The Temporal Variability of Soil Moisture and Surface Hydrological Quantities in a Climate Model

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ABSTRACT

The variance budget of land surface hydrological quantities is analyzed in the second Atmospheric Model Intercomparison Project (AMIP2) simulation made with the Canadian Centre for Climate Modelling and Analysis (CCCma) third-generation general circulation model (AGCM3). The land surface parameterization in this model is the comparatively sophisticated Canadian Land Surface Scheme (CLASS). Second-order statistics, namely variances and covariances, are evaluated, and simulated variances are compared with observationally based estimates. The soil moisture variance is related to second-order statistics of surface hydrological quantities. The persistence time scale of soil moisture anomalies is also evaluated.

Model values of precipitation and evapotranspiration variability compare reasonably well with observationally based estimates. Soil moisture variability is compared with that simulated by the Variable Infiltration Capacity-2 Layer (VIC-2L) hydrological model driven with observed meteorological data. An equation is developed linking the variances and covariances of precipitation, evapotranspiration, and runoff to soil moisture variance via a transfer function. The transfer function is connected to soil moisture persistence in terms of lagged autocorrelation. Soil moisture persistence time scales are shorter in the Tropics and longer at high latitudes as is consistent with the relationship between soil moisture persistence and the latitudinal structure of potential evaporation found in earlier studies. In the Tropics, although the persistence of soil moisture anomalies is short and values of the transfer function small, high values of soil moisture variance are obtained because of high precipitation variability. At high latitudes, by contrast, high soil moisture variability is obtained despite modest precipitation variability since the persistence time scale of soil moisture anomalies is long. Model evapotranspiration estimates show little variability and soil moisture variability is dominated by precipitation and runoff, which account for about 90% of the soil moisture variance over land surface areas.

1. Introduction

Land surface models (LSMs) provide the connection between the atmosphere and the land surface in general circulation models (GCMs). LSMs range in complexity from the simple “bucket” model with fixed water-holding capacity (Manabe 1969), to models of intermediate complexity, which implicitly represent some effects of soil properties and vegetation (e.g., Noilhan and Planton 1989), to more sophisticated soil–vegetation–atmosphere–transfer (SVAT) schemes that include explicit treatment of vegetation (e.g., Sellers et al. 1986; Dickinson et al. 1986). The SVAT approach emphasizes both the varying properties of the soil and the direct role of vegetation in determining the surface energy and water balance, particularly by taking into account the physiological properties of vegetation [leaf area index (LAI) and stomatal resistance]. The SVAT approach may be enhanced to include the coupling between photosynthesis and stomatal conductance (Sellers et al. 1996b; Dickinson et al. 1998) and the dynamic treatment of vegetation (e.g., Foley et al. 1996; Arora 2002).

The role of LSMs is to mediate the fluxes of energy, moisture, and momentum that connect the atmosphere to the underlying land surface. Soil moisture is the dominant quantity affecting these surface fluxes. Shukla and Mintz (1982) highlight the role played by soil moisture in the extratropics and make an analogy with energy in the ocean. The ocean stores some of the radiational energy it receives in summer and uses it to heat the
atmosphere in winter. The soil stores some of the precipitation it receives in winter and uses it to humidify the atmosphere in summer. The role of soil moisture in influencing near-surface atmospheric variability (Delworth and Manabe 1988, 1989; Manabe and Delworth 1990), in affecting the persistence of atmospheric circulation (Namias 1952, 1959), and the effect of anomalies of soil moisture and temperature on atmospheric circulation (Charney et al. 1977; Walker and Rowntree 1977; Rowntree and Bolton 1983; Yeh et al. 1984; Dickinson and Henderson-Sellers 1988; Shukla et al. 1990; Koster and Suarez 1995; Aiviss and Liu 1996) have been investigated by many researchers. Realistic spatial distributions of soil moisture have been shown to improve rainfall patterns, reduce errors in near-surface temperatures, and improve interannual variability in simulated climate (Fennessey and Shukla 1999; Dirmeyer 2000; Douville and Chauvin 2000).

