Asymptotic matching between weather and climate models

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ABSTRACT

The Deep Numerical Analysis for Climate (DNA-Climate) is a pilot project to develop an earth system model on a kilometer-scale horizontal mesh. The acronym “DNA” is based on the analogies between the hierarchical structures of atmospheric phenomena and living organisms. The multiscale structure of clouds and circulations may be analogous to the multiscale structure of cells and organs organized according to the blueprint, deoxyribonucleic acid (DNA). Whereas global cloud-resolving models (CRMs) can produce better solutions on shorter-time scales that are decisively governed by the initial conditions, global climate models (GCMs) may generate reliable solutions on longer-time scales that are largely determined to balance energy inputs and outputs. Our challenge is to build a physically valid model that consistently bridges the shorter- and longer-time scale solutions in the intermediate time scales. Research topics of DNA-Climate are configured in consideration of the structural similarity between the climate modeling and the technique of matched asymptotic expansions in mathematics. The central question is whether a single modeling framework using only either global CRM or GCM will work adequately at all time scales of climate, or whether a multiscale modeling framework combining several models, of which each is only valid for limited time scales, will be needed. A multiscale modeling is an attractive framework for advancing climate modeling and would be an intriguing topic to be studied in parallel with global CRMs and GCMs.
Deep Numerical Analysis for Climate (DNA-Climate), started in 2020, is a pilot project to endorse diverse approaches of climate modeling. The acronym “DNA” was chosen in consideration of the analogies between the hierarchical structures of atmospheric phenomena and living organisms (Held 2005). Similar to the multiscale nature of atmospheric phenomena, biological organisms have the multiscale structure of organelles, cells, and organs organized according to the blueprint, deoxyribonucleic acids (DNA).

DNA-Climate is focused on laying the groundwork for the development of Digital Earths, in which global cloud-resolving models (CRMs) will be one of the key ingredients. The Digital Earth is a coupled atmosphere-land-ocean-cryosphere model that covers the entire globe with a mesh of at most several kilometers in size. In global CRMs, solving equations of the cloud microphysics, which allows a thermodynamic energy exchange between hydrodynamic and hydrologic processes at the scale of a single grid cell, may lead to proper spontaneous organizations of weather systems composed of turbulence, convection, and cloud systems. However, it is nontrivial whether the calculations of clouds based on equations closer to the first principles, i.e., using a cloud microphysics scheme instead of a cumulus parameterization, will increase the reliability of climate simulations. The temporal scope of global CRMs might extend as supercomputers become faster, analogous to the increase in the size of organisms responding to environmental changes (Troyer et al. 2022). However, there might also be a limit to the temporal scale at which global CRMs can adequately represent the climate system, just as there seems to be a limit to the size of life forms (Blanckenhorn 2000).

The first global CRM was developed on the “Earth Simulator” launched from Japan Agency for Marine-Earth Science and Technology in the early 2000s. Taro Matsuno
dreamed for a global climate model (GCM) with a horizontal mesh size finer than 10 km and without any cumulus parameterization. The cumulus parameterization is a statistical theory of cloud ensembles, which has been considered as one of the largest sources of uncertainties regarding clouds in GCMs (Randall et al. 2003; Stevens et al. 2019). At that time, the term “cloud-resolving model” was widely accepted to refer to such models that allows direct representation of the growth and decay of individual clouds although a mesh of O(1 km) or even O(1 m) is still too coarse to fully represent physical processes involved in clouds (Bodenschatz et al. 2010).

It was Hirofumi Tomita who overcame the seemingly impossible order of Matsuno by developing the first global CRM. Tomita adopted the Arakawa A-grid staggering, following Williamson (1969), on the icosahedral mesh, once studied in the 1960s and revisited by the groups at Japan Meteorological Agency in the 1980s and at Colorado.

Fig. 1. Images of outgoing longwave radiation from the global 3.5-km mesh simulations of (a) Tomita et al. (2005) and (b) Miura et al. (2007b). The timing of the figures are (a) 5 days and (b) 4 days after the initial time, respectively. Areas corresponding to clouds are displayed by (a) gray to white gradient and (b) white opacity gradient. Clearer white represents higher clouds.
State University in the 1990s. His originality in optimizing the hexagonal-pentagonal mesh has enabled a stable calculation of the fluid dynamics at quasi-second-order accuracy (Tomita et al. 2001). Tomita named his model Nonhydrostatic Icosahedral Atmosphere Model (NICAM; Tomita and Satoh 2004) and performed the first global O(1 km) mesh atmospheric simulation under an ideal aquaplanet condition (Fig. 1a; Tomita et al. 2005). Then, a simulation employing realistic initial and boundary conditions followed (Miura et al. 2007a).

The essence of the innovation from GCMs to global CRMs is in the representations of hydrodynamic transports of energy and momentum. In GCMs, cumulus, shallow cumulus, and gravity wave parameterizations perform non-local mixing of heat, moisture, and momentum within a vertical column quasi-instantaneously, whereas in global CRMs, the fluid dynamics directly coupled to cell-scale local representations of microscale phenomena (e.g., cloud microphysics, physical turbulence, and artificial diffusions) build up local transports between cells sequentially. A global simulation with realistic-appearing clouds has been made possible by this coupling of large-scale circulation and small-scale convection (Fig. 1b; Miura et al. 2007b), but it has not yet been verified whether this sequential, accumulative approach of transport can produce reliable climate simulations as well.

