Mechanisms of Global-Mean Steric Sea Level Change

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ABSTRACT

Global-mean sea level change partly reflects volumetric expansion of the oceans because of density change, otherwise known as global-mean steric sea level change. Owing to nonlinearities in the equation of state of seawater, the nature of processes contributing to recent observed global-mean steric sea level changes has not been well understood. Using a data-constrained ocean state estimate, global-mean steric sea level change over 1993–2003 is revisited, and contributions from ocean transports and surface exchanges are quantified using closed potential temperature and salinity budgets. Analyses demonstrate that estimated decadal global-mean steric sea level change results mainly from a slight, time-mean imbalance between atmospheric forcing and ocean transports over the integration period: surface heat and freshwater exchanges produce a trend in global-mean steric sea level that is mainly offset by the redistribution of potential temperature and salinity through small-scale diffusion and large-scale advection. A set of numerical experiments demonstrates that global-mean steric sea level changes simulated by ocean general circulation models are sensitive to the regional distribution of ocean heat and freshwater content changes.

1. Introduction

Global-mean sea level rise is one of the most publicized aspects of ongoing climate change. Over the 1993–2003 decade, global-mean sea level rose at an average observed rate of 3.1 ± 0.7 mm yr\(^{-1}\) (Bindoff et al. 2007, hereafter B07). The interpretation of observed changes is typically in light of a sea level budget comprising a steric component, reflecting density changes at shallow, deep, and abyssal ocean depths, and a mass component, due to exchanges of freshwater with other components of the climate system (e.g., Leuliette and Willis 2011). Steric and mass terms are thought to have contributed 1.6 ± 0.5 and 1.2 ± 0.4 mm yr\(^{-1}\) to the 1993–2003 global-mean sea level budget, respectively (B07). [See Church et al. (2011) for a more recent assessment of global-mean sea level changes and their partitioning between mass and steric terms.]

While the contemporary global ocean observing system has facilitated increasingly accurate estimates of global-mean steric sea level change (e.g., Levitus 2005; Lyman et al. 2010; von Schuckmann and Le Traon 2011; Levitus et al. 2012), the nature of processes contributing to the observed change remains opaque. The interpretation of observations is complicated by the nonlinear nature of the equation of state of seawater; for example, since the thermal expansivity of seawater increases with temperature, salinity, and pressure, a mass of warmer, saltier subtropical seawater will expand more in response to a given temperature increase than an equal mass of cooler, fresher subpolar seawater. This suggests that global-mean steric changes might be sensitive to the details of regional temperature and salinity changes, prompting a number of questions: Are global steric changes mainly related to sea surface forcing of heat and freshwater, or does internal redistribution of temperature and salinity by small-scale diffusion or large-scale advection contribute? How well can contributions from advection, diffusion, and forcing be constrained?

The nature of processes contributing to global-mean steric sea level change has been considered to some degree in idealized or climatological contexts: Schanze and Schmitt (2013) used most available atmospheric surface products over the ocean and determined that climatological buoyancy forcing implies a positive global-mean steric sea level tendency between 3.3 and 14.7 mm yr\(^{-1}\); Gille
2. Materials and methods

a. Ocean state estimate framework

We use an ECCO state estimate, generated by constraining the Massachusetts Institute of Technology general circulation model (Marshall et al. 1997) to observations (satellite radar altimetry, historical hydrography, Argo profiles, etc.) in a physically consistent manner using nonlinear least squares (Wunsch and Heimbach 2007). The solution (version 2, iteration 216) has been used in a number of previous studies of regional and global sea level changes (e.g., WPH07; Piecuch and Ponte 2011; Fukumori and Wang 2013). Interested readers are referred to WPH07 for a detailed presentation of the solution and its caveats.

Sufficient for the present purposes is a brief description of some salient aspects. The solution is nearly global and defined over 1992–2004 on a 1° horizontal grid with 23 vertical levels. The estimate is driven by surface fluxes based on reanalysis fields from the National Centers for Environmental Prediction (NCEP)—National Center for Atmospheric Research (NCAR) (Kalnay et al. 1996). Parameterization schemes are used for physics not permitted by the coarse resolution (Redi 1982; Gent and McWilliams 1990; Large et al. 1994; Griffies 1998). Model formulation is Boussinesq, insofar as the velocity field is nondivergent, but some compressibility effects are retained (e.g., see Table 1), and a linear free surface is used. Given the focus of previous work (e.g., B07), we consider the solution over 1993–2003.

