REVIEW

The Parameterization of Radiation for Numerical Weather Prediction and Climate Models

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ABSTRACT

This paper presents a review of the various methods used to compute both the fluxes and the rate of heating and/or cooling due to atmospheric radiation for use in numerical models of atmospheric circulation. The paper does not follow, step by step, the solution to the relevant radiative transfer problem but rather concentrates on providing the reader with the physical basis underlying the various methods. The paper discusses, separately, the various parameterizations for the absorptions by water vapor, carbon dioxide and ozone and for the scattering and absorption associated with cloud (and hazes) and also provides some indication of their accuracy.

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1. Introduction

The object of any parameterization of atmospheric radiation for use in an atmospheric circulation model is to provide a simple, accurate and fast method of calculating the total radiative flux profile within the atmosphere. These calculations must supply

i) the total radiative flux at the surface to calculate the surface energy balance and

ii) the vertical and horizontal radiative flux divergence to calculate the radiative heating and cooling rates of an atmospheric volume.

The parameterization should include the combined effects of absorption and scattering by the trace gases of H$_2$O, CO$_2$ and O$_3$ together with cloud and haze particles. The main problem is that the atmosphere is not specified by the model as well as we might wish and, as a consequence, we are left with the problem of defining the radiative transfer characteristics in terms of parameters that are (or will be) readily supplied by a circulation model.

The requirement of computational speed implies that a high level of approximation is necessary and one is then faced with a trade-off between accuracy and speed. Both the level of approximation and the associated accuracy criteria determine the type of interactions between radiation and dynamics that will be ignored. These aspects are therefore central to the overall design of any parameterization scheme. Yet accuracy criteria are extremely difficult to define a priori. Some factors that complicate matters are

i) radiation may simultaneously affect the dynamics in several different ways and the accuracy required of the radiation computations depends on which process is important to the given dynamical problem.

ii) The dynamics respond to the total heating fields (the sum of the radiative, latent heating and sensible heating components). These heating components are not always independent of each other and as a result radiation may influence the dynamics in a complex nonlinear manner which is often difficult to assess a priori.

iii) The heating and cooling by radiation can vary considerably as do the temperature variations caused by various dynamical factors. As an example, Table 1 from Kondratyev (1972), summarizes a number of different temperature changes observed in the atmosphere under different synoptic conditions. These temperature changes vary in magnitude and can be of the same order or several times larger than those due to radiation (which, for clear skies, are $\sim 2$ K day$^{-1}$). On this basis, one might conclude that radiative processes may be important in certain situations (such as for the tropical easterly wave) but not in others (such as for subsidence warming).

These few examples aptly demonstrate that the design and accuracy requirements of a radiation parameterization are highly problem dependent. While this paper presents an overview of the various approaches that can be used to parameterize atmospheric radiation and some idea of the error introduced by the use of these techniques, it is left to the individual researcher to establish the accuracy requirements for the given problem at hand and to establish if these requirements are compatible with the practical limitations imposed by the structure of the dynamical model.

The purpose of this paper is to review the various techniques that are available to calculate the radiative fluxes and radiative heating rates as applied in general circulation models. The emphasis throughout is to provide the reader with the physical basis underlying a variety of techniques which range in sophistication and to highlight the practical advantages and disadvantages associated with each. The paper does not trace the various methods through step-by-step since these details can be found in cited references. The paper concentrates only on the parameterization of radiative transfer in the atmosphere and not on the coupled, and more difficult problem, of defining relevant atmospheric properties from insufficient model information. In particular, the paper does not address the problem of defining the presence of clouds in the model given only broadscale velocity, temperature and moisture information.

It is assumed throughout that the reader is familiar with certain of the general definitions and concepts of atmospheric radiation and radiative transfer (e.g., see texts on the subject like Kondratyev (1969), Paltridge and Platt (1976), Coulson (1975), Liou (1980) among others).

In the following section, the difficulties associated with the calculation of clear sky longwave fluxes are described and the well-known parameterization techniques that overcome these difficulties are outlined. This section is followed by a discussion of the parameterization of longwave flux transfer in clouds and the similarities with the clear sky techniques are highlighted. Section 4 presents an overview of the methods that are used to calculate solar fluxes in a clear sky. This is followed by the discussion of parameterization of solar radiation within clouds and in cloudy skies.

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Table 1. Observed rates of temperature change $\Delta T/\Delta t$ associated with various dynamical situations (Kondratyev, 1972).

<table>
<thead>
<tr>
<th>Synoptic situation</th>
<th>$\Delta T/\Delta t$</th>
<th>Frequency of occurrence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chinook, foehn</td>
<td>10 K h$^{-1}$</td>
<td>Sporadically</td>
</tr>
<tr>
<td>Cold wave</td>
<td>20 K day$^{-1}$</td>
<td>Monthly</td>
</tr>
<tr>
<td>Cyclone at 850 mb</td>
<td>8–10 K day$^{-1}$</td>
<td>Weekly</td>
</tr>
<tr>
<td>Midlatitude summer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cyclone</td>
<td>7–8 K day$^{-1}$</td>
<td>Weekly</td>
</tr>
<tr>
<td>Subsiding air in high</td>
<td>23 K day$^{-1}$</td>
<td>Once or twice a week</td>
</tr>
<tr>
<td>Equatorial waves</td>
<td>0–3 K day$^{-1}$</td>
<td>Once or twice a week</td>
</tr>
</tbody>
</table>
making specific mention of the treatment of partial cloudiness and overlapping clouds. Section 6 reviews some of the schemes used in existing GCMs summarizing the radiative processes included in the schemes, the equations solved and some indication of computational efficiency. This section can be considered as a summary since the various schemes described collectively employ most of the parameterization techniques described throughout the paper. The paper concludes with an overall summary and a discussion of various aspects where significant improvements are required and of certain areas that are unsatisfactorily tested. A list of all symbols employed in this paper can be found in Appendix A.

2. The parameterization of longwave fluxes in a cloud free atmosphere

Examples of the spectral distribution of longwave flux are well documented in existing literature, and it is sufficient to say that the general problem of parameterizing longwave radiative transfer in the clear atmosphere requires the suitable treatment of absorption and simultaneous emission by the 9.6 μm ozone band, the rotation and vibration bands of water vapor, the continuum absorption in the atmospheric window between 8 μm and 14 μm and the absorption by the 15 μm carbon dioxide band which overlaps a portion of the rotation band.

a. The longwave radiative flux equations

The parameterization of longwave flux in a cloud free atmosphere is usually based on the solution of the radiative transfer equation with some simple parameterization of the absorption factors contained within the equation. The equations appropriate for longwave flux are (e.g., see Liou, 1980, p. 93 for the complete derivation of these equations).

\[ F'(z) = \int_0^\infty \pi B(z) \tau_z(z, z = 0) d\nu + \int_0^\infty \int_0^\infty \pi B(z') \frac{d\tau_{z'}}{dz'} (z, z')dz'd\nu, \]  

(1)

\[ F'(z) = \int_0^\infty \int_0^\infty \pi B(z) \frac{d\tau_{z'}}{dz'} (z, z')dz'd\nu, \]  

(2)

where \( F'(z) \) and \( F'(z) \) are the upward and downward longwave fluxes through level \( z \), \( B \) is the Planck function and \( \tau_{z'} \) the diffuse transmission function defined by the hemispheric integral

\[ \tau_{z'}(z, z') = 2 \int_0^\infty \tau_z(z, z', \mu) d\mu, \]  

(3)

where \( \mu (= \cos \theta) \) is the cosine of the zenith angle and

\[ \tau_z(z, z', \mu) = \exp \left[ - \frac{1}{\mu} \int_{\mu(z)}^{\mu(z')} k_\nu(p, T)d\mu \right], \]  

(4)

where \( k_\nu(p, T) \) is the absorption coefficient and \( u \) is the concentration of the attenuating gas defined along the path from \( z \) to \( z' \). The transmission function \( \tau_{z'} \) is often referred to as the slab transmission function as it describes the transmission of the hemispheric radiative flux along the path \( u \). There are several ways of defining the path quantity \( u \) and each defines a specific form of absorption coefficient (or total extinction coefficient when scattering is also involved). Some of the more popular definitions of \( u \) (and hence \( k_\nu \)) and relevant conversion factors are summarized in Appendix B.

The above flux equations can be combined and differentiated directly to obtain an equation of flux divergence and thus of radiative cooling (e.g., Rodgers and Walsh, 1966). This has some practical advantages in that a study of the divergence equations reveals the more important terms which, in some circumstances, provide an approximation to the flux equation itself (the cooling-to-space term is such an example and this is discussed later). However, for the purposes of use in an operational GCM, it is desirable to calculate the fluxes at each model level and then evaluate the flux divergence for the layer between two levels in a finite difference form. This approach automatically supplies fluxes at those levels where a radiation budget is required (such as at the surface, the tropopause or at the top of the model atmosphere).

Nested within the equations for longwave flux are four integrals which refer to i) the summation of contributions over all zenith angles [Eq. (3)], ii) the summation of the contributions to the flux at level \( z \) from all atmospheric layers \( dz' \) (including the ground), iii) summation over all spectral intervals \( d\nu \) containing the relevant absorbing bands and iv) the integration over absorption path \( d\nu \) (4).

The first of these integrals is adequately approximated by

\[ \tau_{z'}(z, z') \propto \tau_z(z, z', 1/\beta), \]  

(5)

where \( \beta = 1.66 \) and is the well known diffusivity factor. This approximation states that the transmission of flux through a slab between \( z \) and \( z' \) can be represented by the transmission of a beam along the direction defined by the representative zenith angle \( \theta = \cos^{-1}(1/\beta) \). We will see below that this is a very convenient approximation as it allows the flux to be determined using the parameterizations of the intensity transmission function which have a path length adjusted by the \( \beta \) factor.

\[ \text{Unauthenticated} | \text{Downloaded 07/30/22 08:51 PM UTC} \]
The remaining integrals are discussed in more length below, but it is relevant to comment that the traditional problem of parameterizing the clear sky longwave flux revolves around the suitable approximation of both frequency and path integrals.

\section*{b. Integration over optical path}

The integration of absorption over optical path is complicated by the dependence of $k_{1}$ on both pressure and temperature. This path dependence of absorption introduces a twofold problem. The first is that atmospheric paths along which the fluxes are to be determined generally involve large variations in both pressure and temperature, and it is not appropriate to assume constant values of $k_{1}$. The second is that most absorption data are collected in the laboratory at fixed pressures and temperatures, and one is left with the problem of applying these data to atmospheric absorption problems. This latter problem is alleviated to some extent by measuring the absorption data at pressures and temperatures which are not greatly different than those expected in the atmosphere.

The two commonly used approximations employed to overcome these problems both assume that the absorption along a nonhomogeneous path can be approximated by the absorption along some homogeneous path with some appropriately adjusted value of $p$ and $u$ (we denote these hereafter as $\tilde{p}$ and $\tilde{u}$). These approximations are known as

\begin{enumerate}
\item \textbf{The Scaling Approximation}

The necessary conditions for the existence of the scaling approximation is well described by Goody (1964a, Section 6.1). The approximation assumes that the absorption coefficient is separable into two factors: one depending only on $\nu$ and the other on $p$ and $T$. That is the optical path $\delta_{e}$ (see Appendix B for the definition of $\delta_{e}$) is

$$
\delta_{e} \approx k_{e}(p_{0}, T_{0}) \int \frac{\phi(p, T)}{\phi(p_{0}, T_{0})} \, du \approx k_{e}(p_{0}, T_{0}) \tilde{u},
$$

(6)

where $p_{0}$ and $T_{0}$ are the reference pressure and temperature. Thus only $\tilde{u}$ is required to determine the optical path and, together with $k_{e}(p_{0}, T_{0})$ all other absorptive properties. The usual form of $\tilde{u}$ is

$$
\tilde{u} = \int \left( \frac{p}{p_{0}} \right)^{m} \left( \frac{T_{0}}{T} \right)^{n} \, du.
$$

(7)

Table 2 lists the currently accepted values of $n$ and $m$ applicable to the major absorption bands of both long- and short-wave absorption spectra.

The scaling approximation is the most commonly employed simplification to the path integral problem. This popularity is based on the overall simplicity of the method as it is only necessary to replace $u$ with $\tilde{u}$ in simple one parameter absorption schemes (e.g., as in the emissivity method or the empirical functions described later). However, it has long been considered that there is little justification for the assumptions contained in (6). Chou and Arking (1980) demonstrate that the factorization of $k_{e}(p, T)$ as in (6) is possible for Lorentz line absorption provided that the absorption occurs at some distance from the line center. It is illustrated in Fig. 2 that most of the radiative cooling results from the exchange of energy through the wings of strong lines and, under these conditions, the scaling method is likely to provide a reasonable approximation to the problem of absorption along nonhomogeneous paths. It is, however, difficult to isolate the errors in IR cooling rate that result from the application of the scaling method. It suffices to say that the errors attributable to the scaling approximation are likely to be larger than those of the two parameter approximation. Qualitatively, the approximation is most unsatisfactory in the upper atmosphere where absorption tends to be dominated by the line centers.

\item \textbf{Two Parameter Approximation}

With the application of transmission parameterizations which express absorption as a function of more than one dependent variable (e.g., band models), it is possible to employ more sophisticated and accurate approximations to the path integral. The common higher order approximation was introduced independently by Curtis (1952) and Godson (1954) and earlier by Van de Hulst (1945). The VCG approximation relates the absorption along a nonhomogeneous path to that of a corresponding homogeneous path by adjusting the path $u$ and pressure $p$ according to

$$
\tilde{p}u = \int p \, du,
$$

$$
\tilde{u} = \int du.
$$

(For those interested see Goody (1964b) and Armstrong (1968) for a more extensive discussion on this topic.)

The accuracy of the VCG cannot be defined in absolute terms as it depends on the strength of the absorption and on the distribution of absorber amount with pressure. Since the definitions in (8) were derived on the basis of matching the absorption in the strong
and weak limits, it is to be expected that the VCG is least satisfactorily in the intermediate absorption range. Walshaw and Rodgers (1963) demonstrated that the VCG is most satisfactory when applied to the 15 μm CO₂ band with a maximum percentage error of 2.6% in radiative cooling. The approximation is tolerable for water vapor cooling rates but errors in the range of 20–40% or more are likely in O₃ cooling above 100 mb since this emission is dominated by intermediate absorption. A more sophisticated approach (like the three parameter method described by Goody, 1964b) seems mandatory for this band.

c. Integration of absorption over frequency

The problem of simplifying the integration over frequency in (1) and (2) is unfortunately more complex than the rather simple and obvious task of averaging \( k_\nu \) over some broad interval \( \Delta \nu \). Complications arise because there are four different, and distinguishable frequency scales contained within the integral. (These are schematically illustrated in Fig. 1.) The first refers to the relatively slow variation of the Planck function \( B_\nu \) with frequency (upper panel). The second scale is that of the unresolved contour of the absorption band (second panel). For gases other than water vapor, the Planck function can be treated as a constant with frequency for each band. However, for water vapor it is necessary to subdivide the absorption bands into smaller sub-intervals. The next scale is that associated with the separation of individual lines (exploded scale in Fig. 2) while the finest frequency scale is that on which Lambert’s law of absorption applies (i.e., a sub-line scale on which the exponential transmission function and hence on which (1) and (2) are valid). The total broadband longwave flux then results from the convolution of the very fine frequency scale of line absorption with the coarser Planck scale (this flux is shown schematically as the shaded area in the lower panel of Fig. 2). There are basically two different ways of calculating this integrated flux quantity. The first is to resolve the variation of the Planck function directly by dividing the spectrum into a number of discrete intervals and then to define the mean absorption characteristics for each interval. The second method convolves the absorption and Planck functions and integrates this quantity over the entire longwave spectrum. Some techniques that employ the first of these approaches are now described.

1) Band models

The finest frequency scale of absorption, that of the individual line, is described by a simple analytic function given by the Lorentz line absorption profile, at least for altitudes below about 40 km, (e.g., see details in Liou, 1980, or any other standard radiation text). Unfortunately a single absorbing line cannot be considered in isolation from neighboring lines and it is not meaningful to average \( k_\nu \) over a group of lines in a simple linear manner. This led to the concept of a band model which enables the averaging of the absorption properties for bands of lines with properties (such as line strength, separation, position, etc.) that are specified by well-defined statistical relationships. There are numerous ways of defining these relationships (e.g., Elsasser, 1938; Godson, 1954; Malkmus, 1967) each resulting in a specific form of transmission function expressed as a function of pseudo line parameters (such as a “mean” line intensity \( \tilde{S} \), a “mean” half-width \( \tilde{\alpha} \) and a “mean” line spacing \( \tilde{d} \)). Perhaps the most widely used band model in atmospheric flux calculations is that of Goody (1952) which considers that all lines are randomly distributed. The resulting transmission functions for the spectral interval \( \Delta \nu \) is

\[
\tau_{\nu} = \exp \left[ - \frac{\tilde{S} \tilde{\nu} \tilde{\beta}}{\tilde{d}} \left( 1 + \frac{\tilde{S} \tilde{\nu} \tilde{\beta}}{\pi \tilde{\alpha}} \right)^{-1/2} \right],
\]

where the overbar denotes an “average” over \( \Delta \nu \). A particular advantage of this form of transmission func-
tion is that the mean transmission for a band of lines resembles that for an individual line. Note that the diffusivity factor $\beta$ has been included in (9) and it is a simple matter to incorporate the VCG approximation by replacing $\mu$ with $\bar{\mu}$ and $\alpha$ with $\bar{\alpha}(\beta/p_0)$ ($p_0$ is the pressure at which the band parameters $S$, $\bar{\alpha}$ and $\bar{\mu}$ apply). Rodgers and Walsh (1966) provide the band parameters $S/\bar{\mu}$ and $\pi\bar{\alpha}/\bar{\mu}$ for the two major water vapor absorption bands and the 15 $\mu$m CO$_2$ band. Goldman and Kyle (1968) provide these quantities for the 9.6 $\mu$m O$_2$ band while Wu (1980) present upgraded values for the water vapor 6.3 $\mu$m vibration band.

