) (the non-L term), nonlinear terms between low and high modes \( \sum_{m=1}^{10} \sum_{n=11}^{50} (M_{mn} + M_{nm}) A_{m} A_{n} \) (the NLH term), and nonlinear terms among high modes \( \sum_{m=11}^{50} \sum_{n=11}^{50} M_{mn} A_{m} A_{n} \) (the NHH term). Here, we further define the sum of the NLH and NHH terms as a non-H term.

4. Results

a. Eastern Pacific blocking

Averaging PCs over 52 blocking-peak dates with its centroid in the eastern North Pacific longitudes in the range 180°–150°W, the normalized sectorial blocking index can be defined as

\[
I(EP) = -0.092A_1 + 0.853A_2 - 0.005A_3 - 0.103A_4 \\
+ 0.026A_5 + 0.239A_6 - 0.235A_7 + 0.220A_8 \\
- 0.055A_9 + 0.299A_{10} 
\]

by truncation at 10 modes (Table 1). Though the coefficients of each blocking episode deviated from the
averaged coefficient [Eq. (16)], the PC2 magnitude characterized the eastern Pacific blocks (Fig. 4a). The coefficients of PC2, PC6, PC8, and PC10 were nonzero at a 5% significant level. The composite map for a peak of 500-hPa geopotential height shows a very strong anticyclone in western Alaska (Fig. 4b). The anomaly attained a height of 255 m with the center of action located at 57.5°N, 162.5°W. A weak cyclone anomaly resided south of this. This composite pattern was a typical dipole blocking pattern around this area.

Using the mode-decomposed Eq. (12), we diagnosed the blocking formation in the eastern Pacific. The lin-L term contained a large counteraction between the A and B terms (Fig. 5a). This term was obviously uncorrelated with blocking tendency and was nearly consistent with the budget analysis in Holopainen and Fortelius (1987). In addition, the lin-H term contributed somewhat to blocking development, though it contained some semi-diurnal noise (Fig. 5b). The A-term contribution was dominant in the lin-H term during the blocking development. Figure 6a shows the lead–lag composite for 52 blocking episodes in the eastern Pacific and (b) 48 blocking episodes in the central Atlantic. The color shading is as per the reference in the bottom right. Components that are nonzero with a statistical significance at 5% are masked out. The diagonal components are basically nonzero, because $M_{nn}A_n^2$ would follow the $\chi^2$ distribution, assuming that PCs followed the normal distribution.
1 week led to the peak. Throughout the period in this composite analysis, the lin-L term was quite small and was statistically insignificant around the peak time. The non-L term was mostly negative and nearly constant and was also uncorrelated with blocking tendency. In contrast, the non-H term was well correlated to blocking development and decay, which is evidence that these interactions contributed to the increase in blocking intensity before the peak and the decrease in blocking decay after the peak. The NLH term accounted for most of the non-H term.

The composite of the nonlinear interaction terms between two modes averaged over data lagged 4 days from the peak time is shown as a lower triangular matrix (Fig. 7a), which is done by considering that a nonlinear operator for \(m\) mode and \(n\) mode can be written as \(M_{mn} + M_{nm}\) where \(m \neq n\). This plot showed an interesting feature: few mode-to-mode interactions significantly contributed to blocking development. Whereas most of the diagonal components in low modes contributed to a negative constant in the non-L term, components related to high modes contributed to blocking development (cf. Fig. 6a). Among them, the nonlinear interaction between the second and seventh modes (Figs. 1b,g) was the largest and positive during the 4 days before the peak (Fig. 8a). In this term, the VFC from the second-mode wind vector and the seventh-mode vorticity was spatially correlated with the blocking structure in the eastern Pacific (Fig. 9a). The second mode indicates a quasi-stationary block itself, with its magnitude synchronized to blocking intensity. In contrast, PC7 changed sign from negative to positive near the peak time, corresponding to westward movement of low-frequency eddies (not shown). In blocking development, northerly in the seventh mode followed a dipole vorticity pattern from the second mode, which brought negative and positive VFCs that effectively acted to force the block in the eastern Pacific. The second largest term in the 2 days before the peak time was the nonlinear interaction between the third and seventeenth modes, which acted as a forcing to blocking development (Fig. 8a). The seventeenth mode projected a set of meridionally elongated highs and lows alternately arranged across the basin, whereas the third mode occupied a monopole vortex over the basin (Figs. 1c, 9b). Notably, the seventeenth-mode vorticity pattern resembled an idealized picture of transient eddies propagating along the blocked flow (Shutts 1983; Luo 2000, 2005). These interactions created a negative VFC in the south of Alaska that had...
large projection onto the vorticity tendency in the blocking development. Other interaction among the top 10, typically where \((m, n) = (1, 1)\), were almost always positive constants, uncorrelated to blocking development (Fig. 8a).

