A Nonlinear Multiscale Theory of Atmospheric Blocking: Eastward and Upward Propagation and Energy Dispersion of Tropospheric Blocking Wave Packets

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(Manuscript received 27 May 2020, in final form 2 September 2020)

ABSTRACT: In this paper, a nonlinear multiscale interaction model is used to examine how the planetary waves associated with eddy-driven blocking wave packets propagate through the troposphere in vertically varying weak baroclinic basic westerly winds (BWWs). Using this model, a new one-dimensional finite-amplitude local wave activity flux (WAF) is formulated, which consists of linear WAF related to linear group velocity and local eddy-induced WAF related to the modulus amplitude of blocking envelope amplitude and its zonal nonuniform phase. It is found that the local eddy-induced WAF reduces the divergence (convergence) of linear WAF in the blocking upstream (downstream) side to favor blocking during the blocking growth phase. But during the blocking decay phase, enhanced WAF convergence occurs in the blocking downstream region and in the upper troposphere when BWW is stronger in the upper troposphere than in the lower troposphere, which leads to enhanced upward-propagating tropospheric wave activity, though the linear WAF plays a major role. In contrast, the downward propagation of planetary waves may be seen in the troposphere for vertically decreased BWWs. These are not seen for a zonally uniform eddy forcing. A perturbed inverse scattering transform method is used to solve the blocking envelope amplitude equation. It is found that the finite-amplitude WAF represents a modified group velocity related to the variations of blocking soliton amplitude and zonal wavenumber caused by local eddy forcing. Using this amplitude equation solution, it is revealed that, under local eddy forcing, the blocking wave packet tends to be nearly nondispersive during its growth phase but strongly dispersive during the decay phase for vertically increased BWWs, leading to strong eastward and upward propagation of planetary waves in the downstream troposphere.

KEYWORDS: Blocking; Dispersion; Eddies; Nonlinear dynamics; Planetary waves; Potential vorticity

1. Introduction

Among atmospheric phenomena, the blocking situation is one of the most important low-frequency modes with time scales ranging from 10 to 20 days (Berggren et al. 1949; Rex 1950; Diao et al. 2006). Because of its importance in winter cold extremes and summer heat waves, the study of the characteristics and underlying mechanism of blocking has attracted a great deal of attention (Yeh 1949; Rex 1950; Egger 1978; Charney and Devore 1979; Tung and Lindzen 1979; McWilliams 1980; Kalnay-Rivas and Merkin 1981; Frederiksen 1982; Shutts 1983; Colucci 1985; Haines and Marshall 1987; Holopainen and Fortelius 1987; Tsou and Smith 1990; Nakamura et al. 1997; Luo 2000, 2005; Luo et al. 2014, 2019; Huang and Nakamura 2016; Nakamura and Huang 2018; Zhang and Luo 2020; Luo and Zhang 2020).

While some studies indicated that the formation of the blocking situation is related to large-scale topographic forcing (Egger 1978; Tung and Lindzen 1979; Charney and Devore 1979; Kalnay-Rivas and Merkin 1981), many more studies have revealed that the establishment and maintenance of quasi-stationary blocking are mostly due to the forcing of transient synoptic-scale eddies (Berggren et al. 1949; Shutts 1983; Colucci 1985; Haines and Marshall 1987; Holopainen and Fortelius 1987; Tsou and Smith 1990; Nakamura et al. 1997). Shutts (1983) noted that the eddy straining is important for the maintenance of blocking. However, the life cycle of blocking with a time scale of 10–20 days cannot be described by the previous models. Based on a nonlinear multiscale interaction (NMI) model, Luo (2000, 2005), Luo et al. (2014, 2019), and Luo and Zhang (2020) suggested that the presence of preexisting synoptic-scale eddies is critical for the evolution (growth, maintenance and decay) of blocking with a period of 10–20 days, whereas the eddy straining or cyclonic wave breaking (CWB) is more likely a result of the feedback of intensified blocking on preexisting synoptic-scale eddies (Luo et al. 2014, 2019; Zhang and Luo 2020; Luo and Zhang 2020). In this NMI model, the eddy-induced potential vorticity (PV) flux divergence by preexisting synoptic-scale eddies must be required to match the PV anomaly of an incipient blocking in order for it to grow, whereas their opposite match leads to the decay of blocking (Luo et al. 2014). The most important merit of this NMI model is that it not only captures the eddy straining or CWB during the blocking episode, but it also reflects the temporal evolution (growth, maintenance and decay) of a blocking wave packet associated with a meandering westerly jet stream with a time scale of 10–20 days. To some extent, the life cycle of blocking is a result of the natural evolution of preexisting synoptic-scale eddies with a specific spatial structure.
Some studies using reanalysis data have showed that the sudden stratospheric warming occurred about several days to 2 weeks after the onset of blocking, indicating that a blocking might serve as the initial trigger for the upward propagation of planetary waves (O’Neill and Taylor 1979; Quiroz 1986; Martius et al. 2009; Colucci and Kelleher 2015). Thus, blocking events are, to some extent, precursors to sudden stratospheric warming (SSW) events (Martius et al. 2009) via increasing upward wave activity pseudomomentum to the stratosphere (Lubis et al. 2018; Nakamura et al. 2020). The weak background westerly wind has been shown to favor the upward propagation of planetary waves associated with SSW events (Charney and Drazin 1961). However, the underlying dynamics, especially under what condition the planetary waves associated with the blocking decay can propagate strongly upward during the block, remain elusive. Many studies also revealed that the association between tropospheric blocking and the upward propagation of planetary waves is region dependent (Castanheira and Barriopedro 2010; Nishii et al. 2011; Kodera et al. 2013), suggesting that in some specific regions the blocking does not allow the upward propagation of tropospheric planetary waves. Consequently, it is inferred that there are some conditions that allow or not allow tropospheric planetary waves associated with the blocking evolution to propagate upward. This problem was not well understood in the previous studies because it cannot be investigated by using the previous other theoretical models of blocking (Shutts 1983; Haines and Marshall 1987). In contrast, the NMI model can avoid this difficulty. In this paper, the NMI model of eddy-driven blocking in a three-dimensional basic flow recently proposed by Luo and Zhang (2020) is used to examine the propagation and energy dispersion of an eddy-driven blocking wave packet in vertically varying basic westerly winds (BWWs). Through this study, we present a new finding that the vertical structure of the weak BWW and the zonal locality of eddy forcing in the troposphere are important for whether the excited planetary waves in the downstream region of blocking due to the blocking decay propagate upward.

This paper is organized as follows: In section 2, we describe the NMI model in a baroclinic basic flow without the effect of stratospheric processes. The baroclinic basic flow is assumed to have a slow vertical variation to obtain the analytical solution of the NMI model. In section 3, we present a new one-dimensional (1D) finite-amplitude local wave activity flux based on the NMI model. In section 4, we describe the numerical results about the wave activity associated with the blocking evolution in different basic flows using the new 1D finite-amplitude local wave activity flux. It is found that under local eddy forcing, the intensified planetary wave anomaly resulting from the decay and disappearance of blocking due to enhanced energy dispersion can propagate upward when the weak BWW is stronger in the upper troposphere than in the lower troposphere. In section 5, a perturbed inverse scattering transform method is used to obtain the temporal evolution equations of the amplitude, zonal wavenumber, group velocity, and phase speed of the blocking wave packet structure when the blocking wave packet is considered as an envelope soliton. The solutions of the evolution equations show that for vertically increased BWWs, the blocking wave packet can strongly disperse its energy toward downstream and upper troposphere to result in enhanced upward propagation of tropospheric planetary waves. The conclusions and discussions are summarized in section 6.

2. Nonlinear multiscale interaction model of eddy-driven blocking wave packet in a baroclinic basic flow and its solution

In this section, we describe the NMI model of blocking and its solution used in this paper. For simplicity we consider a pure baroclinic basic zonal flow $U(z)$ that satisfies the basic flow streamfunction $\Psi = -U(z)y$. In this case, the three-dimensional (3D) quasigeostrophic baroclinic PV equation of the disturbance streamfunction $\psi$ on a $\beta$ plane can be written as

$$\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x}\right) \psi + J(\psi, q) + \nabla \cdot \bar{\nabla} \psi = s,$$

where $PV = \bar{\nabla}q = \beta_0 - (1/p_0)(\partial/\partial z)[p_0(f_0^2/N)^2(\partial U/\partial z)]$, $\bar{\nabla}^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$, $J(a,b) = (\partial a/\partial y)(\partial b/\partial x) - (\partial a/\partial x)(\partial b/\partial y)$; $f_0$ and $\beta_0$ are the Coriolis parameter and its meridional gradient at a given reference latitude $\theta_0$, respectively; $N$ is the Brunt-Väisälä frequency of reference state atmosphere, and $p_0$ is the reference state atmospheric density with a tropospheric height $H$ (~10 km); $x, y, z,$ and $t$ are the zonal, meridional, vertical, and time coordinates, respectively; $s$ represents the sources and/or sinks of the disturbance PV. We assume that the blocking disturbance has a periodic boundary in the vertical domain (0, $H$) because the blocking has an equivalent barotropic structure, even though the vertical structure of blocking varies with the height. Such an assumption requires that the stratospheric process be ignored. Instead, here we only examine the propagation and energy dispersion of a blocking wave packet in the troposphere. In fact, whether planetary waves propagate from the troposphere to stratosphere is mainly determined by if the planetary waves can propagate upward in the troposphere. Thus, the above treatment is reasonable.

When the 3D PV Eq. (1) is scaled by the characteristic horizontal velocity $U$ (~10 m s$^{-1}$), length $L$ (~10$^6$ m) and height $H$ (~10$^5$ m), a nondimensional 3D PV equation for the nondimensional disturbance streamfunction $\bar{\psi}$ in the vertical domain (0, 1) similar to Eq. (1) can be derived. As in Luo and Zhang (2020), one can obtain a nondimensional two-dimensional (2D) PV equation in a nondimensional basic westerly wind $\bar{U}(z)$ when $\bar{\psi} = \psi(z)/\psi$ and $\psi(z) = 1 + Z_b \sin(\pi z)$ are assumed, where $Z_b$ denotes the vertical variation part of the nondimensional blocking or eddy amplitude, $\psi$ represents the nondimensional planetary-scale streamfunction of the blocking disturbance with zonal wavenumber $k$, and $\psi$ is the nondimensional synoptic-scale eddy streamfunction with zonal wavenumbers $k_j$ ($j = 1, 2$). For $\psi(z) = 1 + Z_b \sin(\pi z)$ and $Z_b > 0$, the baroclinic blocking shows largest amplitude in the midtroposphere ($z = 0.5$) and a
decreasing away from \( z = 0.5 \), when the planetary-scale disturbance streamfunction \( \psi \) is independent of height. The vertical position of the maximum amplitude of the baroclinic blocking becomes somewhat different if the disturbance streamfunction \( \psi \) of blocking is slowly varying in the vertical direction. Even for \( z = 0 \), the baroclinic blocking still has vertical variation in a baroclinic basic westerly wind \( U(z) \) but has no vertical variation for a uniform basic zonal flow that corresponds to a barotropic case.