It is thus a simple matter to replace the integral over $dv$ in (1) and (2) by a summation over a finite number of spectral intervals that span the longwave absorption spectrum. Rodgers and Walsh (1966) used 21 such intervals and employed the Goody band-model transmittance functions. The longwave fluxes and cooling rates calculated by this approach were, until recently, considered as the benchmark calculation against which other simpler schemes were tested. The general band model approach to the solution of (1) and (2) is not often used in operational models because of the computational effort involved.

Figure 2 presents a layer-by-layer plot of the tropical clear-sky cooling rates for the 20 spectral intervals employed by Wu (1980). Each plot represents the averaged cooling over a 3 km layer and only takes water vapor absorption into consideration. In general, the cooling in the very high opacity spectral regions, such as center positions of strong lines or in the very weak absorption regions, do not contribute to the true net energy exchange which determines the cooling of the layer. The cooling mostly occurs in the wings of strong lines, the central portions of moderately weak lines and in the continuum region. It is also apparent that the spectral contributions to the cooling shifts with altitude. The cooling of the lower layer (at least for the tropical atmosphere) is dominated almost entirely by the continuum absorption (which is described in more detail below) while the cooling in the lower frequency end of the station band is more dominant in the upper layers.

2) TRANSMISSION IN THE WATER VAPOR ABSORPTION CONTINUUM

Within the "atmospheric window," between 8 and 14 $\mu$m, absorption varies more or less continuously with wavelength, and it is a relatively simple matter to define an averaged transmission function for the entire spectral region. While this absorption is often considered weak, it is important to model the transmission accurately as it is a region where significant amounts of radiant energy are exchanged between the earth's surface, cloud and space.

The continuum absorption is overlain by many weaker absorption lines of H$_2$O, CO$_2$ and O$_3$ and it has proved difficult to isolate the precise nature of the absorption mechanism. Bignell (1970) discovered from laboratory measurements that the continuum absorption coefficient was dependent on water vapor partial pressure $e$ (hence the name $e$-type absorption). As it turns out, the influence of this absorption on IR cooling rates, satellite SST retrieval and calculations of the surface energy budget are considerable in a moisture laden atmosphere. Fig. 2 readily attests the importance of this absorption on lower tropospheric cooling in the tropical atmosphere. Stephens et al. (1980) claim that the downward longwave fluxes at the surface can be in error by more than 40 W m$^{-2}$ if the absorption is neglected. (See Cox, 1973, for a discussion of other relevant effects of this absorption.)

The absorption coefficient can be usefully written as (Roberts et al., 1976)

$$k_c(p, T) = \frac{\Phi(T)}{\Phi(T_0)} \Psi(v, T_0)e,$$  \hspace{1cm} (10)

where

$$\Phi(T) = \exp(1800/T)$$  \hspace{1cm} (11)

![Fig. 2. Heating rates (°C day$^{-1}$) of the 20 spectral intervals used in the calculation by Wu (1980). Each line represents the spectral heating rate of the given 3 km thick layer. A tropical model atmosphere was used to derive these curves.](image-url)
and $\psi(T_0) = 4.18 + 5578 \exp(-0.00787\nu) \text{ g}^{-1} \text{ cm}^2 \text{ atm}^{-1}$ with $T_0 = 296 \text{ K}$ and $\nu$ in cm$^{-1}$.

The mean diffuse transmission function can therefore be analytically derived using (10) and (11) in (3) and (4) (see Stephens, 1976) for an example of this. Another useful aspect of the absorption coefficient expressed in the form of (16) is that the emissivity (and temperature) dependencies are explicitly defined and thus approximations like (7) or (8) can be avoided.

3) OVERLAPPING WATER VAPOR AND CARBON DIOXIDE ABSORPTION

The transmission of two different gases that absorb the radiation along a given path length in the same specified spectral interval can be determined from

$$\tau_{\Delta l}(u_1, u_2) = \tau_{\Delta l}(u_1)\tau_{\Delta l}(u_2)$$

(12)

which is just the product of the mean transmittances along that path length with an amount of absorbing gas $u$, for the first gas and $u_2$ for the second gas. Unfortunately, this condition only applies to a restricted number of transmission functions; the random band model being one. It is not often realized that the simpler absorption parameterizations, such as the emissivity approximation discussed below, are inconsistent with (12). This is an important aspect as it is necessary to calculate the longwave cooling in the spectral region centered on 15 $\mu$m in which both water vapor and carbon dioxide contribute. The importance of this effect is demonstrated in Fig. 3 in which the vertical profiles of the 15 $\mu$m cooling rates for $\text{CO}_2$ alone, $\text{H}_2\text{O}$ alone and the combined cooling of $\text{H}_2\text{O}$ plus $\text{CO}_2$ (based on random band model scheme) are shown. It is clear that, through most of the troposphere, the combined cooling is substantially less than the cooling rate based on the simple addition of the individual components and some sort of overlap correction is required. The profiles shown in the diagram support the contention that this overlap correction can be neglected high in the atmosphere where cooling is principally due to $\text{CO}_2$ emission, but that the correction is important in the troposphere.

One remark is relevant at this stage. The longwave radiative cooling in the stratosphere is almost entirely dominated by the emission of the 15 $\mu$m $\text{CO}_2$ band. In recognition of this, a large number of accurate parameterizations of $\text{CO}_2$ emission exist in the literature for use in stratospheric circulation models. Most of these are based on calculating the cooling rates via a spectrally integrated flux equation. Some examples are Ramanathan (1976), Ellingson and Gille (1978) and Fels and Schwartzkopf (1981). The algorithms of the latter are a little different from the others as the integrated transmission functions are tabulated for actual inhomogeneous paths. It is unfortunate that these highly accurate $\text{CO}_2$ algorithms do not provide equally accurate cooling rates in the troposphere since there, the precision of the cooling rate calculation is limited by the accuracy of the water vapor transmission and the method of overlapping the two gases.

4) THE $k$ DISTRIBUTION METHOD

An alternative approach has been employed more recently to approximate the frequency integration in the flux equations. The method is demonstrably faster and more accurate than the band model approach described above. This method also has the added advantage that it is straightforward to treat molecular absorption and the scattering by cloud droplets in a self-consistent fashion (see below). The method makes use of the fact that for a homogeneous atmosphere, the transmission within a relatively wide spectral interval is independent of the ordering of the value of $k$ with respect to $\nu$, but depends only on the fraction of the interval that is associated with a particular value of $k$. This fraction is just a probability density function defined such that $f(k)dk$ is the fraction of the frequency interval for which the absorption coefficient lies between $k$ and $k + dk$. The basic idea of grouping frequency intervals according to line strengths dates back at least to Ambartsumian (1936). It has been used

![Fig. 3. The infrared cooling rate profiles in a tropical model atmosphere in the 15 $\mu$m band region showing the contribution by $\text{CO}_2$ alone, the overlapped contribution by $\text{H}_2\text{O}$ plus $\text{CO}_2$ and the contribution by $\text{H}_2\text{O}$ alone (from Ohring and Joseph, 1978).](image-url)

The \( k \) distribution is formally related to the transmission function defined along a homogeneous path as

\[
\tau_k(z, z') = \frac{1}{\Delta v} \int_{\Delta v} e^{-k\nu} d\nu = \int_0^\infty f(k) e^{-k\nu} dk.
\]  

(13)

Chou and Arking (1980) use this technique to calculate the radiative cooling by the major water vapor bands and employ a relationship like (7) to account for pressure variations along the absorption path. Hansen et al. demonstrated that the \( k \) distribution provides an alternate and more accurate treatment of vertical inhomogeneity of the atmosphere by noting that \( k \) distributions at all altitudes are correlated in frequency space, i.e., the strongest (and weakest) absorption occurs at the same frequencies at all altitudes. The pressure effects are then included explicitly in the choice of \( k \) used to evaluate the \( k \) integral in (13).

Figure 4 shows a schematic illustration of the \( k \) distribution method. Panel (a) is a schematic of an absorption line spectrum at two different pressures. The greater the pressure \( (p_2) \), the broader the spectral features. Panel (b) illustrates examples of the two probability density functions \( f(k) \) associated with (a). The shaded area depicts the strongest absorption which refers to the same spectral interval. These two panels illustrate the essence of the \( k \) distribution approach; the integration over \( \nu \) space of a complex function \( k_\nu \) is replaced by the integration of a more smoothly varying function over \( k \) space.

Chou and Arking found that they could obtain accurate calculations by dividing each spectral interval into only two \( k \) types—in the vicinity of band centers where absorption is strong and in the band wings where absorption is moderate to weak. They present precomputed tables of \( f(k) \) for these \( k \) types and choose 34 intervals that span from 20 to 2020 cm\(^{-1}\). These functions are based on the absorption line parameters for water vapor as compiled by McClatchey et al. (1973) but exclude CO\(_2\) and e-type absorption. The transmission for each of these 34 intervals is weighted with the Planck function and integrated to provide an integrated absorption quantity which is tabulated for different temperatures and water vapor path \( u \) (this is in fact the equivalent to the emissivity method described below). Results of their calculations are shown in Figs. 5a and b as a cooling rate difference using a highly accurate line-by-line spectral integration scheme as a standard. These differences are also compared with the band model approach described above. The errors of the \( k \)-distribution method are not only about a factor of two smaller (compare a maximum error of 4\% to an 11\% error) but the method is an order of magnitude faster. It would seem that this approach is ideally suited for use in numerical circulation models but before it can be adopted, it is necessary to extend the method to include other gases as well as the effects of gas overlap.

d. The broadband flux emissivity method

It is obviously a great practical advantage to reduce the number of spectral intervals required to evaluate the broadband longwave fluxes and cooling rates to an absolute minimum. The band model method is restricted by the need to define an interval that is narrow enough to treat the Planck function as a constant across the interval. It is also restricted by the fact that for broad spectral intervals the transmission function is no longer exponential in character thus making it difficult to fit a pure exponential to the mean transmission. A function which, for the most part, overcomes these problems is the emissivity. The function is derived by integrating the absorption over frequency and weighting it in some way with the Planck function. This "integrated" absorption is expressed in terms of one parameter, \( u \) (or \( \bar{u} \) for nonhomogeneous paths). This function therefore has the obvious advantage of

![Fig. 4. A schematic illustration portraying the essence of the k-distribution method. A schematic of absorption line spectra at two different pressures are shown in (a). In (b) the two probability density functions \( f(k) \) associated with (a) are illustrated. The shaded area depicts the strongest absorption (i.e., largest \( k \)) for the same spectral interval (i.e., \( f(k) \) for different pressures are correlated). Integration of \( f(k) \) over \( k \) replaces the integration of \( k_\nu \) over \( \nu \) (modified from Hansen et al., 1985).](image)
removing once and for all, the frequency integral in the longwave flux equations but does so at the expense of computation accuracy.

1) THE LONGWAVE FLUX EQUATIONS IN THE BROADBAND EMITTANCE FORM

We can rewrite the flux equations (1) and (2) in the form

\[ F^i(z) = \int_0^\infty B_i[1 - A_i(z, 0)]dv \]

\[ + \int_0^\infty \int_0^z \pi B_i(z') \frac{dA_i}{dz'}(z', z')dz'dv \]

(14)

for upward flux and a similar equation for downward flux

\[ F^l(z) = \int_0^\infty \int_z^\infty \frac{dA_i}{dz'}(z, z')\pi B_i(z')dz'dv, \]

(15)

where \( A_i \) \((=1 - \tau_i)\) is the absorptivity of the gas. If we define the quantity \( \epsilon \) along the path \( z \rightarrow z' \) as

\[ \epsilon(z, z') = \frac{1}{\sigma T^4} \int_z^{z'} A_i(z, z')\pi B_i(T)dv, \]

(16)

then the flux equations follow as

\[ F^i(z) = \sigma T^4(1 - \epsilon(z, 0)) \]

\[ + \int_0^z \sigma T^4(z') \frac{d\epsilon}{dz'}(z, z')dz' \]

(17)

\[ F^l(z) = \int_z^\infty \frac{d\epsilon}{dz'}(z, z')\sigma T^4(z')dz \]

For homogeneous paths, (15) is the classical definition of emissivity relating the emission of a column of gas to that of a blackbody at the same temperature. This quantity has tangible physical significance and can, in principle, be measured although these measurements are very much ambiguous, often involving overlapping emission of other gases.

For the case of overlapping emissions by multiple absorbers in the same frequency interval \( \Delta \nu \), (such as for a combination of H\( _2 \)O and CO\( _2 \) absorption in the spectral region from about 580 to 750 cm\(^{-1}\)) the flux emissivity is defined as

\[ \epsilon_{\Delta \nu}(u_1; u_2) = \frac{1}{\sigma T^4} \int_{\Delta \nu} [1 - \tau_i^f(u_1)\tau_i^c(u_2)]B_i dv, \]

(18)

where \( u_1 \) and \( u_2 \) refer to the optical paths of H\( _2 \)O and CO\( _2 \) respectively. This can be further rearranged (see Staley and Jurica, 1970, for details) as

\[ \epsilon_{\Delta \nu}(u_1; u_2, p, T) = \epsilon_{\Delta \nu}(u_1, p, T) + \epsilon_{\Delta \nu}(u_2, p, T) \]

\[ - \Delta \epsilon_{\Delta \nu}(u_1, u_2, p, T) \]

(19)

such that the emission by the mixed gases is just the sum of the emission by the individual gases less some “overlap” correction. Eq. (18) and thus (19) are only strictly applicable for the individual flux transmissions expressed in a suitable multiplicative form [see (12)]. Unfortunately, the laboratory transmission data upon which most emissivity parameterizations are based, are rarely in such a form and the common practice of employing (19) to treat overlapping absorption should be viewed with caution.
2) ALTERNATE FORMS OF "EMISSIVITY"

While $\epsilon(z, z')$ is a quantity most generally understood as emissivity, it is perhaps not always the most appropriate when applied to the calculation of atmospheric fluxes since it is generally difficult to obtain expressions of $\epsilon(z, z')$ as a function of $u$ that are accurate enough to evaluate $d\epsilon(z, z')/du$ [or equivalently $d\epsilon(z, z')/dz$] with reasonable precision. This problem is overcome if we integrate (17) by parts to produce

$$F'(z) = \int_0^\infty B_{\nu}(z = 0)dv$$
$$+ \int_0^\infty \int_0^z A(z, z') \frac{dB_{\nu}}{dz'}(z')dz'dv.$$  \hspace{1cm} (20)

$$F''(z) = \int_0^\infty \int_z^\infty A(z, z') \frac{dB_{\nu}}{dz'}(z')dz'dv.$$  \hspace{1cm} (21)

It is therefore appropriate to define quantities like

$$\epsilon'(z, z') = \frac{R(z, z')}{4\sigma T^4} = \int_0^\infty A(z, z') \frac{dB_{\nu}}{d\sigma T^4(z')} dv.$$  \hspace{1cm} (22)

where $R(z, z')$ is the mean absorptivity parameter as defined by Elsasser and Culbertson (1960). The flux equations then follow as

$$F'(z) = \int_z^\infty \epsilon'(z, z') \frac{d\sigma T^4(z')}{dz'} dz'.$$  \hspace{1cm} (23)

$$F''(z) = \sigma T^4 + \int_0^z \epsilon'(z, z') \frac{d\sigma T^4(z')}{dz'} dz'.$$

While $\epsilon'$ bears no direct relationship to the actual emissivity $\epsilon$, it does not differ greatly from it since the shape of $d\epsilon'/dT$ is similar to that of $B_{\nu}$. This aspect is shown in Fig. 6 in which both $\epsilon$ and $\epsilon'$, taken from a variety of sources, are shown as a function of water vapor path $u$ for various specified temperatures.

The relationship between $\epsilon'$ and $\epsilon$ has significant practical implications. It is necessary in most schemes to truncate the top of the model at some level other than $z = \infty$ in which case the blackbody function is nonzero. As a result, there will be some source of downward longwave flux at the top of the atmosphere. It is therefore necessary to include an additional term in the downward flux equation that allows for this radiation. The relevant flux equation is

$$F'(z) = \sigma T^4(z) \epsilon(z, z')$$
$$+ \int_z^{\infty} \epsilon'(z, z') \frac{d\sigma T^4(z')}{dz'} dz'.$$  \hspace{1cm} (24)

where $z_T$ refers to the top of the model and $T(z_T)$ is the characteristic radiation temperature of the atmosphere above the model. The calculation of longwave flux using this formulation thus requires the use of both $\epsilon$ and $\epsilon'$. Ramanathan et al. (1983) define a relationship between $\epsilon$ and $\epsilon'$

$$\epsilon = \frac{\epsilon'}{0.847u^{0.222}}$$

thus requiring only the specification of $\epsilon'$ in the algorithm.

