A Linear Stability Analysis of Coupled Tropical Atlantic Variability

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ABSTRACT

A linear stability analysis of an intermediate coupled ocean–atmosphere model reveals that the tropical Atlantic has two types of coupled modes: a meridional mode at the decadal time scale and a zonal mode at the interannual time scale. The meridional mode, which manifests itself as an interhemispheric SST fluctuation, is controlled by the thermodynamical feedback between winds, latent heat flux, and SST, further modified by ocean heat transport. The zonal mode, which manifests itself as an SST fluctuation in the eastern equatorial basin, is dominated by the dynamical feedback between winds, thermocline, upwelling, and SST. The relative strength of thermodynamical versus dynamical feedback determines the behavior of the coupled system. When the thermodynamical feedback dominates, the meridional mode is the leading coupled mode; when the dynamical feedback dominates, the zonal mode leads all other coupled modes. Interestingly, a nonoscillatory regime exists for the leading mode when both feedbacks are comparable in strength, suggesting a destructive interference between the meridional and zonal modes.

1. Introduction

Various studies of the tropical Atlantic variability (TAV) show that two modes of variability, a meridional and a zonal mode, dominate the fluctuations at interannual-to-decadal time scales (see Xie and Carton 2004; Kushner et al. 2006; Chang et al. 2006, and references therein). Both modes have significant impact on the strength and position of the intertropical convergence zone (ITCZ), hence affecting the rainfall over the surrounding continents.

The meridional mode, featuring a dipole pattern in precipitation and an anomalous interhemispheric SST gradient, is believed to be a coupled mode intrinsic to the tropical Atlantic (Servain 1991; Mehta and Delworth 1995; Nobre and Shukla 1996; Tourre et al. 1999; Servain et al. 2000; Sutton et al. 2000; Chang et al. 2000, 2001). It is most prominent in boreal spring when the ITCZ moves farthest south. The mechanism involves the air–sea coupling between the interhemispheric SST gradient and the cross-equatorial winds, such that an anomalous SST gradient drives a meridional wind anomaly, weakening (strengthening) the trade winds in the warmer (colder) hemisphere, which in turn strengthens the initial SST gradient by altering the latent heat fluxes (Carton et al. 1996; Chang et al. 1997). This positive feedback, often referred to as the wind–evaporation–SST (WES) mechanism, has been demonstrated to work at least within simple coupled climate models (Xie and Philander 1994; Xie 1999; Chang et al. 2001; Kushner et al. 2002) and in atmospheric general circulation model (AGCM) simulations (Chang et al. 2000). In this study, we simply refer to the WES feedback as the thermodynamical feedback (Chang et al. 2006).

The zonal mode, featuring a series of warm and cold events in the eastern equatorial ocean, is regarded as the Atlantic counterpart of the Pacific El Niño–Southern Oscillation (Zebiak 1993). It has maximum amplitude during boreal summer when the ITCZ moves farthest north (Carton et al. 1996; Chang et al. 2000; Ruiz-Barradas et al. 2000; Sutton et al. 2000). The underlying mechanism is thought to be the Bjerknes feedback (Bjerknes 1969) in which the relaxation of the trade winds, in response to a warm SST anomaly in the east, decreases the equatorial upwelling and weakens the west-to-east thermocline slope, causing further
warming in the eastern equatorial ocean. This positive feedback between winds, thermocline, and SST is considered as a dynamical feedback because it involves dynamical adjustment of the upper ocean to changes in the winds (Chang et al. 2006).

However, analyses of observations show that the variability of the tropical North and South Atlantic are uncorrelated to a large extent (Mehta 1998; Enfield et al. 1999; Dommenget and Latif 2000; Häkkinen and Mo 2002; Baquero-Bernal et al. 2002), raising the question whether the meridional mode is a true physical identity or a statistical artifact. Furthermore, the two modes of variability demonstrate a certain degree of correlation after the 1970s climate shift (Servain et al. 1999, 2000; Murtugudde et al. 2001). It is yet to be physically determined whether this correlation truly indicates the interaction between coupled modes or merely reflects the influence of a common trigger.

To shed light on these questions, we perform a linear stability analysis of the tropical Atlantic ocean–atmosphere system represented by a simple coupled model. The procedure is very similar to the stability studies carried out largely for the Pacific ENSO (e.g., Hirst 1986, 1988; Neelin 1991; Jin and Neelin 1993; Thompson and Battisti 2000, 2001; Fedorov and Philander 2001; An et al. 2004). This type of analysis has proven instrumental in our understanding of the physics of ENSO. It demonstrates that in the tropical Pacific dynamical coupling between the ocean and atmosphere can generate a breed of new modes whose characteristics depend on the strength of air–sea interaction and the adjustment time of the tropical ocean. The well-known delayed oscillator of ENSO belongs to this family of coupled modes (Suarez and Schopf 1988; Battisti and Hirst 1989).

In the tropical Atlantic, however, a systematic linear stability analysis for both dynamical and thermodynamical feedbacks is still lacking, even though direct modal analysis has been sought before (Xie et al. 1999).

The objective of this paper is to fill this gap in TAV studies. We first construct a linear coupled model of the tropical Atlantic ocean–atmosphere in section 2, then seek the physical modes through a linear eigenanalysis in section 3, and further investigate the sensitivity of the leading modes to changes in the relative strength of thermodynamical and dynamical feedbacks in section 4. Finally, we present our discussion and conclusions in section 5.