When using \( \phi(z) = 1 + Z_0 \sin(\pi z) \) and integrating the nondimensional disturbance streamfunction \( \phi(z) \psi \) equation multiplied by \( \phi(z) \) from 0 to \( z \) as done in Luo and Zhang (2020), the nondimensional 3D PV equation can be reduced to a 2D PV equation when the basic zonal flow \( U(z) \) with a constant \( N^2 \) is assumed to be slowly varying in the vertical direction. Under the zonal-scale separation assumption \( k \ll k_p \) as used in Luo (2000, 2005) and Luo et al. (2007), the PV equations of the nondimensional planetary- and synoptic-scale disturbance streamfunctions \( \psi \) and \( \phi \) during their interaction in a \( \alpha \)-plane channel with a nondimensional width of \( L_\alpha \) can be obtained as

\[
\frac{\partial}{\partial t} + U \frac{\partial}{\partial x} q + \Theta J(\psi, q) + PV \frac{\partial \phi}{\partial x} = -\Theta \nabla \cdot (\mathbf{\nu} q'_p), \tag{2a}
\]

\[
\frac{\partial}{\partial t} + U \frac{\partial}{\partial x} q' + PV \frac{\partial \phi'}{\partial x} = -\Theta J(\psi', q') - \Theta J(\psi, q') + \nabla \phi'_s, \tag{2b}
\]

where \( PV \beta = (1/\rho_0)(\partial/\partial z)[\rho_0(\hat{L}_f^2/\hat{H}^2 N^2)(\partial U/\partial z)], \) \( q(x, y, z) = \nabla \cdot \psi - \Phi \psi, \) \( \Phi = \beta_0 \hat{L}_f^2 \hat{U}, \Theta = \{(1 + (6/\pi)Z_0 + (3/2)Z_1 + (4/3)\pi Z_2)/(1 + 4Z_0/\pi + Z_2/2), F = F_0 + Z_0 \phi_0^1 \}, \) \( \Theta = (\partial/\partial x, \partial/\partial y), \)

\( F = \hat{L}_f^2 \hat{U} \) and \( \nabla \phi \) have been assumed to be a synoptic-scale wavemaker for the derivation of Eq. (2), which maintains preexisting synoptic-scale eddies before blocking onset. In Eq. (2a), \( \mathbf{\nu} = (-\partial \phi'/\partial y, \partial \phi'/\partial x) \) is the horizontal wind vector of synoptic-scale eddies, whereas the subscript \( P \) of \( \nabla \cdot (\mathbf{\nu} q') \) represents that \( -\nabla \cdot (\mathbf{\nu} q') \) has the same wavenumber structure as the PV anomaly \( q(x, y, z) \) of blocking flow \( \psi \) (Luo 2000, 2005; Luo et al. 2014). For \( \rho_0 = \rho_0 e^{-z} \), \( \rho_0 \) is the reference atmospheric density at Earth’s surface \( (z = 0) \), one can have \( PV \beta = (1/\rho_0)(\partial/\partial z)[\rho_0 \hat{L}_f^2 \hat{U} \phi' \phi_0^1 \] \( \phi_0^1 \) is the ratio of \( U(z) \) and \( PV \) of \( U(z) \) is slowly varying in the vertical direction, the streamfunction \( \psi \) and \( \phi \) fields of blocking and synoptic-scale eddies are dependent on the height \( z \), though Eq. (2) is a 2D horizontal form.

Equation (2) can be considered as a simplified form of the NMI model in a 2D baroclinic basic flow obtained by Luo and Zhang (2020) because \( U(z) \) considered in this paper is only dependent on \( z \). This model can be solved by using the Wentzel–Kramers–Brillouin (WKB) method when \( \psi' = \psi'_1 + \psi'_2 \), is used as in Luo (2005), where \( \psi'_1 \) is the second-order solution of \( \psi \) and \( \psi'_2 \) is its first-order solution, which satisfies a linearized PV Eq. (2b) without forcing. In \( \psi' = \psi'_1 + \psi'_2, \) \( \psi'_1 \) represents the prespecified preexisting synoptic-scale eddies having a meridional monopole structure \( m/2 \) and zonal wavenumbers \( k_1 \) and \( k_2 \) with the corresponding frequencies \( \omega_1 \) and \( \omega_2 \), respectively, whereas \( \psi'_2 \) represents the deformed eddies produced by the feedback of intensified blocking on preexisting synoptic-scale eddies that can be derived from the second-order equation of Eq. (2b). Using the solution of Eq. (2), one can examine the wave activity associated with the temporal evolution of eddy-driven blocking wave packet. Below, we present the analytical solution of the NMI model.

We further assume that the carrier wave of the blocking wavy anomaly has zonal and meridional wavenumbers \( k \) and \( m \), respectively, whereas its envelope amplitude is slowly varying in the zonal direction and time and also in the vertical direction when the weak BWW shows a slow vertical variation. The analytical solution of Eq. (2) can be obtained based on the method of Luo (2005) and Luo et al. (2007). Naturally, the solution of the nondimensional total streamfunction of eddy-driven blocking wave packet can be expressed as \( \psi_F = \Psi + \psi = -U(z)y + \phi(z)(\psi + \psi') \), where \( \Psi \) is a nondimensional form of \( \Psi \); \( U(z) \) is a nondimensional baroclinic basic westerly wind; and \( z \) and \( y \) are nondimensional vertical and meridional coordinates, respectively. As in Luo and Zhang (2020), the analytical solution of the nondimensional total streamfunction field \( \psi_F \) of eddy-driven blocking wave packet in nondimensional fast variable coordinates \( x, y, z, t \) can be approximately obtained as

\[
\psi_F = -U(z)y + \phi(z)(\psi + \psi') = \psi_F + \psi'_0, \tag{3a}
\]

\[
\psi_F = -U(z)y + \phi(z)\psi = -U(z)y + \psi_F + \psi'_m, \tag{3b}
\]

\[
\psi_F = -\phi(z)\theta |B|^2 \sum_{n=1}^{\infty} Q_n g_n \cos(n + 1/2)my, \tag{3c}
\]

\[
\psi_{1} = \phi(z)\psi' = \phi(z)(\psi'_1 + \psi'_2) = \psi_{1} + \psi_{2}, \tag{3d}
\]

\[
\psi_{2} = \phi(z)\psi' = \phi(z)(\psi'_1 + \psi'_2) = \psi_{2}, \tag{3e}
\]

\[
\psi_{3} = \phi(z)\psi' = \phi(z)(\psi'_1 + \psi'_2) = \psi_{3}, \tag{3f}
\]

