An Empirical Model for Estimating Soil Thermal Conductivity from Soil Water Content and Porosity

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ABSTRACT

Soil thermal conductivity $\lambda$ is a vital parameter for soil temperature and soil heat flux forecasting in hydrological models. In this study, an empirical model is developed to relate $\lambda$ only to soil volumetric water content $\theta$, and soil porosity $\theta_s$. Measured $\lambda$ values for eight soils are used to establish the empirical model, and data from four other soils are used to evaluate the model. The new model is also evaluated by its performance in the Simple Biosphere Model 2 (SiB2). Results show that the root-mean-square errors (RMSEs; ranging from 0.097 to 0.266 W m$^{-1}$ K$^{-1}$) of the new model estimates of $\lambda$ are lower than those (ranging from 0.416 to 1.006 W m$^{-1}$ K$^{-1}$) for an empirical model of similar complexity reported in the literature earlier. Further, with simple inputs and equations, the new model almost has the accuracy of other more complex models (RMSE of $\lambda$ ranging from 0.040 to 0.354 W m$^{-1}$ K$^{-1}$) that require additional detailed soil information. The new model can be readily incorporated in large-scale models because of its simplicity as compared to the more complex models. The new model is tested for its effectiveness by incorporating it into SiB2. Compared to the original SiB2 $\lambda$ model, the new $\lambda$ model provides better estimates of surface effective radiative temperature and soil wetness. Owing to the newly presented empirical model’s requirement for simple, available inputs and its accuracy, its usage is recommended within large-scale models for applications where detailed information about soil composition is lacking.

1. Introduction

Soil thermal conductivity $\lambda$ is a primary property related to soil heat flux (Bristow 2002). For a given type of soil, its magnitude depends largely on the soil volumetric water content $\theta$, porosity $\theta_s$, bulk density $\rho_b$, texture, and
mineral composition (de Vries 1963; Campbell 1985). Several $\lambda$ models depending on these soil parameters have been presented. Kersten (1949) proposed an empirical $\lambda$ model with only one input parameter, soil bulk density. The model has limited applicability at lower soil water contents. De Vries (1963) presented a physically based model that has been applied in numerous studies (e.g., Campbell et al. 1994). The model treats soil as a mixture of ellipsoidal particles in the continuous media of air and water and requires various input parameters (Tarnawski and Wagner 1992; Bachmann et al. 2001). In the de Vries (1963) model, the selection of critical water content and shape factors affects $\lambda$ estimations drastically (Horton and Wierenga 1984; Ochsner et al. 2001). Johansen (1975) proposed the concept of normalized thermal conductivity. The Johansen (1975) model with four parameters, the degree of saturation $S$, soil bulk density, soil porosity, and mineral composition, can provide reliable estimations of $\lambda$ for various soils (Farouki 1981, 1982; Tarnawski and Wagner 1992). Based on measurements reported by McInnes (1981), Campbell (1985) proposed an empirical $\lambda$ model with five input parameters. Based on the Johansen (1975) model, Côté and Konrad (2005) developed an improved model by introducing an empirical relationship between the normalized thermal conductivity and degree of saturation. The thermal conductivity of dry soil $\lambda_{dry}$ is a function of soil porosity instead of soil bulk density that is used in Johansen (1975). Later, based on the Johansen (1975) model, Lu et al. (2007) described a relationship between thermal conductivity and volumetric water content of soil and presented thermal conductivity equations for “coarse” and “fine” textured soils. Recently, Lu et al. (2014) presented a model for estimating $\lambda$ from soil volumetric water content, porosity, bulk density, and texture, while Nikoosokhan et al. (2015) developed a $\lambda$ model with soil volumetric water content, sand fraction $f_s$, and dry soil specific weight $\gamma_d$.

All $\lambda$ models require various soil specific parameters. However, some soil parameters are difficult to determine, which limits the use of these $\lambda$ models in large-scale land surface models. For practical purposes, empirical $\lambda$ models that depend on only a few readily obtainable soil parameters warrant consideration for inclusion in land surface models. The Simple Biosphere Model 2 (SiB2; Sellers et al. 1996) uses a $\lambda$ model taken from Camillo and Schmugge (1981) that only has two input parameters: soil porosity and surface soil layer wetness. However, with the availability of recent observations of $\lambda$ values, the possibility exists for developing an improved simple $\lambda$ model that outperforms the Camillo and Schmugge (1981) model.