b. Atlantic blockings

Averaging PCs over 48 blocking-peak dates with the centroid in the central Atlantic longitudes 60°–30°W, the normalized sectorial blocking index (Table 1) was defined as

\[
I(\text{AT}) = 0.792A_1 + 0.081A_2 + 0.264A_3 - 0.073A_4 \\
+ 0.128A_5 + 0.386A_6 + 0.003A_7 - 0.209A_8 \\
+ 0.189A_9 - 0.216A_{10}.
\]  

(17)

The Atlantic blocks were characterized by PC1 magnitude (Fig. 4c). The coefficients of PC1, PC3, and PC6 were nonzero at a 5% significance level, and those of PC8, PC9, and PC10 were nominally so. The composite map for a peak of 500-hPa geopotential height shows a very strong anticyclone over Greenland and the North Atlantic (Fig. 4d). The anomaly attained a height of 266 m at the center of action located at 61.25°N, 36.25°W.

A cyclone anomaly was straddled in the south and over Europe, part of which might not be related to blocking.

Using the same technique as for the eastern Pacific blocks, we examined blocking formation in the central Atlantic. Figure 6b shows the lead–lag composite for 48 blocking episodes of each category in the mode-decomposed equation [Eq. (12)]. The tendency term \(dI(\text{AT})/dt\) [lhs of Eq. (12)] was positive in the 1-week-lagged data from the peak time and negative in the 1-week-lead data after the peak. The lin-L term was too small over the lead–lag time to effectively act as a forcing to blocking, owing to a counteraction between the \(A\) and \(B\) terms with a large variety in blocking episodes (Fig. 5c), which was similar to the pattern in the eastern Pacific blocks. In contrast, the non-L term was significantly positive during the blocking development and diminished after the peak time, whereas the non-H term was almost always negative throughout the period and was not related to blocking tendency. In contrast with the eastern Pacific blocking, the NHH term played a significant role in blocking decay. This feature of the nonlinear interaction terms was quite different from the case in the eastern Pacific blocks. The lin-H term slightly contributed to blocking development, with the \(A\) term dominant during the blocking development (Fig. 5d).

Figure 7b displays the composited nonlinear interaction averaged over 4 days lagged data from the peak time. Consistent with Fig. 6b, most of the non-L components showed a significant positive value. Among nonlinear interactions, the third-mode wind vector and the first-mode vorticity markedly increased in the 4 days before the peak (Fig. 8b). The VFC was negative in the south of Greenland, which was partly correlated with blocking structure in the Atlantic (Fig. 10a). The third-mode
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to third-mode contribution was also dominant in the blocking development (Fig. 8b). A physical mechanism inherent in blocking formation in the Atlantic has been proposed (Nakamura et al. 1997; Michelangeli and Vautard 1998), and our results also suggest that low-mode nonlinear interaction encouraged blocking formation through the VFC by the blocking pattern itself. In contrast, nonlinear interactions related to high modes were not dominant in the Atlantic (Fig. 8b). An exception is the nonlinear interaction between the twelfth (Fig. 11) and thirtieth modes, where the VFC has large projection onto the Atlantic blocking (Fig. 10b). The difference of nonlinear eddy actions between the eastern Pacific and the Atlantic was suggested by Nakamura et al. (1997), but it should be noted that our method of analysis led to the same conclusion without any time filtering.

c. Blocking in other longitudes

We finally summarized the mode-decomposed equation diagnosis for sectorial blocking episodes with its peak’s centroid in 0°–30°E, 30°–60°E, . . . , and 30°W–0°, on the basis of the sectorial blocking indices listed in Table 1. We identified 52 episodes in 180°–150°W, 48 episodes in 60°–30°W, and so on (Fig. 11). The longitudinal distribution of blocking frequency is consistent with that found in previous studies (Tibaldi and Molteni 1990; Pelly and Hoskins 2003). Just as for the eastern Pacific and Atlantic longitude ranges, we calculated the linear and nonlinear terms averaged over the 4 days before the peak composited for each sectorial blocking. The low-mode linear term was not a dominant term except for infrequently in 120°–90°W. Thus, the nonlinear interaction (whichever non-L or non-H terms were essential) played an important role in blocking development in most places in the Northern Hemisphere. The importance of high-mode linear and nonlinear terms encouraged development in the Eurasian longitudes, as with the eastern Pacific blocks. In contrast, the development of Atlantic blocking was promoted by nonlinear interactions among low-frequency modes.

5. Discussion

It seems interesting that the results obtained in this paper differ from some analyses with spatial-scale decomposition of variables related to blocking events, although the approach in previous studies of separating low-frequency eddies from high-frequency eddies is obviously different from our mode decomposition. For example, Holopainen and Fortelius (1987) emphasize high-frequency eddy forcing in blocking development in the analysis of a European block during
16–25 February 1979. This block was also identified in our analysis with its center in the 0°–30°E band at the peak time (Table 2). A positive tendency of the blocking index for this band during 12–17 February 1979 was mostly explained by non-H terms with comparable contributions from the NLH and NHH terms (not shown). Our analysis revealed that this event was a typical block in the longitudes from the view of vorticity balance (Fig. 11). Using a different scale separation, Tsou and Smith (1990) accentuate the self-interaction of synoptic-scale eddies and synoptic-to-planetary-scale interaction in a blocking development during 19–21 January 1979. We also detected this event, which had its peak in the central Atlantic band, and the non-L term was dominant in the vorticity budget diagnosis (not shown). This discrepancy might be related to our separation scale being larger than theirs. Colucci (2001) repeated the suggestion in Tsou and Smith (1990), examining another blocking event over Europe. This event was not well detected in our analysis but was consistent with our composite analysis (Fig. 11).

