The Relationship between Precipitation and Precipitable Water in CMIP6 Simulations and Implications for Tropical Climatology and Change

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ABSTRACT: It is well documented that over the tropical oceans, column-integrated precipitable water (pw) and precipitation (P) have a nonlinear relationship. In this study moisture budget analysis is used to examine this P–pw relationship in a normalized precipitable water framework. It is shown that the parameters of the nonlinear relationship depend on the vertical structure of moisture convergence. Specifically, the precipitable water values at which precipitation is balanced independently by evaporation versus by moisture convergence define a critical normalized precipitable water, pwnc. This is a measure of convective inhibition that separates tropical precipitation into two regimes: a local evaporation-controlled regime with widespread drizzle and a precipitable water–controlled regime. Most of the 17 CMIP6 historical simulations examined here have higher pwnc compared to ERA5, and more frequently they operate in the drizzle regime. When compared to observations, they overestimate precipitation over the high-evaporation oceanic regions off the equator, thereby producing a “double ITCZ” feature, while underestimating precipitation over the large tropical landmasses and over the climatologically moist oceanic regions near the equator. The responses to warming under the SSP585 scenario are also examined using the normalized precipitable water framework. It is shown that the critical normalized precipitable water value at which evaporation versus moisture convergence balance precipitation decreases as a result of the competing dynamic and thermodynamic responses to warming, resulting in an increase in drizzle and total precipitation. Statistically significant historical trends corresponding to the thermodynamic and dynamic changes are detected in ERA5 and in low-intensity drizzle precipitation in the PERSIANN precipitation dataset.

KEYWORDS: Climate change; Drizzle; Convective parameterization; Coupled models

1. Introduction

Understanding and quantifying the effects of global warming on regional hydrological cycles is one of the most important problems in climate science because of the societal implications. At global scale, atmospheric moisture increases with temperature under global warming at a rate that follows the Clausius–Clapeyron relationship of approximately 7% K^{-1}, while global precipitation increases at a much slower rate of approximately 2% K^{-1} (Held and Soden 2006). This difference between the responses of precipitation and atmospheric moisture content is related to the constraint imposed by the atmospheric radiative cooling due to the increased temperature and humidity, which limits the precipitation change through the global energy and water balances (Allen and Ingram 2002; Pendergrass and Hartmann 2014). Regional differences of the precipitation response are much less clear, particularly over the tropics, where spatial and temporal shifts related to dynamic and thermodynamic responses to warming can influence regional hydrology. This challenge is further exacerbated by the persistent precipitation distribution biases in generations of climate models, undermining our confidence in model projections of future changes in precipitation. The biases that are well documented in the simulations from phases 3 and 5 of the Coupled Model Intercomparison Project (CMIP3 and CMIP5, respectively) include excessive precipitation over oceanic regions off the equator (Hirota and Takayabu 2013; Fiedler et al. 2020), weak South Asian monsoon rainfall (Hagos et al. 2019), and weak Amazon precipitation (Yin et al. 2013). Understanding the origin of these biases and how they relate to regional-scale projections of precipitation changes is critical for building confidence in the projections.

Several studies suggest that model precipitation biases are related to the representation of convection. For example, precipitation biases in coupled climate simulations can be reproduced in uncoupled Atmosphere Model Intercomparison Project (AMIP)-style simulations (Zhang et al. 2007; Chikira 2010) while some modifications to the parameterization of...
convection, specifically the sensitivity of convection to environmental humidity, appear to mitigate the biases (Song and Zhang 2009; Hirota et al. 2011; Emori et al. 2001). The nonlinear relationship between convection and environmental humidity over the tropical oceans has been well documented (Bretherton et al. 2004; Rushley et al. 2018; Ahmed and Schumacher 2015, and references therein). Several studies empirically approximate the relationship using an exponential function with a “pick-up" precipitable water value at which precipitation starts to increase rapidly with precipitable water (Igel et al. 2017; Sahany et al. 2014). Conceptually this relationship provides a measure of the effectiveness of environmental air in diluting rising plumes in moist convection (Peters et al. 2009; Holloway and Neelin 2009, etc.) and the moistening effect of convection on the environment. Analyzing the nonlinear relationship between precipitation and precipitable water in the South Asian monsoon region and the equatorial Indian Ocean, Hagos et al. (2019) identified the normalized precipitable water in the equatorial Indian Ocean as an important metric for understanding model biases in simulating South Asian monsoon precipitation and the intermodel spread in future precipitation projections.