\[
\phi(z) = \phi(z) \theta B |B|^2 \sum_{n=1}^{\infty} Q_n g_n \cos(n + 1/2)my. \tag{3g}
\]
where $B$ represents the envelope amplitude of the blocking wave packet that is described by a nonlinear Schrödinger (NLS) equation with an eddy forcing $-\nabla \cdot (\nabla' \phi' )_p$, $[\phi'_1 = \nabla^2 \phi'_1 - F\phi'_1$, and $\psi'_1 = (-\partial \phi'_1/\partial z, \partial \phi'_1/\partial x)]$ induced by preexisting synoptic-scale eddies $\phi'_1$ and $B''$ in $|B|^2 = B''^2$ denotes the complex conjugate of $B$ and $c c$ denotes the complex conjugate of its preceding term. The term $- \nabla \cdot (\nabla' \phi')_p$ represents a local eddy forcing when $f_0(x)$ is zonally localized. In Eq. (3c), $k$ represents a constant zonal wavenumber of the preexisting planetary wave before the blocking onset, which will be slowly varying as a function of slow time, zonal and vertical coordinates when the envelope amplitude $B$ is changed with the local eddy forcing in different vertically varying basic flows. Also in Eq. (3c), $\omega = UK - PV, k (k'^2 + m^2 + F)$ is the frequency of linear Rossby wave whose envelope amplitude $B$ is slowly varying, where $m = -2\pi /L_y k = 2k_0$ and $k_0 = 1/(6.371 \cos \phi_0)$. In Eq. (3h), $C_g = \partial \omega / \partial k = \partial - PV (m^2 + F - k^2 (k^2 + m^2 + F)^2)$ is the linear group velocity, $\delta = (\delta G / PV, \delta_0 = ([3(m^2 - F - k^2 k^2 + m^2 + F)^2) is the linearity strength, $G = \theta G_B, \omega = \omega_0 - \omega_1$ and $-\delta \omega = PV \omega_1 (k^2 (k^2 + m^2 + F) - k_1 (k_1^2 + m^2 + F) - k (k^2 + m^2 + F))$ for $\Delta k = k - (k_2 - k_1) = 0$ or $k = k_2 - k_1$. In Eq. (3d), $q_B = q_{0B} / PV_0$ is the nonlinear self-interaction frequency of each mode for preexisting synoptic-scale eddies $\psi'$. In Eqs. (3f)–(3h), $f_0(x) = a_0 e^{-\mu x^2 + 1/t_x^2}$ represents the zonal distribution of the slowly varying eddy amplitude of preexisting synoptic-scale eddies with a maximum intensity $a_0$ located at $x = -x_t, t < 1.0$ and $\mu > 0$. The descriptions of $r_j, r_J, s_k, h_j, Q_j, D_j, \delta, q_{0B}, g_n, \text{ and } G_B$ can be found in Luo and Zhang (2020). In the NMI model, the type of preexisting synoptic-scale eddies that favor blocking has been discussed in Luo (2000, 2005), Luo et al. (2014, 2019), and Luo and Zhang (2020). An important condition of preexisting synoptic-scale eddies leading to the blocking formation is that $-\nabla \cdot (\nabla' \phi'_p) / \partial t \sim -\nabla \cdot (\nabla' \phi'_p) \sim -\nabla \cdot (\nabla' \phi'_p)$ during the initiation stage of blocking must match the PV anomaly $q$ of the incipient blocking in order for an incipient blocking to grow into a typical blocking circulation for given preexisting eddy strength and initial blocking structure, when the preexisting synoptic-scale eddies satisfy $m = -2\pi /L_y$ and $\alpha = -1$ (Luo et al. 2014; Luo and Zhang 2020). While blocking is excited by preexisting synoptic-scale eddies $\psi'_s$, its intensity, movement, and lifetime are also influenced by the background condition (mainly the magnitude of $PV_0$). Clearly, in the NLS equation there is a $A = 1/0 \sim PV_0$, which suggests that there is an inverse ratio rule between the linear dispersion and nonlinearity strength for a blocking wave packet in the NMI model. When $PV_0$ is small, especially in high latitudes, the linear dispersion is weak and the nonlinearity is strong, thus implying that the blocking can maintain for a long time (Luo et al. 2019). As shown in Luo and Zhang (2020), $\psi'_B$ and $\psi'_R$ in Eq. (3) represent planetary- and synoptic-scale parts of the blocking streamfunction field, respectively, whereas $\psi'_B$ denotes the blocking wave anomaly part and $\psi'_m$ is the blocking-induced mean zonal wind change due to the nonlinear self-interaction of blocking anomaly $\psi_B$. In Eq. (3f), preexisting synoptic-scale eddies $\psi'_s$ that satisfy $m = -2\pi /L_y$ and $\alpha = -1$ are assumed to be located in the upstream side of incipient blocking (Holopainen and Fortelius 1987), whereas $\psi'_s$ denotes deformed eddies induced by the feedback of intensified blocking because it includes the blocking envelope amplitude $B$. The deformed eddies $\psi'_s$ reflects the presence of CWB or eddy straining during the blocking episode (Luo et al. 2014). When blocking disappears, no CWB or eddy straining can take place. Thus, it is inferred that the eddy straining or CWB during the blocking life cycle is likely a result of the blocking intensification and maintenance, which contradicts the eddy straining theory of Shutts (1983). In this paper, in order to examine the wave activity (propagation and energy dispersion) of eddy-driven blocking wave packet for different baroclinic basic zonal flows, we use the same high-order split-step Fourier scheme used in Luo and Zhang (2020) and perturbed inverse scattering transform (PIST) method (Okamawari et al. 1995) to solve Eq. (3h) for given initial value and parameter conditions. 3. Finite-amplitude wave activity flux of blocking of a baroclinic basic flow Although atmospheric blocking is generally driven by high-frequency synoptic-scale eddies, it may be approximately considered as a free quasi-stationary wave (Holopainen and Fortelius 1987). When blocking has small amplitude, the linear small-amplitude theory is approximately accepted. Thus, the previous small-amplitude wave activity flux can be used as diagnosing the wave activity of small-amplitude blocking (Plumb 1985; Takaya and Nakamura 2001). But such a small-amplitude wave activity flux is not appropriate for diagnosing the wave activity associated with the blocking mature stage because it has large amplitude. Our NMI model provides a new way for solving this problem, though highly idealized. In what follows, we use Eq. (3) to construct a 1D finite-amplitude local wave activity flux to examine the wave activity of eddy-driven blocking evolution, even though the blocking may be strongly nonlinear. Here, we consider the case of $\Delta k = k - (k_2 - k_1) = 0$, but the result is similar for $\Delta k \neq 0$. From Eq. (3h), one can obtain

$$i \left( i \frac{\partial |B|^2}{\partial t} + C_g \frac{\partial |B|^2}{\partial x} \right) + \lambda \frac{\partial |B|^2}{\partial x} + \delta |B|^2 + B \cdot \frac{\partial B^*}{\partial x} \exp [-i(\Delta k x + \Delta \omega t)] = 0,$$

$$+ G_{d}^0 \left[ B^* \exp (-i \Delta \omega t) - B \exp (i \Delta \omega t) \right] = 0,$$
are defined for \( \text{PV} \neq 0 \), one can easily obtain the following conservation relation in the horizontal direction from Eq. (4):
\[
\frac{\partial A}{\partial t} + \nabla \cdot \mathbf{F} = \frac{i}{\text{PV}} G \int_0^t \left[ B^* \exp(-i \Delta t \omega) - B \exp(i \Delta t \omega) \right],
\]
where \( \mathbf{F} = C_e A + F_R = (F_x, 0), F_R = C_e A + F_{R_x} \) and \( C_e \) is the horizontal group velocity vector.

In Eq. (5), \( A = |B|^2/\text{PV} \) is the wave-activity pseudomomentum as in Takaya and Nakamura (2001), and \( F_R \) reflects the effect of the zonal nonuniform variation of the blocking envelope amplitude \( B \). When the blocking envelope amplitude \( B \) has the solution of \( B = |B|e^{i \theta} \) (where \( \theta \) is the phase of the blocking envelope amplitude \( B \) and \( |B| \) is its modulus amplitude), we have \( F_{R_x} = 2 \omega_0 |B|^2 (\partial \theta/\partial x) \). Clearly, the wave activity flux \( F_R \) is induced by the zonal nonuniform phase \( \theta \) of blocking envelope amplitude and proportional to the square of its modulus amplitude \( B \), which also reflects the zonal variation of the phase of the blocking envelope amplitude \( B \) and the change of the modulus amplitude \( |B| \) due to local eddy forcing. Thus, \( F_R \) is referred to as a local eddy-induced wave activity flux hereafter. We find \( F_{R_x} = 2 \omega_0 |B|^2 (\partial \theta/\partial x) \approx 0 \) when \( |B| \approx 0 \) even for \( \partial \theta/\partial x \neq 0 \). This suggests that there are \( F_R \approx 0 \) and \( F = C_e A + F_R \approx C_e A \) for a small-amplitude blocking similar to the case discussed by Takaya and Nakamura (2001), implying that the linear energy propagation is dominant for a small-amplitude blocking. But when the blocking has large amplitude, \( F_R \) can become a nonnegligible term. In this case, one cannot simply use \( F \approx C_e A \) to examine the wave activity during the blocking episode. Thus, the wave activity flux (WAF) \( F = C_e A + F_R \) obtained in this paper may be considered as an extension of the previous small-amplitude WAF by Plumb (1985) and Takaya and Nakamura (2001).

For an unforced case \( (f_0 = 0) \), we can have \( \partial \mathbf{F}/\partial t + \nabla \cdot \mathbf{F} = 0 \), which means that the wave-activity pseudomomentum \( A = |B|^2/\text{PV} \) is conserved even for a large-amplitude blocking flow. Thus, it is thought that \( \mathbf{F} \) is a new finite-amplitude local wave activity flux different from the finite-amplitude local wave activity flux formalism of Huang and Nakamura (2016) and Nakamura and Huang (2018), who found that the finite-amplitude WAF in the zonal direction consists of linear WAF and nonlinear part related to the square of the finite-amplitude local wave activity. But, in our paper this \( \mathbf{F} \) flux consists of two parts: linear WAF \( (C_e A) \) and nonlinear part \( F_R \) associated with the blocking envelope amplitude \( B \) and its nonuniform zonal variation because the zonal component of \( F_R \) is proportional to \(-i[B^*(\partial B/\partial x) - B(\partial B^*/\partial x)]\). The conservation relationship (5) still holds in the different vertical layers because \( U \) and \( B \) are the slowly varying functions of \( z \).

When \( f_0 = 0 \), the envelope soliton solution of Eq. (3h) can be obtained as (Luo 2000)
\[
B = M_0 \text{sech} \left[ \frac{\delta}{2 \lambda} \left( x - C_g t \right) \right] e^{-i(kz+2\lambda t)},
\]
where \( M_0 \) is a constant amplitude of \( B \) at \( x = 0 \) and \( t = 0 \) for a uniform BWB.

Clearly, \( |B| = M_0 \text{sech} \left[ \sqrt{(\delta/2\lambda)} \left( x - C_g t \right) \right] \) is the zonal-dependent modulus amplitude of blocking envelope amplitude \( B \), which shows a solitary wave structure in the zonal direction and propagates with the linear group velocity \( C_g \). Equation (6) can also be rewritten as \( B = |B|e^{i \theta} \), where \( \theta = -i \delta M_0^2/2 \) is the phase of the blocking envelope amplitude and independent of \( x \). Thus, \( B^*(\partial B/\partial x) - B(\partial B^*/\partial x) \) or \( F_R \) vanishes because of \( \partial \theta/\partial x \) being zero. In this case, there is \( \mathbf{F} = C_e A + F_R = C_e A \). In other words, the small-amplitude WAF, \( \mathbf{F} = C_e A \), is still correct when the blocking is an unforced wave packet with a zonal uniform phase \( \theta \). But the WAF, \( \mathbf{F} = C_e A \), does not hold for finite-amplitude wave activity associated with the blocking evolution when blocking is a local eddy-forced flow and has large amplitude.

The above analyses lead us to infer that during the life cycle of eddy-driven blocking, \( F_R \approx 0 \) and \( \mathbf{F} = C_e A + F_R \approx C_e A \) hold during the initial and disappearance stages of blocking for the blocking amplitude being small, thus implying that the linear energy propagation process is dominant. But \( F_R \) is important for the wave activity during the blocking mature phase because the blocking amplitude is relatively large. Nevertheless, when \( C_e \) and \( B \) are slowly varying functions of \( x, y, \) and \( z \), \( B \) becomes a 3D WAF if blocking has a wave solution in the meridional and vertical directions. Thus, using \( \mathbf{F} = C_e A + F_R \) one can crudely examine the wave activity during the blocking evolution by solving Eq. (3h) for given initial value and parameter conditions using the numerical method and perturbation theory.

