Atmospheric Quasi-Biennial Oscillations

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ABSTRACT

A detailed analysis has been made of a quasi-biennial oscillation (QBO) in the tropospheric ultralong waves of the Southern Hemisphere and interrelationships with the QBO in zonal mean westerly winds of the equatorial stratosphere. In spite of the fact that highly significant spectral peaks at a period close to 26 months occur in both phenomena, results of cross-spectral analysis reveal that the two QBO's are unrelated. The manner in which these QBO's relate to a recently discovered QBO in the 500 mb zonal mean westerly winds throughout the Southern Hemisphere is also considered.

1. Introduction

The presence of a quasi-biennial oscillation (QBO) in the atmosphere is widely recognized and it has generally been assumed that QBO's in different parameters and different regions of the atmosphere are related in some way. This paper challenges that view by presenting evidence which indicates that there are at least two quasi-biennial oscillations that do not appear to be related in spite of the fact that both have a spectral peak, significant at the 99.9% level, at a period close to 26 months. The first QBO is in the zonal winds of the equatorial stratosphere, and the second is in a regional meridional index of the sea level atmospheric circulation in the Southern Hemisphere whose fluctuations correspond mainly to those in wavenumber 3.

There is very clear evidence for the QBO in the equatorial stratospheric zonal winds (e.g., Coy, 1979) and a theory has been developed which explains most of its features (Holton and Lindzen, 1972; Wallace, 1973; Plumb, 1977). The largest amplitude of the QBO occurs in the tropics, but it is a pervasive phenomenon which is coherent throughout the stratosphere although with systematic phase changes with latitude and height (Newell et al., 1974; Tucker, 1979). Owing to the zonal symmetry, the QBO in the equatorial stratosphere can be well represented by a single station (Wallace, 1973) and the station with the longest record is Balboa.

Evidence for QBO's in the troposphere is also widespread. Landsberg (1962) and Lamb (1972) provide reviews of evidence for quasi-biennial phenomena in the atmosphere and other geophysical phenomena, some dating back thousands of years. Many other studies have more recently produced evidence of a QBO in the troposphere (Angell and Korshover, 1964, 1968, 1974, 1975; Miller et al., 1967, 1974; Brier, 1968; Wright, 1968; Angell et al., 1969; Murray and Moffitt, 1969; Darrt and Belmont, 1970; Kutzbach, 1970; Pittock, 1971; Wagner, 1971; Newell et al., 1974; Gordon and Wells, 1975; Ebdon, 1975; Trenberth, 1975, 1976a). Some studies (e.g., Berlage, 1957) have suggested that the Southern Oscillation exhibits a QBO, but cross-spectral analysis shows that the time scale of the Southern Oscillation is primarily 3–6 years (Trenberth, 1976b) and that the QBO is a separate (but not necessarily independent) phenomenon. [Note that the period of the Southern Oscillation appears to be double that of the QBO (e.g., Trenberth, 1976b).]

In spite of the above evidence, it is not clear whether there is only one QBO or multiple QBO's in the troposphere, and no coherent pattern can be discerned. Tropospheric QBO's remain an enigma without an accepted theoretical explanation. Many studies have therefore assumed a link between tropospheric QBO's and the QBO in the equatorial stratosphere, although such a link is not clearly indicated by current theories. For instance, Ebdon (1975) attempted to isolate aspects of the QBO in the troposphere by compositing patterns of sea level pressure and 500 mb contours for mid-season months when the phase of the equatorial stratospheric winds was either strong easterly or westerly. He found significant differences in only January and July, which Madden (1976) found were above the noise level. However, in general, the picture of tropospheric QBO's that emerges is rather confused. For short periods a relationship with the stratospheric QBO can be determined, but the relative phase and patterns of progression do not appear to be consistent throughout the available record.

Another consideration is the importance of any
tropospheric QBO and this has not been clearly indicated. For the most part the amount of variance explained by a QBO has not been shown to be very large. Brier (1968) found a QBO to explain only ~2% of the monthly variance in a sea level zonal wind index in the Northern Hemisphere. Many analyses do not assess the significance of the QBO at all, but appear to assume its presence and enhance it by filter analysis. However, most studies do indicate less significance in midlatitudes of the Northern Hemisphere than elsewhere.

