On the Use of Double-Mass Analysis for Testing the Consistency of Meteorological Records and for Making Required Adjustments

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The problem of correlating meteorological elements is basic to many research studies. In such studies it is important that the records used be homogeneous, that is, comparable throughout the period of observations. Double-mass analysis, which deals with accumulated values rather than with incremental data, provides a means of determining the consistency of observations collected over a long period of time. It also provides a means of adjusting the early records to conform to those being collected at the existing site of a station which has been moved. This method of analysis, first described by Merriam, is particularly applicable to precipitation data.

The general equation of a linear regression expressing the relation between two variables $X$ and $Y$ takes the form $Y = mX + b$, where $m$ and $b$ are constants. For some meteorological elements, such as temperature, the constant $m$ is nearly unity. In other words, the mean monthly temperature at one site tends to be a constant number of degrees ($b$) higher or lower than that at a nearby site. In the case of precipitation, however, the amounts observed at two adjacent sites tend to be proportional (i.e., $b = 0$).

If $Y = mX$, then accumulated values of $X$ and $Y$ plot as a straight line. When comparing the monthly, seasonal, or annual precipitation as observed at two nearby sites, the function is not exact and, consequently, the broken line connecting adjacent plotted points can normally be expected to weave back and forth across the mean straight line in a random manner. If, however, one of the stations has been moved during the period of record, the constant of proportionality $m$ may have changed. In this event, the points of the double-mass plotting tend to lie along two line segments of different slope (Fig. 1). If the record for station $X$ in Figure 1 is assumed to be consistent throughout, the adjustment factor for the early records of station $Y$ is equal to the ratio of the slopes of the two line segments (1.38). That is, the early records, when multiplied by 0.87/0.63 are essentially equivalent to those collected at the present site.

A plotting of the type shown in Figure 1 is not in itself conclusive, since it does not show which of the two station records is inconsistent. This deficiency can be practically overcome by plotting the record to be analyzed against a base derived from averaging the observations for a large group of stations in the surrounding area (Fig. 2). Generally speaking, the precipitation base should be developed from the records of stations within a rather limited area so that the storm histories at the several stations will be reasonably similar. On the other hand, the base should include a sufficient number of stations so that any individual inconsistency in the record will be negligible in the group average. Thus, if the area is too small, the number of stations available will be so limited as to impair the stability of the base. Therefore, optimum areal size is directly related to the station density of the network.

Double-mass analysis can be expected to yield conclusive and useful results only if: (1) there is a high degree of correlation between the factors

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being compared, (2) the factors are directly proportional, and (3) one of the factors (the base) is comprised of observations comparable throughout the period of record. It is often possible, however, to test and adjust records by a modified procedure even though the basic data do not qualify under condition (2). The adjustment of streamflow data by comparison with precipitation records will serve to illustrate one such modification.

While there is a high degree of correlation between seasonal or annual streamflow and average precipitation over a drainage basin, the two factors are seldom proportional. Even in cases where the relation is linear, the \( Y \)-intercept \((b)\) is often too large to be ignored. The steps in the analysis are:

1. Plot a scatter diagram of incremental values of streamflow \((Y')\) vs precipitation \((X)\) and fit a smooth mean curve.
2. Compute values of streamflow \((Y'')\) from the chart developed in step (1) by entering with observed precipitation amounts.
3. Plot a double-mass curve of computed vs observed streamflow and fit straight-line segments.
4. Compute slopes and adjustment factors considering values of \(Y''\) as the base data.

This type of double-mass analysis is based on the assumption that any changes during the period of record have been such that the streamflow observed during one period is directly proportional to the volume which would have been observed under the conditions of any other period. Although minor refinements may be necessary, the analysis outlined briefly here constitutes a practical method of converting the base to a form such that the values will be proportional to those of the factor being tested.