4. Zonal and vertical propagation and energy dispersion of eddy-driven blocking wave packet in the NMI model

In this section, because our attention is focused on examining the wave activity (propagation and energy dispersion) of blocking evolution in a baroclinic basic zonal flow, we may allow the strength and distribution of BWW to vary in the vertical direction but fix other parameters, though the strength \( a_0 \) of preexisting synoptic-scale eddies can influence the behavior of eddy-driven blocking (Luo et al. 2014, 2019). For example, the preexisting synoptic-scale eddies are too weak to create a typical blocking from a small-amplitude incipient blocking (not shown). In this paper, the fixed parameters are \( N^2 = 2 \times 10^{-4} \text{s}^{-1}, \ k = 2 \omega_0, \ k_1 = 9 \omega_0, \ k_2 = 11 \omega_0, \ k_0 = 1/[6.371 \text{cos}(\varphi_0)], \ a_0 = 0.12, \mu = 1.2, \epsilon = 0.24, \nu = 28.7/2, \varphi_0 = 55^\circ N, \lambda_g = 5, \) and \( \varphi_g = 0.2 \). Then, there are \( F_x = 0.72 \) and \( F = 0.82. \) Here, \( B(x, z, 0) = 0.3 \text{sech} [\sqrt{(\delta/2\lambda)}(0.3x)] \) is chosen as the initial amplitude of blocking, which represents a small-amplitude local incipient blocking. Moreover, preexisting local synoptic-scale eddies are assumed to be located in the up-stream side of this small incipient blocking. Calculations demonstrate that the basic features obtained from the NMI model are similar for different initial conditions and eddy parameters (not shown). Here, we only present the result of a weak BWW that allows \( \omega \) in Eq. (3c) to be near zero because a strong BWW does not allow a typical blocking to occur (Kalnay-Rivas and Merkine 1981).

a. Upward propagation of quasi-stationary waves associated with blocking evolution and its dependence on the vertical structure of baroclinic basic zonal flow

As an example, we consider \( U(z) = u_0 + \Delta u t z^2 + \Delta t z^2 \) as an idealized BWW, where \( u_0 \) is the uniform basic westerly wind
part, $\Delta u_1$ represents a constant first-order vertical shear of $U(z)$ on $z = 0$ as a linear vertical shear and $\Delta u_2$ denotes a constant second-order vertical shear of $U(z)$ as a nonlinear vertical shear. When $\Delta u_1 > 0$ and $\Delta u_2 = 0$ ($\Delta u_2 > 0$), the BWW shows a linear (nonlinear) vertically increasing with the height and is referred to a linear (nonlinear) vertically increased BWW. But when $\Delta u_1 < 0$ and $\Delta u_2 = 0$ ($\Delta u_2 > 0$), the BWW decreases linearly (nonlinearly) with the height and is referred to a linear (nonlinear) vertically decreased BWW. For a fixed $u_0$, the values of $\Delta u_1$ and $\Delta u_2$ are assumed to be small to allow the WKB method. In this case, $U(z)$ is weak for a small $u_0$. Figure 1 shows different vertical profiles of BWW (Fig. 1a) and PV$_y$ (Fig. 1b) for $u_0 = 0.5$ and the different values of $\Delta u_1$ and $\Delta u_2$. In this paper, we only present the result of a fixed $u_0 = 0.5$, whereas the results for the other moderate values of $u_0$ are similar and not shown here. Clearly, in Fig. 1b PV$_y$ is larger for vertically increased BWWs than for vertically decreased BWWs.

We first examine the case of a nonlinear vertically increased BWW ($\Delta u_1 = 0.2$ and $\Delta u_2 = 0.2$), which corresponds to BWW being stronger in the upper troposphere than in the lower troposphere (Fig. 1a). For such a BWW, Fig. 2 shows the daily fields of instantaneous planetary-scale streamfunction $\psi_p$, blocking wavy anomaly $\psi_B$, synoptic-scale eddy streamfunction $\psi'_y$ and total streamfunction $\psi_T$ for an eddy-driven baroclinic blocking in the midtroposphere ($z = 0.5$) obtained from the NMI model solution (3). It is clearly seen that the NMI model can better capture the life cycle of a blocking event with a lifetime of 10–20 days. The planetary-scale streamfunction $\psi_p$ (Fig. 2a) and wavy anomaly $\psi_B$ (Fig. 2b) fields show the growth, maintenance and decay of an eddy-driven antisymmetric dipole blocking whose mature structure resembles a modon solution (McWilliams 1980; Haines and Marshall 1987). Along with the growth and maintenance of this blocking dipole (Figs. 2a,b) the preexisting synoptic-scale eddies (Fig. 2c for day 0) are intensified and split into two branches around the blocking region (Fig. 2c for days 3, 6, and 9). Such an eddy deformation corresponds to CWB or eddy straining. Thus, the CWB or eddy straining may be thought of as being a result of the blocking intensification and maintenance. Moreover, we see that the instantaneous total streamfunction field of eddy-driven blocking shows a life cycle of intensified meandering westerly jet streams (Fig. 2d), consistent with the observational result (Berggren et al. 1949) and the theoretical result of Luo (2000, 2005) and Luo et al. (2014), which also resembles the traffic jam in the jet stream noted by Nakamura and Huang (2018). Thus, our NMI model cannot only describe the spatial structure and time scale of blocking, but also a large meandering of westerly jet streams. Similar blocking circulation patterns can also be obtained from solution (3) for other basic flow parameters (not shown). The above results suggest that the NMI model is appropriate for representing observed blocking structures and their life cycle. Thus, we can apply the NMI model to examine the wave activity during the blocking life cycle to explain under what condition quasi-stationary waves associated with the blocking evolution can propagate upward and downstream.

To examine what type of BWW favors the upward propagation of quasi-stationary waves during the blocking life cycle, it is useful to show the instantaneous zonal-vertical profiles of the modulus amplitude $|B|$ of blocking envelope amplitude $B$ and the anticyclonic anomaly of blocking wavy anomaly streamfunction $\psi_B$ at $y = 3.75$ during the blocking life cycle in Fig. 3 for a nonlinear vertically increased BWW ($\Delta u_1 = 0.2$ and $\Delta u_2 = 0.2$) and a nonlinear vertically decreased BWW ($\Delta u_1 = -0.2$ and $\Delta u_2 = 0.2$). The nonlinear vertically decreased BWW represents that its strength decreases (increases) with the height below (above) the midtroposphere ($z = 0.5$). It is noted that for a nonlinear vertically increased BWW the large value region of the blocking modulus amplitude $|B|$ is shifted to the upper troposphere in the downstream region of blocking during the decay and disappearance stages.
FIG. 2. Instantaneous (a) planetary-scale streamfunction $\psi_p$ (CI = 0.15), (b) blocking wavy anomaly $\psi_B$ (CI = 0.2), (c) synoptic-scale eddy streamfunction $\psi'_e$ (CI = 0.3), and (d) total streamfunction field $\psi_T$ (CI = 0.3) in the midtroposphere ($z = 0.5$) of eddy-driven baroclinic blocking ($Z_b = 0.2$) during the blocking life cycle for an initial amplitude $B(x, z, 0) = 0.3 \text{sech}[\sqrt{0.3/(2\lambda)}0.3]$. The abscissa (ordinate) is the nondimensional $x$ ($y$) axis, the dashed (solid) line represents the cyclonic (anticyclonic) anomaly and the red (blue) shading in Fig. 2d represents the high (low) pressure.
FIG. 3. Instantaneous zonal–vertical profiles of (a),(c) the modulus amplitude $|B|$ (CI = 0.05) of the blocking envelope amplitude $B$ and (b),(d) wavy anomaly streamfunction $\psi_B$ (CI = 0.1) at $y = 3.75$ of eddy-driven blocking during the blocking life cycle for the initial value $B(x, z, 0) = 0.3 \text{sech}[\sqrt{\delta/(2\lambda)} 0.3x]$ in the different basic flow $U = u_0 + \Delta u_1 z + \Delta u_2 z^2$ for (a), (b) nonlinear vertically decreased BWW with a smallest intensity in the midtroposphere ($\Delta u_1 = 0.2$ and $\Delta u_2 = 0.2$) and (c), (d) nonlinear vertically increased BWW ($\Delta u_1 = 0.2$ and $\Delta u_2 = 0.2$) with a fixed $u_0 = 0.5$. The value of $u_0$ is the same as below.
(Fig. 3c for days 15, 18, and 21). For this case, the upward migration of the large value region of the blocking wavy anomaly \( \psi_b \) at \( y = 3.75 \) seems more evident than that of the blocking modulus amplitude \( |B| \) (Fig. 3d for days 15, 18, and 21). This suggests that the planetary waves can propagate eastward and upward in the troposphere mainly in the downstream region of blocking during the decay/disappearance phase of blocking, which is consistent with the data result of Kodera et al. (2013), who noted that for the North Atlantic blocking, an upward-propagating planetary wave occurs in the east side of the blocking region. The upward propagation is suppressed even for vertically increased BWWs if \( u_0 \) is too small (not shown). For the same parameters as in Figs. 3c and 3d, upward- and eastward-propagating planetary wave can still occur during the blocking decay phase for a relatively weak local eddy forcing with \( a_0 = 0.08 \) (Fig. S1 in the online supplemental material), though the upward propagation is slightly weaker than for \( a_0 = 0.12 \). We can also see eastward- and upward-propagating planetary waves during the blocking decay phase even for a uniform initial amplitude \( B(x, z, 0) = 0.3 \) (not shown). In contrast, the upward propagation of the blocking anticyclonic anomaly disappears for a constant \( f_0 \) (or a zonally uniform eddy forcing) (not shown). On the other hand, we note that for a large-amplitude incipient blocking with \( B(x, z, 0) = 0.5 \text{sech} \left[ \sqrt{(\delta/2\lambda)} \cdot 0.5x \right] \), a more intense upward-propagating planetary wave is easily seen during the blocking decay phase for large-amplitude blocking events (Fig. S2) formed under the same condition as in Fig. 2. For a uniform barotropic westerly wind (\( u_0 = 0.5, \Delta t_1 = 0, \) and \( \Delta t_2 = 0 \)), the upward propagation of planetary waves disappears during the blocking decay phase even for a locally eddy forcing (Fig. S3). In addition, we find that the meridional structure of the barotropic basic zonal flow \( U(y) \) cannot trigger the upward propagation of tropospheric planetary waves during the blocking decay phase even for a locally eddy forcing. For example, for \( U = u_0 + \Delta u_1 e^{-0.3(x-1.5)^2}, u_0 = 0.5, \) and \( \Delta u_1 = 0.2 \) (Fig. S4a), one cannot see the upward propagation of planetary waves even during the blocking decay phase (Figs. S4bc). Thus, the vertical structure of BWW rather than its meridional structure is critical for whether the upward propagation of planetary waves occurs during the blocking decay/disappearance phase. For a nonlinear vertically increased BWW (red line in Fig. 4b) than a linear vertically increased BWW (black line in Fig. 4b). Such a shorter local lifetime means that the blocking has a stronger energy dispersion in this case. It is interesting to see that for vertically increased BWWs the maximum anticyclonic anomaly moves slowly during the growth and mature periods (before day 12), but moves rapidly eastward (black and red lines in Fig. 4c) and upward (black and red lines in Fig. 4d) during the decay/disappearance phase (from days 15 to 21) of blocking, thus indicating that the energy of quasi-stationary waves associated with the decay and disappearance of blocking can propagate strongly upward and eastward. In contrast, for vertically decreased BWWs the blocking moves slowly upstream (blue and green lines in Fig. 4e) during the blocking growing phase, but its decay does not accompany the upward propagation of planetary waves (blue and green lines in Fig. 4d). Instead, it may allow planetary waves to propagate downward when the BWW is weaker in the upper troposphere than in the lower troposphere (blue line in Fig. 4d). Crudely speaking, westward-propagating blocking does not allow the upward propagation of planetary waves but may allow the downward propagation of planetary waves. Thus, the above results reveal that in weak basic westerly winds whether quasi-stationary or slowly eastward-propagating planetary waves associated with the blocking decay propagate upward depends on the vertical structure of BWW.