Extending the geographic focus from the South Asian monsoon region to the entire tropics, we aim to address two questions in this study: What are the origins of biases in CMIP6 precipitation climatology over the tropics? What are the implications of the biases for uncertainties in the projected changes? To this end, we first derive the precipitation–precipitable water relationship discussed above from the moisture budget equation and examine its representation in the historical and future climate simulations of the newest generation of models that participated in CMIP6 and in a global reanalysis. Then the relationship is normalized such that key parameters that control the model behavior in both historical and projection simulations are identified and their physical meanings in the context of model representation of convection are discussed.

2. Tropical precipitation climatology in CMIP6 simulations

We analyzed 17 historical simulations and 10 future simulations following the Shared Socioeconomics Pathway (SSP585) from the CMIP6 model archive. The models were selected based on availability of daily precipitation, precipitable water, and evaporation output, as required by the analysis. Where it is available vertically integrated moisture convergence is directly used; otherwise, it is estimated from the above listed variables using the moisture budget equation. We also analyzed data from the ERA5 dataset (C3S climate services 2017), two observational precipitation datasets, and an observational evaporation dataset. They are Precipitation Estimation from Remotely Sensed Information using Artificial Neural Networks (PERSIANN; Ashouri et al. 2015) and Tropical Rainfall Measuring Mission (TRMM 3B42; Huffman et al. 2010) and the Woods Hole Objectively Analyzed Flux Project daily evaporation data (OAFlux; Yu et al. 2008). All data are remapped to uniform 2° grid spacing. The study is focused on the tropics between 20°S and 20°N. We analyzed two 15-yr periods: 2000–14 for the historical simulations and 2086–2100 for the future simulations.

In Fig. 1, we compare the mean precipitation from the 17 CMIP6 models for the historical period to the mean observed precipitation, computed as the average of TRMM and PERSIANN precipitation. Compared to observations, the CMIP6 multimodel mean shows excessive precipitation over the tropical oceans off the equator and drier conditions over the tropical landmasses. Figure 2 shows the climatological mean of the daily low-intensity precipitation, or drizzle, and evaporation. For reasons that will be apparent in the next section, drizzle is defined as precipitation rates less than 0.13 mm h$^{-1}$ (3.12 mm day$^{-1}$). The models have particularly excessive low-intensity precipitation, which has been well documented for previous generations of models (Sillmann et al. 2013; Liu et al. 2014; Stephens et al. 2010; Dai 2006). The corresponding evaporation fields are shown in the right column of Fig. 2. Excess evaporation is also a common bias in previous generation of models. In the tropics, this bias is driven primarily by wind biases (e.g., Small et al. 2019), but also possibly related to biases in downwelling shortwave radiation and model resolution (Demory et al. 2014). The spatial similarities between the biases in total precipitation (Fig. 1c), drizzle (Fig. 2c), and evaporation (Fig. 2f) are striking and will be discussed later.

The effects of tropical oceanic rainfall and 850-hPa wind biases on precipitable water over tropical landmasses are shown in Fig. 3. Excessive CMIP6 rainfall over the tropical oceans diverts moisture away from the tropical lands, weakening the moisture transport to South Asia, Africa, and the Amazon, resulting in dry biases over land regions. As the circulation biases may be induced by precipitation biases over the oceans through diabatic heating, we hypothesize that better representation of the climatology of precipitation over the oceans could reduce the perennial dry biases over land in generations of models (Sperber et al. 2013). This hypothesis will be examined in the next section.