b. Diagnosing closed potential temperature and salinity budgets

Potential temperature \( \theta \) and salinity \( S \) changes, which cause global-mean steric sea level changes, can be interpreted in light of conservation principles, resulting from exchanges with the atmosphere or divergences in ocean transports. The relative roles of surface exchanges and ocean transports can be quantified by closed budgets. As described, for example, by Piecuch and Ponte (2011), the tracer \( c \in \{ \theta, S \} \) evolves actively according to a conservation equation.\(^1\)

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\(^1\)To be consistent with the model numerics, the tracer budget in Eq. (1) has been written for a Boussinesq fluid with a linear free surface and a salt flux boundary condition. These same assumptions are made by a number of coupled climate models used for sea level projections (e.g., Table 1 in Yin et al. 2010). Slight modifications to the tracer equation would need to be made for the case of a non-Boussinesq fluid or a model framework using a nonlinear free surface or a freshwater flux boundary condition (e.g., Table 2 in WPH07).
Table 1. Five combinations of continuity equation and equation of state commonly employed by general circulation models [see Adcroft et al. (2013, section 1.5.1.4) for more details]. Here \( \rho \) is the density, \( p \) is pressure, \( \theta \) is potential temperature, \( S \) is salinity, and \( \mathbf{u} = (u, v, w) \) three-dimensional velocity; these variables are to be understood as fully variable functions of \( x, y, z, \) and \( t \). Quantities \( \rho_0 \) and \( p_0 \) denote horizontally and temporally constant vertical background profiles of density and pressure. The quantity on the left-hand side of the continuity equation determines the hydrostatic pressure and is dynamically active. In Cartesian coordinates, \( D/Dt = \partial/\partial t + \mathbf{V} \cdot \mathbf{V} \) with \( \mathbf{V} = i\partial/\partial x + j\partial/\partial y + k\partial/\partial z \). The general circulation model used here employs the “semicompressible Boussinesq” configuration (see 3) with \( p_0 = -g\rho z \) (\( \rho \) is a constant reference density and \( g \) is gravitational acceleration) and with the equation of state described by Jackett and McDougall (1995).

<table>
<thead>
<tr>
<th>No.</th>
<th>Label</th>
<th>Continuity condition</th>
<th>Equation of state</th>
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<tbody>
<tr>
<td>1</td>
<td>Non-Boussinesq</td>
<td>( \rho^{-1}D\rho/D\rho + \mathbf{V} \cdot \mathbf{u} = 0 )</td>
<td>( \rho = \rho(\theta, S, p) )</td>
</tr>
<tr>
<td>2</td>
<td>Fully compressible Boussinesq</td>
<td>( \rho_0^{-1}D\rho_0/D\rho_0 + \mathbf{V} \cdot \mathbf{u} = 0 )</td>
<td>( \rho = \rho(\theta, S, p_0) )</td>
</tr>
<tr>
<td>3</td>
<td>Semicompressible Boussinesq</td>
<td>( \mathbf{V} \cdot \mathbf{u} = 0 )</td>
<td>( \rho = \rho(\theta, S) )</td>
</tr>
<tr>
<td>4</td>
<td>Incompressible Boussinesq</td>
<td>( \mathbf{V} \cdot \mathbf{u} = 0 )</td>
<td>( \rho = \rho(\theta, S, p_0) )</td>
</tr>
<tr>
<td>5</td>
<td>Quasi-Boussinesq (i.e., anelastic)</td>
<td>( \mathbf{V} \cdot (\rho_0\mathbf{u}) = 0 )</td>
<td>( \rho = \rho(\theta, S, p_0) )</td>
</tr>
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</table>

where \( \mathbf{u} = (u, v, w) \) represents the three-dimensional velocity field, \( \rho_r \) is a constant reference density, and \( Q^c \) contains (potentially vertically penetrating) surface \( c \) fluxes. In the estimate, the diffusive flux \( \kappa \mathbf{V} \) connotes parameterized mesoscale and other small-scale physics. Radiative and turbulent air–sea heat exchanges (divided by specific heat capacity) are represented by \( Q^S \) whereas \( Q^S \) contains the action of evaporation less precipitation and runoff. The right-hand side terms are referred to as (from left to right) advection, diffusion, and forcing. The consistency of the state estimate allows for the diagnosis of all tracer budget terms.