Double-mass analysis is particularly valuable in determining adjustment factors when the dates of changes are known, e.g., when the records show that the station was moved on a specific date. While the plotings will indicate changes which may not have been recorded, random variations can be misinterpreted as breaks in the slope. Therefore, a great deal of caution should be exercised in constructing the line segments. An apparent break in slope, supported by less than five consecutive points and not attributable to a known cause, can lead only to questionable conclusions. Neglecting to make a questionable adjustment is a sounder course than adjusting records which may, in reality, be comparable as observed.
The Relation Between Fronts and Jetstream *

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ABSTRACT *

T HE speaker mentioned that Wexler (Monthly Weather Review, 65: 225, 1937) and Schmidt (Kon. Ned. Meteorologisch Instituut, nr. 125; Mededeelingen en Verhandelingen, Serie B, 1, nr. 4) in discussing the formation of thermal highs and lows had in fact developed a theory of large-scale circulations of the air caused by the production of a solenoidal field. However, they had neglected the influence of friction which he could bring qualitatively in the process, but which he would not discuss now.

He demonstrated the simple theory of Wexler for an ideal case of frontogenesis along a snow-covered coastline bordering an open sea. The production of solenoids considered perpendicular to the coastline is here not a linear function of the distance measured along this line, but a function of the form of a "hyperbolical tangent" (see Figure 1). This means that at the surface over the continental coastline a zone of high pressure will develop and over the sea coastline a zone of low pressure (see Figure).

Proceeding from land toward sea, at the surface we will encounter in succession: a weak zone of cyclonic shear, a strong zone of anticyclonic shear, a strong zone of cyclonic shear, and a weak zone of anticyclonic shear.

Vertical circulations will develop, causing two cloud systems, a major one connected with the main "frontal" cyclonic shear zone over the sea, and a minor one over the land. Above the "front" in the lower layers we have the jet in the higher layers. The circulation may be so strong that it even penetrates into the stratosphere and causes the cooling over the oceanic side of the jet and warming over the continental side (compare Palmen, Journal of Met., 5: 20, 1948, and Riehl, Trans. Am. Geoph. Un., 29: 175, 1948).

The "front" is therefore a vertical circulation system (cf. Raethjen, Met. Zeit., 54: 393, 1937 and Ann. Hydr., 46: 97, 1938); the real front being the well-known vertical zone of solenoid concentration. When the front moves out over the ocean a turbulence or convection inversion is developed which often erroneously is considered as a frontal surface.

The speaker demonstrated the complex structure of the front also noticed by Bjerknes (Meteor. Off., Geoph. Mem. No. 50) by showing several autographic traces of temperature, pressure, and wind during frontal passages in De Bilt, Holland; main and secondary cyclonic shear zones separated by an anticyclonic shear zone with downward motion are found. (Important to consider cloud height variation for aviation.) On weather maps the double structure of the frontal circulation system can often be detected.

Some remarks also were made with regard to the development of fronts and jets along the borders of heated continents.

With reference to the model of the "frontal" circulation as presented we have to revise our textbook picture of the land and sea breeze circulation, since also here the heating is not linear in the cross coast direction. During the day we may have a zone of high pressure over the sea, of low pressure over the land; during the night the opposite is the case. The anticyclonic shear zone may sometimes pass the coast with a sharp backing of the wind (anticyclonic phase of the sea breeze). Such
cases have been described by Bleeker and Postma (Hemel en Dampkring, 42: 32, 1943); an example is demonstrated. He mentioned the large scale land and sea breeze in the higher layers, parallel to the coastline, found in Dutch pilot balloon observations (Bleeker, Kon. Ned. Meteorologisch Instituut, nr. 102; Mededeelingen en Verhandelingen, nr. 44), and compared this system with the jet.

A detailed study of fronts as vertical circulation systems and their relations to the jetstream will appear in the future.

Associations between the Vorticity of the Large-scale Air Currents and the Occurrence of Subsidence and Convection

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In view of the widespread use of vorticity considerations in current discussions on the behavior of the large-scale pressure systems of the free atmosphere, it might be useful to record some statistical relationships that have been found to exist between the relative vorticity of the large-scale currents and the occurrence of subsidence and various types of convection. For a more complete discussion, reference is made to the papers cited below [1, 2].