3. The relationship between precipitation and precipitable water

a. Derivation

To investigate the origins of model precipitation biases, we first examine the processes behind the observed relationship between precipitation and precipitable water. Consider the vertically integrated moisture budget equation:

\[
\frac{\partial (pw)}{\partial t} = E - P - \frac{1}{g} \int_{p_t}^{p_s} \nabla \cdot (\mathbf{v} q) \, dp ,
\]

(1)

where

\[
pw = \frac{1}{g} \int_{p_t}^{p_s} q \, dp
\]

(2)

is the column-integrated water vapor or precipitable water; \(E\) and \(P\) are evaporation and precipitation respectively, while \(p_s\) and \(p_t\) are the pressures at the surface and top of the atmosphere, respectively. To obtain a relationship between precipitation and precipitable water, the moisture convergence
term has to be represented as a separable function of $P$ and $pw$. To do that we introduce a variable, normalized moisture flux convergence (NMFC), as

$$\text{NMFC} = \frac{-\frac{1}{\rho_s} \nabla \cdot (\mathbf{v} q) \, dp}{pw}$$  \hspace{1cm} (3)$$

such that the moisture budget equation can be written as

$$\frac{\partial (pw)}{\partial t} = E - P + (\text{NMFC})pw.$$  \hspace{1cm} (4)$$

As can be seen from its definition, NMFC is related to the vertical structure of wind divergence in relation to moisture profile. Therefore, by continuity it is related to vertical velocity.

Over the tropical oceans tropospheric temperature gradients are weak and diabatic heating is primarily balanced by adiabatic cooling (Sobel and Bretherton 2000; Sobel et al. 2001), leading to a relationship among vertical velocity, static stability, and diabatic heating (see the appendix). Using conservation of moisture, mass, and energy, the relationship between moisture flux convergence and precipitation was obtained in Hagos et al. (2019). Here we focus the discussion on the physical interpretations and implications for subsequent analysis. As is apparent from (4), NMFC plays an important role in moisture variability so its physical interpretation deserves some discussion. Figure 4a shows the relationship between wind divergence (contours) and moisture normalized by the column integrated precipitable water [i.e., $(q/pw)(dp/g)$; shading] over the tropical oceans between 20°S and 20°N based on ERA5. For precipitation less than about 0.13 mm h$^{-1}$, there are strong upper-level convergence and near-surface divergence indicating subsidence and convective inhibition. For precipitation greater than about 0.13 mm h$^{-1}$, near-surface convergence (dashed green contours) increases and deepens, and the height of maximum divergence (Fig. 4a, red contours) increases. The profile of water vapor mixing ratio (i.e., the moisture mixing ratio normalized by precipitable water) is displayed in shadings. This profile shows little change with increasing precipitation. Increases in the height level of maximum divergence as well as an increase in low-level moisture convergence mean that a smaller fraction of moisture diverges with increasing precipitation. Therefore, NMFC represents the strength of convection not only in the sense of the magnitude of vertical velocity but also in its effectiveness in converging moisture by virtue of its vertical profile.

The relationship between the divergence profile and precipitation implies that NMFC increases with precipitation, and the relationship is approximately linear as demonstrated in Fig. 4b. Figure 4c shows the linear relationship derived by calculating the mean NMFC values within equally sized precipitation bins to find the intercept (0.13 mm h$^{-1}$) at which, in a mean sense, evaporation balances precipitation [Eq. (4)]. The value of $pw$ at which this balance occurs, $pw_0$, which is an important parameter in the subsequent discussion, is also calculated in a similar manner (Fig. 4d). Defining the inverse of the slope of the regression line in Fig. 4c as the precipitable water limit $pw_{lim}$ and the intercept as $E_0/pw_{lim}$, the following equation is obtained:

$$\text{NMFC} = \frac{P - E_0}{pw_{lim}}.$$  \hspace{1cm} (5)$$

Before moving the analysis further, a brief discussion of NMFC in the context of previous work is warranted. The normalization introduced in Eq. (3) is also relevant to the concept of gross moist stability (GMS) put forward by Neelin and Held (1987) and further developed by Raymond et al. (2009) and many others. The relationship between these two concepts is made more explicit in the appendix. As a measure of the depth of the moisture convergence field, NMFC is also related to temperature and saturation. For example, if the circulation is shallow, moisture can converge and diverge close to the surface with little saturation and precipitation as the near-surface temperature is warmer than the temperature at upper levels, but the circulation could also be deep enough to allow significant saturation and condensation.
Substituting Eq. (5) into Eq. (4), the moisture budget equation can be written as

\[
\frac{\partial (pw)}{\partial t} = \left( E - \frac{pwE_0}{pw_{lim}} \right) - P \left( 1 - \frac{pw}{pw_{lim}} \right) \quad (6)
\]