2. An intermediate model

The model used here is a modified version of the intermediate coupled model (ICM) developed by Chang (1994) and Wang et al. (1995), which consists of the linear reduced-gravity model of thermocline dynamics, the mixed layer thermodynamics, and the empirical atmospheric model. Similar models have been applied to study TAV at interannual-to-decadal time scales with reasonable performance (Zebiak 1993; Chang et al. 1997). For reference, the model equations are given in appendix A.

a. Linear coupled model of the tropical Atlantic

The original ICM is a nonlinear model. To determine the stability and identify the physical modes, we first linearize the ICM around an appropriate mean state. To be consistent, we set the mean state to be the steady solution of the ocean model forced by observed mean winds and heat flux. In other words, we turn off the coupling and weather noise in (A4) and then integrate the model until it reaches a steady state \((\mathbf{u}, \mathbf{w}, \mathbf{T})\). In this way, the major features of observed mean SST and surface currents are fairly reproduced by the ICM, such as the warm water residing in the eastern ocean and the upwelling/entrainment concentrating along the equator. While the primary model mean bias is the lack of the season cycle, our sensitivity tests (e.g., perturbing the mean fields \(\mathbf{u}, \mathbf{w}, \mathbf{T}, H, H_{\text{mix}}\) respectively) indicate that including the annual cycle in the mean ocean state will unlikely change our results fundamentally. As a first-order approximation, we will linearize the ICM around the annual mean state of the tropical Atlantic and only consider the effect of seasonality in modulating the ocean–atmosphere coupling strength (see section 4c).

Decomposing SST \((T)\) and ocean current \(\mathbf{u}\) into a mean and perturbation, that is, \(T = \bar{T} + T', \mathbf{u} = \bar{\mathbf{u}} + \mathbf{u}'\), and assuming that perturbations are small and higher-order terms are negligible, the linearized form of Eqs. (A1) and (A3) can be written as

\[
\begin{align*}
\frac{\partial T'}{\partial t} &= \mathcal{L}_1 T' - \mathcal{L}_2 \cdot \mathbf{u}' + \mathcal{L}_3 h' + \frac{\eta}{\rho_0 C_H H_{\text{mix}}} - \frac{\mathbf{E} \cdot \mathbf{k}}{\rho_0 H} \\
\frac{\partial \mathbf{u}'}{\partial t} &= \frac{\mu}{\rho_0 H} \nabla^2 T' + (\gamma \nabla^2 - \alpha - f \mathbf{k} \times ) \mathbf{u}' - g' \mathbf{v} h' + \frac{\xi}{\rho_0 H} \\
\frac{\partial h'}{\partial t} &= -H \nabla \cdot \mathbf{u}',
\end{align*}
\]

where \(\mathbf{k}\) is the unit vector along the vertical axis (see appendix A for the notation of variables and parameters). Note that the Ekman dynamics (A2) and air–sea feedback (A4) are implicitly built into the linear model (1), which consists of four dependent variables \((T', u', v', h')\) and four equations. All operators \(\mathcal{L}_i\) are linear and time invariant:
\[ \mathcal{L}_1 = \kappa \nabla^2 - \lambda - \vec{u} \cdot \nabla - \frac{0.5}{H_{\text{mix}}} \overline{w_0} \mathcal{H}(\overline{w_0}) + \frac{\nu}{\rho_0 C_p H_{\text{mix}}} \overline{\mathcal{B}} - \frac{\mu}{\rho_0 H} \mathcal{E} \]

\[ \mathcal{L}_2 = \nabla \mathbf{T} + 0.5 \mathcal{H}(\overline{w_0})(\overline{T} - T_{50}) \nabla \]

\[ \mathcal{L}_3 = \frac{2}{H_{\text{mix}}} \overline{w_0} \mathcal{H}(\overline{w_0}) \frac{\partial T_{50}}{\partial \mathcal{E}} \]

where operator \( \mathcal{E} \) relates to Ekman divergence and Ekman drift

\[ \mathcal{E} = \frac{H - H_{\text{mix}}}{H_{\text{mix}}} \frac{1}{r_s^2 + f^2} \left( \nabla \cdot \left( r_s \mathbf{v} \times \mathbf{k} \right) + \frac{g(r_s \overline{w_0})}{2} (\overline{T} - T_{50}) \left( \left( r_s \mathbf{v} + \frac{2f \beta_0}{r_s^2 + f^2} (r_s \mathbf{j} - f \mathbf{i}) - \beta_0 \mathbf{i} \right) \cdot -f \mathbf{v} \cdot \mathbf{k} \times \right) \right). \]

Two nondimensional parameters are introduced: \( \mu \), the relative strength of anomalous wind stress, and \( \nu \), the relative strength of anomalous heat flux. From (A4), it can be seen that they, in fact, control the sensitivity of the atmosphere to SST anomaly, with \( \mu \) measuring the intensity of wind–SST feedback and \( \nu \) the heat flux–SST feedback, respectively. Hereafter, we refer to \( \mu \) as the dynamical coupling parameter because the winds affect SST through currents and the thermocline, whose changes involve dynamical ocean adjustment, and \( \nu \) as the thermodynamical coupling parameter because surface heat flux directly changes SST through thermodynamical processes. For the sake of convenience, we normalize the coupling parameters \( \mu \) and \( \nu \) such that \( \mu = 1 \) and \( \nu = 1 \) correspond to the air–sea feedback strengths that best fit the observation. Naturally, setting \( \mu \) to zero means no coupling between wind stress and SST, and setting \( \nu \) to zero means no coupling between heat flux and SST. In the ICM, we further assume that, compared to the influence of zonal winds, the effect of meridional winds on dynamical coupling can be neglected, while its effect on thermodynamical coupling is incorporated in the wind-induced latent heat flux.