The objectives of this study are to 1) develop a new empirical model of $\lambda$ based on porosity and volumetric water content for general use in large-scale models where only information of porosity and volumetric water content of soil are available, 2) evaluate the new model by comparing it to the simple model of Camillo and Schmugge (1981) and to some other more complex $\lambda$ models (de Vries 1963; Johansen 1975; Côté and Konrad 2005; Lu et al. 2007; Lu et al. 2014; Nikoosokhan et al. 2015), and 3) test the new model in the SiB2.

2. Material and methods

a. Datasets

Measured soil thermal conductivity values used in this study (Table 1) are obtained from Lu et al. (2007). Table 1 lists texture, particle size distribution (PSD), and porosity (i.e., $\theta_c$; equivalently saturated volumetric water content) of 12 soils. Soils 1–8 include almost all of the soil types in the 12 soils, so that they can be used as an integrated dataset, and the other four soils (soils 9–12) then can be used as a verification dataset. In the following sections, measurements from soils 1–8 are used to parameterize the new model, and then the new model is examined with soils 9–12. Soil samples were air dried, ground, and sieved through a 2-mm screen, and soil PSD was determined with the pipette method (Gee and Or 2002). Soil was packed into columns (50.2-mm inner diameter and 50.2-mm high) to desired bulk density, and a thermo–time domain reflectometry (thermo-TDR) probe (Ren et al. 1999) was used to measure the thermal conductivity of each packed soil column. Thermo-TDR probes measure thermal properties with a heat-pulse method. Details on thermo-TDR measurements are presented in Ren et al. (1999), Ochsner et al. (2001), and Ren et al. (2003). Lu et al. (2007) provide details on the heat-pulse measurements.

<table>
<thead>
<tr>
<th>Soil No.</th>
<th>Texture</th>
<th>Sand (%)</th>
<th>Silt (%)</th>
<th>Clay (%)</th>
<th>$\theta_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sand</td>
<td>94</td>
<td>1</td>
<td>5</td>
<td>0.405</td>
</tr>
<tr>
<td>2</td>
<td>Sand</td>
<td>93</td>
<td>1</td>
<td>6</td>
<td>0.432</td>
</tr>
<tr>
<td>3</td>
<td>Sandy loam</td>
<td>67</td>
<td>21</td>
<td>12</td>
<td>0.419</td>
</tr>
<tr>
<td>4</td>
<td>Loam</td>
<td>40</td>
<td>49</td>
<td>11</td>
<td>0.456</td>
</tr>
<tr>
<td>5</td>
<td>Silt loam</td>
<td>27</td>
<td>51</td>
<td>22</td>
<td>0.483</td>
</tr>
<tr>
<td>6</td>
<td>Silt loam</td>
<td>11</td>
<td>70</td>
<td>19</td>
<td>0.479</td>
</tr>
<tr>
<td>7</td>
<td>Silty clay loam</td>
<td>19</td>
<td>54</td>
<td>27</td>
<td>0.491</td>
</tr>
<tr>
<td>8</td>
<td>Silty clay loam</td>
<td>8</td>
<td>60</td>
<td>32</td>
<td>0.507</td>
</tr>
<tr>
<td>9</td>
<td>Clay loam</td>
<td>32</td>
<td>38</td>
<td>30</td>
<td>0.522</td>
</tr>
<tr>
<td>10</td>
<td>Silt loam</td>
<td>2</td>
<td>73</td>
<td>25</td>
<td>0.554</td>
</tr>
<tr>
<td>11</td>
<td>Loam</td>
<td>50</td>
<td>41</td>
<td>9</td>
<td>0.489</td>
</tr>
<tr>
<td>12</td>
<td>Sand</td>
<td>92</td>
<td>6</td>
<td>1</td>
<td>0.415</td>
</tr>
</tbody>
</table>
made on oven-dried soil samples and on moist soil samples.

b. Models

1) SIMPLE \( \lambda \) MODEL

In the SiB2 (Sellers et al. 1996), \( \lambda \) is determined with an equation presented by Camillo and Schmugge (1981):

\[
\lambda = \frac{1.5(1 - \theta_s) + 1.3 \theta_s S_r}{0.75 + 0.65 \theta_s - 0.4 \theta_s S_r} (0.4186),
\]

where \( S_r \) is the degree of saturation (\( \theta / \theta_s \)). With \( \theta \) and \( \theta_s \) as input parameters, \( \lambda \) can be estimated with Eq. (1).