To understand the physical meanings of \(pw_{lim}\) and \(E_0\), consider Eq. (6) as \(pw\) approaches \(pw_{lim}\). In that case \(pw\) becomes less and less sensitive to precipitation. On the other hand, combining Eq. (3) and Eq. (5) yields the following:

\[
-1 \int_{\Sigma} \frac{pw}{E_0} \mathbf{v} \cdot \mathbf{d}p = (P - E_0) \left( \frac{pw}{pw_{lim}} \right) \quad (7)
\]

As \(pw\) approaches \(pw_{lim}\), the loss of moisture by the net precipitation is compensated for by the gain through moisture convergence. Thus, \(pw_{lim}\) represents an asymptotic limit for \(pw\) in the \(P\)-\(pw\) relationship at which precipitation is balanced by moisture convergence. On the other hand, \(E_0\) represents the evaporation at grid points where there is drizzle but no moisture convergence. It is considered constant over the domain and the value is given for each model (legend in Fig. 6).

The above derivations and analyses show that the moisture budget can be used to understand the relative control of moisture convergence and evaporation on precipitation in the low-intensity and high-intensity regimes. Here we will demonstrate that the \(P\)-\(pw\) relationship can be derived from the moisture budget using the physical quantities \(pw_{lim}\), \(pw_0\), and \(E_0\) defined above. Under steady state Eq. (6) can be written as

![Fig. 2. (left) 15-yr (2000–14) mean drizzle precipitation (precipitation < 3.12 mm day\(^{-1}\)) from (a) TRMM-3B42 and PERSIANN (mm day\(^{-1}\)) and (b) multimodel mean of the 17 CMIP6 historical simulations, and (c) the difference of (b) minus (a). (right) 15-yr (2000–14) mean evaporation from (d) Woods Hole OA Flux Project evaporation (mm day\(^{-1}\)) and (e) multimodel mean of the 17 CMIP6 historical simulations, and (f) the difference of (e) minus (d).]
Equation (8) provides a $P$--$pw$ relationship but it is not unique since it depends on evaporation. Of all the $P$--$pw$ curves that arise from Eq. (8), the one that goes through $(pw = pw_0, E = E_0)$ is of most interest to us as it represents the mean $P$--$pw$ relationship when variability in evaporation and precipitable water tendencies are neglected. Therefore, the numerator in (8) becomes $E_0(1 - pw_0/pwl_{im})$. After collecting $(pw_{lim} - pw_0)$ and $(pw - pw_0)$ from the numerator and denominator respectively, Eq. (8) can be written as

$$P = \frac{E - \frac{pw}{pwl_{im}}E_0}{1 - \frac{pw}{pwl_{im}}}.$$  

(8)

$$P = \frac{E_0}{1 - \left(\frac{pw - pw_0}{pw_{lim} - pw_0}\right)}.$$  

(9)

It should be noted that (9) is not prognostic; rather, it should be interpreted as a state defined by precipitation drying, precipitable water, and the vertical structure of moisture convergence about which $P$ and $pw$ fluctuate under the prescribed evaporation $E_0$.

The analysis of Fig. 4 is repeated with all 17 CMIP6 historical simulations (Fig. 5) and the 10 SSP585 simulations (not shown), showing that the linear relationship in Fig. 4b based on ERA5 holds for the CMIP6 models. The model results are shown in blue, green, and red based on the value of $pwl_{im}$.
asymptotic pw limit, falling in the lower, middle, and higher terciles, respectively, of the multimodel ensemble. This color scheme is used in all subsequent plots. The parameter \( pw_{\text{lim}} \) varies widely among models. Similarly, the \( x \)-intercept value \( E_0 \) for which evaporation at NMFC equals zero [Eq. (5)] is shown in Fig. 6a for each CMIP6 historical simulation. Note that \( E_0 \) has a range of values between 0.15 to 0.19 mm h\(^{-1}\), all higher than that for ERA5, which is 0.13 mm h\(^{-1}\). We set the threshold for drizzle precipitation as the value of \( E_0 \) derived from ERA5 (\( E_0 = 0.13 \) mm h\(^{-1}\) or 3.12 mm day\(^{-1}\)). Recall that \( E_0 \) corresponds to the evaporation (and precipitation) rate when NMFC is equal to zero. Rain rates below this \( E_0 \) threshold, defined as low-intensity precipitation, are primarily controlled by evaporation rather than by moisture convergence. Note that this threshold is smaller than 0.3 mm h\(^{-1}\) typically used as the drizzle threshold in an operational context (e.g., AMS Glossary).