**b. Empirical model for wind and heat flux**

In the coupled model (1), the SST-forced surface wind stress and heat flux are determined by the atmosphere model \( \mathcal{A} \) and \( \mathcal{B} \) that yet have to be specified. Although simplified dynamical models for surface winds (Gill 1980; Lindzen and Nigam 1987) and thermodynamical models for surface heat flux (Seager et al. 1988, 1995) have been used widely in theoretical ENSO studies, these models may not gain the same success in the tropical Atlantic as in the tropical Pacific. Chung et al. (2002) compared the performance of the simple dynamical models with that of an empirical model based on rotated principal component analysis of SST and surface winds in the tropical Atlantic. Interestingly, they found that the empirical model generally outperforms the dynamical models in modeling the meridional and zonal winds across the equatorial Atlantic sector. The outperformance is even more striking outside the equatorial region where the empirical model does a fair job while the simple dynamical model performs poorly. One reason for such disparity is that the extratropical atmospheric response to the tropical Atlantic SST anomalies is largely barotropic (Okumura et al. 2001), while the simple dynamical models are formulated to only consider the baroclinic components and boundary processes.

In this study, we follow the approach of Chang et al. (1997, 2001) to construct an empirical atmospheric model based on singular value decomposition (SVD) of SST, wind stress, and heat flux fields. The basic idea is to isolate patterns of covariability between the ocean and atmosphere by maximizing the covariance; then the atmospheric response to an SST anomaly would be given by a linear combination of these covarying patterns weighted by the SST projection coefficient time constants [see Bretherton et al. (1992) and Syu and Neelin (1995), for details; a technical description is also given in appendix B]. By default, such an SVD analysis does not distinguish between SST–force–atmosphere and atmosphere–force–SST scenarios; therefore, further tests are needed to confirm the identified covariability as coupled phenomena (Frankignoul 1999). In the tropical Atlantic, observational analyses give conflicting indications about the existence of the positive WES feedbacks. Simultaneous correlations show that the surface latent heat flux indeed provides positive feedback in the tropical Atlantic sector (Ruiz-Barradas et al. 2000, 2003; Tanimoto and Xie 2002), whereas the lagged covariance of monthly data indicates that the heat flux feedback is largely negative except in the deep tropics of the North Atlantic (Frankignoul and Kestenare 2005). As the time scale of the atmospheric adjustment of the SST anomalies is on the order of weeks rather than months or is instantaneous, the former (latter) analysis tends to overestimate (underestimate) the
positive air–sea coupling. Meanwhile, several modeling studies all suggest a robust atmospheric response to prescribed SST anomalies but with positive WES feedback either confined to the deep tropics (Chang et al. 2000) or extended well into the extratropics (Okumura et al. 2001). In view of the modeling discrepancies and the lack of weekly heat flux observations, the empirically derived atmospheric model of Chang et al. (2001) appears to a good choice for present study.

Now, the linear coupled model (1) can be put in a nominal form

$$\frac{dx}{dt} = Ax + f,$$  \hfill (3)

with the state variable $x = (T', u', v', h')^T$. The linear operator $A$ incorporates all the coupled ocean–atmosphere dynamics and thermodynamics of the tropical Atlantic basin. The forcing term $f$ denotes the external components of surface wind and heat flux, that is, the components that are not directly driven by the tropical Atlantic SST variations but are remotely generated by the processes in other tropical oceans or mid-latitudes. ENSO and the North Atlantic Oscillation (NAO) are the two main candidates for external forcing, and will be further discussed in the companion paper. The present paper mainly focuses on the coupled stability of the tropical Atlantic, which is solely determined by $A$ in (3).

c. Numerical details

The differential operator $A$ is cast into a matrix form by finite differencing on a staggered C grid with a resolution of $\Delta x = 2^\circ$, $\Delta y = 1^\circ$ within the tropical Atlantic basin from 20$^\circ$S to 20$^\circ$N, which gives a dimension of 3462 $\times$ 3462. The boundary conditions are no flux for SST and the thermocline, no normal for flow, and free slip for ocean currents. For the thermodynamical equation $1^\circ$ sponge layers are added along the southern and northern boundaries, while for dynamical equations $10^\circ$ sponge layers are added. The observations are taken from the monthly Comprehensive Ocean–Atmosphere Dataset (COADS) (da Silva et al. 1994) for 1950–90 with $2^\circ \times 2^\circ$ resolution and projected onto the model grid by simple meridional interpolation.

In our statistical atmosphere, the SST-forced component is composed of the first three singular vectors, which explain 27% of the variance. The remaining variance is the atmospheric variability independent of the tropical Atlantic SST. This, in general, agrees well with the estimate of Sutton et al. (2000) for the tropical Atlantic atmosphere: external variability dominates the fluctuations of the wind stress and heat flux while SST-forced variance is low (30%). Finally, all computations are made possible by the FORTRAN library, LAPACK (Anderson et al. 2000).

3. Coupled ocean–atmosphere modes in the tropical Atlantic

For the linear system (3), eigenmodes of the operator $A$, in which the boundary conditions are embedded, represent the true physical modes of the tropical Atlantic ocean–atmosphere system. To ease the discussion, we adopt the common usage here: all modes are sorted in ascending order in terms of their damping rate; that is, the first mode is the least damped (most unstable) mode. A complex conjugate pair of eigenvalues/vectors, which form an oscillatory mode, is regarded as possessing two degrees of freedom. Real eigenvalues/vectors are referred to as stationary modes that may grow or decay exponentially. Furthermore, an oscillatory mode may split up into two stationary modes, indicating degeneracy in parameter space.

a. Free modes

Without coupling ($\mu = \nu = 0$), (1) has two sets of eigenmodes: SST modes with no dynamical components and dynamical modes with passive SST components, which represent the free modes of the SST equation and the shallow-water equation, respectively.