2) COMPLEX \( \lambda \) MODELS

(i) De Vries model

De Vries (1963) proposed a model to calculate soil thermal conductivity of soil as a weighted sum of the conductivities of the constituents:

\[
\lambda = \frac{k_g \theta_s \lambda_g + \sum_{i=1}^{n} k_i \theta_i \lambda_i}{k_g \theta_s + \sum_{i=1}^{n} k_i \theta_i},
\]

where \( k \) is the weighting factor; \( \theta \) is the volume fraction; \( n \) is the number of soil constituents; and subscripts \( g, i \), and \( w \) indicate the phases of gas, solid, and water, respectively. The solid phase includes sand, silt, clay, and organic matter (not considered in this study because of its scarcity). Following De Vries (1963), the thermal conductivities of sand, silt, clay, and air are set as 8.53, 2.93, 2.93, and 0.025 W m\(^{-1}\) K\(^{-1}\), respectively. The thermal conductivity of the gas phase (i.e., \( \lambda_g \)) is given by

\[
\lambda_g = \lambda_a + \frac{h_f f_w L_w \Delta \rho D_w}{P_a - e_s},
\]

where \( \lambda_a \) is the thermal conductivity of air, \( h_f \) is the relative humidity in the soil, \( L_w (44100 \text{ J mol}^{-1}) \) is the latent heat of vaporization, \( \Delta [145 \text{ Pa (°C)}^{-1}] \) is the slope of the saturation vapor pressure, \( \rho (41.4 \text{ mol m}^{-3}) \) is the molar density of air, \( D_w (2.42 \times 10^{-5} \text{ m}^2 \text{s}^{-1}) \) is the vapor diffusivity for soil, \( P_a (101 \text{ kPa}) \) is the atmospheric pressure, and \( e_s (2340 \text{ Pa}) \) is the saturation vapor pressure. The second term in Eq. (8) is the latent heat term, which is responsible for almost all of the temperature dependence of soil thermal conductivity. Values of \( L_w, \Delta, \rho, D_w, \) and \( e_s \) are provided by Campbell and Norman (1998, their Tables A.1, A.2, and A.3); \( P_a \) is 101 kPa; and \( h_f \) is assumed to be zero when \( \theta < \theta_c \); \( \theta_c \) is the critical water content, which is estimated as the water content at 33-kPa water pressure head and to be unity when \( \theta \geq \theta_c \). The value of \( \theta_c \) is approximately 0.03 m\(^3\) m\(^{-3}\) for coarse-textured soils and between 0.05 and 0.10 m\(^3\) m\(^{-3}\) for fine-textured soils (de Vries 1963). This study
assumed the values of $\theta_c$ to be 0.075 m$^3$ m$^{-3}$ for soils 9 and 10 and 0.03 m$^3$ m$^{-3}$ for soils 11 and 12. For $\theta \geq \theta_c$, the shape factor of air $g_a$ is computed following de Vries (1963), as

$$g_a = 0.035 + \frac{\theta}{\theta_c}(0.333 - 0.035).$$

(9)

For $\theta < \theta_c$, the value of $g_a$ is given by

$$g_a = 0.013 + \frac{\theta}{\theta_c}(g_{a_{dc}} - 0.013).$$

(10)

where $g_{a_{dc}}$ is the value of Eq. (9) at $\theta_c$. When $\theta < \theta_c$, a linear interpolation of $\lambda$ versus $\theta$ between its dry value and the value at $\theta_c$ is recommended, and $\lambda_{dry}$ is computed by multiplying the value of Eq. (2) with 1.25 at $\theta = 0$ (de Vries 1963).