b. Wave activity of quasi-stationary planetary waves associated with the blocking life cycle

As seen from the expression of \( F = C_A + F_R \), the finite-amplitude local wave activity flux \( F \) does not only depend on the modulus amplitude \( |B| \) of the blocking envelope amplitude \( B \), but also on its real and imaginary parts \( (B_R \) and \( B_I) \) of \( B = B_R + iB_I \) because \( F_{R,R} = 2u_0|B|^2(\partial \psi_b/\partial x) \cos \theta_B = B_B |B|^2 \) and \( |B|^2 \) represents the blocking energy. Thus, it is useful to show...
the time-zonal evolutions of $|B|$, $B_R$, and $B_I$ on $z = 0.5$ during the blocking life cycle in Fig. 5 for nonlinear vertically decreased ($\Delta u_1 = -0.2$ and $\Delta u_2 = 0.2$) and increased ($\Delta u_1 = 0.2$ and $\Delta u_2 = 0.2$) BWWs. It is seen from Fig. 5 that the blocking energy $|B|$ always propagates eastward for the two types of basic flows (Figs. 5a,d), which represents the group velocity of the blocking wave packet being positive rather than the eastward propagation of the blocking wavy anomaly. We note that the blocking wavy anomaly $\theta_B$ propagates slowly westward (is almost immobile) during the blocking growth stage for a nonlinear vertically decreased (increased) BWW (Fig. 4c). On the other hand, we also see that $B_R$ (Figs. 5b,e) and $B_I$ (Figs. 5c,f) have different spatial structures and show different temporal evolutions in different basic flows, thus leading to different distributions of $F_{R/\theta}$. Below, we use the spatiotemporal variations of $|B|$, $B_R$, and $B_I$ to calculate the wave activity flux in different basic flows.

Defining $F_L = C_L A = (C_L A, 0)$ as a linear WAF, one can have $\nabla \cdot F = \nabla \cdot F_L + \nabla \cdot F_{R/\theta}, \nabla \cdot F_L = \partial (C_L A) / \partial x$, and $\nabla \cdot F_{R/\theta} = \partial F_{R/\theta} / \partial x$. We show the wave activity flux vector $F_{R/\theta}$, $F_L$, and $F$ as well as their corresponding divergence terms $\nabla \cdot F_{R/\theta}, \nabla \cdot F_L$, and $\nabla \cdot F$ averaged over the growth (from days 0 to 6), mature (from days 7 to 13), and decay (from days 15 to 21) periods of the blocking life cycle in Fig. 6 (growth phase), Fig. 7 (mature phase), and Fig. 8 (decay phase) for nonlinear vertically decreased ($\Delta u_1 = -0.2$ and $\Delta u_2 = 0.2$) and increased ($\Delta u_1 = 0.2$ and $\Delta u_2 = 0.2$) BWWs. It is found that for a linear WAF, $F_L$ (Figs. 6b,e), during the blocking growth period the length of its arrow is increased (decreased) with $x$ in the upstream (downstream) side of $x = 0$ near the blocking center. Correspondingly, the divergence (convergence) of $F_L$ for

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**Fig. 4.** (top) Time series of (a) daily blocking intensity $\psi_N$, as defined by the maximum amplitude of daily blocking anticyclonic anomaly $\theta_B$ at $y = 3.75$ and (b) daily domain-averaged blocking intensity $\psi_A$ of streamfunction anomaly $\theta_B$ averaged over the region ($-1.5 \leq x \leq 0.5, 3.5 \leq y \leq 4$) during the blocking life cycle. (bottom) Temporal variations of (c) zonal ($x$) and (d) vertical ($z$) positions of the maximum anticyclonic center of the blocking wavy anomaly streamfunction $\theta_B$ at $y = 3.75$ of eddy-driven blocking in the different basic flows $U = u_0 + \Delta u_1 z + \Delta u_2 z^2$ for linear vertically increased BWW ($\Delta u_1 = 0.2$ and $\Delta u_2 = 0$; black line), nonlinear vertically increased BWW ($\Delta u_1 = 0.2$ and $\Delta u_2 = 0$; red line), linear vertically decreased BWW ($\Delta u_1 = -0.2$ and $\Delta u_2 = 0$; blue line) and nonlinear vertically decreased BWW ($\Delta u_1 = -0.2$ and $\Delta u_2 = 0$; green line).
$\nabla \cdot \mathbf{F}_L > 0 \ (\nabla \cdot \mathbf{F}_L < 0)$ with a zero line near $x = 0$ and $x = -0.3$ on $z = 0$ for nonlinear vertically decreased and increased BWWs, respectively, are seen in the upstream (downstream) region of blocking (Figs. 6b,e). During the blocking growth period, $\nabla \cdot \mathbf{F}_R < 0 \ (\nabla \cdot \mathbf{F}_R > 0)$ mainly appears in the upstream (downstream) region from $x = -3.0$ to $x = 0 \ (x = 0$ to $x = 3.0)$ as shown in Figs. 6a and 6d. At the same time, the strength of $\nabla \cdot \mathbf{F} > 0 \ (\nabla \cdot \mathbf{F} < 0)$ is weakened in the upstream (downstream) side of the blocking center, whereas its zero line is shifted westward and located at $x = -0.2 \ (x = -0.7)$ for a nonlinear vertically increased (decreased) BWW (Figs. 6c,f). Thus, the presence of the local eddy-induced wave activity flux $\nabla \cdot \mathbf{F}_R$ tends to reduce the divergence (convergence) of $\mathbf{F}_L$ in the upstream (downstream) side of the blocking center near $x = 0$ and leads to a small upstream displacement of $\nabla \cdot \mathbf{F} < 0$ to prevent the weakening of upstream planetary wave activity and the intensifying of downstream planetary wave activity, thus inhibiting the linear downstream energy dispersion of blocking wave packet to favor blocking. Such a feature can still be seen during the blocking mature period from a comparison of Fig. 7 with Fig. 6.

During the blocking decay phase (from days 15 to 21) (Fig. 8) the strong convergence (divergence) of $\mathbf{F}_L$ on $z = 0$ is seen in the downstream (upstream) region of $x = 2.5 \ (x = 2)$, which shifts eastward (westward) with the height in the troposphere (lower troposphere) for a nonlinear vertically increased (decreased) BWW as shown in Fig. 8e (Fig. 8b). The strongest convergence ($\nabla \cdot \mathbf{F} < 0$) of $\mathbf{F}$ is located in the lower troposphere and in the east of $x = 2.5$ for a nonlinear vertically decreased BWW (Fig. 8c), but in the upper troposphere and in the east of $x = 4$ for a nonlinear vertically increased BWW (Fig. 8f). It is also seen that $\nabla \cdot \mathbf{F}_R$ is negative in the region from $x = 1$ to $x = 3.5 \ (x = 1$ to $x = 4)$ on $z = 0$ for a nonlinear vertically decreased (increased) BWW (Figs. 8a,d). A comparison with Figs. 8b–f shows that $\nabla \cdot \mathbf{F}_R < 0$ tends to strengthen the negative $\nabla \cdot \mathbf{F}_L$ in the upper (lower) troposphere of the blocking downstream region for a nonlinear vertically increased (decreased) BWW. Especially, for a nonlinear vertically increased BWW, $\nabla \cdot \mathbf{F}_R < 0$ shifts eastward with height and reaches the upper troposphere in the east of $x = 5$, which enhances the planetary wave activity in the upper troposphere in the downstream region of blocking. Clearly, the linear wave activity flux $\mathbf{F}_L = C_g \mathbf{A}$ related to the group velocity of $C_g$ plays a major role in the increased upward planetary wave amplitude in the upper troposphere in the downstream region of blocking during the blocking decay phase, whereas the local eddy-induced
wave activity flux seems to play a secondary role. However, because the linear wave activity flux $F_L = C_a A$ is related to the blocking envelope amplitude $B$ through the relation $A = |B|^2 / PV_y$, it is inferred that the linear wave activity flux is also associated with the local eddy forcing via the change of eddy-driven blocking envelope amplitude. Thus, during the blocking decay stage the upward propagation of planetary waves in the downstream region of blocking is likely related to enhanced

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**FIG. 6.** Wave activity flux vectors (a), (d) $F_R$ (blue arrow), (b), (e) $F_L$ (blue arrow), and (c), (f) $F = F_L + F_R$ (blue arrow) and their corresponding divergence terms $\nabla \cdot F_R$, $\nabla \cdot F_L$ and $\nabla \cdot F$ [contour interval (CI) = 1.0 and the unit of contour line is 0.001] averaged during the blocking growth period (from days 0 to 6) of eddy-driven blocking for (a)–(c) a nonlinear vertically decreased BWW ($\Delta \mu_1 = -0.2$ and $\Delta \mu_2 = 0.2$) and (d)–(f) a nonlinear vertically increased BWW ($\Delta \mu_1 = 0.2$ and $\Delta \mu_2 = 0.2$). The negative (positive) value represents the convergence (divergence).
linear wave activity and intensified local eddy-induced wave activity originating from the upstream blocking flow.