The SST modes are the eigenfunctions of operator $L_1$ defined in (2), which are determined by the advection and dissipation processes (Neelin 1991; Jin and Neelin 1993; Neelin and Jin 1993). In the case of the subtropical gyre, the modes typically contain a set of purely damped modes and a set of propagating modes with phase speed $U$ and period $T \sim L/U$, where $L$ and $U$ are the characteristic length and velocity scales of ocean circulation (Wang and Chang 2004). However, the equatorial current system of the Atlantic is too complex to form a closed loop. Therefore, the eigenspectrum of these SST modes is now a continuum rather than a discrete set.

For an unlimited basin, the free modes of the shallow-water equation are the equatorial Kelvin, Rossby and inertia–gravity waves (see Philander 1990). While short high-frequency waves dissipate very quickly in the ocean once generated, the gravest baroclinic waves could survive a long time to exert influence on interannual variability. For a zonally bounded basin, however, free Kelvin and Rossby waves are no longer the eigen-solutions. Instead, a new set of eigenmodes appears, known as the basin modes (Cane and Moore 1981; Neelin and Jin 1993; Jin 2001). Such modes are composed of
the Kelvin and long Rossby waves with periods determined by the sum of the basin-crossing time, which is about 4 months in the tropical Atlantic (Fig. 1). In these modes, the SST variations are merely passive responses to the thermocline and advection changes, realized through $L_2$ and $L_3$ in (1).

Although none of those free modes directly explains the observed interannual-to-decadal variability of the tropical Atlantic, they are of relevance to the formation of the meridional and zonal modes, especially when the active air–sea feedbacks are weak (see section 4b).

b. Coupled ocean–atmosphere modes

It is well known that when the atmosphere and ocean are coupled through energy and momentum transfer, new modes emerge from the large-scale air–sea interactions (e.g., Philander 1990). In previous coupled stability analysis relevant to ENSO, primary attention is paid to the dynamical coupled modes. For TAV, thermodynamical and dynamical coupling are equally important. Therefore, we will first analyze their characteristics separately and then discuss how they may interact.

1) Pure thermodynamical coupled mode

Without dynamical coupling, that is, setting $\mu = 0$, the leading modes are pure SST modes in the sense that there is no corresponding signal in the $u$, $v$, and $h$ fields. For a realistic thermodynamical coupling strength $\nu = 1$, the first mode is an oscillatory mode with a period of 25 yr, an $e$-folding time of 10 months, centers of action at 15°S and 15°N, and a noticeable southward propagation (Fig. 2). The main feature of this decadal oscillation resembles the meridional mode described in the literature (Carton et al. 1996; Nobre and Shukla 1996; Chang et al. 1997, 2000, 2001; Sutton et al. 2000; Ruiz-Barradas et al. 2000). By eliminating the dynamical terms one by one from the SST equation in (1), we further show that this decadal mode originates from the thermodynamical air–sea feedback. In other words, the thermodynamical coupling $B$ itself supports a similar oscillatory mode but with a prominent southward propagation and a shorter period ($\approx 10$ yr). A comparison with the mode in Fig. 2 reveals that ocean advection (northward on average) slows the southward propagation and enables a longer period oscillation ($\approx 25$ yr). This observation, in general, agrees with the previous study on the role of ocean advection in sustaining the period of the meridional mode (Chang et al. 1997; Xie 1999).

Since the decadal mode is identified by applying SVD analysis to COADS observations, one may argue that the influence of ocean dynamics has been prebuilt into the system; hence one may wonder whether the meridional mode is truly generated by the thermodynamical feedback. To address this issue, we turn to an AGCM experiment in which the Community Climate Model version 3 (CCM3) (Kiehl et al. 1998) is coupled to a slab ocean, hereafter CCM3-ML. The only possible feedback in such an experiment is the thermodynamical feedback: heat flux changes SST, which in turn perturbs atmosphere circulation and then changes heat flux.

The same SVD analysis is applied to a 100-yr CCM3-
ML run. The dominant coupled mode, using the first 10 singular vectors, depicts a 4-yr westward and equatorward propagating oscillation with larger loading in the Northern Hemisphere (Fig. 3). To explain physically why such an oscillation may exist in a coupled slab ocean–atmosphere system, we invoke the following WES argument. For a warm SST anomaly in the tropical North Atlantic, the atmosphere responds with an anomalous westerly to the west and an anomalous easterly to the east that, with the mean easterly trade winds, cause less evaporation on the west side and more evaporation on the east side. Hence, the induced change of latent heat flux acts to move the initial SST anomaly westward, whose speed and period depend on the strength of WES feedback and the phase difference between SST and wind anomalies. The same argument

FIG. 2. First normal mode of tropical Atlantic for realistic thermodynamical coupling ($\nu = 1$) and zero dynamical coupling ($\mu = 0$): (a) phase 0, (b) phase $\pi/4$, (c) phase $\pi/2$, and (d) phase $3\pi/4$. Positive SST anomalies are indicated with solid lines, and negative anomalies are indicated with dashed lines. The contour interval is 0.2°C.

FIG. 3. As in Fig. 2, but for the thermodynamical coupled mode in the CCM3 slab ocean run. The contour interval is 0.1°C.
can be used to explain the equatorward propagation as well. Experiments with a Gill-type atmosphere coupled to a slab ocean also exhibit similar westward propagating SST modes (Xie 1996; Zhou and Carton 1998), lending support to the above hypothesis.

2) Pure Dynamical Coupled Mode

Without thermodynamical coupling, our linear model (1) has similar dynamics to that of ENSO (Cane and Zebiak 1985; Zebiak and Cane 1987). For a realistic coupling parameter \( \mu = 1 \) of the tropical Atlantic, it produces an equatorial mode with a period of 3 yr and an e-folding time of 6 months. Figure 4 shows the half-period evolution of SST and thermocline anomalies associated with this mode. Phase 0 represents the structure of a mature cold episode; phase \( \pi \) the warm episode. The cold SST anomaly extends to 30°W with a maximum located near the eastern boundary. We also see the slight eastward migration of positive anomalies from the western region during retreat of the cold event and rapid westward migration from the eastern region during growth of warm event. On the other hand, the maximum thermocline variability (about 8 m) occurs along the western boundary, while the eastern tropical Atlantic experiences a weaker thermocline perturbation (less than 4 m). However, because of the shallow mean thermocline over the eastern equatorial Atlantic, a relatively small perturbation in thermocline can induce a significant change in SST via modulation of subsurface upwelling.