Figure 7 is included to illustrate the important, practical aspect that a temperature factor is required when either $\epsilon$ or $\epsilon'$ is defined for a narrow spectral interval. Shown on this diagram are the contributions to $\epsilon$ by the individual water vapor absorption bands. These individual band contributions are strongly temperature dependent as a result of the variation of $B_{\nu}/\sigma T^4$ with temperature. It is rather fortuitous that the sum total is, more or less, independent of temperature as the ratio of $B_{\nu}/\sigma T^4$ for wavelengths in the rotation band
increases with decreasing temperature and this increase approximately cancels the reverse trend for wavelengths in the vibration band. For this reason, it is most inconvenient to calculate the longwave radiative flux in a few discrete spectral bands using the emissivity method.

The solution of the flux equations in emissivity form [e.g., (17) or (22)] is straightforward given $\varepsilon$ or $\varepsilon'$ as a function of $u$ (the reader is referred to Charlack and Herman, 1976, Sasamori, 1968, for specific examples). Staley and Jurica (1970) tabulate $\varepsilon$ for water vapor and carbon dioxide absorption as a function of $u$. Sasamori gives convenient formulas for $\varepsilon$ and $\varepsilon'$ for H$_2$O, CO$_2$ and O$_3$ absorption based on the absorption data of Elsasser and Culbertson (1960). In a slightly different approach, Rodgers (1967) provide analytic formula for $\varepsilon$ and $\varepsilon'$ for both H$_2$O and CO$_2$ absorption based on the band model data summarized by Rodgers and Walsh (1966). Stephens and Webster (1979) offer a parameterization of the contribution that can be added to $\varepsilon$ and $\varepsilon'$ by the e-type absorption in the atmosphere continuum while Garand (1983) outlines an alternate method for the inclusion of this absorption into the emissivity formulation.

Rodgers (1967) demonstrated that the emissivity method provides best results when $\varepsilon'$ is used to calculated upward flux and $\varepsilon$ is applied to downward flux. Fig. 8, modified from Rodgers, illustrates this point by comparing upward and downward fluxes using both $\varepsilon$ and $\varepsilon'$ against some standard (which in this case is the Rodgers and Walsh, 1966, scheme). These comparisons are shown as flux differences. Also listed on the diagram are the rms deviations and the average deviations of the emissivity derived fluxes from the standard averaged over all levels and over some 200 different atmospheres. The average deviation provides some indication of systematic error; errors of the order of 10 W m$^{-2}$ are typical when either $\varepsilon$ or $\varepsilon'$ are used to calculate longwave fluxes while the error in the cooling rate can be as large as 0.5°C day$^{-1}$ (e.g., Stone and Manabe, 1968; Fels and Schwarzkopf, 1975).

e. Summation of contributions by all atmospheric layers

So far the approximations described above are, accuracy requirements apart, independent of the host dynamical model. The integration of the flux equation to include the contributions by all atmospheric layers to the flux and flux divergence at some reference layer depends on the actual vertical resolution of the model. Two closely related questions arise in relation to this integration. These are i) how many layers are required to carry out the integral adequately and ii) to what height must the calculations be taken? Answers to both of these questions are a problem and thus model dependent (whether or not temperature and moisture discontinuities are involved for example), and they really require assessment on a trial and error basis by individual models. It suffices to say that provided there are no temperature and moisture discontinuities along the atmospheric path length, then it would seem that...
the calculation of longwave flux and flux divergence can be handled by around a dozen or so layers. As an example, Rodgers and Walsh (1966) use 13 layers to calculate the cooling profile for an atmosphere with a smooth temperature profile and compared this profile to one based on a 25 layer atmosphere with the same temperature and moisture profile. The differences between the two were about 0.01–0.03 °C day\(^{-1}\). Manabe and Strickler (1964) indicated that they could obtain, without any appreciable loss of accuracy, representative radiative equilibrium temperature profiles using their 9 layer model rather than their full 18 layer version. 

The integration of the flux equation over all atmospheric layers is a laborious process requiring the largest outlay of computational effort. As the computational effort varies quadratically with the number of model layers, the longwave flux calculations can easily occupy a large percentage of the total time required to run a GCM, especially one with several vertical levels. An obvious and fruitful exercise is to simplify this integration in some fashion. This can be achieved by considering the structure of the equations of longwave flux and flux divergence. Green (1967) formalized the equation of radiative flux divergence into two major terms which are schematically illustrated in Fig. 9. The first is the cooling-to-space term (CTS) which is that part of emission from a reference layer A that is directly transmitted to space. The next major component is that of the internal exchange between layers. An example is the radiation emitted by layer A, transmitted as far as layer B, then absorbed by B less the radiation emitted by B and absorbed by A. The relative importance of this exchange process rapidly diminishes as the distance from the reference layer increases due to the exponential drop-off of transmission. This point is well demonstrated in Figs. 10a and b in which the contribution by various layers to the radiative cooling of a reference layer is shown.

The diagram is modified from the work of Wu (1980) who considered the total longwave clear sky cooling of 3 km thick layers in a model tropical atmosphere. The cooling of the reference layer is separated into two major components similar to, but not the same as, the cooling to space and exchange terms described before. The components considered and illustrated in Fig. 10a are the direct cooling of the reference layer itself (shown as the shaded bar on Fig. 10a) and the heating induced by absorption of radiation incident on the reference layer from the immediate layers above and below. It is evident from Fig. 10a that the contribution to the radiative cooling by radiation exchange between distant layers (more than 3 km from the reference) is negligibly small. Fig. 10b shows a spectral breakdown of the cooling by the reference layer for the 20 spectral intervals employed by Wu (1980). Most of the energy exchanged between adjacent layers is through the wings of strong lines, the continuum and, to a lesser extent, the centers of moderately weak lines.

Figures 10a and b serve to indicate, at least for the case shown, that it is not necessary to sum the contributions to the radiative flux divergence from all atmospheric layers either side of the reference level but rather to sum only those contributions from adjacent layers (for the case shown, a 3 km thick layer either side of the reference is sufficient). It also follows that only the top few layers will be influenced by decision of where to place the top of the model. It is important to emphasize, however, that more serious errors can arise by failing to include the atmosphere above the top level in computing the path quantity \( \mu \) in the calculations.

Unfortunately, the heating from distant layers is not always as negligible as Fig. 10a would lead us to believe. The effects of temperature discontinuities in distant layers or the warming of the ozone layer as a result of the warmer underlying surface are two examples.

A number of schemes have exploited the separation of the flux divergence equation into the cooling to space component and the exchange between layers component (e.g., Joseph and Burstyn, 1976; Fels and Schwarzkopf, 1975). Fels and Schwarzkopf demonstrated that techniques suitable for the evaluation of one of the components are not necessarily best for the other. They proposed a hybrid scheme, that of calculating the internal exchange terms via an emissivity method and a band model approach to evaluate the CTS component. This hybrid scheme is demonstrably more accurate than the standard emissivity methods and it can be achieved with little additional computational effort.

3. The parameterization of longwave fluxes in cloudy atmospheres

The longwave cooling rates within clouds are crudely calculated in the majority of dynamical models by
assuming that clouds radiate as black bodies. These cooling rates represent the average radiative cooling of a model layer when the cloud is wholly contained within the layer. However, when the cloud fills more than one adjacent layer, it becomes necessary to distribute this cooling between layers in some fashion. It is therefore necessary to parameterize the longwave transmittance of clouds not only in order to calculate the cooling profile through thick clouds but also to define the transmittances properties of thin clouds possessing emittances that are much less than unity (cirrus for instance, Platt and Dilley, 1979). Although such parameterizations have only been recently incorporated into GCMs (refer to Section 6 for some examples) the need for an accurate specification of cloud emittance in these models is genuine (e.g., Ramanathan et al., 1983).

The following section outlines a viable method for parameterizing the longwave transmission (and emission) properties of clouds, in terms of gross cloud parameters that are (or will be) predictable a priori. In developing such a parameterization, it is necessary to look in some detail at the interaction of radiation and cloud microphysics. While one is formally faced with the problem of solving the complete equation of transfer in a multiple scattering–absorbing medium, the dominance of liquid water absorption in the infrared over scattering by droplets simplifies the problem somewhat. This simplification allows us to approximate the cloud transmission characteristics in terms of simple approximations analogous to those described above for gas absorption.

### a. The absorption approximation for clouds

If we assume that cloud droplet absorption is dominant then the absorptivity along the path \((z, z')\) inclined at an angle \(\theta = \cos^{-1} \mu\) is defined as [cf. (4)]

\[
A_z(z, z') = 1 - \exp[-\delta_z(z, z')],
\]

where \(\delta_z\) is the absorption optical thickness

\[
\delta_z(z, z') = \frac{1}{\mu} \int_z^{z'} \kappa_z(z'')dz''.
\]

The volume absorption coefficient \(\kappa_z\) is defined for spherical droplets as

\[
\kappa_z(z'^*) = \pi \int_0^{\infty} n(r, z'^*)r^2Q_{abs}(\nu, r)dr,
\]

where \(r\) is the radius of the water droplet, \(n(r)\) is the droplet's size distribution and \(Q_{abs}\) is the absorption efficiency of a droplet of radius \(r\). This efficiency is obtained from Mie theory for spherical water droplets (see Van de Hulst, 1957, for an excellent discussion of Mie theory).
The above formulas are not of great practical use since the droplet absorption coefficient \( k_s \) varies along the path \( z' \) (through the dependence of \( n(r) \) on \( z' \)) and as well contains the particle size distribution which is a quantity that is not readily (or may never be) supplied by general circulation models.

b. Mass absorption coefficient and the liquid water path

We can convert \( k_s \) to a mass absorption coefficient \( k_s^c \) according to Appendix B by dividing \( k_s \) by the density of liquid water in the cloud, i.e.,

\[
k_s^c(z') = \frac{k_s(z')}{w(z')} = \frac{3}{4} \int_0^\infty n(r, z') r^2 Q_{abs}(v, r) dr
\]

where \( w(z') \) is the liquid water content of the cloud at \( z' \). It is well known (e.g., Van de Hulst, 1957, p. 113) that \( Q_{abs} \) increases linearly with particle radius when the size parameter \( x = (2\pi r/\lambda) \) is small. That is

\[
Q_{abs}(v, r) = c(v)r \quad \text{for} \quad r > r_m,
\]

where \( c \) is some frequency factor and \( r_m \) is the radius limit to which this linear relation is applicable. An example of such a relationship is shown in Fig. 11 for \( \lambda = 9.5 \, \mu \text{m} \) (i.e., \( v = 1053 \, \text{cm}^{-1} \)) for which \( r_m \approx 12.5 \, \mu \text{m} \).

It is indeed fortunate that such an approximation applies since (26) then reduces to

\[
k_s^c = \frac{3}{4} c(v),
\]

and the absorptivity follows as

\[
A_s(z, z') = 1 - \exp(-k_s^c W(z, z') \mu^{-1}),
\]

where \( W(z, z') \) is the liquid water path along the geometric path \( (z, z') \) defined as

\[
W(z, z') = \int_z^{z'} w(z'')dz'',
\]

where the limits of the integral are usually defined from cloud top (or base) to the depth \( z' \) under consideration. This expression has been derived by a number of workers who all made use of the relationship between \( Q_{abs} \) and \( r \) (e.g., Platt 1976; Chylek 1978; Pinnick et al., 1979; among others).

Equation (29) is only valid for clouds composed of droplets that are similar than \( r_m \). Despite the fact that real clouds contain some droplets that exceed this size limit, it has been illustrated repeatedly (both from ob-

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3 In this context it is relevant to compare (27) with the factorization in (6) and then compare the effects of both on the path integrals in the relevant definitions of transmissivity (absorptivity).
which is conveniently approximated by
\[ \epsilon_f' \approx 1 - \exp(-\beta k_f W), \]  
(32)
where \( \beta (=1.66) \) is the diffusivity factor as defined in relation to (5). Since, for the purposes of computational efficiency, we are interested in broadband emissivities over some interval \( \Delta \nu \) it is convenient to define the broadband flux emissivity
\[ \epsilon_f' = \int_{\Delta \nu} B_\nu(T)\epsilon_f' d\nu \left[ \int_{\Delta \nu} B_\nu(T) d\nu \right]^{-1}. \]  
(33)

The variation of \( \epsilon_f' \) with frequency (or of \( k_f' \) with frequency) is quite smooth owing to the more continuous nature of liquid water absorption compared to molecular absorption. In this context, the grey approximation is more readily invoked, viz.,
\[ \epsilon_f' = 1 - \exp(-\beta k_f W), \]  
(34)
where \( k_f \) is the mass absorption coefficient averaged over the interval \( \Delta \nu \). The appropriateness of (34) for a variety of water cloud types is again supported by both observation and theory.

Figure 12 shows the various contributions to the total broadband emittance as a function of \( W \) for three spectral regions chosen to coincide with those of Fig. 7. These contributions were calculated from (23) and (33) using Mie theory and a particle size distribution which could be considered as representative of a stratocumulus cloud type. The similarities in the curves shown in Fig. 12 and those for water vapor absorption attest to the effect of the temperature variation in the ratio \( \int_{\Delta \nu} B_\nu d\nu/\sigma T^4 \) in the definition of both emissivities. The total cloud emissivity, which is just the sum total of the individual band contributions, are shown in Fig. 13 for three given temperatures. It is clear from the temperature variation depicted in Fig. 12 that it is not possible to define an unambiguous narrow-band value of \( \epsilon_f' \) and that the approximation employing a single value of emissivity is best suited only to the description of the total broadband emittance properties of clouds.

d. Estimating broadband values of \( k_f \)

If we assume that droplet absorption is the dominant longwave process in clouds, then the longwave radiative flux through a cloud layer is simply obtained by the emittance form of the flux equations. The vertical integrals in these equations are trivial given a knowledge of the variation of liquid water path with \( z \). For example, the fluxes through some level \( z \) in an isothermal cloud layer are
\[
\begin{align*}
F'(z) &= F'(z_b)\left[1 - \epsilon(z_b, z)\right] + \epsilon(z_b, z)\sigma T_c^4, \\
F'(z) &= F'(z_t)\left[1 - \epsilon(z_t, z)\right] + \epsilon(z_t, z)\sigma T_c^4,
\end{align*}
\]  
(35)
where \( z_b \) and \( z_t \) are the cloud base and top altitudes and \( T_c \) is the cloud temperature. These equations have been extensively applied to broadband measurements or to rigorous theoretical calculation of longwave flux. The cloud emissivity can be determined from the measurement of the flux profile through a cloud and the cloud temperature by
\[
\begin{align*}
\epsilon'(z) &= \epsilon(z_t, z) = \frac{F'(z_t) - F'(z)}{F'(z_t) - \sigma T_c^4(z)}, \\
\epsilon'(z) &= \epsilon(z_b, z) = \frac{F'(z_b) - F'(z)}{F'(z_b) - \sigma T_c^4(z)}. \end{align*}
\]  
(36)
(37)

A major aspect of these definitions is that the emissivity varies with the different spectral nature of the upward and downward fluxes that are incident on the relevant cloud boundaries. Table 3 lists various values of \( k_f' \) obtained by fitting (36) and (37) with both measured fluxes and fluxes determined from rigorous radiative
transfer theories. These data are taken from various sources for boundary layer stratiform cloud and for upper level cirrus cloud and are based on both theory and observations. The general agreement between theory and observations support the application of (34) and the assumptions embodied within this parameterization.

As a final remark, the measured fluxes used in (36) and (37) to obtain values of $\epsilon'$ and thus of $k_0^2$ include the combined effects of scattering plus droplet absorption as well as absorption by the trace gases. The emissivities so derived are often called "effective" emissivities for this reason (e.g., Cox, 1976). Stephens (1980) demonstrated that the reflection of longwave flux from the base of a cold cloud over a warm surface (as for high cirrus in a tropical atmosphere) leads to values of $\epsilon'$ that substantially exceed unity. The form of (34) precludes such a possibility and this general parameterization approach may not be sufficiently accurate for general use in numerical models. This problem, in addition to that associated with the small particle assumption implied in (34), highlights the need for an alternate parameterization scheme for high cloud.

e. The calculation of longwave fluxes in an atmosphere containing cloud

If we make the standard blackbody assumption for clouds in the IR, then the calculation of longwave flux through the entire atmosphere is not overly complicated by their presence. Black clouds simply add extra boundaries to the clear sky flux equations; one at the cloud base representing an absorbing lid to the up-welling radiation from below and the other at cloud top forming another surface which radiates to space. The calculation of clear sky longwave flux then proceeds in the usual manner. The following equations, modified from (22), are offered as examples of the flux equations above and below a cloud layer with a cloud top altitude of $z_t$ and cloud base altitude of $z_b$.

\[
F'(z) = \sigma T^4(z) [1 - \epsilon(z, z_t)] \\
+ \int_{z}^{z_t} \sigma T^4(z') \frac{d\epsilon(z, z')}{dz'} dz' \quad \text{for} \quad z > z_t
\]

\[
F'(z) = \sigma T^4(z_b) [1 - \epsilon(z_b, z)] \\
+ \int_{z}^{z_b} \sigma T^4(z') \frac{d\epsilon(z, z')}{dz'} dz' \quad \text{for} \quad z < z_b
\]

The fluxes within the cloud can also be calculated from these equations by interposing $z_t$ and $z_b$ and employing the appropriate cloud emissivity. In this case these general equations include the sum of the contributions by all cloud layers between the reference level and the relevant cloud boundary. Since clouds often occur within a single layer of the host dynamical model it is generally not possible to carry out the summation on this "subgrid" scale. The integral term is then replaced by a single layer term similar to that of (35).