(ii) Lu et al. model

Lu et al. (2014) proposed the following exponential function to express the nonlinear behavior of $\lambda$ as a function of $\theta$, texture, and $\rho_b$:

$$\lambda = \lambda_{dry} + \exp(\beta - \theta^{-\alpha}) \quad \theta > 0,$$

(11)
where \( \lambda_{\text{dry}} \) is the thermal conductivity of an oven-dried soil sample (W m\(^{-1}\) K\(^{-1}\)) and parameters \( \alpha \) and \( \beta \) are shape factors of the \( \lambda(\theta) \) curve, which are related to soil texture and \( \rho_b \). The \( \lambda_{\text{dry}} \) is estimated from \( \theta_s \) by the linear equation of Lu et al. (2007):

\[
\lambda_{\text{dry}} = -0.56\theta_s + 0.51. \tag{12}
\]

A linear relationship between \( f_{\text{cl}} \) and \( \alpha \) is established as follows:

\[
\alpha = 0.67f_{\text{cl}} + 0.24. \tag{13}
\]

The parameter \( \beta \) is estimated by \( \rho_b \) and \( f_a \) with the following multiple regression equation:

\[
b = 4.57 - 9.89\theta_s. \tag{22}
\]
Therefore, when soil porosity, texture (i.e., $f_{cl}$ and $f_a$), and $\rho_b$ data are available, $\lambda$ can be estimated as a function of $\theta$ using Eqs. (11)–(14).

(iii) Nikoosokhan et al. model

Nikoosokhan et al. (2015) used the normalized thermal conductivity $\lambda_e$ proposed by Johansen (1975) in their model:

$$\lambda = \lambda_e (\lambda_{\text{sat}} - \lambda_{\text{dry}}) + \lambda_{\text{dry}},$$

where $\lambda_{\text{sat}}$ is the thermal conductivity of saturated soil.

Two sets of linear relationships of $\lambda_{\text{sat}}$ and $\lambda_{\text{dry}}$ to sand fraction (i.e., $f_a$) and dry soil specific weight (i.e., $\gamma_d$) were proposed:

$$\lambda_{\text{sat}} = 0.53f_a + 0.1\gamma_d$$

and

$$\lambda_{\text{dry}} = 0.087f_a + 0.019\gamma_d.$$  

For a wide range of sand fractions ($0 < f_a < 1$), the dry soil specific weight falls within the range $11 < \gamma_d < 20$. When $\gamma_d$ has units of kilonewtons per cubic meter and $\rho_b$ has units of grams per cubic centimeter, they relate as

$$\gamma_d = 9.8\rho_b,$$

where $9.8 \text{ (m s}^{-2}\text{)}$ is gravitational acceleration. To relate $\lambda_e$ to $S_r$, an equation proposed by Côté and Konrad (2005) is used:

$$\lambda_e = \frac{\kappa S_r}{1 + (\kappa - 1)S_r},$$

where $\kappa$ is a texture-dependent parameter and changes linearly with $f_a$:

$$\kappa = 4.4f_a + 0.4.$$
Detailed descriptions of the models by Johansen (1975), Côté and Konrad (2005), and Lu et al. (2007) can be found in Lu et al. (2007).

3) THE NEW EMPIRICAL MODEL

Investigators (e.g., de Vries 1963; Camillo and Schmugge 1981; Oechsner et al. 2001; Jury and Horton 2004; Lu et al. 2007; Chen 2008; Smits et al. 2010; Lu et al. 2014) have reported that θι and θ are sensitive input parameters for λ determination, and there often is an exponential relationship between λ and θ. Lu et al. (2007) described a normalized $\lambda \sim S_i$ relation with an exponential equation. After examining previously proposed $\lambda$ equations, we were inspired to investigate the practicality of a new, simple equation for use in large-scale surface process models. Our newly proposed empirical $\lambda \sim \theta$ model is based on Jury and Horton (2004) as follows:

$$\lambda(\theta) = a - b \exp(-c\theta).$$  (21)

We assume that $a$ and $b$ are empirical parameters that are related to $\theta_s$. Jury and Horton (2004) set the parameter $c$ to be 4 for their example soil. To quantify the $c$ value for soils 1–8, we did the following:

$$\lambda_i = a - b \exp(-c\theta_1)$$

$$\lambda_2 = a - b \exp(-c\theta_2)$$

$$\lambda_3 = a - b \exp(-c\theta_3)$$

$$\lambda_4 = a - b \exp(-c\theta_4)$$

$$\lambda_5 = a - b \exp(-c\theta_5)$$

$$\lambda_6 = a - b \exp(-c\theta_6)$$

$$\lambda_7 = a - b \exp(-c\theta_7)$$

$$\vdots$$

$$\lambda_i = a - b \exp(-c\theta_i).$$  (21a)