c. Defining steric sea level and its constituents

Proceeding on the basis of the hydrostatic assumption, Gill and Niiler (1973) quantified the sea level influence of temperature and salinity changes by defining steric sea level \( \xi_\rho \) as

\[
\frac{\partial \xi_\rho}{\partial t} = -\mathbf{u} \cdot \mathbf{V}_c + \mathbf{V} \cdot (\kappa \mathbf{V}_c) + \frac{1}{\rho_r} \frac{\partial Q^c}{\partial z},
\]

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local temperature and salinity changes according to the chain rule:

\[
\frac{\partial \rho}{\partial t} = \sum_{c \in \{\theta, S\}} \frac{\partial \rho}{\partial c} \frac{\partial c}{\partial t}.
\]

Definitions for \( \xi_\theta \) and \( \xi_\rho \) follow naturally: after Piecuch and Ponte (2011), substitute the full summation on the right-hand side of Eq. (3) for the density tendency integrand on the right-hand side of Eq. (2) and subsequently bring the summation outside of the integral:

\[
\frac{\partial \xi_p}{\partial t} = \sum_{c \in \{\theta, S\}} \left( \frac{1}{\rho_r} \int_H^0 \frac{\partial p}{\partial c} \frac{\partial c}{\partial t} dz \right) \leq \sum_{c \in \{\theta, S\}} \frac{\partial \xi_c}{\partial t}.
\]

As defined here, the thermosteric sea level represents the sea level change under arbitrary potential temperature \( \theta \) change at constant salinity \( S \) and pressure \( p \) deriving from consistent application of the chain rule to a nonlinear equation of state (e.g., J. G. Tagg and McDougall 1995). The halosteric sea level term is to be interpreted analogously. We note that our partition of \( \xi_p \) into \( \xi_\theta \) and \( \xi_\rho \) contributions is mathematically exact; it is distinct from approximate partitions based on linearization (e.g., Yin et al. 2010). In the case of small \( \theta \) and \( S \) changes, approximate linear formulations converge to our formulation.

3. Estimating and attributing global-mean changes

a. Estimation

As a necessary context for studying processes contributing to global-mean steric sea level changes, we first consider estimated global-mean changes in potential temperature, salinity, and steric sea level (cf. WPH07). The time series of global-mean potential temperature \( \langle \theta \rangle \) and salinity \( \langle S \rangle \) from the state estimate over 1993–2003
are shown in Figs. 1a and 1b, respectively. Here and
throughout, brackets are used to denote volumetric av-
erage of a quantity \( u \) over the global ocean:
\[
\langle u \rangle = \frac{1}{V} \int_A \int_{-H} u \, dz \, dA,
\]
where \( V \) is a fixed reference value for the global ocean volume, and \( A \) is the ocean surface. The \( \langle \theta \rangle \) and \( \langle S \rangle \) curves exhibit variability at annual, interannual, and
decal time scales, with the ocean gradually warming and freshening. Ocean warming is captured by the \( \langle \theta \rangle \) linear trend\(^3\) of roughly \( 1.4 \times 10^{-3} \text{C yr}^{-1} \); this trend is equivalent to an average heat flux of about \( 0.4 \text{ W m}^{-2} \) applied over the surface of the planet and is in agree-
ment within uncertainty with the contemporaneous trend of \( 0.50 \pm 0.18 \text{ W m}^{-2} \) reported by B07. Ocean freshening is partly described by the \( \langle S \rangle \) linear trend of \( -3.1 \times 10^{-6} \text{ psu yr}^{-1} \); this trend qualitatively accords with observational inferences of long-term global ocean freshening (B07).

Ocean warming and freshening together suggest global-mean steric sea level rise. To investigate sea level changes, the time series of global-mean steric sea level \( \zeta_p \) diagnosed from the estimate over 1993–2003 is shown in Fig. 1c. Here, the overbar denotes the areal mean of a quantity \( \varphi \) over the ocean surface:
\[
\overline{\varphi} = \frac{1}{A} \int_A \varphi \, dA,
\]
where \( A \) is a fixed reference value for the ocean surface area. Considering Eqs. (2), (5), and (6), and using Leibniz’s rule, we see that the tendency of \( \zeta_p \) is proportional to the global mean of the local density anomaly tendency:
\[
\frac{\partial \zeta_p}{\partial t} = -\frac{1}{\rho_r A} \frac{\partial \rho}{\partial t} - \frac{V}{\rho_r A} \langle \varphi \rangle.
\]
The \( \zeta_p \) time series is characterized by seasonal and interannual variations as well as long-term (decadal) changes (Fig. 1c). The \( \zeta_p \) linear trend of \( 0.9 \text{ mm yr}^{-1} \) (the constituent \( \zeta_T \) and \( \zeta_S \) trends are 0.89 and 0.03 mm yr\(^{-1}\), respectively) is smaller than the contemporaneous rate of \( 1.6 \pm 0.5 \text{ mm yr}^{-1} \) given by B07.\(^4\)

b. Attribution

What are the causes for the global-mean changes shown in Fig. 1? We address this question by diagnosing closed budgets as discussed in section 2. Contributions from advection, diffusion, and forcing to global-mean

\( ^3 \) All quoted trends represent slopes of linear least squares fits to deseasoned time series over 1993–2003.