The question arises as to what cloud temperature should be used to evaluate this term. The effective emission temperature or "center of gravity" temperature occurs at some distance from the reference level $z$. This temperature is typically around 50–100 m above or below the reference level in water clouds (e.g., Platt, 1976) for which case the local reference temperature is probably most adequate. However, a problem is likely for transparent cirrus cloud with center of gravity temperatures occurring several kilometers from the reference level.

1) PARTIALLY BLACK CLOUD

The latter point highlights one of the problems immediately encountered for clouds that are partially opaque to longwave radiation. Another problem arises from the need to define a transmittance along paths that have sections in both clear and cloudy sky. If we employ the customary greybody approach for both gaseous and cloud absorption, then the emission along a path with overlapped absorption is often approximated by

\[
\epsilon_{\text{overlap}} \approx 1 - (1 - \epsilon_{\text{cloud}})(1 - \epsilon_{\text{gas}})
\]

which is the equivalent of (19). This overlapped emissivity was employed by Griffith and Cox (1979) and is implied in the formulas of Ramanathan et al. (1983) among others. Eq. (39) is, at best, an approximation to the problem of absorption overlap and the errors associated with its application have not been assessed. If these errors prove to be intolerable, then some alternative method of overlapping cloud and gaseous absorption is required (e.g., such as that of Fiedelson, 1970, or that described as follows in relation to shortwave parameterization).

2) PARTIAL CLOUDINESS

The usual approach for calculating the radiative fluxes in an atmosphere containing partial cloudiness is to calculate separately the fluxes for completely overcast and clear skies and combine the two in some manner proportional to cloud amount. For example, the upward flux at level $z$ above some layer that contains a fractional area of cloud cover $A_c$ is determined by linearly weighting the clear and cloud sky fractions.

\[
F'(z) = (1 - A_c) F'(z)_{\text{clear}} + A_c F'(z)_{\text{cloudy}}
\]

As a result of recent theoretical studies on the effect of cloud shape on longwave transfer, it is now possible to test the validity of (40) for certain hypothetical cloud types. Harshvardham and Weinman (1982) demonstrated with a laboratory arrangement of black square blocks over some radiating surface that the upwelling flux from this arrangement is given by (40) when the actual cloud amount $A_c$ is replaced by
where \( a \) is the aspect ratio of the block (ratio of depth to width). (Note that \( A_e \rightarrow A_c \) when \( a \rightarrow 0 \), the horizontally infinite cloud case.) Fig. 14 illustrates the relationship between \( A_e \) and \( A_c \) for cloud with several different values of \( a \). For large \( a \), the shape and radiative characteristics are dominated more by the sides of clouds (provided that they are separated far enough from their neighbors as is the case for small \( A_e \)) and the net emission by the layer containing these cloud arrays is enhanced substantially. The effect of the different specifications of \( A_e \) on calculation of the longwave radiative cooling is shown in Fig. 15. Three cooling profiles are shown for a model tropical atmosphere i) the clear sky cooling profile, ii) the cooling profile for a cloudy sky with a 10% coverage of planiform cloud in the layer from 905 mb to 860 mb (i.e., \( a = 0 \), \( A_e = A_c = 0.1 \)) and iii) the cooling profile of a cloudy sky with a 10% areal coverage of broken cloud with \( a = 1 \). The radiative cooling of the atmospheric layer that contains the cloud is shown as an average over that layer, and it is readily seen that the layer with broken cloud radiatively cools at a rate that is substantially greater than the layer that contains a partial, but solid, cloud cover.

4. The parameterization of shortwave transfer in the clear sky

The spectral distribution of solar irradiance received at sea level through a cloud and haze free atmosphere is shown in Fig. 16. A natural division exists for this irradiance each side of \( \lambda \approx 0.7 \) \( \mu \text{m} \) as a result of the spectral differences in the absorption by ozone in the visible \((0.5 \mu \text{m} \leq \lambda \leq 0.7 \mu \text{m}) \) and in the ultraviolet \((\lambda \leq 0.3 \mu \text{m}) \) and by water vapor in the near IR region \((0.7 \mu \text{m} \leq \lambda \leq 4.0 \mu \text{m}) \). The major absorption bands are shown as solid areas in Fig. 16. The absorptions by \( \text{O}_2 \) and \( \text{CO}_2 \) are substantially less than either that of water vapor or ozone and their contribution can often be neglected. The formal division of the solar spectrum is further reinforced by noting that Rayleigh scatter is dominant only for the shorter wavelengths (in fact roughly proportional to \( \lambda^{-4} \)) and that liquid water absorption in cloud occurs only in the region \( \lambda > 0.75 \mu \text{m} \).

\[ a. \quad \text{Shortwave flux equations in a nonscattering atmosphere} \]

The transfer of solar radiation through an atmosphere in which the effects of scattering are neglected is described by the well-known equation

\[ S'(z, \mu_0) = \mu_0 \int_0^{\infty} S(\infty) \tau(z, \infty, \mu_0) dv, \quad (42) \]

where \( S'(z) \) is the downward irradiance through level \( z \) for a collimated beam of solar irradiance \( S(\infty) \) at the top of the atmosphere inclined at a zenith angle

\[ \text{FIG. 14. The effective cloud fraction for radiatively black cuboidal clouds (\( A_e \)) as a function of true cloud fraction (\( A_c \)) for aspect ratios marked on the curves. Full lines are given by (41) in the text, measured points refer to laboratory measurements and theory refers to radiative transfer theory based on single non-(radiatively) interacting clouds (from Harshvardham and Weinman, 1982).} \]

\[ \text{FIG. 15. The longwave cooling rate profiles in a tropical atmosphere for i) clear sky (dotted curve), ii) a sky covered by 10% planiform (i.e. solid) cloud (dashed curve), and iii) a sky covered by 10% broken (cuboidal) clouds (solid curve).} \]
\(\theta_0 (\text{or } \mu_0 = \cos \theta_0)\). The monochromatic transmittance function for this collimated beam is

\[
\tau_\nu (z, \infty, \mu_0) = \exp \left( -\frac{1}{\mu_0} \int_z^\infty k_\nu du \right). \tag{43}
\]

The transfer of solar radiation defined by (42) is less complex than the longwave counterpart as it is not necessary to consider the complicated problem of simultaneous absorption and emission from layer to layer in the atmosphere. While equivalent problems are encountered in the path and frequency integrals, the latter is also less involved as the spectral variation of the solar flux [i.e., of \(S_\nu (\infty)\)] can be defined \textit{a priori} and the pressure and temperature effects on \(k_\nu\) only complicate matters for water vapor absorption. It thus remains to define the mean transmission function

\[
\tau_\nu (z, \infty, \mu_0) = \frac{1}{\Delta \nu} \int_{\Delta \nu} \exp \left( -m_\nu (\mu_0) \int_z^\infty k_\nu du \right) du, \tag{44}
\]

where we introduce the relative air mass factor \(m_\nu (\mu_0)\) in place of \(1/\mu_0\) in (43). This factor is different only from \(1/\mu_0\) for large solar zenith angles (\(\theta_0 \lesssim 75^\circ\)) when it is necessary to account for the effects of earth’s curvature and atmospheric refraction. In most cases \(m_\nu = 1/\mu_0\) except for ozone for which

\[
m_\nu = 35 \mu_0 (1224 \mu_0^2 + 1)^{-1/2}. \tag{45}
\]

The clear-sky downward solar flux transmitted to level \(z\) along \(\theta_0\) is

\[
S'(z) = \mu_0 \sum_{i=1}^{N} S_\nu (\infty) \tau_\nu (u_i). \tag{46}
\]

where the integral in (44) is replaced by the summation over \(N\) subintervals and where \(u\) is used for the path length from \(z\) to \(\infty\) along \(\theta_0\). The upward solar flux received at level \(z\) by reflection from the ground is similarly defined as

\[
S'(z) = \mu_0 \sum_{i}^{N} \alpha_{s_i} S_\nu (\infty) \tau_\nu (u^*_i), \tag{47}
\]

where \(\alpha_{s_i}\) is the surface albedo for the \(i\)th subinterval. The path length \(u^*_i\) is the effective total absorber amount traversed by the diffusely reflected radiation and this can be approximated usefully by (Lacis and Hansen, 1974)

\[
u^*_i = m_\nu (\mu_0) u_0 + (u_0 - u) \bar{m}. \tag{48}
\]

In this formula \(u_0\) is the total absorber amount of the vertical atmospheric column and \(\bar{m}\) is an effective magnification factor for diffuse radiation analogous to \(\beta\) in (5). Lacis and Hansen suggest that \(\bar{m} \sim 1.9\) is appropriate for ozone and 5/3 for water vapor.

The mean transmittance is commonly defined as a convolution of the transmission function and the extraterrestrial solar flux \(S_\nu (\infty)\). One can then define a
transmittance function averaged over the entire solar spectrum as

\[ \tau(z, \infty, \mu_0) = \frac{1}{S(\infty)} \int_0^\infty S_\nu(\infty) \tau(\nu, z, \infty, \mu_0) d\nu, \]  

(49)

in which case the downward solar flux at level \( z \) is defined by

\[ S'_\nu(z) = \mu_0 S(\infty) \tau(z, \infty, \mu_0). \]  

(50)

The use of (49) and (50) has proved to be the most popular method of calculating solar fluxes within the atmosphere since it is only necessary to perform the solar flux calculations once to obtain total broadband solar fluxes (in the same way that greybody emissivity reduced the complexity of the longwave flux calculations). However, there are a number of reasons why the discrete band approach of (46) is preferable. First, it is possible to choose one (or more) discrete band(s) to correspond to certain of the satellite radiometer channels (such as the 0.5–0.7 \( \mu \)m channel of the TIROS N satellite). Satellite measurements provide a convenient way of calibrating the radiation parameterization. This is naturally achieved at the expense of computational efficiency and one must assess the trade-off between decreased computational speed and increased information. The major disadvantage of (50) is that different parameterizations of \( \tau \) for a given gas can vary with the choice of the extraterrestrial solar flux \( S(\infty) \) even when the same absorption data are used. Therefore each new estimate of the solar constant \( S(\infty) \) requires a new parameterization of \( \tau \). It is demonstrated below that this is not a trivial disadvantage of the method.

b. Parameterization of transmission (absorption) functions

1) WATER VAPOR

Absorption of solar radiation by water vapor is the major source of solar heating in the lower atmosphere. The parameterization of this absorption is more complicated than that of ozone because i) the absorption coefficient of water vapor is more strongly frequency dependent and accurate absorption data are not available for the entire spectrum, ii) the absorption is complicated by the pressure and temperature dependences and iii) both water vapor and liquid water absorption by clouds overlap thus precluding the simple reflecting schemes discussed below for ozone absorption.

There are numerous formulations for water vapor absorption. Most notable are those of Korb et al. (1956), McDonald (1960), Yamamoto (1962), Sasaki et al. (1972) and Lacis and Hansen (1974). Most, however, are based on the measurements of Howard et al. (1956) which are usually augmented by the measurements of Fowle (1915) for the weak bands near 0.7 and 0.8 \( \mu \)m. Fig. 17 displays the absorption \( A = 1 - \tau \) obtained from a number of these sources as a function of the water vapor path \( u \). The difference between some of the formulations can be explained by the choice of \( S(\infty) \) used to define \( A \). (This certainly applies to the relations of Lacis and Hansen, 1974, compared to those of Fouquard and Bonnet, 1980, or to the relations of Yamamoto, 1962, compared to McDonald, 1960, for example.)

The formulation of Lacis and Hansen (1974) has proved to be a very useful one, namely

\[ A = \frac{2.9u}{(1 + 141.5u)^{0.635} + 5.925u} \]  

(51)

which fits the Yamamoto absorption curve to within 1% for the range 0.001 < \( u < 10 \) cm. The quantity \( u \) is the scaled water vapor amount in cm of precipitable water and is defined by \( (8) \) with \( T_0 = 273 \) K and \( p_0 = 1013 \) mb.

The solar heating rates based on the various different absorptivity formulations are displayed in Fig. 18. This diagram presents the clear-sky water vapor heating rate profiles with \( \theta_0 = 60^\circ, \alpha_0 = 0.07 \) for the two model atmospheres of McClatchey et al. (1972) with \( u_0 = 2.95 \) cm (midlatitude summer) and \( u_0 = 0.50 \) cm (subarctic winter).

The curves apply to calculations that use \( u \) defined with the pressure scaling factor \( n = 1.0 \). The discrepancies apparent in the absorptivity curves in Fig. 17 are also evident in the heating profiles in Fig. 18. McDonald’s absorptivities clearly underestimate the heating by 20–30%, those of Yamamoto and Korb et

\[ 1 = McDonald (1960) \]
\[ 2 = Korb et al. (1956) \]
\[ 3 = Yamamoto (1962), Lacis and Hansen (1974) \]
\[ 4 = Fouquard and Bonnet (1980) \]
\[ 5 = Fowle (1915) \]

\[ \text{FIG. 17. A comparison of some different relationships between total water vapor absorptance and water vapor path } u \text{ based on work cited.} \]

\[ \text{Unauthenticated | Downloaded 07/30/22 08:51 PM UTC} \]
al. are inconsistent for middle to high altitude even though both use the data of Howard et al. The comparisons shown in Fig. 18 demonstrate the importance of defining the absorptivity for relatively small values of $ar{u}$. The different methods for extrapolating the absorption in this small $ar{u}$ region could account for the discrepancies in the upper troposphere and lower stratosphere for the moist midlatitude summer atmosphere or for the discrepancies throughout the troposphere in the drier subarctic winter atmosphere.

The solid and dashed curves shown on Fig. 18 refer to the parameterization of Wang (1976) who employed different absorption data and developed a parameterization of absorptance based on the Goody band model, thus permitting nonhomogeneous path corrections via the VCG approximation rather than the pressure scaling approach. The differences between two pressure scalings with $n = 1$ and $n = 0$ and the VCG approach when applied to the calculation of shortwave heating is shown in Fig. 19 for the two McClatchey et al. model atmospheres with $	heta_0 = 60^\circ$ and $\alpha_\nu = 0.07$. The mean effect of the simple scaling formula (7) is to reduce the heating rates in the lower troposphere and to increase these in the higher atmosphere relative to the VCG approach. The uncertainties associated with the application of (7) for $ar{u}$ and the subsequent choice of $n$ are smaller than those associated with uncertainties in both the absorption data and $S_\nu(\infty)$.

2) Ozone Absorption from 0.28 to 0.65 $\mu$m

The absorption by ozone occurs at those wavelengths where molecular scattering is significant. Fortunately, this absorption occurs high in the atmosphere where there is little scattering owing to the low molecular density. Most parameterizations are therefore generally concerned with the downward direct solar beam although the absorption of the diffusely scattered and reflected upwelling flux cannot be totally excluded. The parameterization of the latter effects are discussed below.

Lacis and Hansen (1974) present convenient analytic formula for the frequency integrated absorption by ozone in both the ultraviolet and visible regions of the spectrum. For visible light, the absorption is by the Chappuis band and is given by

$$A_{\text{ozone}}^{\text{vis}}(\nu) = \frac{0.02118\nu}{1 + 0.042\nu + 0.000323\nu^2}$$  \hspace{1cm} (52)

for the range $0.0001 < \nu < 10$ cm (NTP) with four figure accuracy. The ultraviolet absorption, with an accuracy $<0.5\%$ between 0.0001 < $\nu$ < 1 cm (NTP), can be determined from

$$A_{\text{ozone}}^{\text{uv}}(\nu) = \frac{1.082\nu}{(1 + 138.6\nu^{0.805}) + \frac{0.0658}{(1 + 103.6\nu^{3})}}$$  \hspace{1cm} (53)

and the total absorption (fraction of the incident solar flux) is obtained by the addition of (13) and (14). Fig. 20 presents a comparison of various relationships between $A$ and ozone amount $\nu$. Table 4 summaries the various relationships, the data upon which they are based and the spectral solar flux data used for ozone absorption and water vapor absorption.

It is a rather trivial exercise to show that the solar heating, based on a flux divergence equation which
can be derived from (46) or (47) is proportional to \(dA/dz\). It follows that the average heating prescribed for some broad layer is representative of the heating within that layer provided that \(A\) is a relatively smooth function of either \(u\) or \(v\) across the layer. This presents a particular problem for ozone absorption. The top model layer in many GCMs is usually assumed to extend to the top of the atmosphere and the midpoint of the model layer is often somewhere below the ozone layer. Because substantial \(O_3\) absorption occurs above and within this layer, especially in the UV region, the solar heating at the midpoint of the top model layer can be unrealistically large. It is therefore necessary to define some appropriate midlevel height to obtain the most representative solar heating at that level (e.g., Ramanathan et al., 1983).