Substituting two equations within three interval points, and combining them as an equation set, results in

$$\begin{align*}
\lambda_i - \lambda_{i+3} &= -b[\exp(-c\theta_i) - \exp(-c\theta_{i+3})] \\
\lambda_{i+3} - \lambda_{i+6} &= -b[\exp(-c\theta_{i+3}) - \exp(-c\theta_{i+6})].
\end{align*}$$  (21b)

Dividing one equation in Eq. (21b) by the other equation gives

$$\frac{\lambda_i - \lambda_{i+3}}{\lambda_{i+3} - \lambda_{i+6}} = \frac{\exp(-c\theta_i) - \exp(-c\theta_{i+3})}{\exp(-c\theta_{i+3}) - \exp(-c\theta_{i+6})}$$  (21c)

and

$$0 = (\lambda_i - \lambda_{i+3})[\exp(-c\theta_{i+3}) - \exp(-c\theta_{i+6})]$$

$$- (\lambda_{i+3} - \lambda_{i+6})[\exp(-c\theta_i) - \exp(-c\theta_{i+3})].$$  (21d)

Parameter $c$ can be obtained from Eq. (21c) with three groups of observed thermal conductivity data. Parameter $c$ for soils 1–8 ranges from 2.96 to 6.04, with a mean value of 3.90. The parameter $c$ for all of the soils 1–12 was then fixed as 3.90, which is close to 4, the value of $c$ in Jury and Horton (2004). Parameters $a$ and $b$ in Eq. (21a) are determined by nonlinear regression.

A quadratic function and a linear function are used to describe the behavior of $a$ and $b$ as a function of $\theta_s$:

$$a = a_1 + a_2 \theta_s + a_3 \theta_s^2$$  and  (22)

$$b = b_1 + b_2 \theta_s,$$  (23)

where $a_1, a_2, a_3, b_1,$ and $b_2$ are empirical constants, which can be derived with predetermined values of $a$ and $b$ and observed values of $\theta_s$. Variables $\theta_s$ are the measurements of soil porosities for soils 1–8. The selection of the quadratic and linear formats of Eqs. (22) and (23) is a compromise between accuracy and equation complexity. It should be noted that the new empirical model is restricted to soil porosities ranging from 0.40 to 0.55, which covers the majority of field soil porosities.

c. Model calibration and testing

To calibrate the new empirical model, we use measured thermal conductivity values for eight soils reported by Lu et al. (2007) (soils 1–8 in Table 1). To evaluate the new empirical model, we use measured thermal conductivity values for four additional soils reported by Lu et al. (2007) (soils 9–12 in Table 1). The parameters $a$ and $b$ for the eight soils (soils 1–8) are
Table 3. Bias of the de Vries (1963), Johansen (1975), Côté and Konrad (2005), Lu et al. (2007), Lu et al. (2014), Nikoosokhan et al. (2015), and Camillo and Schmugge (1981) models and the new model in estimating thermal conductivity of four soils at specified water content (i.e., \( \theta \)) range (W m\(^{-1}\) K\(^{-1}\)).

\[
\begin{array}{|c|c|c|c|c|c|c|c|c|}
\hline
\text{Soil No.} & \theta \text{ range} & \text{De Vries model} & \text{Johansen model} & \text{Côté and Konrad model} & \text{Lu et al. (2007) model} & \text{Lu et al. (2014) model} & \text{Nikoosokhan et al. model} & \text{Camillo and Schmugge model} & \text{New model} \\
\hline
9 & <0.10 & -0.052 & 0.046 & -0.061 & -0.018 & -0.046 & -0.151 & -0.016 & -0.108 \\
 & >0.10 & -0.033 & 0.077 & 0.101 & 0.031 & 0.047 & -0.002 & 0.539 & 0.023 \\
10 & <0.10 & 0.027 & 0.074 & -0.037 & 0.005 & -0.033 & 0.013 & 0.001 & -0.294 \\
 & >0.10 & 0.128 & 0.142 & 0.170 & 0.091 & 0.166 & 0.440 & 0.577 & -0.084 \\
11 & <0.10 & -0.027 & 0.095 & -0.005 & -0.090 & -0.110 & -0.084 & 0.067 & 0.124 \\
 & >0.10 & -0.006 & 0.206 & 0.206 & 0.058 & -0.105 & 0.045 & 0.788 & 0.272 \\
12 & <0.10 & -0.117 & 0.007 & -0.064 & -0.076 & -0.142 & -0.136 & 0.271 & -0.035 \\
 & >0.10 & -0.012 & -0.162 & -0.133 & -0.111 & 0.127 & 0.127 & 1.348 & 0.310 \\
\hline
\end{array}
\]