While the role of soil moisture in affecting the near-surface atmosphere has been the focus of a number of studies, very few have focused on the nature and causes of soil moisture variability itself. Delworth and Manabe (1988) describe the variability of soil moisture in a GCM (using a bucket LSM with a field capacity of 15 cm) in terms of the variability of precipitation and the control of potential evaporation. In the light of stochastic theory, they report that the forcing of the land surface system by precipitation (a white noise input variable) will yield soil moisture (the output variable) with a red spectrum, the redness of which is controlled by the dependence of evaporation on soil moisture and potential evaporation (the damping term). They conclude that the increasing redness of soil moisture spectra at high latitudes is a result of declining potential evaporation. Entekhabi and Rodriguez-Iturbe (1994) derive an analytical expression for the space–time power spectrum of soil moisture that is related to the space–time power spectrum of the rainfall through a hydrological gain function. They use a simple soil water balance model in which losses (combined evapotranspiration, surface runoff, and drainage) are represented as a linear function of soil moisture. The space–time power spectrum of rainfall is estimated by assuming that rainbands arrive in space–time according to a homogeneous Poisson process. As with Delworth and Manabe (1988), Entekhabi and Rodriguez-Iturbe (1994) conclude that the hydrological gain function (whose parameters depend on climate and surface hydrological processes) acts as a low-pass filter that preferentially dampens high-frequency space and time fluctuations of rainfall to produce low-frequency and large-scale soil moisture variations.

In this study, we evaluate the variability of modeled precipitation, evapotranspiration, and runoff and investigate the connection between the variability of these moisture budget components and that of soil moisture. Monthly values of moisture budget quantities from the second Atmospheric Model Intercomparison Project (AMIP2) simulation made with the Canadian Centre for Climate Modelling and Analysis (CCCma) third-generation atmospheric general circulation model (AGCM3) are used. The SVAT scheme used in the CCCma AGCM3, the Canadian Land Surface Scheme (CLASS), explicitly models interactions between canopy, snow, and ground (soil) moisture reservoirs. A brief description of AGCM3 and the land surface scheme is given in section 2, which also describes the AMIP2 simulation. Comparisons of the variance of simulated surface hydrological quantities with observationally based variance estimates are presented in section 3. The connection between the variability of soil moisture and other moisture budget components is investigated in section 4, and the results are summarized in section 5.

2. Model and experimental design

a. The CCCma third-generation AGCM

The CCCma third-generation atmospheric GCM (Scinocca and McFarlane 2004) is the latest in the series of AGCMs described in Boer et al. (1984) and McFarlane et al. (1992). The results analyzed here are obtained with a T47 L32 version of the model. Dynamic terms are calculated at triangular T47 spectral truncation, and the physical terms on a 96 × 48 (3.75°) horizontal linear grid. The vertical domain extends to 1 hPa with the thicknesses of the model’s 32 layers increasing monotonically with height from approximately 100 m at the surface to 3 km in the lower stratosphere.

While many of the parameterized physical processes in the third-generation model are qualitatively similar to AGCM2, key new features include 1) a new parameterization of cumulus convection (Zhang and McFarlane 1995), 2) an improved treatment of solar radiation that employs four bands in the visible and near-infrared region, 3) an “optimal” spectral representation of topography (Holzer 1996), 4) a revised representation of turbulent transfer coefficients at the surface (Abdella and McFarlane 1996), 5) a hybrid moisture variable (Boer 1995), and 6) CLASS for treatment of the land surface processes (Verseghy 1991; Verseghy et al. 1993).

b. The Canadian Land Surface Scheme

The CLASS land surface scheme processes heat and moisture via three reservoirs: the ground, the canopy,
and the snow with fractional coverages, $f_c$, $f_s$, and $f_g$, respectively, as shown in Fig. 1a. A GCM grid cell is further divided into four subareas: bare soil ($f_b$), snow-covered ground ($f_{gs}$), canopy-covered ground ($f_{gc}$), and canopy-covered snow ($f_{sc}$), for each of which heat and moisture balances are evaluated separately. The soil is divided into three layers of depth 0.10 m, 0.25 m, and 3.75 m, each with its own prognostic liquid and frozen soil moisture content and temperature.

The condensed moisture budget equation for a system of $N$ reservoirs applied to grid-averaged quantities is written as

$$\frac{dW_n}{dt} = P_n - E_n - \sum_{m=1}^{N} I_{nm}, \quad n = 1, \ldots, N,$$

where $W_n$ is the moisture content of reservoir $n$, $P_n$ is the gain of moisture by direct precipitation input into the reservoir, $E_n$ is the loss of moisture in the form of vapor (the evaporative output including transpiration and sublimation), and $I_{nm}$ is the moisture exchange between reservoir $n$ and $m$ (in practice we use subscripts $c$ for canopy, $s$ for snow, and $g$ for ground rather than numbers).

Here $I_{nm}$ represents the exchange as a loss from reservoir $n$ and a gain by reservoir $m$ so that $I_{nm} = -I_{mn}$ for $n \neq m$. When $n = m$, the term represents a loss (flow out of a grid cell) by some other process. For the ground moisture reservoir $I_{gg} = R$ is the runoff and, hence, river flow. For the snow moisture reservoir $I_{ss} = C$, the conversion of snow to glacial ice, which is parameterized in CLASS as a function of snow mass. The net moisture budget equation,

$$\frac{dW}{dt} = P - E - (R + C),$$

is obtained by summing over all $N$ reservoirs, where unsubscripted variables represent the sum; that is, $W = \sum_{n=1}^{N} W_n$ and $\sum_{n=1}^{N} \sum_{m=1}^{N} I_{nm} = R + C$.