On the other hand, in the Southern Hemisphere, Trenberth (1975, 1976a) found the QBO to be very marked in some circulation indices. Trenberth (1975) originally found the QBO in a time series of coefficients of the second eigenvector P2 of an empirical orthogonal function analysis of sea level pressure fields in the Australasian sector of the hemisphere (10°–60°S, 110°E–150°W) for 1959–72. The P2 pattern has centers of action near Hobart and southeast of Chatham Islands, and appeared to correspond to wavenumber 3 (Trenberth, 1975). Hobart and Chatham Islands are near 43°S at 147½°E and 176°W, respectively, and Trenberth (1976a) found that the simple pressure difference between these stations, denoted the M1 index, was highly correlated with P2 and also exhibited a pronounced QBO. Further, the time series of M1 could be extended back to June 1929, when Chatham Islands station was established. M1 is therefore an index of the meridional flow across New Zealand as part of the ultralong waves in the Southern Hemisphere and it appears to undergo a pronounced and statistically significant QBO of any parameter representing the tropospheric flow. Trenberth (1975, 1976a) performed spectral analysis of P2 and M1. The QBO was responsible for about 10% of the monthly variance for the entire period from June 1929–December 1973, but was more prominent after 1954 and accounted for 17% of the variance for 1954–73.

We have therefore performed a cross-spectral analysis of M1 with the zonal winds at 50 and 30 mb at Balboa in order to consider whether and how the two QBO's may be related. The manner in which these QBO's relate to a recently discovered QBO in the 500 mb zonal mean westerly winds of the Southern Hemisphere (Trenberth, 1979) is also considered. The latter is remarkably coherent throughout the Southern Hemisphere and is manifest as a systematic progression of zonal mean wind anomalies from low to high latitudes with a quasi-biennial frequency. Although the evidence is not conclusive owing to the short record length of hemispheric data available in the Southern Hemisphere, results indicate that the QBO in the zonal mean westerly winds in the troposphere may be related to that in the stratosphere, but that a second independent

![Fig. 1. Normalized power spectra for (a) M1 and (b) Balboa 500 mb zonal wind. The null hypothesis of white noise is plotted along with the 95, 99 and 99.9% significance levels for spectral peaks. The bandwidth (bw) is plotted and corresponds to 18 degrees of freedom.](image)

QBO also exists in the ultralong waves of the Southern Hemisphere.

2. Analysis of the QBO's

In this analysis we have used 28 years (336 months) of data, from September 1950 to August 1978. Monthly values of the zonal wind at 50 and 30 mb for Balboa, Canal Zone (9°N, 80°W), beginning in September 1950, were kindly supplied by Dr. J. Angell. M1 values have been updated and may be derived from the Hobart and Chatham Islands observations available from U.S. Monthly Climatic Data for the World.

Normalized power spectra of M1 and the 50 mb wind at Balboa (henceforth referred to as B50) are shown in Fig. 1. The series were prewhitened by removing the 28-year monthly means and detrending the residuals. The latter has almost no effect since the trends were small (see Fig. 4). Ten percent at
Table 1. Autocorrelations and cross-correlations between B50 and M1.

<table>
<thead>
<tr>
<th>Lag (months)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>B50</td>
<td>1.00</td>
<td>0.93</td>
<td>0.82</td>
<td>0.67</td>
<td>0.50</td>
<td>0.31</td>
<td>0.12</td>
<td>-0.07</td>
<td>-0.24</td>
<td>-0.39</td>
<td>-0.52</td>
<td>-0.68</td>
<td>-0.70</td>
</tr>
<tr>
<td>M1</td>
<td>1.00</td>
<td>0.05</td>
<td>0.07</td>
<td>0.02</td>
<td>0.04</td>
<td>0.10</td>
<td>0.05</td>
<td>-0.10</td>
<td>-0.01</td>
<td>-0.03</td>
<td>-0.09</td>
<td>0.04</td>
<td>0.23</td>
</tr>
<tr>
<td>B50 leads M1</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.03</td>
<td>0.02</td>
<td>0.02</td>
<td>0.03</td>
<td>0.07</td>
<td>0.07</td>
<td>0.06</td>
<td>0.05</td>
<td>0.04</td>
<td>0.05</td>
</tr>
<tr>
<td>M1 leads B50</td>
<td>0.01</td>
<td>-0.01</td>
<td>0.01</td>
<td>-0.01</td>
<td>-0.03</td>
<td>-0.01</td>
<td>0.03</td>
<td>0.00</td>
<td>0.02</td>
<td>0.03</td>
<td>0.02</td>
<td>0.01</td>
<td></td>
</tr>
</tbody>
</table>