estimated by fitting Eq. (21) to the measured \( \lambda \) and \( \theta \) data. The values of \( a \), \( b \), and \( \theta_s \) are obtained to obtain parameters \( a_1 \), \( a_2 \), and \( a_3 \) in Eq. (22) and parameters \( b_1 \) and \( b_2 \) in Eq. (23).

The new empirical model was tested by using \( \lambda \) and \( \theta \) measurements of four soils (soils 9–12). The RMSE and the bias in model estimations are calculated as follows:

\[
\text{RMSE} = \sqrt{\frac{\sum (\lambda_m - \lambda_{est})^2}{m}} \tag{24}
\]

and

\[
\text{bias} = \frac{\sum (\lambda_m - \lambda_{est})}{m} \tag{25}
\]

where \( m \) is the number of measurements and \( \lambda_m \) and \( \lambda_{est} \) represent the measured and estimated values, respectively.

Finally, the RMSE and bias of the new empirical model were compared against the values of more complex models developed by de Vries (1963), Johansen (1975), Côté and Konrad (2005), Lu et al. (2007), Lu et al. (2014), and Nikoosokhan et al. (2015) and the simple model of Camillo and Schmugge (1981).

The new model was also tested by incorporating it into SiB2 to replace the original SiB2 \( \lambda \) model, the Camillo and Schmugge (1981) model. The other settings of SiB2 were the same as those used in Gao et al. (2004). The outputs of SiB2 coupled with the new and the original \( \lambda \) models for surface effective radiative temperature \( T_{\text{eff}} \) and soil wetness were compared with observed values.

3. Results and discussion

The main factors influencing soil thermal conductivity \( \lambda \) are soil water content, porosity, bulk density, mineral composition, and temperature. Water content is the most important factor, and it can vary significantly under field conditions (Lu et al. 2007). Figure 1 shows the \( \lambda (\theta) \) measurements (symbols) and the fitted curves [Eq. (21)] for eight soils (soils 1–8). In general, the fitted curves well represent \( \lambda \) as a function of \( \theta \), with coefficients of determination \( R^2 \) larger than 0.94. There are no significant deviations between the measured and fitted values. The values of \( a \) range from 1.51 (soil 7) to 2.64 (soil 1), and the values of \( b \) range from 1.40 (soil 7) to 2.04 (soil 1), which are quite similar to the values from Jury and Horton (2004), \( a = 1.88 \) and \( b = 1.67 \). Generally, \( \lambda \) increases with \( \theta \), but the increase rate decreases at larger \( \theta \). For a given value of \( \theta \), the value of \( \lambda \) ranges considerably among soils as \( \theta \) varies among the soils. For soils with a relatively large sand fraction, generally sharper increasing rates are found. For example, for soils 1 and 2, the values of \( b \) are 2.04 and 1.85, which are larger than those for the other soils. Soils 3–8 are loamy soils and the values of \( b \) are smaller, varying from 1.40 to 1.84. At large \( \theta \), the sandy soils have larger \( \lambda \) values (about 2.0 W m\(^{-1}\) K\(^{-1}\)) than the loamy and clayey soils (~1.5 W m\(^{-1}\) K\(^{-1}\)). These characteristics are consistent with the findings of Lu et al. (2014).

