Observations and Mixed-Layer Modeling of a Terrain-Induced Mesoscale Gyre: The Denver Cyclone

J. M. Wilczak

NOAA/ERL/Wave Propagation Laboratory, Boulder, Colorado

J. W. Gledenning

Naval Postgraduate School, Monterey, California

(Manuscript received 8 June 1987, in final form 22 January 1988)

ABSTRACT

In northeastern Colorado a frequently observed feature of the surface wind field is a stationary, terrain-induced mesoscale gyre, which is often associated with the formation of severe weather. Because of the gyre's proximity to the Denver metropolitan area, local weather forecasters frequently refer to it as the "Denver Cyclone." The development of one such cyclone, which occurred on 1 August 1985, is documented with mesonet, radiosonde, wind profiler, radiometer and tower data. Mixed-layer model simulations of this event closely agree with the observed gyre structure and indicate that the gyre is associated with a plume of warmer potential temperature air, which originates from a ridge of higher terrain to the south of Denver, and advects northward into the area of gyre formation. A mixed-layer vorticity budget demonstrates that the formation of the gyre results from the baroclinic and slope effects on the turbulent stress divergence profile.

1. Introduction

Northeastern Colorado is a region of sharply contrasting terrain relief where the rolling hills of the high plains abruptly end at the Front Range of the Rocky Mountains (Figs. 1 and 2). The north–south oriented Front Range reaches elevations greater than 3700 m MSL, rising more than 2000 m above the plains in less than 30 km.

Oriented perpendicular to the Front Range are two east–west ridges. The Cheyenne Ridge follows the Colorado–Wyoming border; parallel to the Cheyenne Ridge but 250 km to the south lies the Palmer Ridge. Both ridges reach their maximum heights at their intersection with the Front Range and decrease in amplitude to the east. The maximum height amplitude of the Palmer Ridge is approximately 800 m, while the Cheyenne Ridge is half that.

In the lower elevations between the two ridges lie the cities of Denver, Boulder and Fort Collins, as well as the PROFS (Program for Regional Observing and Forecasting Services) mesonet system (Fig. 1). The mesonet system consists of 22 surface stations, which provide 5 min averages of wind speed and direction, temperature, dew point, solar radiation and precipitation (Pratte and Clark 1983).

With the establishment of the PROFS mesonet system, it has become evident that intense mesoscale vortices often occur along the Front Range (Szoke et al. 1984; Wakimoto 1986; Williams 1985). In particular, a stationary, terrain-induced mesoscale cyclonic gyre is often observed in the lower elevations between the Palmer and Cheyenne ridges when the surface (~850 mb) geostrophic wind is southerly. This wind gyre has a typical diameter of 100 km and has been observed to be a nearly stationary feature for periods on the order of 10 hours. Because of the proximity of this mesoscale cyclone to the Denver area, local weather forecasters often refer to it as the "Denver Cyclone."

Observations of the Denver Cyclone indicate that it is associated with an anomalously high percentage of severe weather events in the Denver metropolitan area. For example, Blanchard and Howard (1986) documented the presence of the Denver Cyclone prior to and during the development of the severe, hail-producing convective storm of 13 June 1984, which caused more than $350 million in damage. In addition, Szoke et al. (1984) traced the development of the tornadoes of 3 June 1981, which caused extensive damage as they traversed sections of metropolitan Denver, to the interaction between a propagating upper level thunderstorm cell and the lower level stationary mesoscale cyclone. Using two years of mesonet data, they found that the frequently observed Denver Cyclone is correlated with a disproportionate number of severe weather events, especially tornadoes.
wind and temperature obtained by a wind profiler and radiometer system operated by NOAA at Denver's Stapleton airport, co-located with the Aurora (AUR) mesonet station (Fig. 1).

The model that we use to simulate the Denver Cyclone is a two-dimensional mixed-layer model, similar to that used by Glendenning (1985) in a study of thermally induced flow in the Los Angeles basin. Other examples of mixed-layer models are those used by Lavoie (1972) to study lake effect snowstorms; by Keyser and Anthes (1977) in a real-data simulation of synoptically active flow in the Middle Atlantic States; by Overland et al. (1979) to investigate terrain channeling of boundary-layer flow; by Anthes et al. (1982) to study the sea-breeze phenomenon; and by Han et al. (1982) in a study of terrain-induced mesoscale motions. The principal differences between these and the present model lie in a new parameterization of the interaction between the mixed layer and the free atmosphere aloft and in the inclusion of a stably stratified inversion layer just above the mixed layer.

A vorticity analysis is applied to the mixed-layer model simulations. First, a vorticity equation is developed for the mixed layer, and the mechanisms that are responsible for the formation of the Denver Cyclone within the model are explored through diagnosis of the vorticity equation in a Lagrangian framework. Second, the effects of Coriolis force, surface drag, baroclinicity, mixed-layer depth and wind speed on the formation of the eddy are investigated by considering a nondimensional form of the vorticity equation.

2. Observations

On 28 July 1985 a slowly moving cold front passed through Colorado and the northern plains states, bringing cooler, more stably stratified air behind it. By 1200 UTC 1 August, a sea-level high developed over Minnesota, with southeasterly surface flow extending from the mountain states to the central plains (Fig. 3). The near-surface (850 mb) geostrophic flow was south-
erly (~190°) over eastern Colorado, becoming southwesterly (~220°) at 700 mb and 500 mb. As often happens in this type of synoptic pattern, a mesoscale cyclone formed along the Front Range.

Tracing the development of the cyclone, we begin at 0900 UTC (0200 MST) on 1 August. At this time the contours of surface potential temperature, $\theta_s$, reflect the terrain contour patterns, but are offset to the north,
At 1800 UTC the southeasterly flow has increased to \( \sim 10 \text{ m s}^{-1} \), while the winds along the foothills have decreased in speed and become northeasterly (Fig. 4d). The warm plume has narrowed in its east–west dimension, and now extends to the Wyoming border, more than 200 km to the north. The convergence zone, which separates the northerly and southerly flow, appears to have become a stationary feature of the wind field.