Each end of the series was tapered with a cosine bell in order to reduce leakage (Bendat and Piersol, 1971). Spectra were calculated using a fast Fourier transform technique and smoothed by applying a running average over nine adjacent harmonics. The bandwidth is plotted and there are 18 degrees of freedom in the resulting spectrum. A white noise null hypothesis is plotted along with the 95, 99 and 99.9% significance levels for spectral peaks. The variance in the prewhitened series was 21.7 mb² for M1 and 73.3 mb² for B50.

In the raw (unsmoothed) spectra of both series, all nine frequencies in the bandwidth from 9 to 17 × (336 months)⁻¹, corresponding to periods of 19.8 to 37.3 months, contributed to the QBO. Major peaks for B50 occurred at periods of 24 and 28 months, whereas for M1 the maximum was at 25.8 months.

Both spectra in Fig. 1 exhibit peaks which are significant at the 99.9% level at the quasi-biennial frequency, with a maximum at a period of 25.8 months. After removing the annual cycle and trend, the fraction of remaining variance associated with the QBO peak was 82.8% for B50 and 13.3% for M1.

To merely note that the QBO in B50 is significant is clearly an understatement. The spectrum of the 30 mb zonal wind at Balboa is very similar, and both are dominated by the QBO peak. The features in the remainder of the spectrum are essentially lost and could be enhanced by a logarithmic plot, but the presentation in Fig. 1 emphasizes the contrast with the M1 spectrum. The latter has two other peaks above the 95% significance level and the power does not fall off markedly for any frequency band.

Trenberth (1976a) previously noted the presence of spectral peaks in P2 (or equivalently, M1) near periodicities of 5 and 2.2 months. Here, there is also a third peak at 2.6 months. All these frequencies are possibly strongly affected by aliasing. Webster and Keller (1975) found strong 18–23 day variations in the kinetic energy of the Southern Hemisphere, and variations with periods of 20–22 days would alias onto periodicities of 2–3 months. Periodicities of 1.833 and 1.625 months strongly influence the power at 2.2 and 2.6 months, respectively, through aliasing, and fluctuations every 1.25 or 0.833 months would alias onto a 5-month period. We do not attach any particular significance to these spectral peaks since there was no a priori expectation for their presence.

In Fig. 1 white noise was used as the null hypothesis. Another possibility would have been to use red noise (persistence), and Table 1 shows the autocorrelations of each series for up to 12 lags. The positive autocorrelations in the first six lags are due to the QBO, and a first-order Markov process is not a good fit to either series. The significance of the QBO is unaffected by the choice of null hypothesis.

Cross-spectral calculations were carried out between the Balboa winds at 50 and 30 mb and M1. The zonal winds at 50 and 30 mb are significantly coherent for all periods greater than seven months, and values > 0.9 in the coherence squared occurred for periods of 20–40 months, with a lead at 30 mb of about four months. The coherence squared for B50 and M1 is shown in Fig. 2 with the 95% significance level plotted. Note the extremely small coherence at the QBO frequencies. This clearly demonstrates the lack of any linear relationship between the QBO's in M1 and the equatorial stratosphere, regardless of phase. Table 1 also shows the cross correlations of the two series, and the highest value is 0.07.

In order to further show the reality of the QBO in each series and the lack of any relationship between them, each series, without the detrending, has been subjected to a low-pass filter with 11 terms. The weights used are shown in Table 2 and the response function is given in Fig. 3. Note that we

Table 2. Weights used in low-pass filter.

<table>
<thead>
<tr>
<th>Weight number</th>
<th>±1</th>
<th>±2</th>
<th>±3</th>
<th>±4</th>
<th>±5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>0.131</td>
<td>0.127</td>
<td>0.114</td>
<td>0.092</td>
<td>0.0645</td>
</tr>
</tbody>
</table>
have deliberately chosen a filter which effectively removes fluctuations with periods < 12 months but includes all others. This is not a band-pass filter which shows up only the QBO, although the output from these series could easily lead one to believe otherwise.