The relative vorticity around a vertical axis is defined by

$$\xi = \frac{V}{r} - \frac{\partial V}{\partial n},$$

where $V$ is the wind velocity, $r$ the radius of curvature of the streamlines ($r$ being positive in cyclonically curved motion), and $n$ measures length along an axis at right angles and to the left of the wind direction. For the purpose of the statistical investigations, the vorticity was classified as cyclonic when $V/r > 0$ and $-\partial V/\partial n > 0$, and as anticyclonic when $V/r < 0$ and $-\partial V/\partial n < 0$. When the two terms were of opposite sign the balance was not sought and the case referred to as "intermediate."

Table 1 shows the association between the vorticity of the currents and the occurrence of widespread and persistent subsidence over the British Isles during a period of 150 days chosen at random and comprising about 300 soundings.

It will be seen that, with very few exceptions, subsidence was associated with anticyclonic vorticity. A similar but reversed relationship was found between the occurrence of convection and the vorticity of the large-scale air currents aloft. For the purpose of the statistical investigation, the convection was classified as "shallow," "deep," or "very deep" according as the tops of the convective clouds were below the 800-mb level, between the 600- and 400-mb surfaces, or above the 400-mb surface, respectively. The results are reproduced in Table 2 which is based upon about 300 soundings.

Table 2. Percentage Frequency of Vorticity in Convective Cases

<table>
<thead>
<tr>
<th>Vorticity</th>
<th>Shallow</th>
<th>Deep</th>
<th>Very deep</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anticyclonic</td>
<td>41</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Intermediate</td>
<td>30</td>
<td>14</td>
<td>6</td>
</tr>
<tr>
<td>Cyclonic</td>
<td>29</td>
<td>85</td>
<td>94</td>
</tr>
</tbody>
</table>

It will be seen that shallow convection shows no marked preference; such convection is apparently determined by the heating from the underlying surface and may occur simultaneously with anticyclonic vorticity and subsidence aloft. On the other hand, the deep and very deep convection are, almost exclusively, associated with cyclonic vorticity aloft, and such convection should not be forecast unless it is expected that the large-scale currents will have cyclonic vorticity.

It is of interest to note that although heating (or cooling) from below is an important factor in the formation of air masses, the type of current is equally important. As far as lapse rate is concerned there is often as much difference between
"anticyclonic" and "cyclonic" air masses as there is between "warm" and "cold" air masses as defined in the classical air-mass classification.

Finally, it should be noted that the above tables represent "one-way" associations and show the frequency with which the various types of vorticity occurs when subsidence (Table 1) and convection (Table 2) are present. It would seem to be of considerable interest to determine the similar associations when types of vorticity are chosen as the primary element.

References


Compensation Method for Thermistor Beads *

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The use of ceramic elements whose electrical resistance is a function of temperature has proven to be very satisfactory for atmospheric temperature measurements. In research work it is often desirable to measure temperatures at a number of levels with separate elements all having the same calibration (resistance vs temperature). In an article by Gordon and Seay published in the January 1948 issue of this BULLETIN, a method is given for compensating a group of Sanborn ceramic-rod temperature elements. The rods, having an initial tolerance of 5%, are brought to the same calibration within 0.25% or 0.1°C by determining suitable values of a series resistor and a parallel resistor to be used with each given element. The validity of their method of obtaining these values depends upon the secant of the temperature vs resistance curve between 10°C and 30°C being parallel to the tangent to the curve at 20°C. With the type of elements employed this assumption is evidently valid.

In many applications, Western Electric thermistor beads are more suitable than ceramic-rod elements, because of their lower lag, smaller size and greater sensitivity. The beads, for example, have a sensitivity of 4 to 5% resistance change per degree centigrade as compared with 2.5% for the rods. Due to this higher sensitivity, the shape of the temperature vs resistance curve of the beads is such that the assumption of parallelism between secant and tangent is no longer valid and a more rigorous method is needed for obtaining the values of the series and parallel resistors.

The following method has proven to be adequate for the purpose and involves no assumptions regarding the shape of the calibration curves.