Pressure differences were computed between 1500 and 0900 UTC (Fig. 5). The pressure tendencies show a clear trend toward falling pressure in the region of the Denver Cyclone to the lee of the Palmer Ridge, relative to increasing pressures on the ridge crests.

Observations of low-level wind and temperature profiles were made from an instrumented 300 m meteorological tower (Kaimal and Gaynor 1983) located at the Boulder Atmospheric Observatory (BAO) at a surface elevation of 1580 m (Fig. 1). The passage of the convergence zone as it moved southward can be seen in the tower winds between 0900 and 1000 UTC (Fig. 6). No apparent changes in temperature occurred with this shift in winds. The northerly winds continued through the morning hours, slowly becoming more northeasterly with time, perhaps due to an upslope component of the flow resulting from surface heating. The height of the convectively driven PBL, defined by the presence of a capping inversion, grew to 300 m AGL at 1700 UTC, with northerly flow above the convective PBL before this time. At \( \sim 1830 \text{ UTC} \) a weak thunderstorm outflow, originating in the mountains to the west, passed the tower, resulting in a \( 1^\circ \text{C} \) drop in temperature and a shift of the winds to the northwest. At times later than 2000 UTC the winds went through a complex series of changes due to multiple thunderstorm outflows.

Continuous profiles of wind and potential temperature were also provided by WPL's 905 MHz radar wind profiler and radiometer (Fig. 7), located 10 km northeast of Denver at Stapleton International Airport at an elevation of 1630 m, near the mesonet station of

![Diagram](image-url)
AUR (Fig. 1). The wind profiles have been filtered and interpolated to a 140 m vertical resolution, with the lowest data level at 300 m AGL. Low-level northerlies do not appear until ~1330 UTC (0730 MST), nearly 4 h later than the northerlies observed at the BAO, 30 km to the north. At 1330 UTC the northerlies extend upward to ~2200 m MSL (or 600 m AGL). Above the northerlies the winds are westerly, backing to southerly further aloft. As time progresses the depth of the northerlies remains nearly constant; immediately above the northerlies the winds become easterly before veering to southwesterly aloft. The height of the capping inversion, estimated from the radiometer temperature profiles, again shows the mixed layer growing through the northerly flow, reaching a height of 2400 m (800 m AGL) by 1900 UTC.

Additional wind and temperature profiles were provided by two balloon-launched sounding systems. The first was located at the Boulder Atmospheric Observatory (Fig. 1, BAO). Soundings were taken at the BAO at 1300, 1500, 1800 and 2000 UTC (Fig. 8a). The 1300, 1500 and 1800 UTC profiles show a 1 km deep (2500 m MSL) pool of very stably stratified ($\gamma = \partial \theta / \partial z = 0.013^\circ$ C m$^{-1}$) northerly flow, through which a well-mixed boundary layer slowly develops. The 2000 UTC profile shows westerly flow associated with the thunderstorm outflow that passed the tower at ~1830 UTC.

The second balloon-launched system was a mobile sounding unit, described by Street and Wade (1986). This unit was set up on the lee slope of the Palmer Ridge, 10 km north of the mesonet station of Elbert (ELB) at an elevation of 2100 m. Soundings were coordinated with those at the BAO. Again, at 1300 UTC a very stably stratified layer exists up to ~2500 m MSL, with a lapse rate of $\gamma = 0.015^\circ$ C m$^{-1}$ (Fig. 8b). The winds near the Palmer Divide are weakly southwesterly near the surface, becoming more strongly southwesterly above 2500 m MSL. At 1520 UTC the mixed layer has grown to a depth of ~400 m (2500 m MSL), with northwesterly winds, indicating that the northerly flow of the Denver Cyclone has penetrated that far south. By 1800 UTC the mixed layer has grown to ~700 m (2800 MSL), and the winds have become southerly again. The shift in the PBL winds indicates that the mobile launch site was very close to the convergence zone separating the northerly and southerly flow regions of the Denver Cyclone.

At 2000 UTC the mobile rawinsonde unit was moved down from the Palmer Ridge to a location well
within the southerly flow, 30 km southwest of the Byers mesonet station (BYE). Here the mixed layer has a well-defined cap at $\sim 700$ m AGL (2400 m MSL), with strong uniform southerly flow within the mixed layer and southwesterly flow aloft.

At $\sim 2015$ UTC the gust front that had earlier passed the BAO tower collided with the Denver Cyclone’s convergence zone, generating new thunderstorms along it. These storms moved off to the northeast, with the cyclonic rotation of the surface winds diminishing as the storms developed and created new low-level outflows.

3. Model characteristics

The basic tenets of the mixed-layer model were first proposed by Lavoie (1972). The concept of mixed-layer models is based on the observation that when turbulence within the PBL is sufficiently energetic, there result vertically homogeneous profiles of potential temperature and horizontal wind. Because of the vertically uniform nature of this layer, it is possible to describe its characteristics with a single level of data. This, together with some assumptions about the interaction of the mixed layer with the free atmosphere aloft, makes possible a two-dimensional simulation of the mesoscale PBL.