Equation (9) implies that one can construct the relationship between \( P \) and \( pw \) from \( pw_{\text{lim}}, pw_0 \), and \( E_0 \), which are already obtained for each model from the above analysis. Figure 7a shows the actual relationship obtained by partitioning the 15 years of daily \( pw \) (tropical ocean points only) into 80 bins and averaging the corresponding daily precipitation in each bin. Figure 7b is the idealized relationship reconstructed from Eq. (9) using values of \( pw_{\text{lim}}, pw_0 \), and \( E_0 \) determined for each model. The idealized relationship captures the main features of the \( P-pw \) relationship and the differences among the models, as demonstrated by the correspondence between the color associated with each model relative to ERA5 in the two panels. Our analysis shows that, to first order, the relationship between precipitation and precipitable water 1) is governed by moisture conservation and 2) can be uniquely defined by two \( pw \) values \( pw_{\text{lim}} \) and \( pw_0 \). When \( pw \) approaches \( pw_{\text{lim}} \), precipitation is balanced by moisture convergence, but when \( pw \) approaches \( pw_0 \), precipitation is balanced by evaporation. Each critical \( pw \) value is set by the vertical structure of moisture convergence and its relation to precipitation (Fig. 4). In many ways, \( pw_0 \) and \( pw_{\text{lim}} \) together play similar roles as \( w_c \) and \( \beta \) of Neelin et al. (2009) in that they determine the “pickup point” and the growth rate of precipitation with \( pw \). As the atmosphere warms up, they both increase such that the \( P-pw \) curves shift and stretch to the right, as one would expect from the Clausius–Clapeyron relation. The idealized \( P-pw \) relationship will be used to analyze model diversity and biases from the CMIP6 simulations next.
b. Normalization of precipitable water

Here we examine the origins of precipitation biases and inter-model spread. Since the \( P-pw \) curves for the diverse models are of the same form, appropriate normalization could yield a general relationship for a straightforward interpretation.

Equation (9) can be written more concisely by defining a normalized precipitable water as \( pw_n = \frac{pw}{pw_{lim} - pw_0} \), such that

\[
P = \frac{E_0}{1 - (pw_n - pw_{nc})}, \tag{10}
\]

where

\[
pw_{nc} = \frac{pw_0}{pw_{lim} - pw_0} \tag{11}
\]

is defined as the normalized critical precipitable water.

Equation (10) implies that precipitation can be estimated using \( pw_n - pw_{nc} \) and \( E_0 \). The raw and normalized forms of the \( P-pw \) relationship are plotted in Figs. 8a and 8c, while the actual and estimated \( P \) frequency distributions are plotted in Figs. 8b and 8d. As predicted by Eq. (10) normalization reduces the spread in the \( P-pw \) relationships and in the \( pw \) frequency distributions. Note, however, that the normalized \( P-pw \) curves diverge near \( pw_n - pw_{nc} = 0.5 \) (Fig. 8c); because of the nonlinearity, small differences at higher \( PW \) lead to larger spread in precipitation than at low \( pw \).
To understand the physical implications of Eq. (10), consider the normalized $P$–$pw$ relationship at its limits. At one extreme with $pw_n \ll pw_{\text{wc}}, P = E_0/(1 + pw_{\text{wc}})$. That is, precipitation is largely controlled by surface evaporation and is essentially independent of the local precipitable water. This regime corresponds to the widespread drizzle over high evaporation areas. In Fig. 4a, $pw_n \ll pw_{\text{wc}}$ corresponds to strong subsidence (left of the blue line) where wind divergence is confined to shallow levels, efficiently transporting moisture out of the column. At the other extreme with $pw_n \rightarrow 1 + pw_{\text{wc}}$ or $pw \rightarrow pw_{\text{lim}}, P/E_0$ becomes very large (Figs. 8a,c) and is much more sensitive to the precipitable water. Thus $pw_{\text{wc}}$ can be thought of as a form of convective inhibition (e.g., the strength of upper-level convergence and subsidence in Fig. 4a) that a moist column must overcome to transition to deep convection. Before discussing the implications of this for model biases and projections the next section, there is one more point worth mentioning. Equation (10) is of the form $y = 1/(1 + x)$, which at first glance appears different from exponential form used in several previous studies to empirically fit observed and modeled $P$–$pw$ relationships. Note however that for small $x, 1/(1 - x)$ can be approximated by $1 + x$, which is also true for $e^x$. That
appears to us to be a mathematical coincidence. The above
analysis suggests that the former is a physically sound rela-
tionship since it follows from conservation of moisture, mass,
and energy under weak temperature gradient.