The 40 teraflops Earth Simulator realized a 10-day integration on a global 3.5-km mesh. Although there was a criticism that coarse O(1 km)-resolution models should be considered “only partly cloudy” (Sperber et al. 2008), there was also an anticipation that it will become a new trend of climate simulations. However, the focus of NICAM has been inclined towards short-term higher resolution simulations, except for a 20-year calculation on a global 14-km mesh (Kodama et al. 2015). The simulation period was two
days on a global 870-m mesh when the 10 petaflops “K computer” came (Miyamoto et al. 2013), and it will be less than half a day on a global 220-m mesh on the 500 petaflops “Supercomputer Fugaku.” After initially proving the capabilities of global CRMs by producing beautiful pictures of two to seven-day simulations (e.g., Fig. 1), NICAM let nearly two decades pass without demonstrating the advantages of global CRMs over GCMs in reproducing longer-time scale phenomena.

One may wonder why we tend to resort to demonstrations of short-duration calculations for results that are easy to understand. The answer to this question may have been foretold by John von Neumann at Conference on the Application of Numerical Integration Techniques to the Problem of the General Circulation, held in Princeton University on October 26-28, 1955:

> It seems quite plausible from general experience that in any mathematical problem it is easiest to determine the solution for shorter periods, over which the extrapolation parameter is comparatively small. The next most difficult problem to solve is that of determining the asymptotic conditions—that is, the conditions that exist over periods for which the extrapolation parameter is very large, say near infinity. Finally, the most difficult is the intermediate range problem, for which the extrapolation parameter is neither very small nor very large. In this case the neglect of either extreme is forbidden. (von Neumann 1958)

von Neumann’s words can be interpreted as classifying the problem of physically representing the time evolution of the atmosphere into three categories according to their
time scales. The first could be interpreted as the weather forecast. Global CRMs have often been used as a tool to promote simulations shorter than a few weeks. However, according to von Neumann, this is the easiest problem because the initial state largely governs the flow evolution. The second refers to the climate simulation. In sufficiently longer-time scales, the details of variability in atmospheric and oceanic flows are presumably filtered out, and thermodynamic equilibrium and its internal fluctuations become our interests. The last problem deals with the atmospheric and oceanic conditions where the energy balance is in the state of quasi-equilibrium (Arakawa and Schubert 1974). The atmosphere fluctuates as an adjustment to eliminate the instability created continuously by the imbalance between the solar and terrestrial radiation. von Neumann stated that it is the most difficult to predict the intermediate time scales that lose much of the information of the initial state and do not settle in a thermodynamic equilibrium state.

There seem to be parallels between these three classifications of the prediction problem and the method of multiple scales in differential equation theory (e.g., Bender and Orszag 1999). A typical example in the atmospheric science is the boundary layer problem for near-surface steady flows. In the region sufficiently far from the surface, one asymptotic solution is obtained by neglecting the viscosity term of the Navier-Stokes equations. In the region near the surface, another asymptotic solution is obtained by treating the viscosity term as significant. The region where an accurate solution is difficult to obtain lies in between. In this intermediate region, an approximate solution is created by compositing the two asymptotic solutions. This technique is called the matched asymptotic expansions or the asymptotic matching.

These three classifications seem applicable to global atmospheric modeling as well. The first extreme is the global CRM, which can produce reasonable cloud and rain
distributions by consuming water vapor given as initial conditions. However, it is not clear yet whether the equilibrium conditions of moisture and clouds created spontaneously would be realistic. On the other extreme is the GCM, which is capable of energetically-balanced climate simulations on a 1000-year time scale. However, there are limitations in the representation of mesoscale phenomena such as squall lines and tropical

![Fig. 2. Schematic illustration of the matching of global CRM and GCM solutions.](image)

The horizontal axis denotes time scales of typical weather and climate phenomena. The vertical axis is a projection of model solutions onto a vector. Global CRM and GCM are represented by the orange and blue lines, respectively. The broken lines signify that the solutions available by current models depart from natural behavior. The intentions of plan A and plan B of our project are to work to get the two solutions closer to each other. The solution obtained by the matching may be an improved approximation of the reality, although of course it is not guaranteed.
cyclone eyewalls. For the time scales in between, a physically adequate model has not been established yet. Our challenge is to build a model that consistently bridges the fine solutions of mesoscale phenomena on shorter-time scales and the energetically balanced solutions on longer-time scales.

DNA-Climate is designed in consideration of the structural similarity between the three classifications of the time scales mentioned by von Neumann and the matched asymptotic expansions in mathematics (Fig. 2). Our strategy is to have two research groups working simultaneously, one for a global CRM and the other for a GCM. We regard global CRM and GCM solutions as two asymptotic solutions and aim to virtually match the two solutions for the intermediate time scales by fostering communication between the two groups. We are trying a new framework to tackle the difficulties of the intermediate range problem, which, according to von Neumann, neglect of either extreme is forbidden.