Figure 1b identifies the inputs, outputs, and exchanges of moisture to, from, and between the snow,
the canopy, and the ground moisture reservoirs. The exchange quantities $I_{\text{ct}}, I_{\text{cs}},$ and $I_{\text{sg}}$ refer to the falling of snow from the canopy onto the snow below, the dripping of rainwater from the canopy to the ground below, and the transfer of moisture from snow to ground due to snowmelt. Moisture input into a reservoir is the sum of direct precipitation input and any positive moisture exchanges. For example, moisture input into the ground moisture reservoir is the sum of direct precipitation that falls on the bare ground ($P_g$), rainfall drip from the canopy ($I_{\text{ct}}$), and snowmelt ($I_{\text{sg}}$). Evaporation loss from the ground moisture reservoir, $E_g$, is made up of two components: transpiration from the ground via the canopy ($E_c$) and direct evaporation from the soil ($E_{\text{soil}}$). Runoff from ground ($R_g$) is also made up of two components: 1) surface (overland) runoff ($R_o$), which is generated when the amount of ponded water exceeds a specified limit and ponds form when precipitation intensity exceeds infiltration capacity, and 2) drainage ($R_d$) from the bottommost soil layer, which is assumed equal to the saturated hydraulic conductivity of the soil. The rate of change of soil moisture in the ground reservoir is the sum of the net moisture input ($G_g$), evapotranspiration ($E_g$), and drainage ($R_g$):

$$\frac{dW_g}{dt} = G_g - E_g - R_g,$$

where $G_g = P_g + I_{\text{ct}} + I_{\text{cs}} - R_g$ and $E_g = E_c + E_{\text{soil}}$. Similar equations can be written for the canopy $dW_c/dt = G_c - E_c - R_c$ and the snow $dW_s/dt = G_s - E_s - R_s$ moisture reservoirs, where $G_c = P_c, R_c = I_{\text{ct}} + I_{\text{cs}}, G_s = P_s + I_{\text{cs}},$ and $R_s = I_{\text{sg}} + I_{\text{soil}}$.

c. The AMIP2 simulation

The Atmospheric Model Intercomparison Project, initiated in 1989, undertakes the systematic validation, diagnosis, and intercomparison of the performance of atmospheric GCMs (Gates et al. 1999). In AMIP2 simulations, an atmospheric GCM is integrated for the 17-yr period (1979–95) with specified lower boundary conditions of observed monthly sea surface temperatures (SSTs) and sea ice concentrations (Fiorino 1996). AMIP2 is an extension of AMIP1, with improvements in experimental design, diagnosis of an expanded set of model output variables, and establishment of new standards and protocols for data analysis.

3. Comparison with observations

In Arora and Boer (2002, 2003) the first-order statistics of moisture budget quantities from an AMIP2 simulation made with the CCCma AGCM3 are analyzed and compared with observations. The mean annual values and the first harmonic of the annual cycle of simulated precipitation, runoff, and other primary water budget components for the globe and selected major river basins are considered and modeled mean annual precipitation and runoff and their latitudinal structures are shown to compare well with observations, although some discrepancies remain in the simulation of regional values of these quantities. Here the second-order statistics (variances and covariances) of the model water budget quantities are analyzed. The variance of simulated monthly precipitation is compared with observationally based estimates and the variance of simulated monthly evapotranspiration with estimates from European Centre for Medium-Range Weather Forecasts (ECMWF) reanalysis. Mean annual values of simulated soil moisture, and its monthly variance, are compared with estimates generated from the Variable Infiltration Capacity-2 Layer (VIC-2L) hydrological model.

a. Precipitation

In Fig. 2a the variance of simulated monthly precipitation values (calculated after removal of the climatological annual cycle) is compared with observationally based estimates from Xie and Arkin (1996). Xie and Arkin (1996) construct global 2.5° gridded monthly precipitation estimates based on gauge data, satellite information from three difference sources, and results from the ECMWF operational forecast model. Estimates for the period from 1979 to 1997 are used. Model precipitation variance compares well with that based on observations, except in Amazonia where the model produces less precipitation (Arora and Boer 2002). The variability of both model and observationally based precipitation values is higher in the Tropics and lower at high latitudes, at least partially as a consequence of
a) Monthly precipitation variance (mm/day)^2

b) Coefficient of variation - monthly precipitation

c) Coefficient of variation - monthly evapotranspiration

d) Soil moisture in top 1m of the soil layer (mm)