Let \( a \) be the resistance in parallel with the thermistor \( r \) and \( b \) the resistance in series with the combination (Fig. 1). \( R_1 \) and \( R_2 \) are the resistance values of the entire network at 30°C and 0°C, and \( r_1 \) and \( r_2 \) the thermistor resistances at 30°C and 0°C respectively.

\[
\frac{ar_1}{a + r_1} + b = R_1, \quad (1)
\]

and

\[
\frac{ar_2}{a + r_2} + b = R_2. \quad (2)
\]

The value of \( R_1 \) is chosen equal to the greatest thermistor resistance \( r_1 \) in the group at 30°C and \( R_2 \) equal to the lowest thermistor resistance \( r_2 \) at 0°C.

When these values have been chosen, one can determine the values of \( a \) and \( b \) for each thermistor.

* Published with permission of the Navy Department.
by solving (1) and (2) simultaneously for $b$:

$$b = \frac{1}{2} \left[ (R_2 + R_1) - (R_2 - R_1) \left( 1 + \frac{4R_1R_2}{(R_2 - R_1)(r_2 - r_1)} \right) \right]. \quad (3)$$

Then from (2)

$$a = \frac{r_2(R_2 - b)}{r_2 + b - R_2}. \quad (4)$$

When a series of beads are to be matched to a single calibration curve, $R_1$ and $R_2$ will be the same for each, thus greatly simplifying the solution of (3). TABLE 1 is a comparison between the method described in this paper and that of Gordon and Seay, using the thermistor data given by them.

<table>
<thead>
<tr>
<th>Element number</th>
<th>Measured values (ohms)</th>
<th>Gordon and Seay</th>
<th>Anderson</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>21,352</td>
<td>22,366</td>
<td>21,352</td>
</tr>
<tr>
<td>50</td>
<td>24,160</td>
<td>22,284</td>
<td>21,352</td>
</tr>
<tr>
<td>48</td>
<td>22,567</td>
<td>22,341</td>
<td>21,352</td>
</tr>
<tr>
<td>Total spread C°</td>
<td>4.9</td>
<td>0.14 Match Point</td>
<td>0.00</td>
</tr>
<tr>
<td>24</td>
<td>at 0°C</td>
<td>15.70</td>
<td>15.46</td>
</tr>
<tr>
<td>50</td>
<td>257.8</td>
<td>253.9</td>
<td>255.5</td>
</tr>
<tr>
<td>48</td>
<td>251.4</td>
<td>253.0</td>
<td>256.4</td>
</tr>
<tr>
<td>223</td>
<td>253.5</td>
<td>255.1</td>
<td>257.4</td>
</tr>
<tr>
<td>Total spread C°</td>
<td>0.57</td>
<td>0.34</td>
<td>0.16</td>
</tr>
<tr>
<td>207</td>
<td>at 20°C, Match Point 15°C</td>
<td>154.7</td>
<td>159.3</td>
</tr>
<tr>
<td>208</td>
<td>152.6</td>
<td>155.1</td>
<td>159.3</td>
</tr>
<tr>
<td>223</td>
<td>149.2</td>
<td>154.7</td>
<td>157.9</td>
</tr>
<tr>
<td>Total spread C°</td>
<td>1.10</td>
<td>0.07</td>
<td>0.20</td>
</tr>
<tr>
<td>207</td>
<td>at 30°C</td>
<td>98.3</td>
<td>102.8</td>
</tr>
<tr>
<td>208</td>
<td>95.0</td>
<td>97.5</td>
<td>102.3</td>
</tr>
<tr>
<td>223</td>
<td>92.7</td>
<td>98.8</td>
<td>102.3</td>
</tr>
<tr>
<td>Total spread C°</td>
<td>1.34</td>
<td>0.34 Match Point</td>
<td>0.00</td>
</tr>
<tr>
<td>Average spread C° at all temperatures</td>
<td>0.98</td>
<td>0.25</td>
<td>0.09</td>
</tr>
</tbody>
</table>

### SENSITIVITY

Any compensation network using fixed resistors has a lower thermal sensitivity than the uncompensated elements. Accordingly, a comparison of compensation methods should include a comparison of sensitivities as well as errors.