\( ^4 \) The trend of \( 0.9 \text{ mm yr}^{-1} \) computed here is larger than the rate of \( 0.5 \text{ mm yr}^{-1} \) computed by WPH07 using the same state estimate. Note that WPH07 study the years 1993–2004, whereas we consider the period 1993–2003 (for reasons stated in section 2). The difference between values is accounted for by the significant drop in estimated global-mean ocean temperature during 2004 (see Fig. 16 in WPH07). This emphasizes the sensitivity of the \( \zeta_p \) trend to the particular time period considered, as mentioned elsewhere (e.g., Cazenave and Llovel 2010).
boundary condition. Similarly, the global-mean salinity term also vanishes because it is subject to a vanishing flux. The global sum of the diffusion over 1993–2003.

Contributions from ocean transports ultimately reflect the nonlinear nature of the equation of the state of seawater: physically, density is not materially conserved in the ocean and transport processes act as interior sources and sinks; mathematically, the global integrals of terms involving advection and diffusion do not vanish in general owing to the weighting by the expansion coefficients [i.e., the $\frac{\partial \rho}{\partial \sigma}$ term in Eq. (8)].

Linear least squares trends fit to the curves in Fig. 2b can be compared to values from previous studies. The forcing trend (9.2 mm yr$^{-1}$) is within the bounds established by Schanze and Schmitt (2013) for the $\overline{z_r}$ change under the climatological surface buoyancy forcing (their Tables 1 and 2) and close to the value given by GG12 for $\overline{z_r}$ change owing to the net surface buoyancy exchanges for a 20-yr period driven by an idealized atmosphere (their Table 1). The diffusion trend ($-6.9$ mm yr$^{-1}$) is similar to the $\overline{z_r}$ change calculated by Gille (2004) for the internal redistribution of initial conditions by vertical diffusion (her Table 1 and Fig. 2b) and near to the value given by GG12 for $\overline{z_r}$ change resulting from subgrid-scale processes under a scenario of repeating annual forcing (their Table 1).

The nature of the $\overline{z_r}$ balance depicted in Fig. 2b depends on the time scale. At the annual period, steric variations are controlled by surface forcing, suggesting that these $\overline{z_r}$ variations might be approximated, for example, by scaling global temperature fluctuations by some bulk expansion coefficient [e.g., Eq. (2) in WPH07]. With respect to long-term behavior, the lowest-order balance is between forcing and diffusion, but the advection trend ($-1.3$ mm yr$^{-1}$) is not negligible relative to the $\overline{z_r}$ change. That the mixing is larger than the advection

changes also are caused only by surface exchanges (not shown).

A steric sea level budget can be formulated based on the methods introduced above: substituting the sum of the advection, diffusion, and forcing terms on the right-hand side of Eq. (1) for the tracer tendency integrand in Eq. (4) yields [e.g., Eqs. (5) and (6) in Piecuch and Ponte 2011; cf. Eq. (41) in GG12]

$$\frac{\partial \overline{z_r}}{\partial t} = \sum_{c \in \{s,f\}} \left\{ - \frac{1}{\rho_f} \int_0^\infty \left[ -\mathbf{u} \cdot \nabla \mathbf{c} + \mathbf{v} \times (\mathbf{k} \nabla) \rho_f \right] + \frac{1}{\rho_f} \frac{\partial Q_c}{\partial z} \Big|_0^\infty \right\} \, dz .$$

Fig. 2. Budget for global-mean (a) potential temperature (°C) and (b) steric sea level (mm; black) in terms of advection (red), diffusion (blue), and forcing (green) from the ocean state estimate over 1993–2003.

5 For the case of the real ocean, in addition to boundary fluxes at the ocean–solid Earth interface along the seafloor that we do not consider (e.g., Kadko et al. 1995), we are ignoring heating owing to mixing friction (e.g., Wunsch and Ferrari 2004) as well as freshening because of deposition in hydrothermal systems. While these contributions are small relative to surface fluxes, strictly speaking they are nonzero.