Figure 9 illustrates the idealized structure of the atmosphere used in the present model, consisting of a mixed layer capped by a potential temperature jump of $\Delta\theta$, an overlying inversion layer and a stably stratified atmosphere aloft. The stably stratified inversion layer and atmosphere aloft have spatially and temporally varying lapse rates that respond to forcing from the mixed layer below.

a. Mixed-layer equations

The derivation of the mixed-layer equations begins with the dry, hydrostatic, Reynolds averaged equations for shallow flow in which the thermodynamic variables have been linearized following Ogura and Phillips (1962), defined in a tangent plane Cartesian coordinate system:

$$ \frac{dv}{dt} = -f k \times v - c_p \theta_0 \nabla \pi' - \frac{\partial}{\partial z} F_\theta + K \nabla^2 v $$  

$$ \frac{d\theta}{dt} = -\frac{\partial}{\partial z} F_\theta + K_\theta \nabla^2 \theta $$  

$$ \nabla \cdot v + \frac{\partial w}{\partial z} = 0 $$  

$$ \frac{\partial \pi'}{\partial z} = -\frac{g \theta'}{c_p \theta_0^2}.$$  

Here $v$ is a horizontal velocity vector and $\nabla$ is the horizontal gradient operator; $F_\theta$ and $F_\theta$ represent the upward vertical fluxes of velocity and temperature; $K$ and $K_\theta$ are eddy diffusion and conductivity coefficients, assumed to be constants, which are used to parameterize horizontal flux divergences; $\pi'$ represents the deviation of the scaled pressure from that of an adiabatic atmosphere of constant potential temperature $\theta_0$; and $\theta'$ is the deviation of potential temperature from $\theta_0$ where $\theta' \ll \theta_0$.

Next, we make the mixed layer assumption that $v$ and $\theta$ are independent of height. The height dependence of the remaining terms in (1)–(3) is then eliminated by applying a vertical averaging operator

$$ \langle \cdot \rangle = \frac{1}{h-s} \int_s^h \cdot \, dz $$

(5)

to (1)–(3) from the surface $s$ to the top of the mixed layer $h$. Treating the mixed-layer top as a material surface except for entrainment results in

$$ \frac{\partial v}{\partial t} = -v \cdot \nabla v - f k \times v - \langle c_p \theta_0 \nabla \pi' \rangle + \frac{[F_\theta(s) - F_\theta(h)]}{D} + K \nabla^2 v $$

(6)

$$ \frac{\partial \theta}{\partial t} = -v \cdot \nabla \theta + \frac{[F_\theta(s) - F_\theta(h)]}{D} + K_\theta \nabla^2 \theta $$

(7)

$$ \frac{\partial w}{\partial t} = -v \cdot \nabla w - D (\nabla \cdot v) + w_e. $$

(8)

Here we define

- $D$ depth of mixed layer = $(h - s)$
- $w_e$ entrainment velocity
- $V, \Theta$ mixed-layer values of horizontal velocity and
potential temperature, \( \langle v \rangle \) and \( \langle \theta \rangle \), respectively.

The turbulent fluxes at the surface and inversion are parameterized as

\[
F_v(s) = -C_D |V| V \quad (9)
\]
\[
F_v(h) = -w_e (v_g - V) \quad (10)
\]
\[
F_\theta(s) = Q_{\text{max}} \cos \left( \frac{t_{\text{max}} - t_{\text{phase}}}{2} \right) \pi \quad (11)
\]
\[
F_\theta(h) = -w_e (\theta^+ - \Theta) = -w_e \Delta \theta \quad (12)
\]

where the superscript \( + \) indicates that \( \theta \) is evaluated just above the mixed layer at \( h^+ \), and where

- \( C_D \) bulk drag coefficient
- \( v_g \) geostrophic wind
- \( Q_{\text{max}} \) maximum surface temperature flux
- \( t_{\text{max}} \) time of maximum surface temperature flux
- \( t_{\text{phase}} \) time from initial surface heating to maximum heating.

The entrainment velocity is parameterized in terms of mixed-layer variables (Driedonks 1982; Tennekes 1973) as

\[
w_e = \frac{\beta F_v(s)}{\Delta \theta} + \frac{2.5 u_L^3}{D \Delta \theta \theta_\theta} + \frac{\partial D}{\partial t} \quad (13)
\]

where the convective entrainment parameter \( \beta \) is taken as 0.2 (Stull 1976) and where the surface friction velocity is defined as

\[u_* = C_D^{1/2} |V| \quad (14)\]

The last term in (13) represents a convective adjustment parameterization that removes superadiabatic lapse rates at the top of the mixed layer \( (\Delta \theta < 0) \) by increasing entrainment until \( \Delta \theta = 0 \) (Keyser and Anthes 1977). Following Driedonks (1982), the entrainment rate is limited to

\[w_e = \left( \frac{C_F}{C_T} \right) \sigma_w \quad \text{where} \quad \sigma_w = \left[ \frac{\theta_0 F_v(s) D + (2.5/C_F) u_*^3}{\theta_0} \right]^{1/3}
\]

\[C_F = 1.5, \quad C_T = 1.5 \]

Additionally, (12) requires an equation for the temperature just above the top of the mixed layer,

\[\frac{\partial \theta^+}{\partial t} = -v^+ \cdot \nabla \theta^+ + \gamma w_e. \quad (15)\]

The velocity just above the mixed layer, \( v^+ \), is not a calculated variable in this one-layer model, so it must be specified independently; the choices \( v^+ = 0, v^+ = V \) and \( v^+ = v_g \) will be explored in section 4.

Finally, the layer-averaged pressure gradient force in (6) is found by taking the horizontal gradient of the integrated (between \( z \) and \( h \)) perturbation hydrostatic equation (4), which gives

\[
c_p \theta_0 \nabla \pi^*(z) = c_p \theta_0 \nabla [\pi^*(h)] - \frac{g}{\theta_0} \nabla [(h - z) \theta'] \quad (16)
\]

where \( \nabla [\pi^*(h)] \) is the horizontal gradient of \( \pi^* \) evaluated along the inversion interface. Applying the vertical averaging operator to (16), the layer-averaged pressure gradient force is

\[-\langle c_p \theta_0 \nabla \pi^* \rangle = -c_p \theta_0 \nabla [\pi^*(h)] + \frac{g}{\theta_0} \left[ \Theta \nabla h + \frac{D}{2} \nabla \Theta \right]. \quad (17)\]

\[b. \text{ Parameterization of the upper-layer pressure gradient force}\]

We now consider the response of the overlying stably stratified atmosphere to forcing from the mixed layer below. This forcing results from motions at the PBL top that produce thermodynamic changes in the upper layer, thereby creating pressure-gradient forces that act upon the PBL. These forces are accounted for in (17) by \(-c_p \theta_0 \nabla [\pi^*(h)]\).