e) Monthly soil moisture variance, top 1m (mm^2)
the large precipitation rates at low compared to high latitudes. The spatial correlation between the model and observationally based precipitation variance estimates is 0.52. The coefficient of variation (cv), given by \( \gamma = \sigma_X / \bar{X} \), where \( \sigma_X \) is the standard deviation and \( \bar{X} \) is the mean, characterizes the variability relative to the mean. The cv represents the fact that the same level of precipitation variability is of less practical consequence in a region of high mean precipitation than in a region of low mean precipitation. Values of cv over land are compared in Fig. 2b. Model values of the coefficient of variation compare well with observations (spatial correlation of 0.70). Although the precipitation variability is higher in the Tropics, the cv values are higher in areas that receive less precipitation on average such as Australia, the Sahara Desert, the Middle East, and the southwestern United States. Precipitation variability is typically of more practical importance in these regions of less precipitation (McMahon 1979). The land-averaged precipitation variance compares well with the observationally based estimate, 1.65 versus 1.59 (mm day\(^{-1}\))^2 in Fig. 2a. The corresponding averaged values of cv are 0.56 versus 0.68 in Fig. 2b, although averages of ratios can be misleading. Here and throughout the rest of the paper Greenland and Antarctica are excluded from the averages.

b. Evapotranspiration

In the absence of an observed evapotranspiration dataset, model evapotranspiration variance is compared with estimates from the ECMWF reanalysis (Gibson et al. 1997) for the period 1979–93. The land surface scheme used in the ECMWF reanalysis consists of four soil layers of thickness of 0.07, 0.21, 0.72, and 1.89 m (Viterbo and Beljaars 1995). Evapotranspiration displays much less variability than precipitation with variance values (not shown) of less than 0.20 (mm day\(^{-1}\))^2 over most of the land area. The variance of monthly evapotranspiration simulated over land is 0.19 (mm day\(^{-1}\))^2 in AGCM3, which is slightly higher than the value of 0.15 (mm day\(^{-1}\))^2 obtained from the ECMWF reanalysis. The coefficient of variation for evapotranspiration is shown in Fig. 2c. The spatial correlation between AGCM3 and ECMWF values is 0.50. Evapotranspiration estimates show higher values of cv over areas characterized by low precipitation, implying that relative to the mean evapotranspiration is more variable in arid and semiarid areas, as is also the case for precipitation. In Fig. 2c, the value of cv averaged over land for the AGCM3 simulated evapotranspiration estimates (0.38) is slightly higher that that for the ECMWF values (0.25).

c. Soil moisture

Observationally based estimates of soil moisture content at global scale are not readily available. Data in the Global Soil Moisture Data Bank (Robock et al. 2000) are mostly over Russia, China, and Mongolia. Model-derived global soil moisture estimates are available from various sources (e.g., Mintz and Serafini 1992; Schmii et al. 1992; Willmott et al. 1985) typically estimated with a bucket hydrological model of field capacity 150 mm. These models do not, however, capture the large observed high values of soil moisture because of the limited depth of the bucket. This results in constant soil moisture values in winter while observations suggest soil moisture varies as much in winter as in other seasons (Robock et al. 1998). Soil moisture information is also available from the Global Soil Wetness Project (GSWP) (Dirmeyer et al. 1999) in which the 10 participating land surface schemes are forced with meteorological data from the International Satellite Land Surface Climatology Project (ISLSCP) (Sellers et al. 1996a) for two years (1987–88). An assessment of GSWP data by Entin et al. (1999) over selected regions (for which global soil moisture data are available) reveals, however, that none of the models do a particularly good job of producing the observed soil moisture values for any of the regions. Of course, the 2-yr duration of GSWP does not provide variability information. Soil moisture values are also available from various reanalyses but these are nudged toward climatology using an artificial moisture input term (e.g. see Roads and Betts 2000).

AGCM3/CLASS simulated soil moisture and its variability is compared with the results of Niijens et al. (2001), who estimate daily soil moisture values for the period from 1980 to 1993 at 2° resolution using the VIC-2L model (Liang et al. 1994). VIC-2L is a somewhat more realistic hydrological model than the bucket model and predicts a larger range of soil moisture. Unlike most land surface schemes, it includes an explicit representation of spatial variability of infiltration capacity that is used for surface runoff generation. It assumes a 1-m soil layer with soil porosity values varying between 0.4 for coarse-textured to 0.48 for fine-textured soil. The maximum water-holding capacity of the soil column in the VIC-2L model averages about 440 mm compared to 150 mm for bucket model. Niijens et al. (2001) force the VIC-2L model with observationally based daily precipitation and temperature data, wind speeds based on the National Centers for Environmental Prediction–National Center for Atmospheric Research (NCEP–NCAR) reanalysis data (Kalnay et al. 1996), and other meteorological variables (ra-
diation and vapor pressure) derived from precipitation and temperature.