In summary, the tropical Atlantic ocean–atmosphere system contains two sets of coupled modes: a meridional mode of decadal time scale and a zonal mode of interannual time scale. The former is caused by the thermodynamical feedback between SST and heat flux,
while the latter is due to the dynamical feedback between winds, SST, and subsurface ocean dynamics. In reality, these two modes coexist; thus they may interact as illustrated in the next section.

4. Interference between the zonal and the meridional modes

a. Modal interference

Setting both thermodynamical and dynamical coupling parameters $\mu = 1$ and $\nu = 1$, the structures of the first two leading modes are shown in Figs. 5 and 6. The least damped mode has meridional appearance with a period of 24.6 yr and $e$-folding time of 10 months (Fig. 5); the second least-damped mode has zonal appearance with a period of 3.4 yr and $e$-folding time of 4 months (Fig. 6). By comparing with the pure thermodynamical mode (Fig. 2) and pure dynamical mode (Fig. 4), we conclude that they are basically the same mode but with modified features. Now, the meridional mode is more confined against the western boundary, while noticeable SST (thermocline) variability appears in the eastern (western) equatorial Atlantic. The zonal mode spreads more meridionally such that its influence reaches beyond 10°N and 10°S. In other words, the meridional and zonal modes are no longer orthogonal; they project onto each other strongly, which is an indication of nonnormality (Farrell and Ioannou 1996). With simple models, Chang et al. (2004a,b) have shown that the ocean–atmosphere coupling, indeed, makes the system nonnormal and enables the mode–mode connection.

![Fig. 5. Meridional mode of tropical Atlantic for realistic thermodynamical and dynamical coupling ($\mu = 1$ and $\nu = 1$): evolution of (left) SST anomalies and (right) thermocline and current anomalies through one-half cycle. Positive anomalies are indicated with solid lines, and negative anomalies are indicated with dashed lines. Contour intervals are 0.2°C for SST and 1 m for thermocline; arrows are ocean currents. Note the left and right columns are scaled differently in latitude.](image-url)
To further illustrate the interaction between the two coupled modes of the tropical Atlantic, we test their sensitivity to changes in the coupling strength. Figures 7 and 8 show the eigenvalues of leading modes as a function of thermodynamical feedback \( \nu \) and dynamical feedback \( \mu \), respectively. As we can see, the two feedbacks have contrasting effects on the strength of the two coupled modes. For example, increasing the thermodynamical feedback while keeping the dynamical feedback constant will make the meridional mode less stable and the zonal mode more stable (Fig. 7). On the other hand, increasing the dynamical feedback while keeping the thermodynamical feedback constant will lead the zonal modes toward instability, but will have little effect on the meridional mode (Fig. 8). Furthermore, the zonal mode interferes with the meridional mode destructively when they are of similar strength (\( \nu = 0.6 \) in Fig. 7 and \( \mu = 2 \) in Fig. 8). Namely, the oscillatory meridional mode degenerates into two stationary modes of distinct spatial patterns: one resembles a meridional dipole and the other a monopole in the North/South Atlantic (not shown), once the zonal mode starts becoming important. Such mode interference surely has an impact on the variability and predictability of the tropical Atlantic, as will be shown in the companion paper.

Here, we offer a pragmatic explanation for the destructive mode interference. For the meridional mode (Fig. 5), the interhemispheric SST gradient induces cross-equator winds with a strong zonal component, which in turn drives large anomalies in thermocline depth and ocean current in the western tropical Atlantic. As a result, the dynamical adjustment of the western equatorial Atlantic correlates less with the eastern equatorial SST, indicating suppression (but not breakdown) of the dynamical feedback loop along the equator. This is manifested in the damping effect of the thermodynamical coupling on the zonal mode (Fig. 7). For the zonal mode (Fig. 6), the influence of equatorial dynamics extends beyond the tropics, causing large SST...
anomalies around 10°S and 10°N. If the zonal mode is strong enough, the SST anomalies in the deep tropics decouples from the subtropics, meaning that the equatorward propagation of the meridional mode is broken by the zonal mode: hence, the oscillatory meridional mode degenerates into two stationary modes (Figs. 7 and 8).

b. An exploration of coupling parameter space

The mode interaction basically reflects the interface between the thermodynamical and dynamical feedback in the tropical Atlantic. An exploration of parameter space would tell us how these feedbacks jointly determine the intrinsic properties of the tropical ocean–atmosphere system. Figure 9 shows the frequency and growth rate of the leading mode as a function of coupling strength \( \mu \) and \( \nu \) while the other parameters are kept constant. As expected, strong coupling usually leads to large growth and eventually the system becomes unstable (Fig. 9a). However, such transition is not of particular interest here because the tropical Atlantic variability is regarded as stable in reality. More useful information can be extracted by dividing the parameter space into three regimes according to the oscillating period of the leading mode (Fig. 9b): 1) the thermodynamical regime in which thermodynamical feedback dominates and the leading mode is a decadal oscillation manifested in the form of meridional mode, 2) the dynamical regime in which dynamical feedback dominates and the leading mode is an interannual oscillation manifested in the form of zonal mode, and 3) the mixed regime in which two feedbacks are of comparable strength and the leading mode is nonoscillatory.