Because the one-layer PBL model does not explicitly resolve motions within the upper layer, \(-c_p \theta_0 \nabla [\pi^*(h)]\) must be parameterized. One approach in previous mixed-layer model studies has been to treat both the mixed layer and the upper layer as fluids of constant but differing potential temperatures, with the upper layer affecting the mixed layer below it but experiencing no changes itself (Han et al. 1982). A second approach has been to parameterize the effects of the upper layer by introducing a prognostic equation for the lapse rate \( \gamma \) that assumes that a horizontally uniform level, i.e., a model “top,” exists where mesoscale perturbations caused by mixed-layer forcing vanish (e.g., Lavoie 1972; Keyser and Anthes 1977; Anthes et al. 1982). In the latter parameterization, mesoscale perturbations are assumed to decrease linearly up to the model top. In the case of northeastern Colorado, the mountain peaks rise nearly 3000 m above the lowest terrain, which would require this overlying layer of influence to be much deeper over the valleys and plains than over the mountain areas. To circumvent this unrealistic situation, we follow Glendenning (1985) and instead assume that mesoscale perturbations everywhere decrease exponentially with height, with an \( e \)-folding length scale \( L \).

For simplicity we first consider the case of an atmosphere that has a single, initially uniform lapse rate \( \gamma_0 \), with a temperature profile given by \( \theta(z) = \theta_0 - \gamma_0(H - z) \), where \( \theta_0 \) is a temporally constant potential temperature at a reference height \( H \), which is high above the mixed layer (Fig. 9). If synoptic scale baroclinicity is present, \( \theta_0 \) will vary horizontally. The exponentially decaying perturbation temperature profile aloft is then given by
\( \theta^*(z) = \theta^*(h^+) \exp[-(z - h)/L] \) \hspace{1cm} (18)

where

\[ \theta^*(h^+) = \theta^+ - [\theta_t - \gamma_0(H_r - h)]. \] \hspace{1cm} (19)

Here \( \theta^*(h^+) \) is the difference between the actual temperature at \( h^+ \) and the temperature that would exist if the mixed-layer depth changed because of entrainment only.

Assuming that any synoptic scale baroclinicity is independent of height, one can then write for \( z > h \)

\[ \theta'(z) = (\theta_t - \theta_0) - \gamma_0(H_r - z) \]

\[ + \theta^*(h^+) \exp[-(z - h)/L]. \] \hspace{1cm} (20)

Substituting (20) into (4), integrating from \( h \) to \( H_r \), assuming \( (H_r - h) \gg L \), and then taking the horizontal gradient eventually leads to

\[ c_p \theta_0 \nabla \pi'(H_r) = c_p \theta_0 \nabla \pi'(H_r) - \frac{g}{\theta_0} \{ \gamma_0(H_r - h) \nabla h \]

\[ - \nabla[(\theta_t - \gamma_0)(H_r - h) + L \nabla [\theta^*(h^+)]] \}. \] \hspace{1cm} (21)

Combining (17) and (21) gives the complete, layer-averaged pressure-gradient force, which with the use of (19) can then be expressed as

\[ -\langle c_p \theta_0 \nabla \pi' \rangle = -c_p \theta_0 \nabla \pi'(H_r) + \frac{g}{\theta_0} \{ \frac{D}{2} \nabla \Theta - L \nabla \Theta \}

\[ + \theta^*(h^+) \nabla h + L \nabla [\theta^*(h^+)] + (H_r - h) \nabla \theta_t \}. \] \hspace{1cm} (22)

An alternative form of (22) is found by assuming geostrophic balance at \( z = H_r \),

\[ c_p \theta_0 \nabla \pi'(H_r) = f \mathbf{k} \times \mathbf{v}_g(H_r), \] \hspace{1cm} (23)

and by employing the thermal wind equation,

\[ -\frac{g}{\theta_0} \nabla \theta_t = f \mathbf{k} \times \frac{\partial \mathbf{v}_g}{\partial z} \] \hspace{1cm} (24)

to obtain

\[ -\langle c_p \theta_0 \nabla \pi' \rangle = f \mathbf{k} \times \mathbf{v}_g(h) + \frac{g}{\theta_0} \{ \frac{D}{2} \nabla \Theta

\[ - \Delta \theta \nabla h + \theta^*(h^+) \nabla h + L \nabla [\theta^*(h^+)] \} \}. \] \hspace{1cm} (25)

c. Inversion layer equations

Early morning soundings over land often indicate the presence of a very stably stratified low-level inversion, resulting from nocturnal radiational cooling. Because mixed growth due to entrainment will occur at a slower rate until the nocturnal inversion is eroded away, the vertical variability of stability above the mixed layer must be taken into account.