In the case of thermistor beads, there is little reduction in sensitivity by either method, 4.5% by the described method and 3.0% by that of Gordon and Seay. In the case of the Sanborn elements, however, the reduction is 26% against 6.5% respectively. Thus it is seen that if greater compensation accuracy is to be attained, it must be at the expense of sensitivity. For some elements, such as Western Electric beads, the sacrifice is negligible, while with the Sanborn elements used by Gordon and Seay, it is more serious.

The choice of compensation methods depends upon the relative magnitude of error and sensitivity for the type of elements to be used. In general it would seem more desirable to sacrifice sensitivity for accuracy, particularly since it is possible to obtain high sensitivity elements.
Note on Nuclei for Ice Crystal Formation

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In the “Second Partial Report on the Artificial Production of Precipitation” by Coons, Jones, and Gunn, mention is made of experiments conducted on cloud modification using “persistent nuclei” of lead oxide and potassium iodide. It is not clear from the report whether the term “persistent nuclei” is intended to signify nuclei for the condensation of water drops or nuclei for the formation of ice crystals. Nuclei for the formation of water drops are abundant in the natural atmosphere and appreciable supersaturation of water vapor with respect to liquid water is rare in meteorology. On the other hand, nuclei for the formation of ice crystals are frequently absent from the atmosphere and, as a consequence, supercooled clouds are a very common occurrence.

Experiments have been made using Schaefer’s technique to find whether these substances are good nuclei for the formation of ice crystals. In their report, Coons, Jones, and Gunn do not specify which oxide of lead was used. In experiments conducted in this laboratory, the common oxides PbO, Pb$_3$O$_4$, PbO$_2$, and KI were used. These substances were dispersed as a smoke in the cold box by evaporating them for 30 seconds from a resistance wire electrically heated bright orange. The air in the cold box was maintained at $-19^\circ$C on the bottom. Following the production of a smoke of each of these materials, breath was blown into the cold box, and the concentration of snow crystals which formed in the supercooled cloud was visually estimated.

Of the four substances tested, PbO produced the highest concentration of ice crystals, about one crystal per cubic centimeter. Pb$_3$O$_4$ produced about 0.1 crystal per cc and PbO$_2$ and KI produced no observable crystals.

For purposes of comparison, silver iodide, which has been found effective for producing ice forming nuclei, was tested under the same conditions with the exception that it was vaporized for only one second instead of thirty. This produced a concentration of ice crystals of the order of 10,000 per cubic centimeter. On the basis of these experiments, it is estimated that silver iodide smokes produced by this method are of the order of at least $10^5$ times more effective than smokes of lead oxide or potassium iodide.

Coons, Jones, and Gunn give no information on the techniques they used in the production of nuclei, so it is possible that the most favorable methods were not employed in these laboratory tests. However, in the absence of knowledge of any superior techniques for producing nuclei of these substances, it is concluded that, unless the temperatures were well below $-19^\circ$C, cloud modification following the use of these materials was probably not produced as the result of their action as nuclei for the formation of ice crystals. The fact that success is reported in partially dissipating a non-supercooled cloud with these nuclei indicates this is probably the case.

On the Energy in a Hurricane

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During the past few years, the energy released by an atomic bomb has caused considerable comment. It is of interest to compare this energy with that transformed by some weather processes. A hurricane is a relatively self-contained atmospheric disturbance. For that reason, an opportunity is given to obtain reasonably accurate estimates of the energy and energy transformations within it.