One interesting observation of Fig. 9 is the selective damping effect of the positive air–sea feedback just discussed. In the thermodynamical regime, the frequency and growth rate of the leading mode increase linearly with the thermodynamical coupling strength but decrease, albeit slightly, with the dynamical coupling strength. In the dynamical regime, the growth rate of the leading mode linearly increases with the dynamical coupling but decreases with the thermodynamical coupling. Most importantly, a destructive consequence occurs during the transition from the thermodynamical regime to the dynamical regime; that is, the decadal oscillation degenerates into two nonoscillatory modes of distinct meridional structure. What happens in this mixed regime, we think, is that the zonal mode induces SST anomalies of same sign in the North and South Atlantic, which in turn weakens the positive thermodynamical feedback associated with the meridional mode (see section 4a for more discussion). As a result, the meridional mode loses its oscillatory characteristics and becomes two stationary modes.

It is also worth noting that the zonal mode tends to conserve its period, which is in sharp contrast to the meridional mode. This implies that the period of the zonal mode is not determined by dynamical feedback strength but is a result of balancing various equatorial
processes (Cane and Zebiak 1985; Zebiak and Cane 1987). Zebiak (1993) demonstrated that the difference in mean current and ocean stratification of the equatorial Pacific and Atlantic offsets the difference in basin size, allowing a similar oscillation of 3-yr period in each ocean.

Furthermore, when both dynamical and thermodynamical feedbacks are weak (e.g., $\mu < 0.5$ and $\nu < 0.5$), the background modes of section 3a become the leading modes with the growth rate largely determined by the linear damping $\lambda$. In this parameter regime, it is convenient to think that the role of the thermodynamical coupling is to give strength to the free SST mode of the meridional pattern and the dynamical coupling is to modify the free basin mode.

Assuming that the realistic coupling strength is around $\mu = 1$ and $\nu = 1$, we conclude from Fig. 9 that the tropical Atlantic ocean–atmosphere system lies in the stable regime, with a decadal meridional oscillation as the first leading mode and an interannual zonal oscillation as the second, even though a slight change in the coupling strength may cause the zonal mode to break the meridional mode and subsequently become the first mode.

c. Seasonal variations in the coupling strength

Until this stage our analysis mainly focuses on the stability of the annual mean states. In the tropical Atlantic, however, variability displays a pronounced seasonal cycle with the meridional (zonal) mode peaking in boreal spring (summer). To explain the different phase locking, seasonal variations in the thermodynamical and dynamical coupling are now described.

In our model, the coupling strength is manifested in the sensitivity of heat flux to SST $\nu$ and the sensitivity of wind stress to SST $\mu$, which can be estimated from the observation through linear regression as in Keenlyside and Latif (2007). The observation data that we used is the Southampton Oceanography Centre (SOC) flux analysis derived from COADS, which includes the $1^\circ$ monthly SST, wind stress, and heat flux from January 1980 to December 2005 (Josey et al. 1998, 1999). The trend and annual cycle are first removed from the data; then the regression coefficients are estimated via a least squares method using simultaneous correlations (lagged correlations would give similar results). Figure 10 shows the regression values as a function of calendar month. As we can see, the response of the latent heat flux to the underlying SST is strongest during February–March in both the North ($3^\circ$–$20^\circ$N, $50^\circ$–$20^\circ$W) and South ($3^\circ$–$20^\circ$S, $30^\circ$–$0^\circ$W) Atlantic (Fig. 10a), whereas the response of the western ($3^\circ$S–$3^\circ$N, $40^\circ$–$20^\circ$W) zonal winds to the eastern ($3^\circ$S–$3^\circ$N, $20^\circ$W–$10^\circ$E) Atlantic SST is strongest during April–May (Fig. 10b). Furthermore, the regression indicates that the heat flux–SST feedback is positive in the North Atlantic and negative in the South Atlantic, creating a thermal gradient feedback that is only positive in boreal spring. These results are consistent with previous studies about the air–sea feedback in the tropical Atlantic (Ruiz-Barradas et al.

![Figure 9](image.png)

**Fig. 9.** Eigenvalue of the first mode as a function of thermodynamical and dynamical coupling strength: (a) Real part is the growth rate and (b) imaginary part over $2\pi$ is the frequency. Shading area in (a) delimits the unstable regime and in (b) depicts the no oscillation (zero frequency) regime.
In principle, seasonal variations in the coupling strength provide a way for the seasonal cycle to modulate the interannual and decadal variability. During the boreal spring, the Atlantic ITCZ moves southward and comes close to the equator in April, implying a symmetric distribution of trade winds in the Northern and Southern Hemisphere, which favors the development of the WES feedback (Okajima et al. 2003; Hu and Huang 2006). This corresponds to the positive season in Fig. 10a and the thermodynamical regime in Fig. 9b, where the meridional mode dominates the variability of interhemisphere SST gradient. During boreal summer, the ITCZ moves northward, the western Atlantic starts to warm up as the easterly starts to intensify, and the thermocline shoals in the Gulf of Guinea, which supports a strong Bjerknes feedback (Keenlyside and Latif 2007). This corresponds to the peak season in Fig. 10b and the dynamical regime in Fig. 9b in which the zonal mode dominates the variability of equatorial SST. In boreal autumn and winter, both the thermodynamical and dynamical feedback, apparently weakened in strength, may coexist in the tropical Atlantic. However, the dominant mode is no longer associated with a single oscillation, indicating that the system is in the mixed regime. In these seasons, the meridional mode degenerates accordingly; however, the zonal mode is still a robust feature of the ocean–atmosphere system, as recently revealed by Okumura and Xie (2006).