The presence of a nocturnal inversion is treated by adding a layer from \( h \) up to to height \( h_i \) with an initially uniform lapse rate \( \gamma_i \) (Fig. 9). For this case one can write

\[ \theta'(z) = (\theta_t - \theta_0) - \gamma_0(H_r - h_i0) - \gamma_i(h_i0 - z) \]

\[ + \theta^*(h^+) \exp[-(z - h)/L] \] \hspace{1cm} (26)

\[ \theta'(z) = (\theta_t - \theta_0) - \gamma_0(H_r - z) + \theta^*(h_i) \]

\[ \times \exp[-(z - h_i)/L] \] \hspace{1cm} (27)

Here \( h_i0 \) is the initial height of the inversion, \( L \) is assumed to be the same in both layers, and

\[ \theta^*(h^+) = \theta^+ - [\theta_t - \gamma_0(H_r - h_i0) - \gamma_i(h_i0 - h_i)]. \] \hspace{1cm} (28)

\[ \theta^*(h_i) = \theta_t - [\theta_t - \gamma_0(H_r - h_i) = \theta^*(h^+) \]

\[ \times \exp[-(h_i - h)/L] + (\gamma_i - \gamma_0)(h_i - h_i0) \] \hspace{1cm} (29)

where \( \theta_t \) is the potential temperature at height \( h_i \).

Substitution of (26) and (27) into (4), and then following the same procedure outlined in section 2b, eventually leads to

\[ -\langle c_p \theta_0 \nabla \pi' \rangle = f \mathbf{k} \times \mathbf{v}_g(h) + \frac{g}{\theta_0} \{ \frac{D}{2} \nabla \Theta - \Delta \theta \nabla h

\[ + \theta^*(h^+) \nabla h + L \nabla [\theta^*(h^+)] \]

\[ + L(\gamma_i - \gamma_0) \nabla (h_i - h_i0) \]

\[ + (\gamma_i - \gamma_0)(h_i - h_i0) \nabla h_i

\[ - (\gamma_i - \gamma_0)(h_i - h) \nabla h_i0 \}. \] \hspace{1cm} (30)

In addition, an equation is required for \( h_i \). Since we assume in our upper-layer parameterization that mesoscale perturbations decrease exponentially with height, we write the vertical velocity at \( h_i \) as

\[ \frac{\partial h_i}{\partial t} = \left( \frac{\partial h}{\partial t} - w_e \right) \exp[-(h_i - h)/L]. \] \hspace{1cm} (31)

Finally, the calculation of \( \theta^+ \) from (15) requires the use of the lapse rate found at the mixed-layer interface. Consistent with our use of an exponentially decaying perturbation temperature profile, the appropriate lapse rate is found by differentiating (20) or (26) with respect to \( z \), which gives

\[ \gamma(h^+) = \gamma_{0,i} - \frac{\theta^*(h^+)}{L}. \] \hspace{1cm} (32)

d. Numerical considerations

For temporal differencing, the model uses a two-level, explicit "forward-backward" scheme (Haltiner and Williams 1980). For spatial differencing of the advection terms, an upstream centered "donor cell" scheme is used (Roache 1976); all other spatial differencing is centered. The equations of motion are written in flux form, so the finite differencing is performed on flux quantities. A staggered grid (Arakawa Type C) is
chosen for the spatial differencing; the variables are not staggered in time.

An artifact of the above scheme is its computational diffusion. The implicit horizontal diffusion coefficient $K$ for the advective mode is approximately $0.5(\bar{u}\Delta x - \bar{u}^2\Delta a)$ (Smolarkiewicz 1983). With $\Delta x = 7$ km, $\bar{u} \approx 10$ m s$^{-1}$, and $\Delta t = 120$ s (typical values for the present simulations), the effective $K$ is of order $3 \times 10^4$ m$^2$ s$^{-1}$. This value is within the range of horizontal diffusion coefficients observed in the PBL (Gifford 1981) and used by others in models with explicit diffusion terms; for example, Keyser and Anthes (1977) used $K = 5 \times 10^3$ m$^2$ s$^{-1}$ in their mixed-layer model. The effect of diffusion on the formation of the Denver Cyclone is calculated explicitly in section 5.

Because the finite differencing grid is staggered and the model equations are written in flux form, boundary conditions are required only for the flux advection terms of the prognostic equations. For outflow boundaries, the flux advection is calculated from “donor cell” (one-sided) differencing in the same manner as elsewhere in the grid. For inflow boundaries, which are far from the region of interest, flux advection is assumed to be zero, except for time steps when a wave is diagnosed as approaching the boundary, when a radiation condition modifies the normal velocity. This boundary condition, first suggested by Orlanski (1976), reduces wave reflection.

The mixed-layer model is applied to the region between 38°12′–42°08′N and 102°00′–106°15′W, which includes northeastern Colorado and portions of Wyoming, Nebraska, and Kansas, covering an area of 360 km $\times$ 437 km (Figs. 1 and 2). An “envelope” topography is used, in which maximum values of half-second data are blocked into a 51 $\times$ 59 grid of 7.1 km $\times$ 7.4 km spacing. For numerical stability, the terrain data are filtered to eliminate wavelengths of twice the grid spacing and then desmoothed once to partially restore the original terrain amplitudes (Shapiro, 1970). To allow application of the radiation boundary condition, an edge of two grid points with heights equal to the perimeter terrain height value is added, increasing the total model domain to 55 $\times$ 63 horizontal grid points.

### 4. Model simulations: 1 August 1985

Model parameters for the simulation are given in Table 1. Values of the geostrophic wind speed and direction are taken from the 1200 UTC NMC analysis. The model is initialized with a “balanced” wind such that the initial acceleration, which is the sum of the geostrophic, Coriolis and surface friction forcings, is zero. The initial temperature jump at the top of the mixed layer is set to $\Delta T = \gamma_0 D/2$, with an initial mixed-layer depth of 50 m. The simulations begin at 0700 UTC, 7 hours before the onset of a positive surface heat flux, which we find allows the PBL winds to become more realistic when the daytime mixed layer begins to evolve. The amplitude and onset of the upward surface heat flux are taken from direct surface-layer turbulence measurements at the BAO. The surface heat flux is assumed uniform over the model domain and reaches a maximum nocturnal value of $0.15^\circ$C m s$^{-1}$, or, equivalently, 150 W m$^{-2}$.