Fig. 4 shows the filtered M1 and B50 series. The regularity of the Balboa series is immediately obvious although there are clear variations in the period of the QBO. M1, while still clearly exhibiting a QBO, tends to undergo larger fluctuations in amplitude and also exhibits variations in period but at different times to those in B50. From 1955–70 the period of M1 averaged 24 months, whereas in B50 the period lengthened considerably in the mid-1960’s. However, M1 failed to change phase from 1970–71 and again from 1975–76. By overlaying the series, we find it possible to line them up for at most four cycles (8 years) before one or other changes phase.

3. Relationships with 500 mb

Recently, nearly 6 years of daily analyses of the Southern Hemisphere have become available from the Australian Bureau of Meteorology. Trenberth (1979) discussed the quality of this data set in some detail and analyzed the interannual variability. This showed the presence of a quasi-biennial oscillation in the zonal mean westerly winds \( \bar{u} \) which exhibited a systematic poleward progression of the anomalies. A latitude-time section of \( \bar{u} \), smoothed with the low-pass filter introduced earlier, is shown in Fig. 5. This is similar to Fig. 9 of Trenberth (1979) but uses the more efficient low-pass filter to emphasize the interannual variability and retains 27.6% of the monthly variance. The QBO shown in Fig. 5 is quite coherent when viewed as a whole, but contains a certain amount of “noise” at individual latitudes. Part of this noise is due to higher frequency meteorological fluctuations but it also undoubtedly arises because of the sparse data coverage and sampling fluctuations.

Trenberth (1979) also suggested that this QBO in \( \bar{u} \) may be related to the QBO in the equatorial stratosphere, but it does not correspond well with the fluctuations in M1. The series are shown in Figs. 6 and 7. Here, and in all figures presented in this section, the series have had the annual cycle for this period (May 1972–January 1978) removed and are filtered using weights given in Table 2 with response function in Fig. 3. Zeros have been added to each end of the series, so that all 69 months can still be plotted, but the first and last five values

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**Fig. 3. Response function of the low-pass filter.**

**Fig. 4. Filtered time series of M1 and Balboa at 50 mb.**
are therefore somewhat unrepresentative of the smoothed trend.

Fig. 6 shows the out-of-phase relationship between Balboa at 50 mb and $\langle \bar{u} \rangle$ at 30–35°S at 500 mb. There is some variation in the relative phase during 1972 and it is possible with a longer series that the two QBO's may be unrelated. Other differences in phase and amplitude emerge if the 30 mb level in the equatorial stratosphere or other latitudes at 500 mb are used. For the present a viable hypothesis is that there is a direct relationship between the zonal mean westerly winds in the equatorial stratosphere and at 500 mb. Neither shows very good agreement with M1 for this period.

It is also reasonable to expect that the 500 mb flow would include fluctuations associated with M1. M1 is a surface pressure difference centered at 166°E near 45°S and Trenberth (1975) suggested that it was related to wavenumber 3 fluctuations. In order to test this, the monthly 500 mb geopotential height field $\bar{z}$ was harmonically analyzed in space at each latitude, i.e.,

$$\bar{z}(t) = \sum_n c_n(t) \cos[n(\lambda - \theta_n(t))],$$

(1)

where $\lambda$ is longitude, and $c_n(t)$ and $\theta_n(t)$ are the amplitude and phase of wavenumber $n$ at time $t$.  

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**Fig. 5.** Latitude-time section of the smoothed departures of the zonally averaged westerly wind $\langle \bar{u} \rangle$ from the long-term mean at 500 mb (m s$^{-1}$). Negative values are shaded.

**Fig. 6.** Filtered time series of $\langle \bar{u} \rangle$ at 500 mb at 30–35°S and Balboa at 50 mb for May 1972–January 1978. The scale for Balboa is given at left and $\langle \bar{u} \rangle$ 500 mb at right.

**Fig. 7.** Filtered time series of M1 and EP-M1 for May 1972–January 1978. The scale for M1 is given at left and for EP-M1 at right.
However, as M1 is an index of southerly flow, it is more appropriate to consider the geostrophic north-south velocity \( \tilde{v} \), where

\[
\tilde{v} = -\frac{g}{f} \sum_n n c_n(t) \sin[n(\lambda - \theta_n)].
\]

A detailed analysis of all planetary waves in the Southern Hemisphere at 500 mb has been reported by Trenberth (1980), who shows that wave 1 accounts for most of the interannual variability in the height field. However, \( \tilde{v} \) is proportional to the wave-number [see Eq. (2)] and waves 1–6 contribute to the interannual variance of \( \tilde{v} \) at 45ºS in the ratio 7:6:14:10:3:2, where wave 6 has been scaled at 2 [using Eq. (2) and Trenberth (1980)]. Thus wave 3 explains most of the variance in the \( \tilde{v} \) field. For \( \lambda = 166^\circ \text{E} \) in (2), we have confirmed that the wave 3 component of \( \tilde{v} \) is highly correlated with M1, but there is no general relationship between \( \tilde{v} \) and M1 for all \( \lambda \), or between M1 and the amplitude or phase of wave 3. Apparently, M1 is not a pure wave-number phenomenon.