5. Perturbation theory of blocking propagation and energy dispersion

Although \( \mathbf{F} = \mathbf{C}_s \mathbf{A} + \mathbf{F}_R \) can tell us what influences the wave activity associated with blocking evolution, it is unable to identify how the wave activity in amplitude, zonal wavenumber, group velocity, phase speed, and energy dispersion changes with the BWW. Thus, it is useful to use a perturbation theory to solve Eq. (3h).

To establish whether the wave activity flux \( \mathbf{F} \) is related to the variant group velocity of the blocking evolution, we may define \( C_{sx} = F_x/A \) in the region of \( A \neq 0 \). Then, \( C_{sx} = C_s + \Delta C_m \) [where \( \Delta C_m = (F_{Rx}/A) = -i \hbar (\partial^2 B^1/\partial x^2) - (\partial B^1/\partial x^2)/|B|^2 \) can be considered as the modified group velocity of an eddy-driven

Fig. 7. As in Fig. 6, but for the blocking mature period (from days 7 to 13), where the unit of the contour line with \( CI = 4 \) is 0.001.
blocking wave packet for $|B| \neq 0$. Here, $\Delta C_m$ is a correction of the linear group velocity $C_g$ as a local eddy-induced group velocity change. It is also noted that $\Delta C_m = -\text{Re} \left[ B^* \left( \frac{\partial B^*}{\partial x} \right) - B^* \left( \frac{\partial B^*}{\partial x} \right) \right] |B|^2$ does not hold in the $A = 0$ or $|B| = 0$ region of blocking. One can also calculate the wave activity flux $\mathbf{F}$ and its divergence $\nabla \cdot \mathbf{F}$ as shown in Figs. 6–8 instead of directly calculating $\Delta C_m$ from Eq. (3h). However, as noted below, the approximate solution of $C_g' = C_g + \Delta C_m$ can be obtained if the analytical solution of the blocking envelope amplitude $B$ in Eq. (3h) can be derived. Using $B = |B| e^{i\theta_E}$, one can have $\Delta C_m = 2\text{Re} \left[ \frac{\partial \theta_E}{\partial x} \right]$. Thus, the local eddy-induced group velocity change $\Delta C_m$ is related to the zonal nonuniform phase $\theta_E$ of the eddy-driven blocking envelope amplitude $B$. As we will see below, the phase $\theta_E$ of the blocking envelope amplitude $B$ is also dependent on the variation of the maximum amplitude of the blocking modulus amplitude $|B|$ due to local eddy forcing.

FIG. 8. As in Fig. 6, but for the blocking decay period (from days 15 to 21), where the unit of the contour line with $CI = 8$ is 0.001.
a. Time-dependent equations of soliton amplitude, zonal wavenumber, group velocity, phase speed, and energy dispersion speed of eddy-driven blocking wave packet

The PIST method used widely in previous studies (Okamawari et al. 1995; Kath 1997) has been an important tool for solving Eq. (3h) if the forced NLS equation possesses a soliton solution. The most important assumption of the PIST method is that the spatial shape of the NLS soliton remains unchanged when the eddy forcing is not too strong. For this reason, it is supposed that the soliton parameters (amplitude, group velocity, wavenumber, and phase) of the NLS soliton vary with time. To some extent, the eddy forcing can be considered as a small perturbation of the NLS soliton.

We assume that the envelope amplitude $B$ of Eq. (3h) has the soliton solution in the form of (Luo 2000)

$$B = M_0(t) \text{sech} \left\{ \frac{\delta}{\sqrt{2\lambda}} M_0(t) [x - C_\parallel t - Z(t)] \right\}$$

$$\times \exp \left\{ -i \frac{K(t)}{\sqrt{2\lambda}} [x - C_\parallel t - Z(t)] + \theta(t) \right\},$$

(7)

where $M_0(t)$ is the instantaneous maximum modulus amplitude of the envelope amplitude $B$, $\theta(t) = -\left[ K(t)/\sqrt{2\lambda} \right] [x - C_\parallel t - Z(t)] + \theta(t)$ and $\theta(t)$ denotes its phase change, whereas $dZ/dt$ [$K(t)$] represents the group velocity (zonal wavenumber) change. Because $U$ and $PV_x$ are slowly varying in the vertical direction, the soliton parameters $M_0(t), Z(t), K(t)$, and $\theta(t)$ of the blocking envelope amplitude $B$ depend on the slow variation of $x$, but are independent of $x$. Clearly, the modulus amplitude $|B|$ of $B$ is $|B| = M_0(t) \text{sech} \left\{ \sqrt{\delta/2\lambda} M_0(t) [x - C_\parallel t - Z(t)] \right\}$ and shows a soliton-like structure in the zonal direction. Thus, $M_0(t)$ may be also referred to as the blocking soliton amplitude. Moreover, one can obtain $\partial \theta / \partial x = -K(t)/\sqrt{2\lambda}$ and $\Delta C_m = -\sqrt{\delta/2\lambda} K(t)$ from Eq. (7).

Following the PIST method of Okamawari et al. (1995), the nonlinear ordinary equations of the blocking soliton parameters $M_0(t), Z(t), K(t)$, and $\theta(t)$ can be obtained as

$$\frac{dM_0}{dt} = GM_0(t) \int_{-\infty}^{x_0} f_0^2(x) \sin \Omega \text{sech}(X) \, dx',$$

(8a)

$$\frac{dK}{dt} = -GM_0(t) \int_{-\infty}^{x_0} f_0^2(x) \cos \Omega \text{sech}(X) \tan(X) \, dx',$$

(8b)

$$\frac{dZ}{dt} = -\sqrt{2\lambda} K + G \int_{-\infty}^{x_0} f_0^2(x) \sin \Omega \text{sech}(X) \, dx',$$

(8c)

$$\frac{d\theta}{dt} = -\frac{1}{2} \left( K^2 - \delta M_0^2 \right) - \frac{K}{\sqrt{2\lambda}} \frac{dZ}{dt} + G \int_{-\infty}^{x_0} f_0^2(x) \cos \Omega [1 - X \tan(X)] \text{sech}(X) \, dx',$$

(8d)

where $X = \sqrt{\delta/2\lambda} M_0(t)x', x' = x - C_\parallel t - Z(t)$, and $\Omega = \Delta o t - K(t)x'/\sqrt{2\lambda} + \theta(t)$.

Using Eq. (7), the blocking wavy anomaly $\psi_B$ during the blocking life cycle can be expressed as

$$\psi_B = 2\phi(z)M_0 \sqrt{2} \frac{\delta}{\sqrt{2\lambda}} M_0 x' \times \cos \left[ kx - K(t)x' + \theta(t) - \omega t \right] \sin \mu,$$

(9)

Clearly, the three-dimensional structure of the blocking streamfunction anomaly $\psi_B$ during the blocking life cycle in a $\beta$-plane channel can be known if the solutions of $M_0(t), Z(t), K(t)$, and $\theta(t)$ are obtained from Eq. (8) for given initial values. Here, we choose $M_0(0) = 0.3, Z(0) = 0, K(0) = 0$, and $\theta(0) = 0$ as the initial value of the blocking envelope amplitude $B$ in Eq. (7), whereas the eddy parameters are the same as those in section 4.

Using Eq. (9), one can obtain

$$k_m = k - K(t)/\sqrt{2\lambda},$$

(10a)

$$C_{gm} = C_g + dZ/dt,$$

(10b)

$$C_{pm} = \omega - \frac{1}{\sqrt{2\lambda}} \frac{dZ}{dt} \frac{d\theta}{dt} - \frac{C_g}{\sqrt{2\lambda}} \left( K + \frac{dK}{dt} \right) / k_m,$$

(10c)

$$C_{gm} = C_{gm} - C_{pm}.$$  

(10d)

In Eq. (10a), $k_m(t)$ represents the modified zonal wavenumber of the carrier wave of eddy-driven blocking wavy anomaly $\psi_B$ due to eddy forcing and reflects the change of the zonal wavenumber of eddy-driven blocking, whereas $C_{gm}$ is the modified group velocity of the blocking wavy anomaly $\psi_B$ by adding a group velocity change $dZ/dt$ to the linear group velocity $C_g$ in Eq. (10b) and referred to as the nonlinear group velocity of blocking. In Eq. (10c), $C_{pm}$ is the modified phase speed and referred to as the nonlinear phase speed of blocking because it includes the blocking amplitude and nonlinearity, whereas $C_{gpm}$ is defined as the modified energy dispersion speed of blocking and also referred to as the nonlinear energy dispersion speed of blocking. It should be pointed out that $C_{pm}$ ($C_{gpm}$) represents the zonal propagation speed of blocking wavy anomaly (blocking energy). The value of $C_{gpm}$ reflects whether the blocking wavy anomaly is dispersive or nondispersive. When $C_{gpm} = 0$, the blocking is nondispersive. In Eqs. (8) and (10), $M_0, k_m, C_{gm}, C_{pm}$, and $C_{gpm}$ represent the blocking soliton parameters describing the change of the blocking wave packet structure under eddy forcing, which cannot be obtained from the small-amplitude wave activity flux formalism (Takaya and Nakamura 2001).

Because the PIST method requires that the eddy forcing is relatively weak, it is easy to obtain $dZ/dt = -\sqrt{2\lambda} K(t)$ from Eq. (8c) for a small $f_0$. Thus, one can have $C_{gm} = C_g + \Delta C_m = C_g + dZ/dt$ because of $\Delta C_m = -\sqrt{2\lambda} K(t)$. This means that $C_{gpm}$ in Eq. (10b) is a good approximation of $C_{gm} = C_g + \Delta C_m$ under a weak localized forcing. Because our NIM model is based on the assumption of weak eddy forcing, the solution (10) obtained by the PIST method can help us to explain the features of wave activity associated with blocking evolution. When the eddy forcing is zonally localized, then $dK/dt \neq 0$. In this case, $K, dZ/dt$, and $\Delta C_m$ are inevitably related to the blocking soliton amplitude $M_0$ of blocking wave packet.
because $dK/dt$ is related to the blocking soliton amplitude $M_0$, though $M_0$ is changed with time under the local eddy forcing. In this case, $k_m$, $C_{gm}$, $C_{pm}$, and $C_{gpm}$ depend on the variation of the blocking soliton amplitude $M_0$. However, one can obtain $dK/dt = 0$, $K = 0$, $dZ/dt = 0$, $\Delta C_{gm} = -\sqrt{2\lambda}K(t) = 0$, and $F_R = 0$, when $f_0(x)$ is a constant or zonally uniform. Thus, the zonal wavenumber $k_m$ and group velocity $C_{gm}$ of eddy-driven blocking are independent of the blocking soliton amplitude $M_0$. Naturally, it is inferred that the wave activity flux $F_R$ in $F = C_gA + F_R$ is mainly induced by the local eddy forcing caused by preexisting synoptic-scale eddies via the changes in the modulus amplitude and phase of blocking envelope amplitude.