Because of the observed strong low-level nocturnal inversion, the model simulations incorporate an inversion layer (30). The lapse rates and depth of the inversion layer are taken from the 1300 UTC sounding at the BAO and from the mobile rawinsonde unit located on the Palmer Ridge. Since the height of the top of the inversion layer is $\sim 2500$ m (MSL) at both locations, we assume this height to be uniform over the model domain, except at terrain elevations greater than 2500 m, where a minimum inversion layer thickness of 300 m is assumed. The lapse rate within the inversion layer is the average of the lapse rates observed at the two sites. The temperature field is initialized so that $\theta = 316$ K at 2500 m MSL (the top of the inversion layer); this temperature is then extrapolated both upward and downward with the specified lapse rates $\gamma_0$ and $\gamma_i$. This forces the initial contours of potential temperature to be parallel to the terrain contours.

An important model parameter is the length scale $L$, which describes the profile of mesoscale perturbations above the mixed layer. For the Denver Cyclone, significant convergence within the mixed layer occurs over a horizontal distance of $\sim 100$ km, the easterly component of the large-scale flow being blocked by the Front Range. For these conditions we have set $L$ to a characteristic mixed-layer depth value, taken to be 400 m (Fig. 11c). This value is based upon observations within the Los Angeles Basin—a region also bounded horizontally by higher terrain, with horizontal mixed-layer variations of the same scale, and with an inversion layer of similar lapse rate above the mixed layer—indicating that the depth of convergence above the mixed layer compensating for divergence within the mixed layer is equal to the mixed-layer depth (Glendenning et al. 1986). The sensitivity of model results to this parameter will be examined later.

Our basic simulation employs $v^+ = V$ for the advection of $\theta^+$ in (15). By 0900 UTC (0200 MST) a region of enhanced southerly flow develops to the lee of the Palmer Ridge (Fig. 10a), in agreement with the 0900 UTC observations at ELB and LIC (Fig. 4a). The strength of the southerlies decreases sharply to the south
as observed at COS, indicative of strong divergent flow near the ridge crest, and decreases more slowly to the north of the ridge, as observed at AUR, BYE, BRI, BAO and BOU, indicative of convergent flow to the lee of the ridge. Farther north, the winds are northerly just east of the foothills, as observed at LGM, LVE and FOR, while to the east the winds are easterly, as observed at BGD, GLY and FTM. The calculated potential temperature fields are in general agreement with the observed fields, showing a slight displacement of the warmest air to the north of the Palmer Ridge (Fig. 10b). There is, however, a tendency for the model to produce too strong a temperature gradient in this region of warmer air. This may be a result of initializing the model with the 1200 UTC sounding. Because of less time for radiational cooling to occur, the nocturnal inversion was probably weaker at 0700 UTC (the start of the simulation), which would have produced a weaker horizontal temperature gradient.

At 1200 UTC, two hours before the development of a positive surface heat flux, a complete gyre has evolved, approximately 70 km in diameter, with its center located 10 km east of AUR (Fig. 11a). The northerly flow along the foothills has moved farther south, with wind speeds of up to 4 m s$^{-1}$. A northeast–southwest oriented line of convergence is evident in the wind field on the lee slope of the Palmer Ridge, and also in the vertical velocity at the top of the mixed layer, which results from mixed-layer convergence (Fig. 11c). The convergence line's orientation and position are in qualitative agreement with the observations (Fig. 5b). The temperature field shows the beginning of the development of a warm plume originating on the Palmer Ridge, with a band of cooler air forming along the foothills (Fig. 11b).

The next three hours of simulation show little change in the size or location of the wind gyre, or in the location of the convergence zone on the lee slope of the Palmer Ridge. Interestingly, at 1500 UTC a weaker cyclonic gyre has formed on the lee slope of the Cheyenne Ridge (Fig. 12a), ~60 km north of Cheyenne. Mixed-layer potential temperature (Fig. 12b) and depth (Fig. 12c) fields at 1500 UTC show the continued development of a relatively deep, warm plume off the Palmer Ridge, in agreement with the observed $\theta$ field (Fig. 4c). The winds on the east side of the gyre apparently advect this plume to the northeast, while the northerly winds on the west side of the gyre advect cooler $\theta$ air from the lower elevations to the northeast toward and along the foothills. Measured and simulated mixed-layer depths and temperatures at the BAO are 400 m, 800 m, 306.0 K and 308.0 K respectively; at the mobile rawinsonde location these values are 450 m, 500 m, 313.5 K and 313.0 K. The simulated perturbation pressure field at 1500 UTC (Fig. 12d) has a low pressure of ~0.35 mb associated with the warm-core cyclone; conversely, high pressures of 1.60 mb and 1.75 mb due to cold advection are found on the upwind slopes of
FIG. 11. As in Fig. 10 except for 1200 UTC; (c) vertical velocity at \( h \) due to mixed-layer convergence \((\mathbf{D} \cdot \mathbf{V})\), with a contour interval of 0.2 m s\(^{-1}\) beginning at 0.1 m s\(^{-1}\). Only regions of \( w > 0 \) are contoured.
Fig. 12. As in Fig. 10 except for 1500 UTC; (c) mixed-layer depth with a contour interval of 200 m; (d) surface perturbation pressure, with a contour interval of 25 Pa.
the Palmer and Cheyenne ridges. The development of high pressures on the upward sides of the ridges and relative low pressure to the lee of the ridges and in the valley between is in qualitative agreement with the observed pressure tendencies of Fig. 6.