We therefore computed empirical orthogonal functions (EOF's) of the 500 mb height anomalies (see Appendix) and compared them with the teleconnection pattern associated with M1. The latter, shown in Fig. 8a, is the correlation between the anomalies in 500 mb geopotential height at each grid point and M1 for 69 months. For 69 independent values, the 95 and 99.5% significance levels are about 0.2 and 0.3, respectively, and values exceeding the latter are shaded. Using the teleconnection pattern, we may then derive the empirical pattern associated with M1, EP-M1, as described in the Appendix. (The 113 points used in this and the EOF analysis were taken every 10º latitude with longitudinal resolution of 20º from 10–50ºS, 40º from 60–70ºS, and 90º at 80ºS.) EP-M1 is shown in Fig. 8b and accounts for 8.9% of the total variance. In the EOF analysis (not shown) the first five eigenvectors, by comparison, accounted for 21.9, 11.0, 10.0, 7.6 and 7.0% of the variance. The first three of these were essentially zonal in character, but EOF’s 4 and 5, when rotated slightly, resemble EP-M1. EP-M1 is correlated 0.52 with EOF4 when the latter is rotated 10º west, and 0.65 with EOF5 rotated 20º east. All three have strong wave 3 components (see Fig. 8b) and EOF4 and EOF5 were similar but rotated 30º (one-quarter wave 3 wavelength) relative to each other.

Fig. 8b shows the main centers of action in EP-M1 to be in the Australasian and Pacific Ocean areas. The time series of EP-M1 is correlated 0.83 with M1 for 69 months, and the smoothed series, shown in Fig. 7, are correlated 0.93.

It is unclear how the temporal variations in EP-M1 or wave 3 at 500 mb are related to those in the zonal mean flow (Fig. 5), and Trenberth (1980) found no significant relationships between fluctuations in the zonal mean flow and the planetary waves, except for wave 1. If we accept the hypothesis that the 500 mb zonal mean flow is related to the zonal wind in the equatorial stratosphere, the results of Section 2 indicate that any QBO in the long waves must be unrelated to the QBO in the zonal flow. However, such relationships need to be confirmed with a longer period of data.

Fig. 8. (a) Correlations of M1 with departures from the long-term mean of the geopotential height at 500 mb. (b) EP-M1, the empirical pattern associated with M1 at 500 mb.
4. Discussion and conclusions

An analysis of a QBO in an index M1 of the ultralong waves, in the troposphere of the Southern Hemisphere, has shown that it is unrelated in any linear sense to the QBO in the equatorial stratosphere. Superposition of the two filtered time series also shows sufficient variations in the relative phase and amplitude that it is difficult to imagine they are even related nonlinearly. Although there are insufficient data to draw concrete conclusions about the QBO at 500 mb in the Southern Hemisphere, indications are that the QBO in the zonal mean flow at 500 mb is correlated with that in the equatorial stratosphere, but both are unrelated to a second QBO in the ultralong waves and M1. Ebdon's (1975) composite patterns of the QBO in the sea level pressure field for the Northern Hemisphere show a large contribution from the zonal mean component of the flow but the significance of the meridional component is not as clear, especially in the light of the above results.

A review of the evidence for a QBO in the troposphere indicates that it may be more marked in the Southern Hemisphere. Physically, the reason for this may be because the Northern Hemisphere is dominated much more by the annual cycle associated with the distribution of land and sea and the seasonal changes in the centers of action. The dynamics of the ultralong waves changes seasonally only in the Northern Hemisphere from thermally direct in summer to baroclinically forced in winter (Holopainen, 1970). This is accompanied by a marked seasonal change in the zonal wind strength in the troposphere which is not present in the Southern Hemisphere (Trenberth, 1979).