6 For the case of a model with a linearized free surface, the advection term does not vanish exactly. (That this term is very small is clearly established by Fig. 2a.) This is because such models must use a “surface correction” term in order to ensure local conservation, the inclusion of which compromises global conservation; that is, there is a small mismatch incurred between the global tracer content and the integral of the surface tracer fluxes. See Campin et al. (2004) for more details.

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and roughly equal and opposite to the forcing is consonant with steady-state expectations that, to first order, the expansion of the ocean because of buoyancy exchanges at the surface is balanced by ocean contraction resulting from the destruction of thermal variance through interior mixing; hence, the interpretation of the budget could be that decadal changes in $\zeta_\theta$ result mainly from a small departure from a steady-state balance wherein buoyancy enters the ocean mainly in warm tropical surface waters, where the thermal expansivity of seawater is relatively high and is redistributed to depth (e.g., through turbulent processes within the mixed layer), or poleward (e.g., by the meridional overturning circulation), where thermal expansivity of seawater is comparatively low (McDougall and Garrett 1992).

4. Interpretation

Results given in the preceding section elucidate processes controlling global-mean changes, but they also prompt a few questions.

a. Comparing the state estimate and observational assessments

Why does the state estimate agree with observational assessments compiled by B07 regarding ocean heat content change but not regarding steric height change? Some clues are given by budget analyses: global ocean heat content depends only on net surface heat exchanges (Fig. 2a), whereas $\zeta_\theta$ is sensitive to regional surface fluxes and ocean transports (Fig. 2b). Hence, the lower steric trend could result, for instance, from not enough heating by surface exchanges or too much cooling by ocean transports in regions where the thermal expansion coefficient $\alpha = -\rho_r^{-1}(\partial \rho/\partial T)$ is highest.

To demonstrate decadal heating and cooling patterns, linear trends in zonal averages of potential temperature $\theta$ are shown in Fig. 3a. This panel can be compared to equivalent quantities computed over the same period using data. For example, Fig. 9 of Willis et al. (2004) and our Fig. 3a share common features, including strong warming of higher northern latitudes, heating of northern tropical waters at a depth of about 200 m, and warming around 40°S that extends to depth. But there are also differences worth pointing out: while Willis et al. (2004) observe cooling of the midlatitude ocean in the Northern Hemisphere, from surface to depth, we show warming; whereas we find cooling of tropical and subtropical surface waters, Willis et al. (2004) reveal warming.

Maps of temporal and zonal means of $\theta$ and $\alpha$ are shown in Figs. 3b and 3c to aid interpretation of the $\theta$ trends. Relative to the Willis et al. results, the state estimate is heating more in colder regions where $\alpha$ is smaller and cooling more in warmer areas where $\alpha$ is larger (Fig. 3). Discrepancies in regional temperature trend patterns could partly explain differences between observed and estimated ocean expansion.

b. Diffusion and sensitivity to boundary layer parameters

What is the nature of the diffusion contribution to the $\zeta_\theta$ trend? Recall that in this context diffusion comprises all of the parameterized physics used by the state estimate. Additional analysis revealed that the diffusion curve in Fig. 2b results wholly from vertical mixing (Large et al. 1994) and that isopycnal mixing (e.g., Redi 1982) contributes negligibly (not shown). That a key budget contribution derives from unresolved physics raises questions regarding the robustness of the $\zeta_\theta$ changes; for example, is the $\zeta_\theta$ curve sensitive to prior specification of
boundary layer parameters required by the Large et al. (1994) vertical mixing scheme?

To assess the sensitivity of the \( \bar{z} \rho \) trend, a set of experiments was performed using the state estimate setup, such that prior specifications of certain parameters were varied, all else being equal. We considered two parameters involved in setting boundary layer depth: the critical bulk Richardson number \( R_i \) (a measure of the ratio of stabilizing stratification to destabilizing shear at which point boundary layer eddies first become stable) and the ratio of the buoyancy frequency in the interior ocean to the material derivative term

\[
\frac{R_i}{\partial \rho / \partial t} \text{ (not shown), demonstrating that in this Boussinesq context the advection term is indeed negligible relative to the material and local derivative terms in the global mean.}
\]