At 1800 UTC the wind gyre has begun to dissipate (Fig. 13a), with the winds along the foothills becoming nearly calm or weakly easterly, in agreement with the observed wind field (Fig. 4d). The center of the warm plume has now advanced ~150 km north of the Palmer Ridge (Fig. 13b), in comparison to an observed distance of ~200 km. The potential temperature differential between the plume center and the cooler air along the foothills is 3–4°C in both the simulation and observations, while in the first 100 km to the east of the plume \( \theta \) decreases by 7–8°C in both the simulation and the observation. The mixed-layer depth field has a maximum value of 1471 m, co-located with the center of the warm plume (Fig. 13c). We note that mixed-layer depths vary by well over a factor of two even in the relatively flat valley between the Palmer and Cheyenne ridges. The height (MSL) of the mixed layer (Fig. 13d) shows a clear deformation, with the greatest heights co-located with the warm plume; height differences of 400 m exist between the foothills and the plume center, with differences of 1000 m between the plume center and 100 km farther east. Observed and simulated mixed-layer depths and temperatures at the BAO are 700 m, 1100 m, 310.5 K and 311.0 K, respectively; at the mobile rawinsonde stations these values are 550 m, 550 m, 316.0 K and 314.5 K.

Comparison of the 2000 UTC mobile sounding with the simulation (not shown) has good agreement in both the mixed-layer depth, 750 m versus 800 m, and potential temperatures, 314.0 K versus 313.5 K.

Although thunderstorms disrupted further development of the Denver Cyclone after 2000 UTC, we include simulations of winds and temperature at 2100 UTC (Fig. 14a, b) and at 2400 UTC (Fig. 15a, b). These show the continued presence of a strong north–south oriented shear zone located ~15 km east of Denver and the continued northward propagation of the warm plume.

The sensitivity of the results to the various model assumptions was tested in further simulations. The first adjustable parameter examined was the vertical length scale \( L \). Simulations using \( L \) in the range of 300 to 1000 m show the same basic development of a Denver Cyclone, with smaller values of \( L \) giving a narrower, north–south elongated vortex with weaker northerly flow, while larger \( L \) values give a nearly circular vortex and stronger northerly flow. For \( L = 100 \) m a closed gyre did not form, while with \( L = 2000 \) m a strong gyre formed and then dissipated very quickly, unlike the observations.

The influence of \( \theta^+ \) advection was examined by setting \( \theta^+ \) in (15) first equal to zero and then to \( \theta_s \). For \( \theta^+ = 0 \), a Denver Cyclone formed approximately 30 km farther south, with a less realistic northward prop-

agation of the \( \theta \) plume. In their mixed-layer modeling study of a sea breeze, Anthes et al. (1982) likewise found that \( \theta^+ = \theta \) gave better results than \( \theta^+ = 0 \). Setting \( \theta^+ = \theta_s \), we find that the cyclone forms approximately three hours later than when \( \theta^+ = \theta \) and ~30 km farther north. Comparison of the \( \theta \) and depth fields to the observations reveals greater differences than when \( \theta^+ = \theta \).

5. Vorticity analysis

To better understand the dynamical mechanisms by which the mixed-layer model forms a wind gyre similar to the observed Denver Cyclone, we derive the vorticity equation for the mixed layer and apply this equation within a Lagrangian framework to parcels with trajectories that constitute the gyre’s cyclonic circulation.

A general equation for the vertical component of vorticity is obtained by taking the curl of the momentum equation (1), yielding

\[
\frac{d\zeta}{dt} = -(\zeta + f)(\nabla \cdot \mathbf{v}) + k \cdot \left( \frac{\partial \mathbf{v}}{\partial z} \right) \nabla \mathbf{w} - k \cdot \left( \nabla \times \frac{\partial}{\partial z} \mathbf{F}_v \right) + K \nabla^2 \zeta \tag{33}
\]

where \( \zeta = \mathbf{k} \cdot (\nabla \times \mathbf{v}) \) is the vertical component of relative vorticity. Thus the vorticity can change due to stretching, tilting, the curl of the stress divergence and diffusion.

Next, employing the mixed-layer assumption provides the vorticity equation for a well-mixed boundary layer. In particular, since \( \mathbf{v} \) is assumed independent of height, the tilting term is eliminated. In addition, an expression for the divergence of the stress profile is found by taking the vertical gradient of (1) and then imposing the mixed-layer assumption, giving

\[
\frac{\partial^2 F_v}{\partial z^2} = -C_p \theta_0 \frac{\partial}{\partial z} \nabla \pi'. \tag{34}
\]

Substituting the expression for \( \nabla \pi' \) given by (16) into (33) then yields

\[
\frac{\partial^2 F_v}{\partial z^2} = -\frac{g}{\theta_0} \nabla \theta'. \tag{35}
\]

Since \( \nabla \theta' \) is constant with height in a mixed layer, the stress profile is parabolic. The curvature of the stress profile is thus uniquely determined by mixed-layer baroclinicity. An essentially identical result was proposed by Arya and Wyngaard (1975) and was applied to the mixed-layer equations by Dempsey and Rotunno (1988). Examples of nearly parabolic stress profiles observed in a convective boundary layer of unknown baroclinicity can be found in Kaimal et al. (1976); to a lesser extent the stress profiles of Lenschow et al. (1980) also show curvature consistent with (35) in a baroclinic boundary layer, although the authors point out that the convective turbulence was not very intense, which may have kept the boundary layer from being nearly well mixed.
Fig. 13. As in Fig. 10, except for 1800 UTC; (c) mixed-layer depth with a contour interval of 200 m; (d) height of mixed-layer top (MSL) with a contour interval of 200 m.
Fig. 14. As in Fig. 10 except for 2100 UTC.