3) UNIFORM MIXED GASES

The analysis of shortwave absorption by Yamamoto indicates that the total absorption by all major water vapor bands is only marginally different from the combined absorptions by water vapor, \(CO_2\) and \(O_2\). The heatings by the \(CO_2\) and \(O_2\) absorption bands are usually added to the heating by the water vapor absorption bands despite the fact that both \(CO_2\) and \(O_2\) absorption overlaps that of water vapor (see Fig. 16). This is not likely to present any major errors since the contributions of both \(CO_2\) and \(O_2\) to solar heating is small. Sasamori et al. (1972) present analytic formula for both \(CO_2\) and \(O_2\) absorption while Fouquart and Bonnel (1980) give a formula for the absorptivity of \(CO_2\).

c. Parameterization of scattering and absorption in the clear sky

The effect of Rayleigh scatter on the transfer of solar radiation in the clear sky produces two definable effects i) the net solar flux is reduced at the top of the atmosphere as a result of the diffuse reflection by molecules and ii) the absorption within a given layer is enhanced as a result of absorption from the upward diffusely reflected solar flux. The wavelength dependence of Rayleigh scatter confines these effects only to the short wavelength ozone absorption. Also this scattering occurs in the lower atmosphere below most of the ozone absorption. These features make it possible to parameterize the effects of this scattering on absorption in some simple \textit{a priori} manner.

The simplest approach is that of Manabe and Strickler (1964) who use the modified form of (50)

\[
S'(z) = \mu_0 S'(\infty) \tilde{r}(z, \infty, \mu_0),
\]

where \(S'(\infty) = 0.93S(\infty)\). This approach, while simple, is not physically realistic as most of the scattering occurs.
Table 4. Summary of absorption data and solar flux data employed in various parameterizations of solar absorption by H$_2$O and O$_3$.

<table>
<thead>
<tr>
<th>Gas</th>
<th>Parameterization</th>
<th>Absorption data</th>
<th>Solar flux data [S(∞)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>H$_2$O</td>
<td>Fouquardt and Bonnel (1980)</td>
<td>Absorption data compiled by McClatchey et al. (1973) including data of Howard et al. (1956)</td>
<td>Handbook of Geophysics (1965)</td>
</tr>
<tr>
<td></td>
<td>Lacs and Hansen (1974)</td>
<td>Howard et al. (1956) and Fowlie (1915)</td>
<td>Labs and Neckel (1968)</td>
</tr>
<tr>
<td></td>
<td>Yamamato (1962)</td>
<td></td>
<td>Labs and Neckel (1968)</td>
</tr>
<tr>
<td></td>
<td>McDonald (1960)</td>
<td>Fowlie (1915)</td>
<td></td>
</tr>
<tr>
<td>O$_3$</td>
<td>Fouquardt and Bonnel (1980)</td>
<td>Absorption data compiled by McClatchey et al. (1973) and include data of Vigouroux (1953)</td>
<td>Handbook of Geophysics (1965)</td>
</tr>
<tr>
<td></td>
<td>Lacs and Hansen (1974)</td>
<td>Inn and Tanaka (1953) and Vigouroux (1953)</td>
<td>Labs and Neckel (1968)</td>
</tr>
<tr>
<td></td>
<td>and Sasamori et al. (1972)</td>
<td>Inn and Tanaka (1953) and Vigouroux (1953)</td>
<td>Johnson (1954)</td>
</tr>
</tbody>
</table>

below the ozone layer. As a result of (54), the absorption by ozone will be underestimated, first by the depletion of the available solar flux incident on the layer by 7% and second by grossly underestimating the absorption of the diffusely scattered and reflected upward radiation.

An alternate approach is that of Lacs and Hansen (1974) which employs the flux equations (46) and (47) (except that the product of $S(\infty)\tau$ is used instead of $\sum S_i(\infty)\tau_i$). The surface albedo $\alpha_s$ is replaced by some $\sum_i$ compositoed lower atmosphere albedo, defined as

\[
\bar{\alpha}(\mu_0) = \alpha_R(\mu_0) + [1 - \alpha_R(\mu_0)]\frac{\alpha_0}{1 - \alpha_R \alpha_0},
\]

\[
\alpha_R(\mu_0) = 0.219/(1 + 0.816 \mu_0),
\]

\[
\alpha_0 = 0.144
\]

(55)

where $\bar{\alpha}(\mu_0)$ is a combination of the surface albedo and the atmospheric diffuse reflection $\alpha_R(\mu_0)$. This composite is defined by the adding scheme (see below). The albedo $\alpha_R(\mu_0)$ is merely a fit to a standard calculation of Rayleigh scattering in the lower atmosphere. For this exercise, the lower atmosphere (which we take as the lowest model layer) can be treated as a non-absorbing layer. It is then straightforward to calculate the absorption above this layer from a combination of (46) and (47) using $\bar{\alpha}(\mu_0)$ instead of $\alpha_s$.

5. The parameterization of the shortwave radiative properties of clouds

It is almost a standard practice in GCMs to assign fixed, predetermined values to the relevant shortwave radiative characteristics of clouds. This technique has severe limitations; not only is there the problem of determining just what values should be assigned to these parameters but also the radiative properties of clouds change with both changing cloud character and changing elevation angle of the sun. It is apparent from extensive observations (e.g., Feigelson, 1970, 1978) that the pre-assigned cloud properties used in general circulation models are often at variance with both observation and theory. It is therefore desirable to develop a parameterization which has some physical basis and relates, in some way, the radiative properties directly to cloud properties. The following sections now outline such a scheme(s) based on the formal solution of the radiative transfer equation which describes the flow of radiation in a scattering and absorbing cloud (or haze) layer.

a. The transfer of solar radiation in an absorbing–scattering medium

The interaction of clouds with solar radiation cannot be described by the simple absorption models discussed above for longwave flux transfer since the attenuation includes important contributions by scattering and absorption by the cloud particles as well as by gas absorption. The problem of solar transfer is thus complex and one is formally faced with the twofold problem of not only characterizing the cloud microphysics but also solving a complex radiative transfer equation which is usually written in the form (see Liou, 1980)

\[
\mu \frac{dI(\delta, \mu)}{d\delta} = -I(\delta, \mu) + \frac{\bar{\alpha}_0}{2} \int_{-1}^{+1} \hat{\beta}(\delta, \mu, \mu') I(\delta, \mu') d\mu' + \frac{S_0}{4\pi} \hat{\beta}(\delta, \mu, \mu_0) e^{-\bar{\alpha}_0 \mu_0} (56)
\]
where it is taken to be understood that all parameters (excluding \( \mu \) and \( \mu_0 \)) are functions of frequency \( \nu \). The quantity \( I(\delta, \mu) \) is the radiance along the angle \( \mu \) through the level defined by the optical depth \( \delta \) in the cloud and \( S_0 \) is the solar flux associated with a collimated beam incident on the cloud top. The difference that sets (56) aside from the flux equations (1) and (21) are:

i) the optical thickness now includes the contributions through scattering (\( \delta_s \)) and absorption (\( \delta_a \)) by the cloud particles as well as by the intervening gas (\( \delta_g \)), i.e.,

\[
\delta = \delta_s + \delta_a + \delta_g.
\] (57)

ii) The single scatter albedo which is a ratio of the scattering optical thickness to the total optical thickness, i.e.,

\[
\tilde{\omega}_0 = \frac{\delta_s}{\delta}.
\] (58)

Thus, \( \tilde{\omega}_0 = 1 \) for a nonabsorbing cloud and \( \tilde{\omega}_0 = 0 \) when scatter is negligible. Also, \( 1 - \tilde{\omega}_0 \) is the fraction of the incident radiation absorbed by the particle.

iii) The scattering phase function \( \tilde{p}(\delta, \mu, \mu') \) which characterizes the angular distribution of the scattered radiation field. For spherical water cloud droplets, this function exhibits an intensely strong peak in the forward direction with special rainbow and glory effects in the back directions (Liu, 1980, Chapter 5).

As it turns out, most clouds are optically thick (i.e., \( \delta > 5 \)) and the exact nature of \( \tilde{p}(\delta, \mu, \mu') \) is not important as multiple scatter tends to smooth out its peaks. It proves useful to define a parameter

\[
g = \frac{1}{2} \int_{-1}^{+1} \tilde{p}(\delta, \mu, \mu')d\mu
\] (59)

where \( \tilde{p}(\delta, \mu, \mu') \) is the \( g \)th order Legendre polynomial. While the representation of the true angular distribution of scattered light by cloud particles may require a few hundred terms in this series, only a few terms are sufficient for an adequate description of hemispheric integrated fluxes. If we use the analytical function (60), it then follows that \( \tilde{p}(\delta, \mu, \mu') \) is the two term expansion of (61) then yields

\[
\tilde{p}(\delta, \mu, \mu') = \tilde{\omega}_0 (1 + 3g\mu\mu'),
\] (62)

\[
\tilde{p}(\delta, \mu, \mu') = \tilde{\omega}_0 \sum_{l=0}^{\infty} (2l + 1)\psi_l P_l(\mu)P_l(\mu'),
\] (61)

b. The two stream solution

The formal solution of (56) is far too complex and time consuming for operational use in any GCM. It is therefore necessary to reduce the complexities associated with the solution of (56) in some way—the two stream approximation offers such a simplification. It is generally considered that the quantitative study of radiative transfer began with the two stream solution of Schuster in 1905. Today, the preponderance of “two stream models” in the literature is all but confusing to even the more familiar reader let alone one requiring only to understand the basics of the method for use as a parameterization scheme. Yet, the underlying principle of the two stream approximation is deceptively simple, namely to represent the total radiation field as two streams—one each for the upward (\( - \)) and downward (\( + \)) directions.

The starting point of the method is to integrate the radiant intensity \( I(\delta, \mu) \) over the respective upward and downward hemispheres in order to define the relevant fluxes

\[
S^+(\delta) = \int_{0}^{1} \mu I(\delta, \pm\mu) d\mu
\] (63)
which are quantities required to determine solar heating rates and planetary albedo. If we perform the hemispheric integral \( \int d\mu \) on each term of (56), it then follows

\[
\begin{align*}
\frac{dS^+}{d\theta} &= \gamma_1 S^+ - \gamma_2 S^- + \frac{S_0}{4} \tilde{\omega}_0 \gamma_3 e^{-\theta/\mu_{\text{eff}}} \\
\frac{dS^-}{d\theta} &= \gamma_2 S^+ - \gamma_1 S^- + \frac{S_0}{4} \tilde{\omega}_0 \gamma_4 e^{-\theta/\mu_{\text{eff}}} 
\end{align*}
\]

provided that we make some explicit assumption about the \( \mu \) dependence of \( I \). This assumption is necessary as it allows us to replace the hemispheric integral \( \int I(\delta, \mu')d\mu' \) by a simple expression involving \( S^\pm \). It is also relevant to note that the \( \gamma_i \) coefficients contain hemispheric integrals of the form \( \int p(\delta, \mu, \mu')d\mu' \). These integrals are also conveniently evaluated given the relationship between \( I(\delta, \mu) \) and \( \mu' \) and some simple form for the scattering phase function like that of (61).

The solution of (64) can be obtained by standard techniques (e.g., Meador and Weaver, 1980) for given boundary conditions (usually zero diffuse radiation at the cloud boundaries). The solutions are expressed as relatively simple analytic formulas which involve only \( \tilde{\omega}_0, \delta \) and \( g \) since the coefficients \( \gamma_1, \gamma_2, \gamma_3 \) and \( \gamma_4 \) are also defined by these three parameters. There are several ways of relating these coefficients to \( \tilde{\omega}_0, \delta \) and \( g \), each defining a different "two stream model." Some of the more common models, the relevant approximations to the integrated quantities \( S^\pm \) and the resulting \( \gamma_i \) coefficients are listed in Table 5.

While the solution of the general radiative transfer equation using two stream methods are very fast and efficient not all schemes are accurate over the entire parametric ranges of \( \delta, \tilde{\omega}_0 \) and \( g \). For example, negative albedos can occur for thin atmospheres using the Quadrature or Eddington schemes. These inadequacies usually arise when an overly truncated phase function is used in the evaluation of the integrals

\[
\beta_i = \frac{1}{2\omega_0} \int_0^1 p(\mu, -\mu')d\mu'
\]

which can produce negative values of \( \gamma_3 \) (see Table 5). This difficulty is simply overcome by employing the full phase function in the evaluation of \( \gamma_3 \) as is done in the modified Eddington and Quadrature schemes.

Another difficulty usually occurs when \( g \) is near 1 (i.e., highly forward scattering as for water and ice clouds). The addition of a delta function to the scattering phase function is an appropriate modification for these cases for which scattering is very asymmetric. For example, Joseph et al. (1976) introduced the phase function which is a modification of (62) as

\[
p(\delta, \mu, \mu') = 2f \delta_{\mu, \mu'} + (1 - f)(1 + 3g'\mu'\mu),
\]

where \( f \) is the fractional scattering into the forward peak (\( f = g^2 \)) and \( \delta_{\mu, \mu'} \) is the Dirac delta function. This phase function simply rescales the two stream equations by replacing the following

\[
\delta \rightarrow \delta' = (1 - \tilde{\omega}_0 f)\delta \\
\tilde{\omega}_0 \rightarrow \tilde{\omega}_0 = (1 - f)\tilde{\omega}_0/(1 - \tilde{\omega}_0 f) \\
g \rightarrow g' = (g - f)/(1 - f)
\]

The \( \gamma_i \) coefficients resulting from this method are also listed in Table 5 under the \( \delta \)-Eddington method. The scaling defined by (66) reduces the highly forward scattering problem to one of isotropic scattering with the addition of a \( \delta \) function (a so called "ball and stick" representation of the phase function).

c. The parameterization of cloud optical properties

The heart of any parameterization based on the simple two stream format rests with the suitable description of the cloud optical properties at \( \tilde{\omega}_0 \) and \( g \) in terms of some appropriate macrophysical variable. These properties are formally determined from complex particle scatter theories such as Mie theory for spherical particles which require a knowledge of the particle size distribution. Examples of the spectral distributions of the volume extinction coefficient and \( g \) are shown in Figs. 22a and b, respectively, for model cloud types with droplet size distributions typical of a cumulonimbus, stratus and stratocumulus types. Factors relevant to the parameterization of these properties are:

i) both \( \delta \) and \( g \) are relatively smooth functions of wavelengths and thus can be specified over broad spectral intervals.

ii) The asymmetry parameter does not differ greatly for different cloud types (perhaps with the important exception of ice crystal clouds) which suggests that this parameter may be specified a priori.

iii) Typically \( \tilde{\omega}_0 > 0.9 \) for visible and near IR radiation (NIR) and \( \tilde{\omega}_0 \approx 0.5 \) for far IR radiation (FIR). The absorption bands of liquid water (shown as the solid line in Fig. 23) occur in the same spectral region as for water vapor bands (illustrated by the dashed curve in Fig. 23). This feature complicates the calculation of shortwave absorption in the presence of cloud since solar radiation is absorbed in the clear sky above the cloud and, once removed this energy can no longer be absorbed in the cloud layer or in the atmosphere below the cloud. It is therefore necessary to know how much absorption occurs above (or within) a cloud in order to calculate the absorption within (or

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2 A second set of two stream approximations are often described in the literature. These approximations base their derivation on a radiative transfer equation in which the collimated solar beam appears as an upper boundary condition rather than as the explicit exponential term that appears in (55) and thus in (63). The early two stream methods were usually of this form but they suffer from the inability to include the effects of solar zenith angle in the solutions and are not discussed here. (See Meador and Weaver, 1980, for a general review).
<table>
<thead>
<tr>
<th>Method</th>
<th>Example reference</th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
<th>$\gamma_3\dagger$</th>
<th>Angular approximation of $R(\delta, \pi\mu)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eddington</td>
<td>Irvine (1968)</td>
<td>$\frac{1}{4}[7 - \hat{\omega}(4 + 3g)]$</td>
<td>$\frac{1}{4}[1 - \hat{\omega}(4 - 3g)]$</td>
<td>$\frac{1}{4}(2 - 3g\mu\delta)$</td>
<td>$\frac{1}{2}(2 \pm 3\mu)S^+ + (2 \mp 3\mu)S^-$</td>
</tr>
<tr>
<td>Modified Eddington</td>
<td>Meador and Weaver (1980)</td>
<td>$\frac{1}{4}[7 - \hat{\omega}(4 - 3g)]$</td>
<td>$\frac{1}{4}[1 - \hat{\omega}(4 + 3g)]$</td>
<td>$\beta_0$</td>
<td>$\frac{1}{2}(2 \pm 3\mu)S^+ + (2 \mp 3\mu)S^-$</td>
</tr>
<tr>
<td>$\delta$-Eddington</td>
<td>Joseph et al. (1976)</td>
<td>$\frac{1}{4}[7 - \hat{\omega}(4 + 3g')']$</td>
<td>$\frac{1}{4}[1 - \hat{\omega}(4 - 3 - g')']$</td>
<td>Either $\beta_0$ or $\frac{1}{4}(2 - 3g'\mu\delta)$</td>
<td></td>
</tr>
<tr>
<td>Quadrature*</td>
<td>Liou (1973)</td>
<td>$[2 - \hat{\omega}(1 + g)]/\mu_1$</td>
<td>$\hat{\omega}(1 - g)/\mu_1$</td>
<td>$\frac{1}{2}(1 - 3g\mu\delta)$</td>
<td>$\mu^{-1}S^+\delta(\mu - \mu_1)$</td>
</tr>
<tr>
<td>Modified Quadrature</td>
<td>Meador and Weaver (1980)</td>
<td>$[1 - \hat{\omega}(1 - \beta_1)]$</td>
<td>$\omega_0\beta_1/\mu_1$</td>
<td>$\beta_0$</td>
<td>$\mu^{-1}S^+\delta(\mu - \mu_1)$</td>
</tr>
<tr>
<td>$\delta$ Quadrature</td>
<td>Schaller (1979)</td>
<td>$U[2 - \hat{\omega}(1 + g)]$</td>
<td>$U\hat{\omega}(1 - g')$</td>
<td>Either $\beta_0$ or $\frac{1}{2}(1 - \sqrt{3g'\mu\delta})$</td>
<td></td>
</tr>
<tr>
<td>Hemispheric*</td>
<td>Coakley and Chylek (1975)</td>
<td>$U[1 - \hat{\omega}(1 - \beta_1)]$</td>
<td>$U\omega_0\beta_1$</td>
<td>$\beta_0$</td>
<td>$US^+$</td>
</tr>
<tr>
<td>Hybrid</td>
<td>Meador and Weaver (1980)</td>
<td>$7 - 3g^2 - \hat{\omega}(4 + 3g) + \omega_0g^2(4\mu_0 + 3g)$</td>
<td>$-1 - g^2 - \hat{\omega}(4 - 3g) - \omega_0g^2(4\mu_0 + 3g - 4)$</td>
<td>$\beta_0$</td>
<td>$\frac{1}{1 - g^2(1 - \mu_0)} \times \left(1 + g^2\delta\left[1 + \frac{3g}{2}S^+ + \left(1 + \frac{3g}{2}\right)S^+\delta + g^2(\mu - \mu_0)S^+\right]\right)$</td>
</tr>
</tbody>
</table>

$\dagger$ For energy conservation $\gamma_3 + \gamma_4 = 1$.