4. Model biases and projections

a. Characterizing model biases and spread

The convergence of the \( P-pw \) relationship (Fig. 8c) and the
frequency distribution of \( pw \) after normalization (Fig. 8d) suggests
that \( pw_{nc} \), a measure of convective inhibition, and \( E_0 \), the evap-
oration under neutral NMFC (i.e., NMFC \( = 0 \)), may be useful
metrics for characterizing rainfall biases in models. Figure 9
shows a scatterplot of these two parameters. In comparison to
the parameters derived from ERA5, all models have higher \( E_0 \),
implying that the models produce more rain in the form of drizzle
than ERA5 in the absence of moisture convergence (Figs. 2 and
6a). Most models also have higher \( pw_{nc} \) than ERA5, implying
stronger convective inhibition than in observations, which means
it takes more moisture to transition from drizzle to intense con-
vection in the models. The fact that most models overestimate
\( pw_{nc} \) implies they have a greater-than-observed frequency of days
with \( pw_0 \) smaller than \( pw_{nc} \) (or \( pw < pw_{nc} \); Fig. 8d). Most of the
models have frequency distribution with respect to \( pw_0- pw_{nc} \)
that is extended to the left compared to ERA5 (Fig. 8d).

The difference in mean drizzle precipitation (daily precipitation <
3.12 mm day\(^{-1}\)) between the five models with lowest and highest
\( pw_{nc} \) is shown in Fig. 10. As predicted by the analysis, the low \( pw_{nc} \)
models have much less drizzle. These models also simulate heavier
total precipitation over Africa, South Asia, and the Maritime

![Fig. 8](image)

(a) Precip vs PW

(b) PW frequency

(c) Precip vs PW (normalized)

(d) Normalized PW frequency

![Fig. 9](image)

(a) Scatterplot of \( E_0 \) vs \( pw_{nc} \) for the 17 CMIP6 models and
ERA5 dataset.
versus the corresponding change in pwnc and fractional change in precipitation under the SSP585 scenario.

3.12 mm day\(^{-1}\), precipitable water (pwnc). (b) As in (a), but for precipitation and bottom five models according to their critical normalized Eq. (9) with respect to change in precipitation, we consider the partial derivative of implication of the normalization of precipitation for projected malized precipitable water framework. To understand the by the collapse of the tropical ocean that limits the moisture transported to the landmasses through geostrophic adjustment.

Continent. These results support the hypothesis put forward in the first section that the dry bias over tropical land may be related to the excess low-intensity precipitation over the high evaporation regions of the tropical ocean that limits the moisture transported to the landmasses through geostrophic adjustmen.

b. Projected change in precipitation

Finally, the response to warming is examined in the normalized precipitable water framework. To understand the implication of the normalization of precipitation for projected change in precipitation, we consider the partial derivative of Eq. (9) with respect to \(E_0\) and \(pwnc\):

\[
\frac{\Delta P}{P} = \frac{\Delta E_0}{E_0} - \left[ \frac{P}{E_0} \right] \Delta pwnc. \tag{12}
\]