The energy released by the atomic bomb that was dropped on Hiroshima has been estimated as equal to 20,000 tons of TNT. On the basis of 1,278 cal released per gm of TNT, this gives the total energy as

$2 \times 10^4 \text{ tons} \times 9.07 \times 10^5 \text{ gm/ton} \times 1278 \text{ cal/gm} \times 4.186 \times 10^7 \text{ ergs/cal} = 9.7 \times 10^{20} \text{ ergs.}$

When water vapor is turned into liquid water in the atmosphere, the latent heat originally used in evaporation is retained by the surrounding air at the rate of 600 cal per gm. If 1 in. of rain falls on 1 sq. mi, the amount of water equals

$2.54 \text{ cm} \times 2.59 \times 10^{10} \text{ cm}^2 = 6.58 \times 10^{10} \text{ gm}$

The energy released is, then,

$6.58 \times 10^{10} \text{ gm} \times 600 \text{ cal/gm} \times 4.186 \times 10^7 \text{ ergs/cal} = 1.7 \times 10^{21} \text{ ergs.}$

Thus the energy released by the atomic bomb is approximately one-half that released when one inch of rain falls on one square mile.

On September 20-23, 1948, a hurricane passed over Florida. Using the daily weather maps issued by the United States Weather Bureau and also additional data supplied privately, I was able to draw approximate isohyet lines for the rainfall for September 22. These gave the rainfall for the 24 hours as 112,000 in. sq. mi. The energy transformed by the condensation of this water is, then, $1.9 \times 10^{26} \text{ ergs}$. This is equal to $2 \times 10^5$ times the energy of the atomic bomb. If we assume that the structure of the hurricane has not changed, so that the rate of fall equals the rate at which water vapor is condensing, energy throughout the hurricane is being transformed at the rate of $2^{1/2}$ atomic bombs a second.

One form into which the energy of the water has been changed is that of the kinetic energy of the wind. An approximation of this energy at any one time can be calculated. The hurricane weather advisory issued during the morning of September 22 stated that within 50 mi. of the center the winds were of hurricane force, with gale winds to 125 mph. If one assumes that these winds persisted through all levels up to the 500-mb surface, the kinetic energy is found to be $1.7 \times 10^{24} \text{ ergs}$. The assumption may be verified only approximately, but the calculation gives the order of magnitude of the energy involved.

The winds of the hurricane represent then only a very small fraction of the energy released by the water vapor. If the winds were to drop suddenly to zero, the energy being transformed could cause them to blow with their initial force in 15 minutes. Most of the heat energy being released by the water vapor is being used to heat the atmosphere directly.

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**Cold Weather and Railroad Operations**

In February 1948 after some very bad weather in the East, a *New York Times* reporter interviewed a New York Central official. The following excerpts from his story, while interesting in themselves, tend to remind us of the many opportunities for meteorological service to industry which might be open to a private meteorologist.

"The last eight weeks have been rough on railroad men, not so much because of the great snowbanks and drifts that have cluttered up the tracks and disrupted schedules, as because of the intense and persistent cold. Operating difficulties begin to pile up as soon as the temperature gets down and stays down well below freezing. George H. Baker, general superintendent of passenger transportation for the New York Central, says 15 degrees above zero is the point where trouble begins."

"When the mercury hovers near zero, or at 25 degrees below zero, which is what the Central has been running through in the Mohawk Valley, more heat is necessary to keep passenger cars and diners comfortable. This draws more steam from the locomotive, and when the steam pressure falls under the increasing demand, the train has to sacrifice its speed for passenger comfort. Then schedules begin to jam. Frequently in such weather, trains are held down to sixty miles an hour to keep the passengers comfortable, although this may confound the timetable."

"Coal freezes in the engine tender and the automatic stokers are apt to balk or break down. Then an unscheduled stop may be necessary to take aboard an extra fireman's helper to break up the coal or even to help hand-fire the boiler. Diesel-electric locomotives also suffer from the cold. It takes more steam to keep the passengers warm; this uses up water faster and extra, unscheduled stops are required to refill the boilers. One cold morning recently the Twentieth Century Limited with 9,000 horsepower in the Diesel-electric on its headend arrived two hours late.

"To make it possible to keep rail travelers comfortably warm and on schedule, some overnight trains have been cut back to fourteen cars. But even on a short train steam lines under extra pressure to combat subzero weather can spring leaks requiring emergency repairs. Frozen water plugs and hot boxes are other frequent causes of delay—and of the hope of most railroaders that spring will soon come."

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