Maximum soil thermal conductivity decreases as soil porosity increases (Farouki 1986; Oechner et al. 2001; Lu et al. 2014). Figure 2 shows the parameter \( a \) obtained from Eq. (6) as a function of \( \theta_s \). Clearly, \( a \) decreases with \( \theta_s \), with \( R^2 = 0.92 \) and \( p < 0.05 \) [the \( p \) value (0.05) indicates that the equation obtained by regression is statistically significant]. The fitted values of \( a_1 \), \( a_2 \), and \( a_3 \) are 19.93, −69.16, and 64.79. Figure 3 shows the \( b-\theta_s \) relationship, with \( R^2 = 0.94 \) and \( p < 0.005 \) [the \( p \) value (0.05) indicates that the equation obtained by regression is statistically significant]. The fitted values of \( b_1 \) and \( b_2 \) are 4.37 and −5.89. In general, the fitted curves well represent the relationships of \( a \) and \( b \) with \( \theta_s \), and...
Fig. 5. Comparison of estimated λ values from the (a) de Vries (1963), (b) Johansen (1975), (c) Côté and Konrad (2005), (d) Lu et al. (2007), (e) Lu et al. (2014), (f) Nikoosokhan et al. (2015), and (g) Camillo and Schmugge (1981) models and (h) the new model vs measured data for soils 9–12 from Lu et al. (2007). The regression equations and $R^2$ are included.
there are no significant deviations between the estimated and fitted values.

Soils 9–12 are used to evaluate the new model. For comparison, the results of more complex models (i.e., de Vries 1963; Johansen 1975; Côté and Konrad 2005; Lu et al. 2007; Lu et al. 2014; Nikoosokhan et al. 2015) and a simple model [i.e., the Camillo and Schmugge (1981) model] are also shown. Figure 4 presents the $\lambda \sim \theta$ measurements along with the eight model estimations. The RMSEs of the model estimations are presented in Table 2. The results of the more complex models agree well with the observed $\lambda$ for these soils, except for the result of the Nikoosokhan et al. (2015) model in soil 10 (RMSE = 0.354 W m$^{-1}$ K$^{-1}$), and the RMSE of the more complex models ranges from 0.040 to 0.203 W m$^{-1}$ K$^{-1}$. The de Vries (1963) model and the Lu et al. (2007) model perform the best, followed by the Lu et al. (2014) model. The RMSE of the Camillo and Schmugge (1981) model is largest, and at higher water contents the modeled values are smaller than the measured values. The new empirical model has RMSEs ranging from 0.097 to 0.266 W m$^{-1}$ K$^{-1}$, which is similar to, but slightly larger than, those of the more complex models. The new empirical model performs much better than the Camillo and Schmugge (1981) model, even though both models used the same simple inputs.

Following Lu et al. (2007), the results are discussed in the ranges $\theta < 0.1$ m$^3$ m$^{-3}$ (referred to as lower water contents) and $\theta > 0.1$ m$^3$ m$^{-3}$ (referred to as higher water contents). Biases of the eight models in the two ranges for soils 9–12 are shown in Table 3. Among the more complex models, the de Vries (1963) model overpredicts $\lambda$ for entire $\theta$ ranges, except for soil 10, where it underpredicts $\lambda$ for the entire $\theta$ range. The Johansen (1975) model underpredicts $\lambda$ for entire $\theta$ ranges. The Côté and Konrad (2005) model, the Lu et al. (2007) model, the Lu et al. (2014) model, and the Nikoosokhan et al. (2015) model generally overpredict $\lambda$ at lower water contents and underpredict $\lambda$ at higher water contents. The Camillo and Schmugge (1981) model underpredicts $\lambda$ drastically at higher water contents. The new empirical model tends to overpredict $\lambda$ at lower water contents, and it tends to underpredict $\lambda$ at higher water contents.

Figure 5 comprehensively compares the estimations from the eight models to the measurements for soils 9–12. It can be seen that, except for the Camillo and Schmugge (1981) model, measured and estimated values
from the models are generally consistent with each other, the slopes of the regression lines are close to unity, and $R^2$ values are larger than 0.90. The $R^2$ value of the regression for the Nikoosokhan et al. (2015) model is much less than those of the other complex models, which is caused by its poor performance for soil 10 with low sand fraction, as shown in Fig. 4. Although the slope of the regression line for the new empirical model deviates more from unity than those for the complex models, the measured and estimated values are generally distributed along the 1:1 line. However, the estimated values of $\lambda$ by the Camillo and Schmugge (1981) model are generally smaller than the measured values with a slope of only 0.168. Therefore, as a model with only simple inputs, the new empirical model performs better than the Camillo and Schmugge (1981) model.