Note that the term in the square brackets is nearly model independent and does not change much with warming as implied by the collapse of the \(P\)–pw curves into one after normalization (Fig. 8c). Figure 11 shows the end of the twenty-first-century fractional change in precipitation under the SSP585 scenario versus the corresponding change in pwnc and \(E_0\). The change in precipitation has a stronger relationship with \(\Delta pwnc\) under warming than with \(\Delta E_0\). Six of the 10 models show a decrease in \(\Delta pwnc\) and they have larger fractional increase in precipitation. As \(pwnc\) is a function of \(pw_0\) and \(pwlim\) [Eq. (11)], its future change represents the combined effect of both thermodynamic change (increase in moisture represented here by increase in \(pwlim\)) and the dynamic change (increase in inhibition or stability represented by increase in \(pw_0\)). Figure 11c shows the changes in both \(pwlim\) and \(pw_0\). Both changes are also depicted in the rightward shift of the \(P\)–pw curve and the pw frequency under warming (Figs. 8a,b, dashed lines). The dominance of thermodynamic change is reflected in the stronger response of \(pwlim\) to warming (and therefore the decrease in \(pwnc\)). Both changes in \(pwlim\) and \(pw_0\) in the models as responses to warming are related to the equilibrium climate sensitivity (ECS) of the models (Fig. 11d). Models with larger climate sensitivity have a larger change in \(pwlim\) as one would expect from the Clausius–Clapeyron relation and a corresponding larger increase in \(pw_0\) as stability increases as well.

c. Historical trends

In the last subsection we show that the change in precipitation in response to warming can be represented by the decrease in \(pwnc\), that is, the net decrease in convective inhibition resulting from the competing effects of increase in moisture (\(pwlim\)) and increase in \(pw_0\) (Fig. 11c). This raises the question of whether trends in \(pwlim\) and \(pw_0\) can be detected in the global reanalysis and the observed precipitation. To address this question the trends in 40 years of \(pwlim\) and \(pw_0\) values are calculated from ERA5. Because of the nonlinear nature of the \(P\)–pw relationship, a slight decrease in \(pwnc\) leads to a shift of many points from the evaporation-controlled drizzle regime to the pw-controlled regime. Thus we expect a rapid increase in precipitation near \(pwnc\). To verify this expectation, we computed the historical precipitation trends using 36 years of PERSIANN daily drizzle precipitation (precipitation < 3.12 mm day\(^{-1}\) as defined above). Figure 12 shows the trends normalized by their corresponding historical mean values. The statistical significances of the trend in ERA5 \(pwlim\) and \(pw_0\) as well as that of the drizzle precipitation trend from PERSIANN are tested using the Kendall’s tau (KT) test (Kornbrot 2014) and the \(p\) values shown in the figure indicate that the trends are statistically significant. Even though there are also increasing trends in the critical normalized precipitable water from ERA5 and total precipitation from PERSIANN (not shown), they are not found to be statistically significant according to the KT test. This is consistent with the competing effects of the changes in \(pwlim\) versus \(pw_0\) on the changes in \(pwnc\), rendering larger uncertainty in estimating the changes in \(pwnc\) and the total precipitation compared to the changes in drizzle precipitation, which are determined by changes in \(pw_0\) alone.

5. Conclusions

Several previous studies have documented the excess precipitation over tropical oceans and dry biases over tropical landmasses in multiple generations of CMIP models. Motivated by these persistent biases and the uncertainties in the projected changes, we developed a new framework to study the moisture budgets of the ERA5 dataset and CMIP6 simulations. This framework reveals that the regional distribution of precipitation, specifically whether precipitation occurs preferentially over high
evaporation areas (which leads to “double ITCZ”) or high precipitable water areas (Fig. 10), is related to the vertical structure of moisture flux convergence and how it relates to precipitation intensity. It is shown that the well-documented relationship between precipitation ($P$) and column integrated precipitable water ($pw$) can be uniquely defined by two critical $pw$ values: the $pw$ where precipitation balances evaporation, and the $pw$ where precipitation balances moisture flux convergence (Fig. 6). When the raw $pw$ is scaled by the difference of these two values, one can define a normalized critical precipitable water, $pw_{nc}$, which is a measure of convective inhibition that separates tropical precipitation into two regimes: a local evaporation-controlled regime with widespread drizzle, and a precipitable water–controlled regime with heavy rainfall. Compared to ERA5, most of the 17 CMIP6 model historical simulations examined have both higher $pw_{nc}$ (convective inhibition) and $E_0$ (evaporation and precipitation balance point), and too much drizzle (Fig. 9). It is also shown that the magnitude of the response to warming is also linearly related to the change in $pw_{nc}$. A robust increasing trend in both the $P$ vs $pw_{nc}$ relationship are detected in ERA5 and the implied trend of low-intensity precipitation is also observed in the PERSIANN precipitation dataset (Fig. 12).