The total depth of three soil layers in the CLASS land surface scheme is 4.1 m compared to 1 m in the VIC-2L hydrological model. Soil moisture in the top 1 m of CLASS is obtained by combining the soil moisture in the top two layers (of thicknesses 0.1 and 0.25 m, respectively) with the soil moisture in the top 0.65 m of the third layer assuming a uniform soil moisture profile \[ W_{g,1,1m} = W_{g1} + W_{g2} + W_{g3} \times (0.65/3.75), \] where \( W_{g} \) (depth units) is the soil moisture content of the \( i \)th layer. The resulting soil moisture values compare well with those from the VIC-2L model in Fig. 2d, although the AGCM3/CLASS estimates are slightly higher in the northern high latitudes and soil moisture estimates from VIC-2L are noticeably smoother. The average values over land compare well (225 versus 219 mm) and the spatial correlation (excluding Greenland and Antarctica) is 0.58.

Figure 2e shows that the AGCM3/CLASS soil moisture variance, extracted for the top 1 m, is qualitatively similar to that of the VIC-2L model although values are generally somewhat larger, except over the Tropics and the Sahara and the Mongolian deserts. The globally averaged value of soil moisture variability in the top 1 m from the AGCM3/CLASS (376 mm\(^2\)) compares well with that from the VIC-2L model (382 mm\(^2\)).

In summary, comparison of the variability of simulated precipitation and evapotranspiration with observationally based estimates and reanalysis data, respectively, shows that the model estimates of the variability of these components are reasonable. The broad spatial patterns of soil moisture variability, and its land-averaged value, are similar to those generated by the VIC-2L model although the AGCM3/CLASS values are somewhat higher. In the absence of observed monthly runoff estimates, comparisons for model runoff variance are not performed. Runoff data from other models are available, but these data are not used given the uncertainties associated with LSM’s runoff estimates (e.g., see Gedney et al. 2000).

4. Analysis of soil moisture variability

We approach the analysis of soil moisture variability by writing Eq. (3) in terms of deviations (or anomalies, indicated by prime) of a variable from its mean annual cycle as

\[
\frac{dW'_g}{dt} = G'_g - E'_g - R'_g. \tag{4}
\]

Multiplying Eq. (4) by \( W' \) on both sides, dropping the subscript \( g \) for simplicity, and averaging yields

\[
\frac{1}{2} \frac{d}{dt} \overline{W'^2} = G \overline{W'} - \overline{EW'} - \overline{RW'}. \tag{5}
\]

Equation (5) is the same as that obtained by Albertson and Montaldo (2003) [their Eq. (7)] without the terms describing the horizontal redistribution of water and by Teuling and Troch (2005) [their Eq. (9)] who, using observationally based catchment data, attempt to understand how the covariances of soil moisture with the input and output terms act to generate or damp soil moisture variance. Not unexpectedly, the \( G \) term acts to generate, and the \( E \) and \( R \) terms act to damp soil moisture variability.

Although Eq. (5) provides a connection between soil moisture variability and the other terms in the soil moisture budget, it is a differential equation involving the rate of change of soil moisture variability. We attempt a somewhat different connection between the variability of monthly soil moisture and other moisture budget quantities. We rewrite (4) applied to daily values, represented by lower case symbols, and averaged over a month (indicated by overbars) to get

\[
\frac{w(t + n\Delta t) - w(t)}{n\Delta t} = \overline{G'} - \overline{E'} - \overline{R'}
= \overline{\Sigma'}, \tag{6}
\]

where \( n = 30 \) days and \( \Delta t = 1 \) day. Here primes indicate deviations from the long-term mean, and uppercase quantities are monthly means. Squaring both sides of Eq. (6) and averaging yields

\[
\sigma_d^2 = \frac{n^2\Delta t^2}{2(1 - r(n))} \left[ \overline{G'^2} + \overline{E'^2} + \overline{R'^2} - 2\overline{G'R'} - 2\overline{G'E'} + 2\overline{E'R'} \right], \tag{7}
\]

where \( \sigma_d^2 = \overline{w'^2} \) is the daily soil moisture variance and \( r(n) \), for \( n = 30 \), is the lag 30-day soil moisture autocorrelation. Equation (7) formally relates the variances and covariances terms of monthly averaged moisture budget quantities to the daily soil moisture variance. The daily variance of soil moisture \( \sigma_d^2 \) is related to the variance of monthly mean soil moisture \( \sigma_w^2 \) for \( W = \Sigma w(t)/n \) as