Generally, the results from the new empirical model deviate more from the measurements than do the complex models. However, the new empirical model requires only soil volumetric water content and porosity as inputs, while the complex models require several inputs (e.g., soil volumetric water content, porosity, bulk density, and particle size and/or mineral composition information). Some of the input information requires laboratory measurements, and the information is not always available at larger scales. Observation of water content is more available than observation of other properties, such as bulk density and mineral composition. Satellite remote sensing can be used to observe soil volumetric water contents for large land areas (Liu et al. 2012; Blunden and Arndt 2014).

To evaluate the usefulness of the new model, we ran SiB2 (Sellers et al. 1996) with the current $\lambda$ model and again with the new $\lambda$ model. SiB2 outputs for $T_{eff}$ and soil wetness were compared with observed values. Figure 6 presents measured and modeled values of $T_{eff}$. It is obvious that SiB2 with the new $\lambda$ model outperforms SiB2 with the original $\lambda$ model, although both sets of calculations tended to overestimate (underestimate) $T_{eff}$ in daytime (nighttime). As shown in Fig. 4, the estimated soil thermal conductivities using the new $\lambda$ model were in closer agreement with the measurements than were the Camillo and Schmugge (1981) estimates. The new model estimates of $\lambda$ values tended to be larger than the original model estimates. Soil temperature amplitude is smaller, approaching the
measurements, when \( \lambda \) is larger. Although \( T_{\text{eff}} \) modeled by SiB2 coupled with the new thermal conductivity model is better than with the original model, it is underestimated at night. The reason for the underestimation may be that the weak turbulent exchange and small surface energy flux at night cause relatively large errors in the \( T_{\text{eff}} \). A scatterplot of the modeled \( T_{\text{eff}} \) by SiB2 with the new and the original \( \lambda \) values against direct measurements is given in Fig. 7. Results based on the new \( \lambda \) model are closer to the 1:1 line than those for the original \( \lambda \) model. The RMSE (2.30 K) of the \( T_{\text{eff}} \) modeled by SiB2 with the new \( \lambda \) model is significantly less than that (4.23 K) modeled with the original \( \lambda \) model. However, the over- and underestimations of \( T_{\text{eff}} \) sometimes cancel each other in the original \( \lambda \) model, so the bias (2.10 K) from the new \( \lambda \) model is slightly larger than that (2.05 K) for the original \( \lambda \) model.

The SiB2 with the new thermal conductivity model also impacted soil wetness estimations, as shown in Fig. 8. It is apparent that the patterns of the time series of the soil wetness modeled by SiB2 with the original and new \( \lambda \) models are similar, but the magnitude of soil wetness calculated when using the new \( \lambda \) model is slightly larger for all three layers compared to the original model. This is true especially after convective rain occurs for the surface zone (0–0.02 m), but not for the other two layers, root zone (0.02–0.30 m) and recharge zone (0.30–1.0 m). The soil wetness for the surface zone (0–0.02 m) and root zone (0.02–0.30 m) change drastically with weather conditions, while the recharge zone (0.30–1.0 m) hardly varies even with rain. We speculate that the lower \( T_{\text{eff}} \) modeled by the SiB2 with the new \( \lambda \) model produced less evaporation from bare soil.

4. Conclusions

A simple empirical model for estimating soil thermal conductivity has been presented. The model requires only two input parameters, soil porosity and volumetric water content. The new empirical model is able to well estimate \( \lambda \) over the whole range of water content for soils of various porosities. The RMSEs of \( \lambda \) estimations for the new empirical model are similar to those of more complex models, but significantly smaller than those of the Camillo and Schmugge (1981) model. Because the new empirical model does not require detailed soil information, it shows larger biases than the more complex models. The new model is tested in SiB2 and provides
better estimates of the $T_{\text{eff}}$ than the original SiB2. The new empirical $\lambda$ model is more practical than the more complex models for use in land surface models for large-scale calculations of surface energy and water fluxes.

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