5. Discussion and conclusions

Eigenanalysis of a linear model reveals that the tropical Atlantic ocean–atmosphere system has two types of coupled modes: a decadal meridional mode and an interannual zonal mode. The meridional mode, characterized as an SST dipole across the north and south tropical Atlantic, originates from thermodynamical feedback between latent heat flux and SST. The zonal mode, characterized as a SST anomaly in the eastern equatorial Atlantic, originates from dynamical feedback between wind, thermocline, upwelling, and SST. The relative strength of thermodynamical versus dynamical feedback determines the strength of the coupled modes: When the thermodynamical (dynamical) feedback dominates, the meridional (zonal) mode becomes the leading mode. These results are generally consistent with previous studies concerning the dominant modes of the tropical Atlantic variability (Zebiak 1993; Chang et al. 1997; Xie 1999; Ruiz-Barradas et al. 2000; Xie and Carton 2004; Frankignoul and Kestenare 2005; Kushnir et al. 2006; Chang et al. 2006).

It is further shown that, due to the offsetting effects of the thermodynamical feedback and the dynamical feedback, the zonal mode interacts with the meridional mode destructively. On the one hand, the meridional mode, involving strong zonal wind and thermocline anomalies in the western equatorial Atlantic, weakens the dynamical coupling between the wind anomaly in the west and SST anomaly in the east, hence suppressing growth of the zonal mode. On the other hand, the zonal mode, inducing large SST anomalies toward 10°N and 10°S, breaks the thermodynamical coupling between the heat flux and SST anomalies in the deep tropics, thus making the meridional mode nonoscillatory. Such mode–mode interaction, an inherent property of a nonnormal coupled ocean–atmosphere system (Chang et al. 2004a,b), provides a theoretical basis for interpreting the observed mode correlation of the tropical Atlantic (Servain et al. 1999, 2000). It is also noted that the intensity of mode interaction is tightly connected to the strength of thermodynamical and dynamical coupling. For the weak coupling case, the two TAV modes could be totally independent, hence uncorrelated. Additionally, a model bias in the thermodynamical
cal and dynamical feedback would result in a spuriously strong interaction between the zonal and meridional modes (Breugem et al. 2006).

Assuming that the seasonal cycle in the mean states mainly modulates the air–sea coupling strength, we may—within the linear framework developed in this study—examine the phase locking of coupled variability in the tropical Atlantic. Associated with the seasonal shift of the ITCZ, the relative strength of heat flux–SST and wind–SST feedbacks change significantly, forcing a seasonal transition between the thermodynamical and dynamical regimes: therefore, the meridional mode peaks in boreal spring and the zonal mode peaks in boreal summer. However, explaining the seasonal variations in coupling strength is not trivial in itself, as illustrated in the seminal paper on ENSO (Zebiak and Cane 1987). An alternative approach is to eliminate the coupling parameters from the model and use seasonally varying mean states instead. With some modifications, such as replacing the statistical atmospheric model with a dynamical one, our linear model can be used to directly investigate the effect of the seasonal cycle on the coupled modes and their interaction in the tropical Atlantic. The results of such a study will be reported in the future as progress is made.

Finally, since the tropical Atlantic ocean–atmosphere is a stable system, external forcing is needed to provide the background energy that triggers the onset of the coupled modes and their subsequent development. ENSO is believed to be an effective trigger in setting the spring warming in the north tropical Atlantic; however, other triggers, like the NAO, may also play a dominant role in the Atlantic sector (Xie and Carton 2004; Chang et al. 2006; Kushnir et al. 2006). The effects of these external forcing will be discussed in the companion paper.

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APPENDIX A

The Model Equations

The oceanic part of the model is an extension of the conventional 1½ layer (a single moving layer above a motionless abyss) shallow-water reduced-gravity model, which includes thermodynamics of the upper ocean.

Assuming that the effects of compressibility and salinity are of secondary importance within the mixed layer, the equation of SST takes the form

\[
\frac{\partial T}{\partial t} + \mathbf{u}_s \cdot \nabla T = k \nabla^2 T - \lambda (T - T_{\text{obs}}) + \frac{Q}{\rho_0 C_p H_{\text{mix}}} - \frac{1}{H_{\text{mix}}} w_e \mathcal{H}(w_e)(T - T_e)
\]

with density \(\rho_0 = 1 \times 10^3 \text{ kg m}^{-3}\), specific heat at constant pressure \(C_p = 4.2 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}\), mean depth of the mixed layer \(H_{\text{mix}} = 50 \text{ m}\), atmospheric heat flux forcing \(Q\), horizontal heat diffusivity \(k = 1 \times 10^4 \text{ m}^2 \text{s}^{-1}\), and a relaxation term \(\lambda = 1/200 \text{ day}^{-1}\) to confine the evolution of SST around the observed mean state \(T_{\text{obs}}\).

In Eq. (A1), the subsurface influence is embedded in the last term on the right-hand side, with entrainment \(w_e\) determined as divergence of surface flow

\[w_e = H_{\text{mix}} \nabla \cdot \mathbf{u}_s,\]

and the temperature of entrained water \(T_e\) taking a linear form

\[T_e = 0.5(T + \bar{T}_{50}) + \frac{\partial T_{50}}{\partial z} h',\]

where \(\bar{T}_{50}\) is the observed mean temperature, \(\partial T_{50}/\partial z\) the vertical gradient of \(\bar{T}\) at 50-m depth, and \(h'\) the fluctuation of thermocline. The use of the Heaviside function \(\mathcal{H}(\cdot)\) is due to the fact that only entrainment changes \(T\) and detrainment just sinks water to the sublayer without modifying the surface temperature.

The horizontal velocity in the surface mixed layer \(\mathbf{u}_s\) is separated into two components: \(\mathbf{u}_s = \mathbf{u} + \mathbf{u}_s(H - H_{\text{mix}})/H\), where \(H = 100 \text{ m}\) is the mean depth of thermocline.