\[
\sigma_w^2 = \overline{W'^2} = \sigma_d^2 \left(1 + \frac{C}{n} \right); \quad C = 2 \sum_{\alpha=1}^{n-1} \left(1 - \frac{\alpha}{n}\right)r(\alpha), \tag{8}
\]

where \( r(\alpha) \) is lag \( \alpha \)-day autocorrelation. Then from (7) and (8) the variance of monthly mean soil moisture is related to the variances and covariances of monthly moisture budget terms as
\[
\sigma_w^2 = B \Sigma^2
\]
\[
= B(G^2 + \overline{E}^2 + \overline{R}^2 - 2\overline{G\overline{R}} - 2\overline{G\overline{E}} + 2\overline{E\overline{R}}),
\]
where \( \Sigma^2 \) involves variances and covariances as shown and
\[
B = \frac{n(1 + C)\Delta t^2}{2[1 - r(n)]}.
\]
Equation (9) symbolically relates moisture budget variance and covariance terms in \( \Sigma^2 \) and the “transfer function” \( B \) to monthly soil moisture variance \( \sigma_w^2 \). The transfer function \( B \), which has units of \( \text{day}^{-2} \), reflects a time scale that is determined by the autocorrelation structure of the daily soil moisture, and hence is a measure of soil moisture persistence. High values of soil moisture variance would be expected when values of the transfer function \( B \) and/or values of \( \Sigma^2 \) are high.

If daily soil moisture variability can be approximated as a first-order autoregressive [AR(1)] process, then \( r(a) = a^n \) where \( a \) is the lag 1-day autocorrelation. In that case
\[
C = \frac{2\alpha[n(1 - a) - (1 - a^n)]}{n(1 - a)^2},
\]
\[
B = \frac{n(1 + C)\Delta t^2}{2(1 - a^2)} = \frac{[n(1 - a^n) - 2(1 - a^n)\Delta t^2]}{2(1 - a^2)(1 - a^n)}.
\]
The “inertia” of soil moisture as measured by \( a \) is large and provides a long-term memory for the surface moisture and energy budgets. Over land the average of model values is \( a = 0.985 \), while \( a(30) = 0.60 \approx a^{10} = 0.63 \), indicating that estimating \( B \) from Eq. (11) is reasonable on average. The average value of soil moisture variance \( \langle \sigma_w^2 \rangle \) (for the entire soil depth) is 1198 mm² when estimated using Eqs. (9) and (11), with \( r(a) = a^n \), compared to value of 1152 mm² evaluated directly.

Figures 3a–f display variances and covariances of the moisture budget terms of Eq. (9). As expected, the spatial structure of the variance of the moisture input term \( G^2 \) in Fig. 3a is similar to that of precipitation (Fig. 2a) but with somewhat smaller values since all precipitation does not reach the ground because of canopy interception losses. The model evapotranspiration from the ground \( E^2 \) (which excludes evaporation of intercepted precipitation from canopy leaves) shows little spatial structure (Fig. 3b), while the spatial structure of \( R^2 \) is similar to that of \( G^2 \) with small values over areas of little runoff (including the Sahara Desert and southwest United States). Since evapotranspiration variability is comparatively weak, the associated covariance terms \( G^2E^2 \) and \( E^2R^2 \) (Figs. 3e,f) are small, and only the \( G^2R^2 \) covariance term (Fig. 3d) is important. The land-average values \( \langle G^2E^2 \rangle \) and \( \langle E^2R^2 \rangle \) are \(-0.03 \text{ and } -0.05 \text{ (mm day}^{-1})^2 \), respectively, which are an order of magnitude smaller than the \( \langle G^2R^2 \rangle \) value of 0.79 (mm day\(^{-1}\))². These variance and covariance terms scaled by \( B \) are expressed as a percentage of the soil moisture variance in terms of global averages, in Fig. 3g, and imply that Eq. (9) may be reasonably approximated retaining only the \( G \) and \( R \) terms as
\[
\sigma_w^2 = B(G^2 + \overline{R}^2 - 2\overline{G\overline{R}}) = \sigma_{w1}^2.
\]
Simulated soil moisture variance in Fig. 3h (\( \sigma_{w1}^2 \)) is compared to that approximated by Eq. (12) in Fig. 3i (\( \sigma_{w1}^2 \)), and shows that (12) applies over most of the land surface. The land average value of soil moisture variance from (12) is about 90% of the direct value. Figure 4 plots the transfer function \( B \) and \( \Sigma^2 \) as well as a measure of soil moisture persistence \( \tau \) (discussed in the next section). Figures 4 and 3h indicate that in the Tropics soil moisture variability is associated with variability in surface hydrological quantities, in particular precipitation and runoff as measured by \( \Sigma^2 \). At high latitudes, however, soil moisture variability is associated with large values of the transfer function \( B \). Conceptually, large variability in moisture budget input/runoff terms is comparatively strongly damped in the Tropics as indicated by low values of \( \tau \), while at high latitudes comparatively weaker variability in input/runoff is less strongly damped. The result is that large soil moisture variance can result in both regions.