* $1/\mu_1$ and $U$ are the effective equivalents of the diffusivity factor $\beta$ in (4). $1/\mu_1$ is set as $\sqrt{3}$ in the quadrature methods and $U = 2$ for the hemispheric method.

$\beta_1$ and $\beta_0$ are taken to represent the relevant hemispheric integrals of the phase function using the full expansion (51) (see text and Meador and Weaver, 1980, for further details).
The integral over cloud depth \( dz \) is the equivalent of the integral over absorber path \( dv \) in (55). The second integral involves the cloud droplet size distribution \( n(r) \) which varies along the path \( dz \). The difficulty in evaluating (67) lies in the specification of both \( n(r) \) and the efficiency factor for extinction \( Q_{\text{ext}} \). The latter quantity is defined as the ratio of the extinction to geometric cross-sectional areas of the particles. For spherical droplets, \( Q_{\text{ext}} \) is obtained from Mie theory and is a function of particle size through the Mie size parameter \( x = \frac{2\pi r}{\lambda} \) and of the refractive index of the particle \( m_n \). For large \( x \), i.e., for shortwave radiation, \( Q_{\text{ext}} \) asymptotes to a value of approximately 2. This feature, together with the definition

\[
 r_c = \frac{\int_0^\infty n(r)r^2dr}{\int_0^\infty n(r)r^3dr} \tag{68}
\]

was employed by Stephens (1978b) to yield

\[
 \delta = \frac{3}{2} W r_c^{-1}, \tag{69}
\]

where \( W \) is the liquid water path (g m\(^{-2}\)) defined by (30).

Stephens (1978) used eight different cloud droplet size distributions to illustrate that \( \delta \) may be usefully expressed in terms of only the cloud-liquid water path. An example of such a relation is shown in Fig. 24 for the spectral range \( 0.75 \leq \lambda \leq 4.0 \). While the parameterization of \( \delta \) according to (37) requires some information about droplet size, i.e., \( r_c \), relationships like that shown in Fig. 24 indicates that \( W \) is probably the most critical parameter. Also, \( r_c \) is intuitively a function of liquid water content and, more or less, liquid water path.

A further point requires special mention. A great deal of effort has concentrated on parameterizing the particle size distribution. The most popular model size distributions are those of Diermendjian (1969) for both clouds and hazes. For additional reading, Hansen and Travis (1974) provide an excellent review of scattering theory especially including the effects of particle size distribution on various Mie scattering parameters.

2) THE PARAMETERIZATION OF SINGLE SCATTER ALBEDO \( \tilde{\omega}_0 \)

The key to the evaluation of cloud absorption using the two-stream method is to specify \( \tilde{\omega}_0 \) accurately. Twomey and Bohren (1980), using arguments related to the theory of geometric optics (Liou, 1980, Chapter 5) derived a relation which can be modified to provide

\[
 \tilde{\omega}_0 = 1 - 1.7k r_c, \tag{70}
\]

where \( r_c \) is defined by (68) and \( k' \) is the complex part of the refractive index of water, \( m = n_r - ik' \). A similar expression

\[
\text{Fig. 22. The spectral variation of (a) volume extinction coefficient (related directly to cloud optical thickness), (b) asymmetry parameter for three model cloud types discussed in Stephens (1978a).}
\]
\[ \tilde{\omega}_0 = (1 + \exp(-2k'r_e)) \] (71)

was used by Fouquart and Bonnel (1980) and requires that liquid water absorption is weak in the solar region (i.e., \(2k'r_e < 1\)).

Another slightly more elaborate approach which could be particularly useful, especially for aerosol parameterization, utilizes the assumption that \(n_e \sim 1\) (which more or less applies for the case of liquid water and perhaps several aerosol types). Then, the "anomalous diffraction" approximation (see Van de Hulst, 1957 Section 11.23) can be applied to yield

\[ 1 - \tilde{\omega}_0 = \frac{1}{2} \left( \frac{4}{3} \rho \tan \Gamma - \rho^2 \tan^2 \Gamma \right), \] (72)

where

\[ \rho = 2x(n_e - 1) \]

\[ \Gamma = \arctan \left( \frac{k'}{n_e - 1} \right) \]

and given that the factor \(4xk'\) is small and that \(Q_{\text{ext}} = 2\). The real advantage of this approach is that expressions for both \(Q_{\text{ext}}\) and \(Q_{\text{abs}}\) (and thus those for \(\delta\) and \(\tilde{\omega}_0\)) can be derived in general terms as functions of size parameter \(x = 2\pi r_{\lambda}^{-1}\) and the refractive index \(m\).

Because of this generality, the formula may be applicable for a variety of different types of scattering particles with varying chemical properties.

The applicability of (70), (71) or (72), when substituted into the two stream solutions has not, apart from Fouquart and Bonnel (1980), been fully tested and some further tuning of these relationships may be necessary. It is particularly desirable to remove, if possible, the size dependent factor \(r_e\) in the expression for \(\tilde{\omega}_0\) in the manner similar for \(\delta\). Stephens (1978) tuned \(\tilde{\omega}_0\), together with the hemispheric integrated phase function parameter \(\beta_1\) (see Table 5) to match values obtained from more accurate numerical solutions and provided empirical lookup tables for both \(\tilde{\omega}_0\) and \(\beta_1\) as functions of \(\delta\) and \(\mu_0\). It must be remarked that these values are only a measure of \(\tilde{\omega}_0\) and \(\beta_1\) within the given context and should not be applied to other
forms of two stream models. Fouquardt and Bonnel derived the following for the spectrally averaged albedo:

$$\tilde{\omega}_0 = 0.9989 - 0.0004e^{-0.158\delta}$$  \hspace{1cm} (73)

by tuning their calculations to a more accurate reference and claim that it is representative (within 5%) for $\delta \leq 20$.

The above discussions and the work of others (e.g., Liou and Wittman, 1979) have shown that the gross shortwave radiative characteristics of cloud can be modeled in terms of liquid water path and the solar zenith angle. Fig. 25 schematically illustrates the relationship between reflection, transmission and absorption of these two parameters for a given model stratus cloud type. While the specific cloud particle size information cannot be totally excluded from such a parametric relationship [this is evident from the $r_e$ dependence in the $\delta$ relation (68)], the effect of cloud microphysical variations is likely to be smaller than variations associated with other cloud properties.

$d$. Parameterizing the combined effects of cloud droplet and water vapor absorption

One of the major problems associated with the overlapping of water vapor and droplet absorption, apart from the effects of scattering, is the treatment of the two different wavelength's scale of absorption. As highlighted in Fig. 23, the absorption by liquid water is a relatively smooth function of $\lambda$ whereas the absorption by water vapor varies strongly with $\lambda$. (This is also the basic difficulty encountered in relation to the problem involving clouds that are transparent to longwave radiation).

The absorption of solar radiation by cloud droplets is often considered negligible in comparison to water vapor absorption. However, this has no foundation as Fig. 26 demonstrates. The diagram presents the solar heating profile in an atmosphere that contains a model stratus cloud layer from approximately 760 to 850 mb. The calculations were performed with and without the

---

**Fig. 25.** Illustration of cloud reflection, transmission and absorption as a function of $W$ and $\mu_0$ for an idealized stratus cloud type (Liou and Wittman, 1979).
inclusion of cloud droplet absorption for two different cloud optical depths $\delta^* = 1$ and $\delta^* = 10$. Obviously the major effect of droplet absorption is to increase the solar heating within the cloud layer (more than a factor of 2 for $\delta^* = 10$). It is relevant to note that even for the case of nonabsorbing cloud droplets the heating within the cloud is enhanced by the effects of droplet scattering which acts to increase the path of a photon through the cloud. Some schemes [e.g., Sasamori et al., 1972] make use of this feature and suggest that the absorption within cloud can be based on the simple gas absorption parameterizations described above with the path increased by a factor of $\eta$ to account for this scattering effect. (They suggest $\eta = 10$ is appropriate). This is an oversimplification of the problem on two counts. First, the liquid water absorption component varies as a function of liquid water amount and thus $\eta$ is not a constant factor but is variable with $\delta$. Second, the absorption by cloud and the absorption by water vapor have reverse dependences on $\mu_0$; the former decreases with increasing solar zenith angle [e.g., see Fig. 25] while water vapor absorption is an increasing function of solar zenith angle.

Thus we require some means of calculating the absorption for cloudy skies which overlaps the droplet and molecular absorptions in a consistent fashion. The basic problem is that scattering methods require that the transmissions are multiplicative, i.e., that they are exponential. While some band models are exponential, they are usually based on the assumption that all photons travel from two points along the same direction.

This is not true for scattering. Two approximations have been developed, the first is based on the $k$-distribution approach where transmission is expressed as

$$
\tau_{\Delta\nu}(\bar{u}) = \int_{\Delta\nu} f(k)e^{-k\delta}dk = \sum_{n=1}^{N} f(k_n)e^{-k_n\delta}. \tag{74}
$$

The sum of exponentials representation is sometimes called the “picket fence” approach since the distribution function $f(k)$ is taken to be only nonzero at discrete $k$ values. Thus if one considers a frequency interval $\Delta\nu$ over which the cloud properties $\delta$ and $\bar{\omega}_0$ are to be taken as constant, then we use the new properties

$$
\delta_n = \delta + k_n\bar{u} \quad \bar{\omega}_n = \bar{\omega}_0\delta/\delta_n \tag{75}
$$

which replace $\delta$ and $\bar{\omega}_0$ in the formal two stream solution. The broadband solar fluxes $S_{\Delta\nu}$ are then obtained from the individual solution of (64) for each $k_n$ and then combined by

$$
S_{\Delta\nu} = \sum_{n=1}^{N} f(k_n)S_n(k_n). \tag{76}
$$

Thus the problem of molecular absorption for the interval $\Delta\nu$ in a scattering cloud layer reduces to one of $N$ “pseudo” monochromatic multiple scattering problems. The above procedure has been employed in many more complex multiple scattering problems as well as with the simpler two stream methods described before [e.g., Lacies and Hansen, 1974; Wiscombe and Evans, 1977; Liou and Sasamori, 1975, among others].

The second method is the so called “photon path distribution” method and the scattering calculations are employed in an a priori manner to determine the distribution $N(l)$ of path lengths traversed by photons, so that a band model may be used for the transmission of a layer, i.e.,

$$
\tau_{\Delta\nu} = \int N(l)\tau(l, \bar{p})dl \approx \sum_{l} N(l)\tau(l, \bar{p}), \tag{77}
$$

where $\tau(l, \bar{p})$ is taken to represent the band model transmission along a path with the scaled quantities $\bar{u}$ and $\bar{p}$. There exists a form of duality between (75) and (77) and both methods are equivalent for an infinite number of terms. In practice the second method requires fewer terms with the additional benefits that scaling approximates like the VCG can be readily applied. However, the method is difficult to apply for the case of no scattering and errors in clear sky calculation are likely. It is relevant to comment that the use of $\bar{\beta} = 1.66$ for transmission of diffuse radiation [e.g., in (5)] is clearly a particular case of the photon path distribution. Examples of the use of this technique can be found in the work of Bakan and Quenzel (1976), Bakan et al. (1978) and Fouquard and Bonnel (1980) among others.
**e. The transfer of solar radiation through a multilayered atmosphere: The adding method**

The two stream solutions described in Table 5 apply only to a single homogeneous cloud layer with fixed values of $\delta, \tilde{\omega}_0$ and $g$. The problem remains to combine several layers each with different values of these optical properties. It is useful to do this in a way that makes it relatively straightforward to include the variable effects of a number of different factors (such as different cloud and haze types). The simplest approach is to consider only the depletion of the directly transmitted solar beam as in (46) or (50) and define an appropriate form of transmittance function. This approach, while computationally simple, suffers from the drawback that it precludes the effects of multiple scattering between layers and that it is difficult to define a transmission function that correctly overlaps the combined effects of gas absorption, droplet absorption and droplet scattering not only in a single layer but also through several adjacent layers. A more precise and consistent approach which overcomes these problems is the so-called adding method. This method forms the basis for the solution of a majority of the more sophisticated multiple scattering problems and has the advantage in that it is simple in concept and easily reduced in complexity for the purpose of operational models. Despite these obvious advantages, the approach is rarely used in this simple context, (three examples are Lacis and Hansen, 1974; Stephens and Webster, 1979; Fouquardt and Bonnel, 1980).

An outline of the procedure can be best described by reference to a simple two layer atmosphere. If we employ the subscripts $a$ and $b$ to refer to the top and bottom layers respectively, then the reflection and transmission at the boundaries of the atmosphere for the combined layers are

\[
\begin{align*}
\alpha_{ab} &= \alpha_a + \tau_a \alpha_b \tau_b / (1 - \alpha_a \alpha_b) \\
\tau_{ab} &= \tau_a \tau_b / (1 - \alpha_a \alpha_b)
\end{align*}
\]

(78)

There are three important factors in (78) that sets the parameterization apart from the standard technique described by (50). These are:

i) The multiplication of transmittance through the combined layers requires the transmission in a form consistent with (12).

ii) Differentiating between the reflections from above ($\alpha_a$) and below ($\alpha_b^*$), (often it can be assumed $\alpha_a = \alpha_b^*$). It is important to provide such a distinction as the reflection of a layer illuminated by a diffuse source is different from the reflectance of layer illuminated by a collimated beam.

iii) The propagator factor (i.e., the denominator) which accounts for the multiple reflections between layers. This factor is especially significant when the reflection between layers is large and this typically occurs when a relatively dense cloud overlies a bright surface (such as another cloud or a snow covered surface).

It is a simple matter to extend (78) to several layers. The reflection and transmission of individual cloud layers can be determined using the simple recursive relations which can be found in either of the above three cited references.

An equivalent but more computationally efficient approach is to consider the atmosphere as a system of $n$ homogeneous layers each with definable reflections and transmission properties which can be either prescribed or determined from a two stream solution. These reflection and transmission properties in fact relate the outgoing solar fluxes from some layer as a linear combination of the incoming solar fluxes. That is, we have for each layer

\[
\begin{align*}
S_0(Z_b) &\quad S_0(Z_a) \\
S'(Z_b) &\quad S'(Z_a)
\end{align*}
\]

\[
\begin{align*}
S_0(Z_b) &\quad S_0(Z_a) \\
\tau_{ab} &\quad \tau_{ab} \\
S'(Z_b) &\quad S'(Z_a)
\end{align*}
\]

(79)

where $Z_a$ and $Z_b$ refer to the cloud top and base heights respectively; $\tau_0$ is the transmission for the parallel beam, $\tau_d$ the transmission for diffuse radiation, $\alpha_d$ the reflection of diffuse radiation and $\alpha_d$ are the diffuse transmission and reflection of the incident parallel beam. We make no distinction between the reflectances and transmittances associated with illumination from above or below (cf. [76]) although this is easily incorporated. It is straightforward to demonstrate that the formal solution of the two stream equations is expressible in this matrix form. The linear matrix form is just a statement of the interaction principle of radiative transfer which is directly tied to the full radiative transfer equation (56) by the principles of invariance (e.g., Chandrasekhar, 1950). Thus for $n$ layers we obtain an equivalent linear system of $3n + 3$ equations in the form

\[
T \cdot S' = S
\]

(80)

where $T$ is a band structured matrix for which elements define the layer transmissivities and reflectivities (which are in turn functions of $\delta, \tilde{\omega}_0$ and $g$), $S$ is a vector of the required outgoing solar fluxes while $S'$ is a vector of incident “source” fluxes. It is interesting to note that the solution of this matrix equation via a gaussian elimination–substitution method produces the equivalent “adding method” algorithms. A practical and potentially useful aspect of this matrix approach or of the adding method is that the computational time is linearly proportional to the number of model layers.