To consider the problem more completely and generally than we have up to this point, assume that \( \rho \) is a nonlinear function of \( S, \theta \), and \( p \) (e.g., Jackett and McDougall 1995) and that \( S, \theta, \) and \( p \) are fully variable functions of space and time. Let \( \theta \) and \( S \) evolve according to the tracer conservation Eq. (1). Using the chain rule, the local density anomaly tendency can be expressed as

\[
\frac{\partial \rho}{\partial t} = \frac{\partial \rho}{\partial p} \frac{\partial p}{\partial t} + \sum_{c \in \{\theta, S\}} \left\{ \frac{\partial \rho}{\partial c} \left[ -\mathbf{u} \cdot \nabla c + \nabla \cdot (\kappa \nabla c) + \frac{1}{\rho_r} \frac{\partial Q^c}{\partial z} \right] \right\},
\]

where we have substituted into the brackets the sum of the right-hand side terms in Eq. (1) for the local \( \theta \) and \( S \) tendencies. Similarly, material density changes can be represented as

\[
\frac{D \rho}{D t} = \frac{\partial \rho}{\partial p} \frac{D p}{D t} + \sum_{c \in \{\theta, S\}} \left\{ \frac{\partial \rho}{\partial c} \left[ \nabla \cdot (\kappa \nabla c) + \frac{1}{\rho_r} \frac{\partial Q^c}{\partial z} \right] \right\},
\]

where again the chain rule has been applied to \( \rho \) and where we have solved for material \( \theta \) and \( S \) derivatives in Eq. (1) and substituted into the brackets. By virtue of Eq. (9), we can equate global integrals of the right-hand sides of Eqs. (10) and (11); canceling out common terms, and multiplying by a factor of \(-\nabla(\rho_A)^{-1}\) in order to express explicitly in terms of \( \bar{z} \rho \) changes [cf. Eq. (7)], we arrive at the expression

\[
\frac{1}{\rho_r A} \left\langle \mathbf{u} \cdot \frac{\partial \rho}{\partial \theta} \mathbf{V} + \mathbf{u} \cdot \frac{\partial \rho}{\partial S} \nabla S \right\rangle = -\frac{1}{\rho_r A} \left\langle \mathbf{u} \cdot \frac{\partial \rho}{\partial p} \mathbf{V} \right\rangle.
\]

While Eq. (12) is mathematically equivalent to the prior statement that \( \left\langle \mathbf{u} \cdot \nabla \rho \right\rangle \) vanishes, the trail of logic leading from Eqs. (9) to (12) suggests a deeper, physically meaningful interpretation: the two sides of Eq. (12) represent two complementary viewpoints or descriptions of how general circulation processes effect \( \bar{z} \rho \) changes. Consider, for instance, vertical heaving driven by wind stress curl. On the one hand, according to the left-hand side, the impact of vertical heaving on \( \bar{z} \rho \) could be described by a collective of Eulerian observers, stationed
at fixed locations, who would perceive advective density changes resulting from the passage of temperature and salinity horizons but would not experience any advective changes in ambient pressure. On the other hand, according to the right-hand side, $\varpi_{\rho}$ changes because of vertical heaving could be described equivalently by a group of Lagrangian observers (e.g., GG12), moving with water parcels, who would experience advective density changes resulting from changing surrounding pressure but would not experience advective variation in potential temperature or salinity.

Equation (12) thus points to fundamental coupling between the effects of salinity, potential temperature, and pressure on density, implicit in the $S$, $\theta$, and $p$ dependencies of each of the various expansion coefficients, revealing necessary conditions for advection processes to effect $\varpi_{\rho}$ change. First, seawater density must show nonlinear dependence on pressure as well as potential temperature or salinity; if this is not the case, the expansion coefficient terms could be pulled out of the brackets on the left- or right-hand side of Eq. (12), leaving global integrals of pure advection terms, which would vanish (e.g., Fig. 2a). Second, there must be flow across isosurfaces of pressure as well as potential temperature or salinity; if this condition is not satisfied, scalar products within the brackets in Eq. (12) would vanish, leaving one or the other side of the equation equal to zero.

These necessary conditions can be established also by way of the simple thought experiment illustrated in Fig. 4. Consider a water column partly composed of two parcels (indicated by the boxes) of equal mass. The first parcel has potential temperature $\theta_A$ and salinity $S_A$ while the second has potential temperature $\theta_B$ and salinity $S_B$. At some initial time $t_0$, the first parcel begins near the surface, experiencing pressure $p_1$, and the second originates at depth, feeling pressure $p_2$ (indicated by the dashed lines). Now imagine that a circulation in the meridional–vertical plane (indicated by the arrows) acts to redistribute the two parcels such that, at some later time $t_f$, the first parcel ends up at depth, feeling pressure $p_2$, and the second winds up near the surface, experiencing pressure $p_1$. Does this hypothetical scenario imply sea level change?