If the solution of Eq. (8) is obtained for given initial values using the fourth-order Runge–Kutta method, the time-dependent solutions of $M_0$, $k_m$, $C_{gm}$, $C_{pm}$, and $C_{gpm}$ can be known. Because $U$ is slowly varying in the vertical direction, $k_m$, $C_{gm}$, $C_{pm}$, and $C_{gpm}$ are also dependent on the slow variation of $U$. These solutions can help us to examine how the propagation and energy dispersion of blocking wave packet depend on the vertical structure of BWW.

b. Spatiotemporal characteristics of the propagation and energy dispersion of eddy-driven blocking wave packet

We show the time-vertical evolutions of $M_0(t)$, $k_m(t)$, $C_{gm}(t)$, $C_{pm}(t)$, and $C_{gpm}(t)$ for an eddy-driven blocking wave packet in Figs. 9–13 during the blocking life cycle by solving Eq. (8) for the initial values $M_0(0) = 0.3$, $Z(0) = 0$, $K(0) = 0$, and $\theta(0) = 0$. It is found that the maximum modulus amplitude $M_0(t)$ of blocking envelope amplitude can be amplified under the local eddy forcing $f_0(x)$, whose intensity changes with time and height and is different during the different phase of blocking (Fig. 9). For a linear (nonlinear) vertically increased BWW (Figs. 9a,b), the amplitude $M_0(t)$ is large during the period from days 7 to 12 (days 7 to 14) and its large-amplitude region is concentrated in the lower troposphere, especially for a
nonlinear vertically increased BWW (Fig. 9b). During the period from days 18 to 21, $M_0(t)$ is larger in the upper troposphere than in the lower troposphere (Figs. 9a,b). For a linear vertically decreased BWW (Fig. 9c), $M_0(t)$ is large during the period from days 9 to 13 and its slightly large-amplitude region is located in the upper troposphere, but the large-amplitude region of $M_0(t)$ is mainly concentrated in the lower troposphere during the period from days 10 to 16 for a nonlinear vertically decreased BWW (Fig. 9d). Thus, the vertical structure of the BWW can influence the evolution and vertical distribution of the blocking energy because $M_0(t)$ can represent the energy strength of blocking wave packet.

It is also seen that the zonal wavenumber $k_m(t)$ of blocking wavy anomaly $\psi_B$ slightly decreases as blocking grows, but largely increases as blocking decays and disappears (Fig. 10). This means that during the blocking decay phase the strong enlarging (lessening) of the zonal wavenumber (wavelength) of blocking wavy anomaly mainly takes place in the upper troposphere for vertically increased BWWs (Figs. 10a,b), but in the lower troposphere for vertically decreased BWWs (Figs. 10c,d). Correspondingly, the modified group velocity $C_{gm}(t)$ of blocking wave packet is decreased during the blocking growth phase, but significantly increased during the blocking decay phase (Fig. 11). We further find that for vertically increased BWWs, $C_{gm}(t)$ increases vertically with time in the downstream region of blocking (Figs. 11a,b), which is more evident for a nonlinear vertically increased BWW (Fig. 11b) than for a linear vertically increased BWW (Fig. 11a). The large value of $C_{gm}(t)$ mainly appears in the upper troposphere between days 15 and 18, which decreases after day 18. Thus, during the blocking decay phase, the eddy-driven blocking can have a large group velocity $C_{gm}(t)$ in the upper troposphere with its maximum strength on $z = 1.0$ for vertically increased BWWs. This feature cannot be detected for vertically increased BWWs.
decreased BWWs (Figs. 11c,d), though a large group velocity can be seen in the lower troposphere (Fig. 11c).

It should be pointed out that whether eddy-driven blocking wave packet shows a strong energy dispersion is mainly determined by the magnitude of \( C_{gpm} (t) = C_{gpm} - C_{pm} \), even if the nonlinear phase speed \( C_{pm} \) of blocking is different in different tropospheric layers during the different stages of blocking (Fig. 12). Although eddy-driven blocking moves westward during the blocking growth phase for vertically decreased BWWs as seen in Figs. 12c and 12d, it satisfies \( C_{gpm} \approx C_{gm} \) during the blocking mature phase because \( C_{pm} \approx 0 \) is approximately satisfied especially in the midtroposphere (\( z = 0.5 \)). Thus, the quasi-stationary eddy-driven blocking can disperse its energy towards downstream with the group velocity of \( C_{gm} (t) \), though \( C_{gm} (t) \) depends on the blocking soliton amplitude \( M_0 (t) \). However, because the modified phase speed \( C_{pm} \) is relatively large in the upper troposphere during the large-amplitude blocking phase (from days 8 to 14 days) for a nonlinear vertically increased BWW (Fig. 12b), one cannot use the magnitude of \( C_{gpm} \) to explain the energy dispersion change of eddy-driven blocking wave packet. Instead, it is appropriate for us to use the magnitude of \( C_{gpm} \) to examine the energy dispersion speed change of eddy-driven blocking wave packet. A similar feature is found for a linear vertically increased BWW (Fig. 12a).

As we can further see from Fig. 13, the nonlinear energy dispersion speed \( C_{gpm} \) is decreased with time due to increased amplitude of the blocking soliton during the blocking growth phase, but increased with time due to decreased blocking amplitude in the most times of blocking decay phase. A similar variation of \( C_{gpm} \) is found for barotropic blocking (Zhang and Luo 2020), though the meridional structure of the barotropic basic westerly wind \( U(y) \) can influence the temporal variation of \( C_{gpm} \). But in a barotropic basic westerly wind with and without meridional shear, the planetary wave associated with

**Fig. 11.** Time–vertical profile of modified group velocity \( C_{gm} \) (CI = 0.06) of eddy-driven blocking wave packet in different basic flows of \( \bar{U} = u_0 + \Delta u_1 z + \Delta u_2 z^2 \) and \( u_0 = 0.5 \): (a) linear vertically increased BWW \((\Delta u_1 = 0.2 \text{ and } \Delta u_2 = 0)\), (b) nonlinear vertically increased BWW \((\Delta u_1 = 0.2 \text{ and } \Delta u_2 = 0.2)\), (c) linear vertically decreased BWW \((\Delta u_1 = -0.2 \text{ and } \Delta u_2 = 0)\), and (d) nonlinear vertically decreased BWW \((\Delta u_1 = -0.2 \text{ and } \Delta u_2 = 0.2)\).
the decay of eddy-driven blocking does not propagate upward (Figs. S3 and S4) because $C_{gpm}$ is uniform in the vertical direction. Compared to the result of $C_{gpm}$ in a uniform barotropic basic westerly wind, the vertically increased (decreased) BWW can largely increase (decrease) the value of $C_{gpm}$ to significantly strengthen (reduce) the energy dispersion of blocking mainly during the blocking decay phase (Fig. S5). Such an enhanced energy dispersion in the blocking downstream region is more pronounced in the upper troposphere than in the lower troposphere for vertically increased BWWs (Figs. 13a,b). For vertically decreased BWWs (Figs. 13c,d), $C_{gpm}$ tends to increase during the period from days 10 to 21, whose maximum center is located in the lower troposphere (Fig. 13e). However, for vertically increased BWWs (Figs. 13a,b) while $C_{gpm}$ can have a large value region in the upper troposphere between days 16 and 18 (days 17 and 19) for a linear (nonlinear) vertically increased BBW, it is decreased after day 18 (day 19).

Overall, during the blocking decay phase, $C_{gpm}$ is largely increased eastward and upward for vertically increased BWWs (Figs. 13a,b), leading to enhanced upward planetary wave activity in the upper troposphere as seen in Figs. 3b–d. In other words, the energy emitted from the blocking region does not only propagate downstream, but also propagate upward, although it depends slightly on the vertical structure of vertically increased BWW. This explains why the decay of eddy-driven blocking can accompany the upward propagation of tropospheric planetary waves in the downstream region of blocking via intensified eastward and upward energy dispersion, when BWW is stronger in the upper troposphere than in the lower troposphere. This result is consistent with that of the ERA-Interim data (Figs. S6 and S7).

Because the blocking wavy anomaly $\psi_B$ has relatively large amplitude in the midtroposphere ($z = 0.5$) during the blocking mature period, it is useful to show the temporal variations of daily $M_d(t)$, $k_m(t)$, $C_{gpm}(t)$, $C_{pm}(t)$, and $C_{gpm}(t)$ on $z = 0.5$ during
the blocking life cycle in Fig. 14 for different BWWs. It is found that the temporal variation of $M_0(t)$ obtained from Eq. (3h) by the PIST method (Fig. 14a) is basically consistent with the numerical result of Eq. (3h) (Fig. 4a) except for a small difference during the blocking decay stage, though Fig. 4a shows the result of $\psi_1(t)$ on $z = 0.5$ that satisfies $\psi_1 = 2M_0(t)\sqrt{2L_c}$. Clearly, the eddy-driven blocking wave packet has larger amplitude for vertically decreased BWWs (blue and green lines in Figs. 4a and 14a) than for vertically increased BWWs (black and red lines in Figs. 4a and 14a). This does not imply that the energy coming from large-amplitude and long-lived eddy-driven blocking can propagate upward. In fact, although the blocking has a shorter lifetime for a linear vertically increased BWW (black line in Figs. 4a and 14a) than for nonlinear vertically increased BWW (red line in Figs. 4a and 14a), the upward propagation of planetary waves associated the blocking decay is still clearly seen for the two types of vertically increased BWWs (black and red lines in Fig. 4d). Thus, the vertical profile of the BWW is crucial for whether the decay of eddy-driven blocking wave packet can accompany the upward propagation of subsequent downstream planetary waves via the energy dispersion of upstream blocking.