**f. Partial cloudiness and the treatment of overlapping clouds**

The usual procedure for treating the effect of a partially cloud filled layer is to calculate the fluxes separately for a clear and completely overcast sky and
then combine the two sets of fluxes in some way [as with (40)] or to combine the respective layer transmissions and layer reflectives in some manner proportional to the cloud fraction. The latter approach is often applied in the calculation of shortwave fluxes and involves only the single solution on the matrix equation (80). Recent theoretical studies (e.g., Welch et al., 1980) of radiative transfer through three-dimensional cloud shapes indicate that the simple linear combination of layer reflectance and transmittance properties are an over simplification. The incorporation of realistic broken cloudness into a parameterization scheme remains as one of the major problems in the area of radiation–cloud parameterization. Some recent studies that address this problem are those of Welch and Zdunkowski (1981).

Overlapping cloud layers are customarily dealt with in the manner described by Manabe and Strickler (1964) in which it is assumed that all cloud layers are independent of one another. That is, the amount of sky covered simultaneously by \( n \) cloud layers is simply determined from the relationship

\[
1 - (1 - A_1) \times (1 - A_2) \times \cdots (1 - A_n)
\]

where \( A_1, A_2, \ldots, A_n \) are the cloud amounts of different layers. According to this approach, it is necessary to perform \( n + 1 \) longwave flux calculations (\( n \) overcast sky and one clear sky calculation) or \( n \) shortwave flux calculations (since it is not necessary to make the cloudy–clear sky separation in this case). These fluxes are then weighted by the cloud cover relevant to the level of interest (this cloud cover is obtained by a combination of the cloud covers for the layers above or below the reference layer). An example for calculating longwave flux using this scheme is shown schematically in Fig. 27. An alternate, and arguably more realistic approach, was offered by Geleyn and Hollingsworth (1979) for which it was assumed that the cloud cover for a combination of adjacent model layers filled by cloud is given by the cloud cover that corresponds to the maximum of the individual layer. This maximum overlap method is computationally faster than the random overlap case since it is unnecessary to perform a full set of \( n \) (or \( n + 1 \)) flux calculations (see Geleyn and Hollingsworth for details). Unfortunately, there is no way yet of rigorously testing the validity of either scheme or any other scheme to handle overlapping clouds and it is difficult to assess whether more elaborate schemes are justified.

6. Some examples of parameterization schemes as used in existing GCMs

Parameterization schemes currently run in GCMs generally use a combination of the different techniques described above. Unfortunately, any version described in existing documentation is undoubtedly at variance with the version actually being run in the model, and a discussion based on documented literature alone is not likely to be fully representative. Despite this, six different schemes are outlined below; the intention is not to review exhaustively all schemes as currently used in GCMs nor to review a handful of schemes critically and in consummate detail, but rather to summarize a few schemes which are typical and which collectively employ the various different techniques reviewed in this paper. The reader can readily assess the relative merits of, the physical basis for and the uncertainties associated with the various different schemes by a cross reference to the relevant sections above.

The six schemes are outlined in Tables 6a to f and are hereafter referred to in terms of the relevant organization that run the host GCM. The following abbreviations apply:

- **KOLN**: radiation scheme developed at the University of Cologne for use in the GCM of the German Weather Service.
- **GLAS**: radiation scheme used in the GCM run at NASA’s Goddard Laboratory for Atmospheric Sciences, Greenbelt, Maryland.
- **ECMWF**: radiation scheme used in the GCM at the European Centre for Medium Range Weather Forecasts, Reading, U.K.
### Table 6a. NCAR Radiation scheme (Ramanathan et al., 1983).

<table>
<thead>
<tr>
<th>Spectral region</th>
<th>Process</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Clear sky</td>
<td>(i) Longwave</td>
<td>Gas absorption</td>
</tr>
<tr>
<td></td>
<td></td>
<td>H₂O</td>
</tr>
<tr>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>H₂O and CO₂</td>
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<tr>
<td></td>
<td></td>
<td>O₃</td>
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<tr>
<td></td>
<td>(ii) Shortwave</td>
<td>Surface emission</td>
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<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>Rayleigh scattering</td>
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<td>Gas absorption</td>
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<td></td>
<td>O₃</td>
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<tr>
<td></td>
<td></td>
<td>H₂O</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CO₂ and O₂</td>
</tr>
<tr>
<td>(b) Cloudy skies</td>
<td>(i) Longwave</td>
<td>Scattering</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Droplet absorption</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Droplet and gas absorption</td>
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<td></td>
<td>(ii) Shortwave</td>
<td>Scattering (albedo)</td>
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<td>Gas absorption</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>Droplet absorption</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Multiple reflection between layers</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Overlapping clouds</td>
</tr>
</tbody>
</table>

GFDL: one of the radiation schemes employed at NOAA's Geophysical Fluid Dynamics Laboratory at Princeton University, Princeton, New Jersey.


NCAR: radiation scheme developed at the National Center for Atmospheric Research, Boulder, Colorado.

Some comments relevant to these tables and additional to those contained in the main sections of this paper are:

i) Three schemes (GLAS, ECMWF and KOLN) employ the analytic two stream solutions for the longwave flux calculations. Two advantages of this approach are 1) cloud droplet scattering and absorption and gas absorption are treated in a consistent manner (only GLAS and ECMWF explicitly treat droplet scattering in the longwave treatment) and 2) the two stream solutions are readily derived for nonisothermal layers thus avoiding the common practice of dividing the atmosphere into a large number of isothermal layers.

The major disadvantage of the approach is that it loses accuracy when optically thin layers are added together via the adding method. This applies particularly to the problem of flux transfer through thin clouds.

ii) The spectral integration of longwave absorption used in the various schemes range from a full emissivity approach (at least for water vapor absorption in the NCAR model), a hybrid emissivity–band model method (GFDL), a fully fledged 20-interval band model calculation (GLAS), the \( k \)-distribution method (GISS), the photon-path distribution method (ECMWF). The
scheme of KOLN employs an optical depth for gas which is parameterized via a nonlinear function of scaled absorber amount $\tilde{u}$. This function is defined once and for all during the initial design of the scheme. This approach thus assumes that the transmission in a broad spectral interval (there are six for the entire longwave spectrum, refer to Table 6f) preserves a pure exponential form. This assumption generally results in significant errors although Hense et al. (1982) claim accuracies of better than 5% and 20% in the worst case for longwave fluxes and cooling rates, respectively.

The photon-path distribution scheme as employed in the ECMWF model has some difficulty in handling the longwave radiation calculations in a cloudless sky and in optically thin cloud, and it is necessary to make certain assumptions (an effective “cooling-to-space approximation”) in order to define the photon path lengths in clear sky situations. The resulting cooling rates become inaccurate ($\sim$10–15%) in the upper troposphere and stratosphere.

The $k$-distribution method employed in the GISS model was either used in conjunction with six spectral intervals, and thus six $f(k)$ distributions (Model I), or used by convolving transmissivity directly with the Planck function in a similar manner to emissivity (Model II). In Model II, only one $k$ distribution for each gas was employed for the entire thermal region with an absorption coefficient spectrally weighted with the Planck function.

iii) All schemes apart from the KOLN, ECMWF and GLAS schemes require some gross simplification in the treatment of gas overlap. The KOLN scheme treats this explicitly in so far as optical depths are simply additive while the GLAS scheme uses a random band model which obeys (12). The ECMWF scheme also employs band transmissivities and gaseous absorption is overlapped by defining an appropriate “overlap” path length distribution function [refer to (77)]. The other schemes overlap CO$_2$, H$_2$O and O$_3$ absorption in some vague manner. This often creates...
### Table 6c. GLAS Radiation Scheme (Wu, 1980; Herman, 1980; Johnson, 1980).

<table>
<thead>
<tr>
<th>Spectral region</th>
<th>Process</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Clear Sky</td>
<td></td>
<td><strong>Gas absorption</strong></td>
</tr>
<tr>
<td>(i) Longwave</td>
<td></td>
<td>Fully fledged band model approach is used. 20 spectral intervals for the entire longwave absorption spectrum. VCG is applied</td>
</tr>
<tr>
<td></td>
<td>H$_2$O</td>
<td>20 spectral intervals, include e-type absorption</td>
</tr>
<tr>
<td></td>
<td>H$_2$O and CO$_2$</td>
<td>Overlap is treated in one of the 20 intervals by multiplication of transmission</td>
</tr>
<tr>
<td></td>
<td>O$_3$</td>
<td>None</td>
</tr>
<tr>
<td>(ii) Shortwave</td>
<td></td>
<td>Lacis and Hansen scheme (see Table 6a for outline)</td>
</tr>
<tr>
<td>(b) Cloudy Skies</td>
<td></td>
<td><strong>Absorption</strong></td>
</tr>
<tr>
<td>(i) Longwave</td>
<td></td>
<td>Black clouds</td>
</tr>
<tr>
<td></td>
<td>Scattering</td>
<td>None</td>
</tr>
<tr>
<td>(ii) Shortwave</td>
<td></td>
<td>Employed a δ-Eddington model (see Table 5) with preset values of $\tilde{\omega}_0$, δ and g. Differentiates between high,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>middle and low clouds.</td>
</tr>
<tr>
<td></td>
<td>Droplet and gas</td>
<td>Explicit treatment through the sum of exponentials (7).</td>
</tr>
<tr>
<td></td>
<td>absorption overlap</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Multiple reflections</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Employed an adding scheme similar to that described in (78).</td>
</tr>
</tbody>
</table>

A related problem that is best illustrated by the following specific example. The application of a single greybody emissivity for water vapor requires the spectrally integrated blackbody function $\sigma T^4$ such as in (17) or (22). Treating the CO$_2$-H$_2$O and the H$_2$O-O$_3$ overlap separately then implies that the spectral integral of the Planck function is different from $\sigma T^4$ containing the additional contributions by these individual intervals. This doubling-up of the contribution by these intervals to the spectrally integrated Planck function then requires some sort of renormalization. While the longwave cooling profiles for H$_2$O and O$_3$ are approximately additive (this does not apply for the H$_2$O-CO$_2$ overlap, refer to Fig. 3), the individual fluxes are not and this “renormalization” is of some importance. Some schemes (e.g., Model I, GLAS) attempt to avoid this by defining explicit intervals which contain the overlap regions. The problem then remains to treat the spectral variation of the Planck function $B$, over these intervals which are most often too broad to treat $B$ simply as a constant with $\nu$.

iv) The schemes outlined in Tables 6a to f have sufficient complexity to allow some form of interaction between the model derived moisture fields and the calculated clear-sky radiative fluxes and cooling/heating rates. (It is probably adequate for most purposes to employ climatological ozone, and aerosol). The radiation calculations for cloudy skies however are generally only partially interactive or wholly prescribed. Three schemes relate the cloud optical properties to liquid water path $W$: ECMWF employ prescribed $\tilde{\omega}_0$ and an optical depth dependent on $W$, the KOLN scheme is more fully interactive with both $\tilde{\omega}_0$ (determined as an empirical function of optical depth $\delta$) and $\delta$ functions of $W$ and the NCAR scheme allows for a variable cloud emissivity only when $W < 10$ g m$^{-2}$. The GISS scheme employs preset values of $\tilde{\omega}_0$ and $g$ based on Mie calculation for specific particle size distributions but empirically adjust $\delta$ according to pressure and temperature. The GLAS scheme uses preset values of $\tilde{\omega}_0$, $\delta$ and $g$ and assumes black clouds. The Manabe and Strickler (1964) approach employs predetermined cloud reflections and transmissions. The NCAR shortwave cloud scheme uses cloud properties that are also preset and not functions of $W$ although some attempt is made to correlate cloud albedo and emissivity in a crude fashion. It is relevant to comment that the long- and short-wave radiative properties of (thin) cloud are related through cloud optical depth and it is unrealistic to vary cloud emissivity while holding cloud albedo (and shortwave absorption) fixed.

While it is desirable to build into any GCM a complete interaction between radiation and dynamics, the requirement of maximum computational speed usually finally decides what kind of parameterization is to be used. It is difficult to draw any firm conclusions from a timing comparison of the different schemes since the various parameterizations are often coupled to the host dynamical model in a way that makes it difficult to separate the radiation calculations unambiguously from the other physical parameterizations. With this in mind Table 7 presents the timing of some of the schemes described in Tables 6a to f. The CPU times are in milliseconds and apply to a single model grid point. Some relevant comments are:
<table>
<thead>
<tr>
<th>Spectral region</th>
<th>Process</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Clear Sky</td>
<td>(i) Longwave</td>
<td>Longwave fluxes are calculated from a two stream formulation of the flux equations. Pressure scaling (7) employed throughout. Gas absorption</td>
</tr>
<tr>
<td></td>
<td>H$_2$O</td>
<td>Six spectral bands from 0 to 2500 cm$^{-1}$ with 2–9K points per interval (Model I), or one broad interval for the entire spectrum with 11K points (Model II). Infrared continuum is included based on (10). (Presumably in Model I).</td>
</tr>
<tr>
<td></td>
<td>H$_2$O and CO$_2$</td>
<td>Overlap in one of the individual intervals is treated by multiplication of transmittance. Model II employs one interval for CO$_2$ with 10K points.</td>
</tr>
<tr>
<td></td>
<td>O$_3$</td>
<td>One interval with 4K points (Model II)</td>
</tr>
<tr>
<td></td>
<td>CO$_2$, O$_3$, N$_2$O, CH$_4$</td>
<td>Weaker bands of CO$_2$ and O$_3$ together with N$_2$O and CH$_4$ are included</td>
</tr>
<tr>
<td></td>
<td>Surface emission</td>
<td>Ocean—function of wind speed Land—based on spectral emissivities of Hoirs and Callahan (1966) for deserts and Wiscombe and Warren (1980) for snow and ice</td>
</tr>
<tr>
<td>(ii) Shortwave</td>
<td>Rayleigh scattering</td>
<td>Six broad intervals are used to cover the entire shortwave spectrum. As in Lacis and Hansen (1974)</td>
</tr>
<tr>
<td></td>
<td>Aerosol scattering</td>
<td>Mie parameters for aerosol based on the aerosol climatology of Toon and Pollack (1970)</td>
</tr>
<tr>
<td></td>
<td>Gas Absorption</td>
<td>5 intervals (5k distributions)</td>
</tr>
<tr>
<td></td>
<td>H$_2$O</td>
<td>3 intervals (3k distributions)</td>
</tr>
<tr>
<td></td>
<td>CO$_2$</td>
<td>2 intervals (2k distributions)</td>
</tr>
<tr>
<td></td>
<td>O$_3$</td>
<td>1 interval (1k distribution)</td>
</tr>
<tr>
<td></td>
<td>NO$_2$</td>
<td>Analytic formula of Lacis and Hansen (1974)</td>
</tr>
<tr>
<td></td>
<td>$^{+}$O$_3$</td>
<td>Spectral albedo for land is a function of vegetation type distinguishing between visible and near infrared</td>
</tr>
<tr>
<td></td>
<td>Surface albedo</td>
<td></td>
</tr>
<tr>
<td>(b) Cloudy Skies</td>
<td>(i) Longwave</td>
<td>Droplet absorption and scattering</td>
</tr>
<tr>
<td></td>
<td>Droplet absorption and gas absorption</td>
<td>Overlapped via $k$-distribution method (Model I)</td>
</tr>
<tr>
<td>(ii) Shortwave</td>
<td>Scattering</td>
<td>Treated explicitly through two stream method modified from original Lacis and Hansen scheme to include effects of solar zenith angle. Mie calculations supply $\omega_0$, $\delta$ and $g$ for each of the six intervals and these are taken to be constant within each interval. $S$ in empirically modified according to pressure and temperature</td>
</tr>
<tr>
<td></td>
<td>Droplet absorption and gas absorption</td>
<td>$k$ distribution (74) and (75) coupled to the two stream model.</td>
</tr>
</tbody>
</table>

i) The timings of the different schemes are very similar, given that the majority were conducted on like computers. The GLAS scheme, employing the full band model treatment, is the most inefficient but the CPU time decreases by roughly a factor of four when the original scheme (see Sommerville et al., 1974) is run.

ii) The shortwave scheme is generally more time efficient than the longwave scheme (0.6 ms compared to $\sim$48 ms for the GLAS model).

iii) Generally the time required for a complete radiation calculation for most models represents a large portion of the total time step.

iv) The time required of various schemes may vary
### Table 6c. The KOLN Radiation Scheme (Hense et al., 1982).