Surface Ekman flow \(\mathbf{u}_s = (u_e, v_e)\) is determined by balancing friction and Coriolis force with wind stress

\[r_s u_e - f v_e = \frac{\tau_x}{\rho_0 H_{\text{mix}}},\]

\[r_s v_e - f u_e = \frac{\tau_y}{\rho_0 H_{\text{mix}}},\]

where the frictional processes in the Ekman layer are idealized to be linear damping \(r_s = (1/1.5) \text{ day}^{-1}\), \(f = \beta_0 y\) is the Coriolis parameter of the equatorial beta-plane approximation with \(\beta_0 = 2.29 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1}\), and \(\tau = (\tau_x, \tau_y)\) is the wind stress at the sea surface.

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The upper-ocean flow $\mathbf{u} = (u, v)$ is governed by linearized shallow-water dynamics

$$
\frac{\partial u}{\partial t} - fu = -g \frac{\partial h}{\partial x} + \frac{\tau_x}{\rho_0 H} + \gamma \nabla^2 u - au
$$

$$
\frac{\partial v}{\partial t} + fu = -g \frac{\partial h}{\partial y} + \frac{\tau_y}{\rho_0 H} + \gamma \nabla^2 v - av
$$

$$
\frac{\partial h}{\partial t} + H \nabla \cdot \mathbf{u} = 0, \tag{A3}
$$

where $g' = g \Delta \rho / \rho_0 = 4.17 \text{ cm s}^{-2}$ is the reduced gravity and $\alpha = (1/2.5) \text{yr}^{-1}$ is the coefficient of linear friction. Note the horizontal eddy viscosity, $\gamma = 2.5 \times 10^4 \text{ m}^2 \text{s}^{-1}$, takes a different value from heat diffusivity $\kappa$.

The atmospheric part of the model is nominally written as the sum of the annual mean (or climatology of the monthly mean when the seasonal cycle is involved), coupled feedback, and atmosphere internal variability

$$
\begin{pmatrix}
\tau \\
Q
\end{pmatrix} = \begin{pmatrix}
\overline{\tau} \\
\overline{Q}
\end{pmatrix} + \begin{pmatrix}
\mu \mathcal{A}(T') \\
\nu \mathcal{B}(T')
\end{pmatrix} + \begin{pmatrix}
\xi
\end{pmatrix}, \tag{A4}
$$

where climatology $\overline{\tau}$ and $\overline{Q}$ are usually prescribed from observations; $\mathcal{A}$ and $\mathcal{B}$ represent coupling between the atmosphere and ocean and are nonlocal functions of SST anomaly; $\mu$ and $\nu$ are the coupling strength. In theory, $\mathcal{A}$ and $\mathcal{B}$ can be written as linear integral operators over the entire domain $\Omega$

$$
\mathcal{A}(T') = (\Psi, T') = \int_\Omega \int \Psi_{x,y,x',y'} T_{x',y'}(x,y') \, dx \, dy',
$$

$$
\mathcal{B}(T') = (\Phi, T') = \int_\Omega \int \Phi_{x,y,x',y'} T_{x',y'}(x,y') \, dx \, dy', \tag{A5}
$$

where $\Psi$ and $\Phi$ are either estimated by linear regression over observational datasets of $T'$, $\tau'$, and $Q'$ or given by a simple dynamical model of atmosphere.

Together (A1), (A2), (A3), and (A4) form a coupled ocean–atmosphere system with nonlinearity only appearing in the SST equation.

**APPENDIX B**

**The Procedure of SVD Analysis**

Let $n \times m$ matrices $\Theta_x$, $\Theta_y$, $Q$, and $T$ denote zonal wind, meridional wind, heat flux, and SST anomalies, respectively. If $n$ is the spatial dimension and $m$ is the temporal dimension, then the cross-covariance matrix is computed as

$$
\mathbf{C}_{m \times n} = \begin{pmatrix}
\Theta_x \\
\Theta_y \\
Q
\end{pmatrix} T^T. \tag{B1}
$$

The singular vector decomposition (SVD) of $\mathbf{C}$ gives pairs of coupled pattern $(\mathbf{u}_j, \mathbf{v}_j)$ and their relative strength $s_j$

$$
\mathbf{C} = [\mathbf{u}_1, \cdots, \mathbf{u}_n] \text{diag}(s_1, \cdots, s_n) [\mathbf{v}_1, \cdots, \mathbf{v}_n]^T. \tag{B2}
$$

Projecting the state matrices $\Theta_x$, $\Theta_y$, $Q$, and $T$ onto singular vectors results in two sets of time series:

$$
\mathbf{x}_j = T^T \mathbf{v}_j; \quad \mathbf{y}_j = \begin{pmatrix}
\Theta_x \\
\Theta_y
\end{pmatrix} \mathbf{u}_j.
$$

Noting SVD maximizes the covariance between $\mathbf{x}_j$ and $\mathbf{y}_j$, we may write $\mathbf{y}_j$ as a linear function of $\mathbf{x}_j$ via least squares regression:

$$
y_j = a_j \mathbf{x}_j + \epsilon \quad \text{where} \quad a_j = \frac{\mathbf{y}_j^T \mathbf{x}_j}{\mathbf{x}_j^T \mathbf{x}_j} = \frac{s_j}{\mathbf{x}_j^T \mathbf{x}_j}.
$$

Transferring back to physical space gives us the linear relationship of ocean–atmosphere coupling:

$$
\begin{pmatrix}
\mathbf{A}_x \\
\mathbf{B}
\end{pmatrix} = [\mathbf{u}_1, \cdots, \mathbf{u}_k] \text{diag}(a_1, \cdots, a_k) [\mathbf{v}_1, \cdots, \mathbf{v}_k]^T, \tag{B3}
$$

where the first $k$ singular vectors are retained. Generally speaking, it is a multivariate linear regression in space spanned by the singular vectors.

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