As seen by comparison with previous studies, our results possibly depend on how the boundary between low- and high-frequency modes is set. We fixed the boundary at the tenth mode as a standard setting in this paper, but this choice was not examined for a conspicuous scale gap in the power spectrum of geopotential height in the extratropics. We tried to verify the robustness of our results to the boundary choice by a sensitivity test for the mode separation boundary (Fig. 12). If the boundary between low and high modes was redefined as between modes 1–7 and modes 8–50, little change was found in the lin-L, lin-H, non-L, and non-H terms in the blocking development composite. This change in the scale separation boundary decreased the difference between a large negative non-L effect and a large positive non-H effect in the eastern Pacific. This means that the scale interaction related to modes 8–10 effectively acted to prevent blocking development. In contrast, in the central Atlantic the contribution of each term was quite similar to the standard setting. Checking another case, using the division of modes 1–15 and modes 16–50, the result was not much different from the standard setting (Fig. 12b), with only a slight increase in the non-L contribution over the Eurasian longitudes and the Atlantic.

The above analysis was based on the composite of blocking episodes objectively identified by the algorithm of Barriopedro et al. (2010). Recently, Woollings et al. (2018) have reported on blocking climatology and climate change projections with the application of three different algorithms. They adopted Schwierz et al. (2004) as an anomaly-based method, Davini et al. (2012) as an absolute method, and Barriopedro et al. (2010) as a hybrid method. To maintain coherency, they implemented the first two methods by slightly modifying the hybrid method. In this paper, following the idea in Woollings et al. (2018), we checked the dependency on the blocking detection algorithm. Figure 13a displays the same analysis as section 4c, but for the blocking episodes identified by the anomaly method, where the local instantaneous block is the point at which 500-hPa geopotential height anomaly exceeded a threshold described in section 2. This method identified more blocking episodes in the Pacific than the hybrid method did. The results were nearly the same as those of the hybrid method, but the non-L term was much weaker in the Atlantic and the lin-H term was emphasized over Eurasian longitudes. Thus, the main result holds even using the anomaly method. Figure 13b shows the results from using the absolute method, where the local instantaneous blocking was identified from the 500-hPa geopotential height itself (not anomalies in that height) satisfying the conditions.
in the latitude $\phi$, which ranged from 45° to 70°N. The blocking frequency was clearly higher in the Atlantic and over Europe than that found by the hybrid method (Fig. 13b). The lin-H and non-H terms contributed more strongly to the blocking development in the absolute method. In spite of this slight difference, this does not necessarily contradict the main result of this paper that nonlinear interactions among low modes were important for Atlantic blocking development and the contribution of high modes was important for eastern Pacific blocking development.

Another problem stems from the composite analysis. A massive amount of data is necessary to specify a mode (for linear contributions) or two modes (for nonlinear contributions) for blocking development. The composite analysis estimates the terms most likely to contribute to the phenomenon, choosing from hundreds or thousands of linear and nonlinear terms. Thus, analyses such as those shown in Figs. 6 and 11, where a few categories are chosen for the mode-decomposed equation, have high confidence. In contrast, because only tens of blocking episodes were considered in this paper, the statistics for the term specifications are not highly stable, even though some are statistically significant. To specify the term contribution by composite analysis, data from more than 3000 years would be needed. A long-term general circulation model experiment like Database for Policy Decision-Making for Future Climate Change (d4PDF) (Mizuta et al. 2017) might realize the analysis, unless the model bias was large.

6. Conclusions

We have diagnosed the development of atmospheric blocking episodes by a newly established method. Owing to the orthogonality of PCs in the geopotential-height EOF, variables such as zonal wind, meridional wind, and vorticity can be decomposed into modes, whose spatial patterns are not always orthogonal. This decomposition enabled us to formulate, without any temporal filtering, a mode-decomposed vorticity equation at 500 hPa [Eq. (12)]. With data of 50 PCs and their accompanying spatial patterns from 150 winter days for the years from 1960/61 to 2016/17 (as archived in the JRA-55 dataset), we diagnosed the blocking development in the eastern Pacific and in the central Atlantic. Because the low-mode linear terms were approximated to zero, blocking intensity was related to nonlinear interactions among several modes. The composite for the dates of development before the peak of blocking episodes suggested that the development of the eastern Pacific blockings was related to nonlinear interactions among high modes, while the growth (decay) of the Atlantic blockings was promoted by nonlinear interactions among low-frequency (high-frequency) modes.

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