There is increased recognition of the role of representation of entrainment and detrainment processes in defining the $P$–$pw$ relationship, not just in the inhibition or promotion of convection by environmental moisture at diurnal and subdiurnal time scales (Derbyshire et al. 2004; Tompkins 2001; Ahmed and Neelin 2018) but also through the impact of convection on the moisture budget of the column itself at longer time scales (Kuo et al. 2017; Singh et al. 2019; Emanuel 2019). In the framework presented in this study, we use daily averages of $P$ and $pw$, and the analytical solutions are derived from a steady-state form of conservation of moisture; thus, they represent a steady-state relationship about which $P$ and $pw$ fluctuate. From the short time scale perspective, the critical normalized precipitable water derived in this study represents a measure of inhibition that the normalized moisture must exceed for deep convection to take place. That inhibition depends on the how rapidly the updraft deepens with increasing precipitation and therefore to what extent the column-integrated moisture convergence compensates for the moisture loss by precipitation. Thus, the normalized precipitable water framework (specifically the critical normalized precipitable water parameter) could be a theoretical tool for interpreting the combined

![Fig. 11.](image-url)

(a) Changes in precip vs change in $pw_{nc}$

(b) Changes in precip vs change in $E_0$

(c) Changes in $pw_0$ and $pw_{lim}$

(d) Changes in $pw_{lim}$ and $pw_0$ vs ECS

FIG. 11. (a) The percentage difference in $P$ vs in $pw_{nc}$ between historical (2000–14) and SSP585 (2086–2100) simulations. (b) As in (a), but for change in $P$ and in $E_0$. (c) SSP585 $pw_0$ and $pw_{lim}$ (mm) vs their historical values. (d) Change in $pw_0$ and $pw_{lim}$ (mm) under the SSP585 scenario vs equilibrium climate sensitivity (ECS).
FIG. 12. (a) The trends in pwlim and pw0 from ERA5 data as well as in the amount of drizzle precipitation (precipitation rate < 3.12 mm day$^{-1}$; see text) averaged over the tropical oceans. All three trends are plotted as deviations from their respective long-term means and pass the Kendall’s tau statistical significance test at the level indicated in the legend.

**APPENDIX**

**Relationship between Normalized Moisture Flux Convergence and Gross Moist Stability**

Using the steady-state form of the dry static energy and moisture equations [Eq. (2.3) in Raymond et al. 2009], normalized gross moist stability can be written as

$$\Gamma_R = -\frac{\int_{ps}^{pt} \nabla \cdot (vs) \, dp}{L \int_{ps}^{pt} \nabla \cdot (vq) \, dp},$$  

(A1)

where $T_R = 300 K$ is a reference temperature, $L$ is the latent heat of condensation, $v$ is horizontal wind, and $s$ is specific moist entropy. Combined with Eq. (3), (A1) can be rewritten as

$$\text{NMFC} = -\frac{\int_{ps}^{pt} \nabla \cdot (vs) \, dp}{L \int_{ps}^{pt} \nabla \cdot (vq) \, dp}.$$

(A3)

Using Eq. (2.1) of Raymond et al. (2009), again for steady state, (A2) can be written as

$$\Gamma_R \text{NMFC} = \frac{T_R(F_s - R)}{L \text{pw}}.$$

(A3)

where $F_s$ is the moist entropy flux due to surface fluxes of heat and moisture and $R$ is the pressure integral of the entropy sink due to radiative cooling. Equation (A3) states that the strength of convection (NMFC) is related to the diabatic heating and stability. This is analogous to the weak temperature gradient representation of the energy conservation (Holton 1992):

$$-S_p \omega = J/C_p,$$

(A4)

where $S_p$ is static stability, $\omega$ is pressure vertical velocity, and $J$ is diabatic heating. Hence it is not surprising that the linear relationship between NMFC with precipitation follows from (A4) (Hagos et al. 2019).

**REFERENCES**