Soil moisture persistence and the transfer function \( B \)

The transfer function \( B \) is a measure of soil moisture persistence through the autocorrelation terms in (11). Persistence is short when anomalies are strongly damped and long when they are weakly damped. There is a clear visual connection between \( B \) in Fig. 4a and soil moisture persistence \( \tau \) in Fig. 4c. Following Liu and Avisssar (1999), the soil moisture persistence \( \tau \) is measured as the average run of months with the same sign of the soil moisture anomaly. The spatial correlation between the transfer function \( B \) and soil moisture persistence is 0.72. The soil moisture persistence \( \tau \) and the transfer function \( B \) are smaller at low latitudes and larger at high latitudes. The persistence varies from \~1–2 months in the Tropics to over 8 months at high latitudes. These results are generally similar to those obtained by Yeh et al. (1984), Gao et al. (1996), and Liu and Avisssar (1999) although, because soil moisture is a derived quantity, the results are not entirely uniform. Our results differ from those of Liu and Avisssar (1999),

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Fig. 3. (a)–(f) Global plots of components of soil moisture variance for the ground moisture reservoir. (g) The contribution of moisture budget variance and covariance terms to soil moisture variance, expressed as a percentage. (h), (i) Simulated soil moisture variance, and the one approximated through Eq. (12).
for instance, over the Sahara Desert (their Fig. 4) where they find high soil moisture persistence while CLASS/AGCM3 gives lower values. Soil moisture persistence is sensitive to rainfall frequency in arid regions where the same amount of rainfall can yield different persistence values depending on its temporal distribution, so persistence values must be interpreted with caution. Comparatively frequent though small rainfall events, and the quick dissipation of resulting soil moisture anomalies due to high evaporative demand, will yield low persistence values and this is apparently the case for CLASS/AGCM3. Less frequent rainfall events, even if the total rainfall is the same, with sufficient time between events so that soil moisture stays at a constant low (dry) value for long periods will yield long persistence times. The CLASS/AGCM3 results are consistent with the latitudinal dependence of soil moisture persistence on potential evaporation (Delworth and Manabe 1988). At low latitudes evaporation rates are high and soil moisture anomalies are quickly damped, in comparison to high latitudes. Koster and Suarez (2001) point out that the autocorrelation structure of soil moisture in climate models is also related to seasonality in the variance of atmospheric forcing, the dependence of evaporation and runoff on soil moisture, and correlation between the atmospheric forcing and antecedent soil moisture conditions, as perhaps induced by land–atmosphere feedbacks.

We attempt to relate the transfer function $B$ to soil moisture persistence assuming that soil moisture variability can be approximated by an AR(1) process. For a white noise time series, von Storch and Zwiers (1999) derive the expression $P(l) = 2^{-l(1+1)}$ for the probability of a run (i.e., a sequence of consecutive elements with the same sign) of length $l$. This formula does not apply to nonwhite AR(1) processes, however, and they estimate the percentage of runs of length $l$ using Monte Carlo methods (their Fig. 10.2). We repeat the experiment for different values of the lag-1 autocorrelation and empirically fit the function

$$P(l) = \theta e^{-\theta l} + (1 - \theta) e^{-\beta l} \quad \text{for}$$

$$\theta = 0.63 - 0.53 A, \quad \beta = 1.5$$

(13)

to the distribution of run lengths, where $A = r(1)$ is the lag-1 autocorrelation. Figure 5 shows the distribution of run lengths obtained from a Monte Carlo experiment with 200,000 realizations (shown as symbols) and the empirical function from (13) (shown as lines). Equation (13) gives a very reasonable fit for $r(1) \leq 0.6$. The expected average run length, which is measure of the soil moisture persistence time scale $\tau$, is obtained from (13) as

$$\bar{\tau}(A) = \int_0^\infty P(l) dl = \frac{1}{\theta} + \frac{(1 - \theta)}{\beta^2}.$$  

(14)