DNA-Climate consists of four research groups including the aforementioned two groups. The first two take traditional approaches, Plan A and Plan B, respectively. Plan A strives to extend the upper limit of simulation periods of a global CRM. The first 10-year integration of NICAM on a global 3.5-km mesh is being performed using updated settings (Takasuka et al. 2023). A coupled atmosphere-ocean simulation is also being performed on a global 14-km mesh with NICAM coupled to a 0.25° mesh ocean model. Plan B seeks to extend the lower limit of the horizontal mesh size of a GCM. Along with research on dynamical cores (Miura 2019), research on climate systems has been conducted (Kohyama et al. 2021; Yamagami et al. 2022) with standard and high-resolution versions of Model for Interdisciplinary Research on Climate (MIROC; Tatebe et al. 2019). The third group conducts its own verification study of NICAM and MIROC to facilitate
communication between the two modeling groups. Climatological features of convective systems have been analyzed in the multi-decadal simulation of NICAM (Suematsu et al. 2022). The last group promotes innovative model developments. The modeling activities of this group include developments of a new 3D dynamical core using the discontinuous Galerkin method (Kawai and Tomita 2023), a model of a falling droplet using the immersed boundary method (Ong et al. 2021), and a superparameterized-MIROC by adding a blockwise coupling technique (Yamazaki 2023) to the superparameterization (Grabowski 2001; Khairoutdinov and Randall 2001).

Here we may notice one important thing. Notwithstanding the view that the climate system consists of a hierarchy of phenomena with different spatial and temporal scales (Fig. 3a), both Plan A (Fig. 3b) and Plan B (Fig. 3c) implicitly assume that everything from clouds to climate can be represented in a unified way within a single model. Of course, we cannot rule out the possibility that a single modeling framework will work. However, it is also plausible that one model alone cannot cover the full range of time scales, as in the technique of asymptotic matching in mathematics. Therefore, it would be interesting to consider other possibilities.

One possibility is the superparameterization that explicitly incorporates the existence of multiple scales in the climate system (Fig. 3d). The benefits of the superparameterization have already been shown, such as improved representation of convectively coupled equatorial waves (Hannah et al. 2020). Furthermore, we anticipate that the intentional scale separation, though artificial, will provide a framework that enables numerical experiments to investigate the stochasticity and memory effects of filtered subscales (Lucarini et al. 2014).
Another possibility is the supermodeling technique (Schevenhoven et al. 2023). We envision that the use of the supermodeling technique to synchronize global CRMs and GCMs will advance the superparameterization, which currently uses CRMs only as quasi-equilibrium generators inside GCM columns (Fig. 3e). If realized, the model would have a structure that closely resembles the structure of the asymptotic matching. We believe that constructing a supermodel of global CRMs and GCMs is a tangible procedure to match the solutions of the two types of models. Establishment of this matching technique might pave the way for building a climate model in the framework of multiscale modeling, which fuses the models of components of climate systems at each spatio-temporal scale one after another (Fig. 3f). Multiscale models will naturally have the ability to quantitatively evaluate interactions between scales, making them a powerful tool to open...
the frontiers of research of nonlinear interactions between scales.

The idea of the multiscale modeling was inspired by the studies of Andrew Majda (e.g., Majda and Klein 2003) and from the Center for Multiscale Modeling of Atmospheric Processes (CMMAP), led by David Randall in the period 2006-2016. Specifically, the parallel approach of global CRMs, GCMs, and superparameterized-GCMs is in line with aspirations of CMMAP (Arakawa et al. 2011). At the 2012 CMMAP Team Meeting in Fort Collins, Colorado, Akio Arakawa said:

*Why does everybody think climate model is completed? To me, it is still in its infancy. The interesting part is yet to come. Why aren’t you doing it? (Akio Arakawa, personal conversation in Japanese)*

There are still myriads of interesting topics to consider. For example, even a fundamental question like the form of the equations of motion should be revisited since the essential dynamics of atmospheric phenomena marginally resolved by global CRMs changes with the resolution. On O(1 km) horizontal meshes, synoptic weather systems dominated by the vertical component of the vorticity vector may be satisfactorily reproduced. Perhaps that is why global CRMs have often been renamed global storm resolving models. On O(100 m) meshes, simulations of fronts, squall lines, and filamentations of potential vorticity and specific humidity would be improved. In these phenomena, one of the two horizontal components of the vorticity vector is relatively significant compared to the other. When the mesh size becomes O(10 m) or less, the three components of the vorticity vector are equally important. Those simulations might...
warrant the use of the term global large eddy simulations. To represent this hierarchical transition in the dominant components of the vorticity vector, the vector vorticity form of the equations of motion (Jung and Arakawa 2008) might be superior to the widely used velocity form.

Much research, from fundamental to application, is needed for the climate models to evolve from their infancy. We believe that climate models are one of the best frameworks to delineate our understanding of the climate system. With accumulation of knowledge of the atmosphere, the atmospheric models should become a reliable foundation for the Digital Earth. The interesting part is yet to come.

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