The answer depends on the equation of state. For a linear equation of state of the form

$$\rho(S, \theta, p) = \rho_r \left[ 1 + \frac{1}{\rho_r} \frac{\partial \rho}{\partial \theta} (\theta - \theta_r) + \frac{1}{\rho_r} \frac{\partial \rho}{\partial S} (S - S_r) + \frac{1}{\rho_r} \frac{\partial \rho}{\partial p} (p - p_r) \right].$$

(13)

where the partial derivatives are in this equation constants and subscript $r$ denotes constant reference value, it is easy to show that between initial and final times the mass-weighted average density of the two parcels does not change, so sea level is not affected. Hence, a nonlinear equation of state must be required for advective redistribution to affect the sea level.

Now consider the more general form of equation of state, allowing for nonlinear effects:

$$\rho(S, \theta, p) = \frac{\rho_0(S, \theta)}{1 - \rho/K(S, \theta, p)}.$$

(14)

where $\rho_0$ and $K$ are general representations of density of seawater at atmospheric pressure and seafloor bulk modulus, respectively (e.g., Jackett and McDougall 1995). The difference between initial and final mass-weighted average densities of the parcels $\delta \rho$ is

$$\delta \rho = \frac{1}{2} \left\{ \rho_0(S_A, \theta_A)[p_1 K(S_A, \theta_A, p_2) - p_2 K(S_A, \theta_A, p_1)] \right\} \left\{ K(S_A, \theta_A, p_2) - p_2 \right\}^{-1}$$

$$+ \frac{\rho_0(S_B, \theta_B)[p_2 K(S_B, \theta_B, p_1) - p_1 K(S_B, \theta_B, p_2)]}{K(S_B, \theta_B, p_2) - p_2 \left\{ K(S_B, \theta_B, p_1) - p_1 \right\}} \right\}.$$

(15)
Considering the form of Eq. (15), we note that if either $\theta_A \neq \theta_B$ or $S_A \neq S_B$ but $p_1 = p_2$ (e.g., flow is along isobaric surfaces) $\delta p$ vanishes identically. Moreover, we observe that if $p_1 \neq p_2$ but $\theta_A = \theta_B$ and $S_A = S_B$ (e.g., motions are along isothermal and isoline surfaces) $\delta p$ is always zero. Thus, for redistribution to have an effect on sea level, there must be motions across pressure surfaces as well as potential temperature or salinity surfaces.

d. Advection and sensitivity to tracer advection scheme

Given the preceding discussion, it is interesting to note that our trend owing to advective temperature and salinity fluxes and the trend given by GG12 relating to vertical motions across pressure surfaces (their Table 1)—which are approximately equivalent quantities according to the arguments above—have comparable magnitude but opposite sign.

One possible reason for the discrepancy might be the distinct advection schemes used for discretizing the thermodynamical equations: our estimate employs third-order upwind-biased advection in the tracer equations, whereas the estimate of GG12 utilizes a multidimensional piecewise parabolic method (Dunne et al. 2012). To gauge whether estimated $\varphi$ changes are sensitive to the discretization of tracer advection in the thermodynamical equations, another experiment was performed, such that the tracer advection scheme was changed from third-order upwind differencing to fourth-order centered differences, all else being equal.

Steric budget time series produced by the two differencing schemes are shown in Fig. 5. Whereas the upwind scheme yields a negative advection trend and a positive $\varphi$ trend ($0.9 \text{ mm yr}^{-1}$), the centered scheme gives a positive $\varphi$ trend ($1.5 \text{ mm yr}^{-1}$) and a positive advection trend. These results might reflect numerical diffusion effects: in coarse resolution models, whereas centered schemes can be more noisy, upwind-biased schemes can be more diffusive, which could partly explain why the upwind biasing produces a negative advection trend whereas centered differencing produces a positive one.

To interpret correctly the results of the experiments, it is important to recall that the evolution of global ocean heat and freshwater content is the same in the two experiments: different $\varphi$ behaviors are a result of different regional ocean heat and freshwater content changes. With regard to shallow tropical regions, for example, the upwind scheme produces weak cooling trends (e.g., Figs. 3a,c), and the centered scheme yields weak warming trends (not shown), consistent with the $\varphi$ changes produced by the respective experiments (Fig. 5). This result suggests that the tropical surface cooling in Fig. 3a could be because of excessive mixing resulting from the overly diffusive nature of the upwind-biased advection scheme.