In order for an independent QBO to exist in the troposphere, it seems that it must have an identity in all seasons, although it may have a different form in each and may have a preferred season for changing phase. An analysis of M1 shows this to be true in the Southern Hemisphere, but it follows that studies which deal with only one month or season will not be able to analyze the QBO properly.

The exact mechanism of an independent QBO in the troposphere is still open to question although a favorite hypothesis centers on interactions between the atmosphere and ocean (Trenberth, 1975; Brier, 1978; Nicholls, 1978). However, the QBO provides some measure of order in the otherwise chaos of interannual variability, and it may still be possible to make useful statistical forecasts in some areas of the globe. I have attempted to use the M1 index for forecasting purposes in New Zealand but it is necessary to take account of the high level of meteorological noise present (see Fig. 1). Although the QBO in M1 explains only 13.3% of the monthly variance, it explains ~38% of the seasonal (3-monthly) variance. However, in order to avoid the complications of the spectral peak near 5 months periodicity, the most skillful forecasts would be expected for 6-month seasons, where the QBO explains 62% of the variance. Further research into the QBO in the Northern Hemisphere troposphere, using data from all months, is currently underway.

Acknowledgments. I wish to thank Roland Madden for providing a copy of his fast Fourier transform program.

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APPENDIX

Empirical Pattern Analysis

One method of analysis which has recently become widely used in meteorology is Empirical Orthogonal Function (EOF) analysis. These functions are not of predetermined form but depend on interrelationships within the data set being analyzed. The first EOF is that linear combination of the original variables which explains the largest fraction of the total variance. Subsequent EOF's explain the largest fraction of the remaining variance and are orthogonal to each other. Associated with each EOF is a time series of coefficients which describe the evolution in time of that function and the time series of all the EOF's are also orthogonal.

A thorough review of EOF analysis is given by Davis (1976), and here we briefly summarize this method to show the relationship between EOF's and a different form of empirical pattern analysis. We have a field of points $P(x, t)$ at $M$ positions $x$, and there are $N$ such fields for each time $t$ given by $P_{jk}, j = 1, M; k = 1, N$ (the $j$th grid point and $k$th time). Then the EOF's correspond to the eigenvectors of the covariance matrix of the $P_{jk}$ points. There are $M$ EOF's $A_{ij}, i = 1, M$ (the $i$th eigenvector) which are associated with a time series $T_{ik}$ such that

$$P_{jk} = \sum_{i=1}^{M} A_{ij} T_{ik},$$

(1)

$$T_{ik} = \sum_{j=1}^{M} P_{jk} A_{ij},$$

(2)

The eigenvectors are orthogonal

$$\sum_{j=1}^{M} A_{ij} A_{ij} = \delta_{ii},$$

(3)

where $\delta$ is the Kronecker delta, and associated with each is an eigenvalue $\lambda_i$ ($\lambda_i > 0$) which is the variance accounted for by that EOF, and by convention the $\lambda_i$ are arranged in descending order. The $\lambda_i$ correspond to the variance of the time series $T_{ik}$.

While this EOF approach provides a very efficient method of compressing data, the functions are
artificially constrained to be orthogonal whereas no such physical constraints exist in the atmosphere, and their time series are not orthogonal at other than zero lag.

Instead, we may determine the empirical pattern EP associated with a particular index of the circulation which may have a clearer physical interpretation. If we have a normalized time series of the index $I_k$, $k = 1, N$, we may compute the correlations $r(I_k, P_{jk})$ for $j = 1, M$ over the $k$ values, which constitutes the teleconnection pattern, and then

$$C_j = r(I, P)\sigma_{P_j}, \quad j = 1, M,$$

(4)

corresponds to the covariance of the $I$ time series with each point, where $\sigma_{P_j}$ is the standard deviation at each point $j$. It therefore corresponds to a departure pattern associated with one standard deviation of $I$. If the $C_j$ are now normalized so that

$$\sum_{j=1}^{M} C_j^2 = 1$$

(5)
in a similar manner to (3), then the $C_j$ will represent an EP similar to one of the $A_{ij}$ EOFs. We can similarly define a time series $S_k$ associated with $C_j$ [compare with Eq. (2)]

$$S_k = \sum_{j=1}^{M} P_{jk}C_j$$

(6)

and the variance of this time series will be the total variance explained by the EP $C_j$.

REFERENCES