While the zonal wavenumber $k_{g0}(t)$ of blocking wavy anomaly $\psi_g$ changes with the blocking evolution (Fig. 14b), its change is small compared to that of $k$ even for vertically increased BWWs. Because the positive $C_{g0}(t)$ in Fig. 14c [negative $C_{pm}(t)$ in Fig. 14d] is small during the blocking growth phase such that $C_{g0} = C_{g0} - C_{pm}$ (Fig. 14e) is nearly zero between days 4 and 9. Thus, the nondispersion of eddy-driven blocking mainly takes place during the blocking growth stage. However, because $C_{g0}(t)$ [Fig. 14c(t)] is large (small) during blocking decay phase, $C_{g0} = C_{g0} - C_{pm}$ is large particularly during the blocking decay period from days 15 to 18 for vertically increased BWWs. But after day 18, the positive $C_{g0}$ is small to allow the

**Fig. 13.** Time–vertical profiles of nonlinear energy dispersion speed $C_{g0} = C_{g0} - C_{pm}$ (CI = 0.04) of eddy-driven dipole blocking wave packet in different basic zonal flows of $U = t + \Delta \mu z + \Delta \mu z^2$ and $t = 0.5$: (a) linear vertically increased BWW ($\Delta \mu_1 = 0.2$ and $\Delta \mu_2 = 0$), (b) nonlinear vertically increased BWW ($\Delta \mu_1 = 0.2$ and $\Delta \mu_2 = 0.2$), (c) linear vertically decreased BWW ($\Delta \mu_1 = -0.2$ and $\Delta \mu_2 = 0$), and (d) nonlinear vertically decreased BWW ($\Delta \mu_1 = -0.2$ and $\Delta \mu_2 = 0.2$).
maintenance of a large-amplitude planetary wave in the upper troposphere (Fig. 3d). Thus, the vertically increased (decreased) BWWs can significantly enhance (reduce) the energy dispersion of blocking wave packet in the upper troposphere during the blocking decay phase (from days 13 to 18) (Figs. 14e and 13 and Fig. S5).

For a zonally uniform eddy forcing (a constant $f_0$) and the same parameters as in Fig. 14, the temporal variations of daily $M_0(t)$, $k_m(t)$, $C_{gm}(t)$, $C_{pm}(t)$, and $C_{gpm}(t)$ on $z = 0.5$ during the life cycle of eddy-driven blocking are shown in Fig. 15. It is noted that while $k_m(t)$ and $C_{gm}(t)$ are time-independent or independent of the blocking soliton amplitude $M_0(t)$ during

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**Fig. 14.** Temporal variations of (a) daily maximum modulus amplitude $M_0(t)$, (b) modified zonal wavenumber $k_m$, (c) modified group velocity $C_{gm}$, (d) modified phase speed $C_{pm}$, and (e) nonlinear energy dispersion speed $C_{gpm}$ at $z = 0.5$ of eddy-driven blocking wave packet for different basic zonal flows $U = u_0 + \Delta u_1 z + \Delta u_2 z^2$ and $u_0 = 0.5$: linear vertically increased BWW ($\Delta u_1 = 0.2$ and $\Delta u_2 = 0$; black line), nonlinear vertically increased BWW ($\Delta u_1 = 0.2$ and $\Delta u_2 = 0.2$; red line), linear vertically decreased BWW ($\Delta u_1 = -0.2$ and $\Delta u_2 = 0$; blue line), and nonlinear vertically decreased BWW ($\Delta u_1 = -0.2$ and $\Delta u_2 = 0.2$; green line).
the blocking life cycle (Figs. 15b,c), during the blocking mature phase (from days 8 to 13) $M_0(t)$ is large (Fig. 15a) and $C_{pm}(t)$ is largely negative (Fig. 15d) so that the energy dispersion of blocking is strong because $C_{gpm}(t)$ is largely positive. Unlike the result of the local eddy forcing shown in Fig. 14, strong energy dispersion does not take place during the blocking decay phase (Fig. 15e from days 15 to 18), instead during the blocking mature phase (Fig. 15e from days 8 to 13). Such an energy dispersion speed change does not allow the planetary waves to propagate upward. Thus, the zonal locality of eddy forcing is also important for whether planetary waves associated with the blocking decay propagate upward.

6. Conclusions and discussion

In this paper, we have used the nonlinear multiscale interaction (NMI) model of eddy-driven blocking in a baroclinic basic zonal flow to examine how the propagation and energy
dispersion of blocking wave packet in the troposphere depend on the vertical distribution of the weak basic westerly wind (BWW) to explain under what conditions the planetary waves associated with the blocking decay can propagate strongly upward in the downstream region of blocking. In this NMI model, the BWW is assumed to be slowly varying in the vertical direction and the envelope amplitude of eddy-driven blocking wave packet is described by the NLS equation with local eddy forcing induced by preexisting upstream synoptic-scale eddies. This model shows that when the weak BWW is larger in the upper troposphere than in the lower troposphere, the energy, as denoted by the modulus amplitude $|B|$ of eddy-driven blocking wave packet, emitted from the blocking region can propagate eastward and upward. In this case, the streamfunction anomaly $\psi_B$ is manifested by the upward migration of large-amplitude planetary waves in the upper troposphere of the blocking downstream region during the blocking decay phase, even though the blocking tends to be nearly nondispersive during the blocking growth phase. Such a feature reflects the upward propagation of planetary waves associated with the blocking decay. In contrast, the blocking decay may allow the planetary waves to propagate downward when BWW is weaker in the upper troposphere than in the lower troposphere (blue line in Fig. 4d). We also find that the upward propagation of the planetary waves during the blocking decay phase is less evident when the eddy forcing is zonally uniform.

Based on the NMI model, we have constructed a finite-amplitude local wave activity flux to examine the planetary wave activity during different stages (growth, mature, and decay) of blocking. It is shown that the new finite-amplitude local wave activity flux $F$ is composed of two parts: 1) linear wave activity flux $F_L$ related to the linear group velocity and 2) local eddy-induced wave activity flux $F_R$ related to the modulus amplitude of blocking envelope amplitude and its zonal nonuniform phase. Using this new finite-amplitude local wave activity flux, we found that during the blocking decay phase the linear wave activity flux $F_L$ plays a major role in the upward migration of enhanced tropospheric planetary wave activity in the downstream region of blocking, whereas the local eddy-induced wave activity flux $F_R$ plays a secondary role that can strengthen the role of $F_L$. Because the amplification of blocking envelope amplitude is induced by local eddy forcing as well as because $F_L$ and $F_R$ include the modulus amplitude $|B|$ of blocking envelope amplitude $B$, $F_L$, and $F_R$ are essentially linked to local eddy forcing. While the spatial distributions of $\nabla \cdot F_L$ and $\nabla \cdot F_R$ can tell us the relative contributions of $\nabla \cdot F_L$ and $\nabla \cdot F_R$ to the planetary wave activity in the troposphere during different stages of blocking, it is difficult to identify the role of eddy forcing in the blocking evolution and associated wave activity through calculating $F_L$ and $F_R$ and their divergence terms because $F_L$ and $F_R$ cannot show the change of the blocking wave packet structure in the modulus amplitude, zonal wavenumber, group velocity, and phase speed. However, this difficulty can be avoided by using the perturbed inverse scattering transform (PIST) method to solve Eq. (3b).

Using the PIST method, the analytical solutions of the blocking structure parameters (maximum modulus amplitude, zonal wavenumber, phase speed, and group velocity) can be obtained. It is found that the temporal variations of the blocking structure parameters can better reflect the planetary wave activity associated with the blocking evolution in different BWWs. Using the analytical solutions, we have verified that the new finite-amplitude local wave activity flux $F$ represents the group velocity change of eddy-driven blocking wave packet, whereas the time-dependent solutions of the maximum modulus amplitude, zonal wavenumber, nonlinear phase speed, and group velocity of the blocking wavy anomaly $\psi_B$ in Eq. (9) can describe the evolution of blocking wave packet. Moreover, it is revealed that during the blocking growing phase, the eddy-driven blocking wave packet tends to be nearly nondispersive and is mainly related to the local eddy-induced wave activity flux $F_R$. But during the blocking decay phase the blocking is strongly dispersive and its linear energy dispersion as represented by the linear wave activity flux $F_L$ is dominant, which shows increased zonal wavenumber, intensified eastward propagation speed, enlarged group velocity, and enhanced energy dispersion speed. In the blocking structure change, the eastward and upward enhancement of the energy dispersion of blocking is evident especially in the upper troposphere when the weak BWW is larger in the upper troposphere than in the lower troposphere, which leads to enhanced eastward and upward propagation of tropospheric planetary waves in the blocking downstream region due to the blocking decay, as seen from the ERA-Interim data (Figs. S6 and S7). Strong upward propagation of tropospheric planetary waves associated with blocking decay is also easily seen for large-amplitude blocking events formed under the same parameter condition. When the eddy forcing is zonally uniform or when the weak BWW is smaller in the upper troposphere than in the lower troposphere, the upward propagation of planetary waves associated with the decay of eddy-driven blocking is suppressed. The suppressed upward propagation is also seen for vertically increased BWWs if the uniform basic westerly wind part $u_0$ is too small. Thus, whether the decay of eddy-driven blocking can drive the upward propagation of downstream planetary waves depends on the vertical structure of weak BWW and the zonal locality of preexisting synoptic-scale eddies. To some extent, intensified (weakened) background westerly winds in the upper troposphere or near the tropopause favor the upward (downward) propagation of planetary waves associated with the blocking decay.

It should be pointed out that because our theoretical model does not include the effect of stratospheric process, our results are appropriate for the propagation of tropospheric planetary waves related to the decay of blocking only in the troposphere rather than for the upward propagation across the tropopause. As have been noted by many previous studies, the downward propagation of planetary waves in the stratosphere (Harnik 2009; Lubis et al. 2016, 2019) associated with the wave reflection or wave trapping as a result of changes in the vertical wind shear can also lead to amplification of tropospheric planetary waves, which in turn produces a favorable condition for blocking (Kodera et al. 2008). In addition, the strength of the background westerly wind in the lower stratosphere is also shown to be important for the vertical propagation of tropospheric planetary waves (Kodera et al. 2013). The effects of these factors on the propagation and energy dispersion of tropospheric blocking wave packet are not considered in our
NMI model, and they deserve further investigation. Moreover, the detailed comparison of our theoretical results with the re-analysis data should be also made in the future.

Acknowledgments. This research was supported by the National Key Research and Development program of China (2016YFA0601802), the National Natural Science Foundation of China (Grants 41790473 and 41430533), and the Chinese Academy of Sciences Strategic Priority Research Program (Grant XDA19070403). The authors thank two reviewers for their constructive suggestions in improving this paper and Dr. Xiaodan Chen for providing Figs. S6 and S7.

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