5. Concluding remarks

Using a physically consistent ocean state estimate, we quantified the influence of ocean transports and surface exchanges on global-mean steric sea level over 1993–2003 by diagnosing relevant budgets. Our results suggest that recent decadal changes in global-mean steric sea level represent small departures from a near balance between atmospheric forcing and ocean transports (Fig. 2), with important contributions from heat and salt advection representing coupling of nonlinear temperature, salinity, and pressure effects on density.

More generally, this investigation provides a local or Eulerian exploration of processes contributing to global-mean steric sea level changes. It complements recent work (GG12) providing an equivalent material or Lagrangian consideration of processes affecting global sea level. Herein we have demonstrated some necessary conditions that must be satisfied in order for advective transports to affect global-mean steric sea level; for instance, seawater must be (at least partially) compressible and (more generally) the equation of state must show nonlinear dependence on pressure and temperature or salinity.

Global-mean steric sea level changes estimated here are not sensitive to subgrid-scale mixing parameterization schemes. This is somewhat different from previous results based on coupled ocean–atmosphere models, which show that the details of subgrid-scale parameterization schemes strongly influence projected ocean thermal expansion (e.g., Weaver and Wiebe 1999; Wiebe and Weaver 1999). This difference could arise because, for coupled models, mixing parameterization schemes
variably affect ocean heat uptake (hence steric sea level), but our results derive from an “ocean only” model, wherein surface fluxes are specified a priori and coupled air–sea interactions are precluded.

Results from numerical experiments described here suggest that ocean general circulation model estimates of global-mean steric sea level change can be sensitive to the discretization of tracer advection in the thermodynamical equations (Fig. 5). Somewhat similarly, results recently reported by Hallberg et al. (2013) reveal that climate model projections of global-mean steric sea level change can be sensitive to whether the ocean vertical coordinate is represented in terms of either \( z \) (i.e., levels) or \( \sigma \) (isopycnals). These numerical uncertainties owing to representational choices might contribute to the spread in climate model projections of future global-mean steric sea level rise (e.g., Yin 2012); coupled with the myriad relevant geophysical processes (e.g., Milne et al. 2009), they emphasize the difficulty of simulating global sea level change.

Although not the focus (and beyond the scope) of this study, given uncertainties in estimates of buoyancy flux into the ocean (Schanze and Schmitt 2013), another source of uncertainty could be the choice of the atmospheric surface products used for the boundary conditions in ocean models. While our study uses surface fluxes produced by iteratively adjusting first guess fields taken from an NCEP–NCAR reanalysis, it is unclear whether our major findings would have been qualitatively different were different initial forcing fields used in the state estimation procedure. Although some studies show that regional sea level simulations can be sensitive to the choice of the wind dataset used for the boundary conditions (Merrifield and Maltrud 2011; McGregor et al. 2012), we are aware of no study that considers the sensitivity of simulated global-mean steric sea level to the choice of surface forcing dataset. This topic would appear to be an important direction for future work.

The state estimate used in this study has some caveats worth emphasizing. For one, as mentioned above, the model is Boussinesq, an aspect shared by many coupled climate models used in sea level projections (e.g., Table 1 of Yin et al. 2010). While, as first pointed out by Greathbatch (1994), the volume-conserving nature of Boussinesq models precludes online prognosis of global-mean steric sea level changes, it is possible to diagnose global-mean steric sea level changes offline (e.g., Yin 2012); this is standard practice for analyses from phase 5 of the Coupled Model Intercomparison Project (CMIP5) simulations (Griffies et al. 2009). Some minor details of our formulation would need to be quantitatively adjusted in a more general, non-Boussinesq framework (e.g., perhaps based on the Eulerian formulations presented, but not explored, by GG12, section 2.2 or appendix D), but we judge that it is unlikely that our major conclusions would undergo qualitative revisions.

Other caveats, discussed in the preceding sections, are that there are some clear discrepancies between observed and estimated regional and global ocean changes and also that, although contemporaneous with consensus observational assessments of ocean climate change (B07), the estimate covers an earlier decade that excludes most of the temperature and salinity data accumulated by the Argo program (Abraham et al. 2013). In spite of such shortcomings, however, the state estimate used in this study suffices for the present purposes of establishing contributions from redistribution by ocean currents to global-mean steric sea level changes. Updated assessments based on contemporary state estimates that incorporate more observational data are left for future study.

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