The lag-1 autocorrelation $A$ of monthly means is related to the lag-1 autocorrelation $a$ of daily values as

$$A = \frac{a(1 - d^2)}{n(1 - a^2) - 2a(1 - a^d)},$$

(15)

as shown in the appendix. Taken together, Eqs. (11), (14), and (15) yield a functional relationship between the soil moisture persistence time scale $\tau$ and the trans-
fer function $B$ for an AR(1) process. This relationship is the curve plotted in Fig. 6. Actual values of $B$ and $\tau$ from the AGCM3 simulation are plotted as diamonds. Longer persistence time scales of soil moisture anomalies are related to larger values of the transfer function, although not linearly, and the first-order relationship can be approximated assuming an AR(1) process. Soil moisture variability is approximated as an AR(1) process in Eqs. (10) and (11) and, although this approximation works well in a global average sense, this may not be the case for individual grid cells, leading to the scatter in Fig. 6. The autocorrelation of soil moisture depends on the manner in which evaporation and runoff are parameterized as functions of soil moisture (Koster and Suarez 2001) and this is not explicitly accounted for in Eq. (11). The fitted empirical relationship between probability of run length $l$ and lag-1 autocorrelation in Eq. (13), which is used to estimate persistence ($\tau$), is also a cause of scatter in Fig. 6.

5. Summary and discussion

Soil moisture and its variability play an important role in atmospheric anomalies that can persist for long periods. The role of potential evaporation (Delworth and Manabe 1988), the variation in evaporation and runoff with soil moisture, and land–atmosphere feedbacks in affecting soil moisture autocorrelation structure and thus its persistence has been highlighted in earlier studies (e.g., Koster and Suarez 2001). Here, we link soil moisture variance to the second-order statistics of precipitation, evapotranspiration, and runoff and evaluate terms using monthly data from an AMIP2 simulation made with the CCCma third-generation atmospheric general circulation model.

The variance of monthly precipitation simulated in the model is compared with observationally based estimates from Xie and Arkin (1996) and the variance of evapotranspiration is compared to ECMWF reanalysis results. Simulated values compare reasonably well with the observationally based estimates. In the absence of
observed gridded soil moisture values, we compare model soil moisture variance with estimates from the VIC-2L hydrological model driven with observationally based meteorological data. AGCM3/CLASS soil moisture variance is qualitatively similar to that of VIC-2L although values are generally somewhat larger. Simulated evapotranspiration variance is small compared to that of precipitation and runoff and soil moisture variance depends primarily on the precipitation and runoff terms. This approximation is acceptable over almost the entire land surface area and accounts for about 90% of the total soil moisture variance.

Soil moisture variance is linked to the variance and covariance of source/sink terms in the moisture budget via a transfer function. In this view, soil moisture variance is connected to the variability of hydrologic quantities and the persistence of soil moisture anomalies. The transfer function is a measure of the persistence of soil moisture anomalies via lagged autocorrelation terms. An expression linking the transfer function and the persistence time scale of soil moisture anomalies is developed, assuming soil moisture variability can be approximated by an AR(1) process.

Model results show that the soil moisture persistence time scale and the transfer function are smaller in the Tropics and larger at high latitudes, as expected from the dependence on evapotranspiration. In the Tropics, damping is comparatively strong and the persistence of soil moisture anomalies is short and values of the transfer function small. Nevertheless, high values of soil moisture variance may result because of high precipitation variability. At high latitudes, characterized by low variability of precipitation and runoff, high soil moisture variability is linked to comparatively weak damping and long persistence time of soil moisture anomalies. Thus, large soil moisture variability can result in both regions but for different reasons.

These results have some implications for soil moisture initialization and the role of soil moisture memory for seasonal climate prediction. Despite the high precipitation variability in the Tropics that creates large soil moisture anomalies, the effect on seasonal climate is likely not pronounced because anomalies are dissipated quickly by high evapotranspiration and runoff (Koster and Suarez 2001). In mid- and high-latitude regions, although the soil moisture anomalies are smaller because of low precipitation variability, their effect on seasonal climate may be more pronounced because of their longer dissipation time scales. Here, low evapotranspiration and runoff rates and soil moisture freezing in winter cause soil moisture anomalies to last longer.

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APPENDIX

Lag-1 Autocorrelation of Monthly Means Related to Daily Values

Derivation of lag 1-month autocorrelation $A$ as a function of lag-1 day autocorrelation $r$ for an AR(1) process is given as

$$A = \frac{W_r W_{r+1}}{\sigma_w}, \quad (A1)$$

$$A = \frac{1}{n^2} \sum_{a, b} a^{a+b} \sigma_{daily}^2 = \frac{1}{n^2} \frac{a(1-a)^2}{(1-a)^2} \sigma_{daily}^2, \quad (A2)$$

$$A = \frac{1}{n^2} \frac{a(1-a)^2}{(1-a)^2} \sigma_{daily}^2 = \frac{1}{n^2} \frac{a(1-a)^2}{(1-a)^2} + \frac{n}{1+C}. \quad (A3)$$

Substitution of expression for $C$ [Eq. (10)] in Eq. (A3) yields

$$A = \frac{a(1-a)^2}{n(1-a)^2 - 2a(1-a)}. \quad (A4)$$

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