<table>
<thead>
<tr>
<th>Spectral region</th>
<th>Process</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Clear Sky</td>
<td>(i) Longwave</td>
<td>Calculation based on the two stream formulation of the flux equations with parameterized optical depths. Pressure scaling (7) is employed.</td>
</tr>
<tr>
<td></td>
<td>Gas absorption</td>
<td>6 intervals cover the spectral region from 25 to 2525 cm(^{-1}). Dimer absorption is overlapped in three of these intervals.</td>
</tr>
<tr>
<td></td>
<td>H(_2)O</td>
<td>Overlapped in two different intervals (500–830 cm(^{-1})) and for the weak CO(_2) band (1250–2525 cm(^{-1})). Overlap handled by simple addition of optical depths (see text).</td>
</tr>
<tr>
<td></td>
<td>CO(_2) and H(_2)O</td>
<td>Overlap in interval 1000–1100 cm(^{-1})</td>
</tr>
<tr>
<td></td>
<td>O(_3) and H(_2)O (continuum)</td>
<td>Overlapped with H(_2)O and CO(_2) in the interval 1250–2525 cm(^{-1})</td>
</tr>
<tr>
<td></td>
<td>NO</td>
<td>Calculations are based on the two stream method employing 4 broad spectral intervals for entire shortwave spectrum. An inversion procedure is employed to define the relevant optical properties ((\Delta), (\delta) and (\phi)) by matching the two stream solutions to more accurate theoretical calculation.</td>
</tr>
<tr>
<td></td>
<td>Rayleigh scattering and aerosol scattering</td>
<td>Included via a parametric expression of optical thicknesses. Four aerosol models are used to obtain (\delta) and (\Delta): a stratospheric aerosol model (p &lt; 100) mb, a mid tropospheric model (100 &lt; p &lt; 500) mb and either a maritime or rural model below (500) mb.</td>
</tr>
<tr>
<td>(b) Cloudy Skies</td>
<td>(i) Longwave</td>
<td>3 spectral intervals</td>
</tr>
<tr>
<td></td>
<td>Droplet Absorption scattering</td>
<td>Emissivity formulation (34)</td>
</tr>
<tr>
<td></td>
<td>Droplet and gas absorption overlap</td>
<td>Ignored—clear sky fluxes are treated as grey body fluxes</td>
</tr>
<tr>
<td></td>
<td>(ii) Shortwave</td>
<td>(\delta) Two stream model based on Schaller (1979) with values of (\Delta) (\delta) and (\phi) defined from (W) ((p) is presumably prefixed) for each of the 4 spectral regions.</td>
</tr>
<tr>
<td></td>
<td>Scattering</td>
<td>No distinction is made between fluxes in cloud and cloud free regions. “Mean” fluxes are determined from a “mean” optical depth using some type of nonlinear averaging procedure.</td>
</tr>
<tr>
<td></td>
<td>Partial cloud cover</td>
<td>Random overlap</td>
</tr>
<tr>
<td></td>
<td>Overlapping clouds</td>
<td></td>
</tr>
</tbody>
</table>

Conclusively depending on the treatment of overlapping clouds. The time required for a random cloud overlap is proportional to \(n + 1\) (\(n\) is the number of cloud layers) whereas the time required for the maximum overlap assumptions is proportional to \(n' + 1\) (\(n'\) is the number of cloud layers separated by clear sky, generally \(n' < n\)). There is no explicit information on the number of cloud layers used in the above timing tests but most calculations probably employ only three cloud layers, high, middle and low.

v) It is relevant to compare the timing of the ECMWF code on the scalar CDC 175 (35 ms) and on the CRAY 1 (2.5 ms) as it is a good example of the effect of optimizing programs for vector processing. Increased use of vector computers in the future will make practices like only using 3 levels of clouds or employing precomputed effects (such as Rayleigh scatter, aerosol absorption, ozone/carbon dioxide absorption) less attractive since table “lookups” are often more inefficient than computerized calculations.

### 7. Summary

This paper presents an overview of the various methods used to compute both fluxes and the rate of heating and or cooling due to atmospheric radiation for use in numerical models of atmospheric circulation. The paper concentrates on providing the reader with the physical basis underlying the various methods and discusses, separately, the parameterizations of the ab-
### Table 6f. The ECMWF Radiation Scheme (Geleyn and Hollingsworth 1979).

<table>
<thead>
<tr>
<th>Spectral region</th>
<th>Process</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Two stream formulation is employed together with photon path distribution method (77). The radiative transfer equation is solved first for all processes except gas absorption which is included afterwards.</td>
</tr>
<tr>
<td>(a) Clear Sky</td>
<td>(i) Longwave</td>
<td>Gas absorption</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Since scattering for clear sky is negligible, some assumption is required to preserve the overall solution procedure (this amounts to a cooling to space assumption).</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$H_2O$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$CO_2$ and $H_2O$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$O_3$</td>
</tr>
<tr>
<td>(ii) Shortwave</td>
<td></td>
<td>2 broad spectral regions are used for the entire shortwave spectrum.</td>
</tr>
<tr>
<td></td>
<td>Rayleigh scattering</td>
<td>Included (no details)</td>
</tr>
<tr>
<td></td>
<td>Aerosol scattering</td>
<td>Based on some climatological aerosol model(s). No details. Both Rayleigh and aerosol effects are incorporated without any additional cost by employing the two stream solution as a linear system solution (79).</td>
</tr>
<tr>
<td></td>
<td>Gas absorption</td>
<td>1 interval</td>
</tr>
<tr>
<td></td>
<td>$H_2O$</td>
<td>1 interval</td>
</tr>
<tr>
<td></td>
<td>$O_3$</td>
<td>1 interval</td>
</tr>
<tr>
<td>(b) Cloudy Skies</td>
<td>(i) Longwave</td>
<td>Droplet absorption and scattering</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Gas absorption and droplet absorption</td>
</tr>
<tr>
<td></td>
<td>(ii) Shortwave</td>
<td>Droplet absorption and scattering</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Droplet and gas absorption</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cloud overlap</td>
</tr>
</tbody>
</table>

**Molecular absorption.** A number of different techniques exist for the calculation of long- and short-wave flux in an atmosphere containing the trace gases $H_2O$, $CO_2$ and $O_3$. The calculation of longwave flux is more complicated than that of shortwave flux (neglecting scattering) because of the necessity to treat the simultaneous effects of absorption and emission. Three major techniques are reviewed in Section 2 (the band model method, the $k$-distribution method and the well-known emissivity method). Of these only the $k$-distribution approach can be readily incorporated into scattering models while the greybody emissivity method is the more computationally efficient scheme. The errors associated with these methods are probably within the uncertainties that can currently be attached to the prediction of moisture by the host circulation model. The emissivity approach is the most inaccurate with approximately 5–20 W m$^{-2}$ errors in atmospheric longwave flux at the surface and 0.2–0.5°C day$^{-1}$ in abs...
### Table 7. Timing comparisons of different schemes (in $10^{-3}$ secs).

<table>
<thead>
<tr>
<th>Radiation scheme</th>
<th>Computer</th>
<th>No. of layers</th>
<th>Approximate CPU time per grid point (milliseconds)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>GLAS</td>
<td>IBM 360/195</td>
<td>9</td>
<td>50</td>
<td>Total shortwave calculations require 0.6 ms, Herman (1980)</td>
</tr>
<tr>
<td>GLAS</td>
<td>IBM 360/195</td>
<td>9</td>
<td>$\sim 12$</td>
<td>Based on original Sommerville et al. (1974) scheme</td>
</tr>
<tr>
<td>KOLN</td>
<td>CDC 7600</td>
<td>9</td>
<td>13</td>
<td>Hense et al. (1982)</td>
</tr>
<tr>
<td>ECMWF</td>
<td>CDC 175</td>
<td>9</td>
<td>35</td>
<td>Geleyn and Hollingsworth (1979) given general formula for their &quot;standard&quot; calculation, $(0.75 + (2n - 1) \times 0.0031)$ ms/spectral interval/level where $n$ is the number of gases considered. CPU time shown is based on 5 spectral intervals (3 longwave and 2 shortwave)</td>
</tr>
<tr>
<td></td>
<td>CRAY1(ECMWF)</td>
<td>9</td>
<td>2.5</td>
<td>Geleyn (private communication)</td>
</tr>
<tr>
<td>GFDL</td>
<td>CDC 7600</td>
<td>9</td>
<td>$\sim 10$</td>
<td>Uses Fels and Schwartzkopf (1975) longwave hybrid method and Lacin and Hansen (1974) shortwave scheme. Based on tests by B. Forgan (private communication)</td>
</tr>
<tr>
<td>Manabe/Mollert</td>
<td>CDC 7600</td>
<td>9</td>
<td>$\sim 10$</td>
<td>Based on tests by B. Forgan (private communication)</td>
</tr>
<tr>
<td>Strickler</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

atmospheric longwave cooling rates. The clear sky shortwave heating rates are limited more by the lack of accurate absorption data and solar flux data than by the methodology of calculation. Uncertainties in shortwave heating in the order of 20% can be expected.

- **Cloud-radiation parameterizations.** The two stream solution of the general scattering radiative transfer equation provides a useful framework for the consistent treatment of the combined effects of clouds, aerosol and molecular absorption. The two-stream methods are now of sufficient economy and accuracy and are currently employed in a number of the more recent radiation schemes developed for GCMs. In the past, the approach has mainly been used to calculate solar fluxes and solar heating rates but some of the more recent schemes have also applied the method to calculate longwave fluxes. The particular advantage of the two stream method for longwave radiation is that it can treat partially transparent cloud without the arbitrary overlap assumptions described in Section 3. Unfortunately, its application is not altogether straightforward because of the inherent inaccuracies of the approach for clear-sky longwave flux calculations. Given that the atmosphere is stratified and horizontally homogeneous, then the accuracy limitations for flux calculations through clouds or aerosol layers lies not with calculation procedure but with the uncertainties of the prescribed optical properties (i.e., $\tilde{\omega}$, $\delta$ and $g$) of the layer in question. These optical properties can be related to the cloud liquid water path $W$. Those schemes which do not carry $W$ as a prognostic variable then resort to either prescribing the cloud reflection and transmission properties or prescribing values of $\tilde{\omega}$, $\delta$ and $g$.

Partial cloudiness is usually handled in the computations by considering separately the clear and cloud calculations in the conventional way. These individual sets of fluxes are then combined in a manner proportional to cloud amount. This approach involves the calculation of two sets of long and shortwave fluxes for a single cloud layer. The computational burden associated with this approach can be shortened by incorporating cloud cover information directly into the two-stream solutions. Certainly the validity of the current methods for dealing with partial cloud cover are highly questionable in the light of several recent radiative transfer studies.

- **Aerosol.** It is difficult to establish a general parameterization of radiative transfer in aerosol layers because of the wide diversity of aerosol types. Since aerosol properties are geographically localized it is difficult to define a meaningful climatology with any useful degree of accuracy. However, aerosol is a minor effect, and can be allowed for to first order (as can all unknown effects) by "tuning" the radiation parameterization to fit such global criteria as overall radiation balance, and agreement with satellite measurements.

There are specific cases for which it may be desirable to allow an interaction between the radiative effects of aerosol and model dynamics (e.g., as for the Saharan Desert, Ackerman and Cox, 1982). The two stream methodology can be readily adopted to handle aerosol in combination with the other radiative processes.

- **Resolution.** The horizontal and vertical resolution
as set by the host dynamical model is most often too coarse. This creates interpolation problems when one wishes to increase the number of levels in the radiation calculations. However, the most important issue relates to the question of subgrid scale processes which are lost. Radiative heating and cooling in clouds is often confined to a few hundred meters from the cloud top or base. The processes of cloud formation and dissipation can be influenced by these relatively sharp gradients of heating and cooling. Since (ultimately) full cloud character must be predictable a priori by such models, the problem of designing a model with a resolution that suitably handles the different vertical scales of radiative heating is a major consideration.

Similarly, the radiative transfer is strongly influenced by the horizontal cloud structure. The horizontal variation in cloudiness is definitely a subgrid scale phenomenon and the variation between two model grid points can be large. Just as it was important to parameterize the subgrid effects of cloud (convection) on the grid point momentum and thermodynamic budgets, so it would seem to be important to parameterize the effects of subgrid scale radiative transfer on the grid scale radiation budget. This problem remains as one of the major challenges for cloud–radiation parameterizations.

Acknowledgments. I gratefully acknowledge the cooperation of Dr. B. Forgan who made several useful comments on the text and who also provided me with some of the timing tests listed in Table 7.

APPENDIX A

List of Symbols

\[
\begin{align*}
A_c & \quad \text{Fractional (by area) cloud amount} \\
A_e & \quad \text{Effective cloud amount} \\
A_v & \quad \text{Monochromatic absorptivity} \\
A & \quad \text{Broadband absorptivity} \\
B_v & \quad \text{Planck function} \\
\bar{d} & \quad \text{"Mean" line spacing of band of absorptivity lines} \\
f & \quad \text{Fraction of scattered radiation into a forward peak} \\
f(k)dk & \quad \text{Probability density function as defined in the } k \text{ distribution method} \\
F & \quad \text{Longwave fluxes} \\
g & \quad \text{Asymmetry parameter} \\
l & \quad \text{Radiance} \\
k & \quad \text{Pseudo absorption coefficient defined in the } k \text{ distribution method} \\
k_v & \quad \text{Monochromatic (mass) absorption coefficient for gas absorption} \\
k_e & \quad \text{Monochromatic (mass) absorption coefficient for cloud absorption} \\
k' & \quad \text{Complex part of the refractive index} \\
l & \quad \text{Photon path length} \\
m & \quad \text{Refractive index of a particle } (m = n, -ik') \\
m_{\text{top}} & \quad \text{Relative airmass} \\
n & \quad \text{Real part of the refractive index} \\
n(r, z) & \quad \text{Numbered particles at level } z \text{ in the radius interval from } r \text{ to } r + dr. \\
p & \quad \text{Scattering phase function} \\
p & \quad \text{Scaled pressure} \\
P & \quad \text{Standard pressure} \\
P_l & \quad \text{Pressure} \\
Q_{\text{abs}} & \quad \text{Absorption efficiency factor} \\
Q_{\text{ext}} & \quad \text{Extinction efficiency factor} \\
r & \quad \text{Particle radius} \\
r_e & \quad \text{Effective radius of a given frequency} \\
\text{distribution of particle sizes} \\
r_m & \quad \text{Radius limit} \\
S & \quad \text{Mean line intensity of a band of absorption lines} \\
S^s, S^l, S^o & \quad \text{Shortwave (solar) fluxes} \\
S_{\text{top}}(\infty) & \quad \text{Monochromatic solar flux at the top of the atmosphere} \\
S(\infty) & \quad \text{Solar constant} \\
S_0 & \quad \text{The collimated solar flux incident on} \\
\text{cloud top} \\
T & \quad \text{Temperature} \\
T_0 & \quad \text{Standard temperature} \\
u & \quad \text{Path length of attenuating gas} \\
u^* & \quad \text{Effective path length for diffuse radiation} \\
u_0 & \quad \text{The total absorber path length of the} \\
\text{vertical atmospheric column} \\
w & \quad \text{Liquid water content} \\
W & \quad \text{Liquid water path} \\
x & \quad \text{Mie size parameter } (=2\pi r/\lambda) \\
z, z', z'' & \quad \text{General altitude variable} \\
z_b & \quad \text{Cloud top altitude} \\
z_l & \quad \text{Cloud top altitude} \\
z_T & \quad \text{Height to the top of the model} \\
\alpha_a & \quad \text{Reflectance of a layer } a \\
\alpha_{ab} & \quad \text{Reflectance of a composite of layers } (a + b) \\
\bar{\alpha} & \quad \text{Mean line half width of a band of absorption lines.} \\
\alpha_g, \alpha_{g}, \alpha_g & \quad \text{Surface albedo} \\
\alpha_R(\mu_0) & \quad \text{Reflection of the atmosphere due to} \\
\text{Rayleigh scattering} \\
\beta & \quad \text{Diffusivity parameter } (=1.66) \\
\beta_i & \quad \text{Hemispheric integral of scattering phase} \\
\text{function for radiation incident along} \\
\text{the direction defined by } \mu \\
\gamma_{1,2,3,4} & \quad \text{Coefficients associated with the two} \\
\text{stream equation} \\
\delta & \quad \text{Monochromatic optical thickness} \\
\delta & \quad \text{(depth)} \\
\delta^* & \quad \text{Total cloud optical thickness} \\
e, \epsilon_{\Delta} & \quad \text{Flux emissivity (gas)}
\end{align*}
\]
APPENDIX B

Extinction Coefficients and Conversion Factors

As discussed in Section 2, the extinction coefficient (which is just the sum of the absorption and scattering coefficients) can be defined in a number of different ways according to the measure of the amount of matter in the path. The following table gives four more commonly used quantities together with the specification of the amount of matter. Note that the product of extinction coefficient and amount of matter defines the unitless quantity $\delta$ referred to as optical path (or more usually optical depth when paths are along the vertical). Table B2 provides the conversion factors between the different forms of extinction coefficient. With volume extinction, the computations use distance as the independent variable. This is generally only used in calculations involving cloudy skies. Laboratory data cannot be presented in this way without specifying gaseous density, and it is more convenient to use one of the other forms. Of these $e_4$ is the most popular choice by many spectroscopists and thus we see the path lengths per centimeter at STP often used in empirical transmission formulas like those presented in Section 4. Two specific examples of conversion highlight the advantage of $e_4$ and the number of molecules per cm as the measure of attenuating gas over the other combinations. From Table B2 we deduce

$$1 \text{ cm of gas at STP} = \frac{M}{2.24 \times 10^4} \text{ gm cm}^{-2},$$

where $M$ is the molecular weight of the gas

$$1 \text{ cm-STP} = 2.69 \times 10^{19} \text{ molecules cm}^{-2}.$$ 

The above expression is valid for all gases. It follows from these two examples that for water vapor

$$1 \text{ g cm}^{-2} (\text{H}_2\text{O}) = 3.34 \times 10^{22} \text{ molecules cm}^{-2}$$

Thus the unit, molecule cm$^{-2}$, is independent of the nature of the absorbing gas and basic to all gases and offers a way of unifying absorber concentration units for all atmospheric constituents.

REFERENCES


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