Fig. 15. As in Fig. 10 except for 0000 UTC 2 August 1985.
Vertically integrating (35) and applying the boundary fluxes \( F_v(s) \) and \( F_v(h) \) provides expressions for the stress,

\[
F_v = -\frac{g}{\theta_0} \frac{(z-h)(z-s)}{2} \nabla \Theta + F_v(h) \frac{(z-s)}{h-s} - \frac{\mathcal{F}_v}{h-s} \frac{(z-h)}{h-s},
\]

and for the stress divergence,

\[
\frac{\partial F_v}{\partial z} = -\frac{g}{\theta_0} \left( \frac{z-D}{2} - s \right) \nabla \Theta + \frac{F_v(h) - F_v(s)}{D}.
\]

Substitution of (37) into (33) and expanding the stress divergence term yields the final form of the mixed-layer vorticity equation,

\[
\frac{d \zeta}{dt} = -(\zeta + f)(\nabla \cdot v) - k \left[ \nabla \times \left( \frac{F_v(h) - F_v(s)}{D} \right) \right] + \frac{g}{\theta_0} k \cdot [\nabla \Theta \times \nabla D] - \frac{k}{\theta_0} [\nabla \Theta \times \nabla s] + K \nabla^2 \zeta
\]

where by assumption \( V \) and \( \Theta \), and therefore \( \zeta \), are uniform with height.

Besides stretching and diffusion, three terms representing the effect of stress divergence are present in (38). The first reflects the effect of the boundary stresses at the top and bottom of the mixed layer. The second and third terms reflect the effect of baroclinicity on the stress divergence. For convenience in interpreting the effects of terrain on vorticity generation, these terms have been written separately into baroclinic depth-gradient (involving \( \nabla \Theta \times \nabla D \)) and baroclinic-slope (involving \( \nabla \Theta \times \nabla s \)) terms. Here we have chosen one of several alternate expressions for the baroclinic influence. The form chosen explicitly includes dependence on depth gradients. This form reduces to a single term, dependent on terrain slope, when depth gradients are small; in our initialization, for example, the depth gradient term is zero. Alternative expressions are \((g/2\theta_0) [k \cdot \nabla \Theta \times \nabla h] + k \cdot [\nabla \Theta \times \nabla s]\) —convenient when height gradients are small—and \((g/\theta_0) [k \cdot \nabla \Theta \times \nabla (h + s)]/2\), which combines the baroclinic term dependence on PBL height and terrain slope into one parameter, that of the PBL mid-height.

Although the diffusion has been written explicitly in the development of the mixed-layer vorticity equation, the present model incorporates diffusion implicitly through the choice of finite difference scheme (section 2d). Even though it is implicit, the diffusion can nevertheless be calculated to a high degree of accuracy by writing \( v \) as a Taylor series expansion (e.g., Smolarkiewicz 1984). Using this method we calculate the diffusion for time-dependent, divergent flow to second order accuracy.

<table>
<thead>
<tr>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 m s(^{-1})</td>
<td>( V_z )</td>
</tr>
<tr>
<td>180°</td>
<td>( AZ_{\gamma} )</td>
</tr>
<tr>
<td>0.01°C m(^{-1})</td>
<td>( \gamma )</td>
</tr>
<tr>
<td>0.0</td>
<td>( Q_{\max} )</td>
</tr>
<tr>
<td>400 m</td>
<td>( L )</td>
</tr>
<tr>
<td>0.004</td>
<td>( C_D )</td>
</tr>
<tr>
<td>500 m</td>
<td>( D_i )</td>
</tr>
</tbody>
</table>

For the purpose of investigating the vorticity budget, a Denver Cyclone is simulated using a simplified atmospheric structure, consisting of a single lapse rate \( \gamma_0 \), which eliminates complexities arising from inclusion of the inversion layer. Other simplifications are the use of a constant drag coefficient, zero heat flux and an initial boundary layer depth of 500 m (Table 2). After nine hours of simulation, a well-developed eddy has formed (Fig. 16a), which is quasi-steady-state, changing only slightly in size and shape between hour 6 and hour 12 of the simulation. The corresponding relative vorticity field at nine hours (Fig. 16b) reaches a local maximum of \( 3.2 \times 10^{-4} \) s\(^{-1}\) on the lee slope of the Palmer Ridge. The mixed-layer depth, potential temperature and surface perturbation pressure at this time are all qualitatively similar to those for the 1 August 1985 simulation (Fig. 12a–d).

The vorticity equation is applied in a Lagrangian framework to a parcel that is released at the start of the simulation 15 km south of the crest of the Palmer Ridge. A Lagrangian framework is used in order to trace the origin of vorticity, which is generated upstream and advects into the region of the gyre. The parcel's trajectory over a 12 hour period is overlaid on the 9-h wind field in Fig. 16a, where the length of each segment corresponds to the distance traveled over a one-hour interval. The parcel's trajectory travels across the crest of the Palmer Ridge and to its lee, where the parcel becomes embedded in the circulation of the Denver Cyclone.

The vorticity and the terms of the vorticity budget (38) for the parcel are shown in Fig. 17. After six hours of simulation the vorticity of the parcel reaches its maximum of \( 1.7 \times 10^{-4} \) s\(^{-1}\) and slowly decreases afterward. The vorticity budget terms show that both drag and diffusion have relatively small effects, tending to decrease the parcels' vorticity. In contrast, the stretching term is a significant sink of vorticity at hour 2 as the parcel enters a divergent region near the crest of the ridge, and it rapidly becomes a large source term as the parcel enters the highly convergent region to the lee of the ridge. Of the two baroclinic stress divergence terms, the baroclinic-depth gradient term is always a sink of vorticity, whereas the baroclinic-slope term is always positive and is the single largest term in the budget. Further, the sum of the two baroclinic terms form a large positive net contribution to the generation of vorticity. Other parcels, released either at later times or at different locations south of the Palmer Ridge, but
which were also caught in the cyclonic circulation of the vortex, all had similar vorticity budgets.

The baroclinic-slope term, which requires intersecting contours of potential temperature and terrain height, is the largest of the vorticity production terms. It, along with the baroclinic-depth and drag terms represents torques generated by horizontal variations of the stress divergence. Fig. 18 depicts the qualitative orientation of terrain height and potential temperature contours, both observed (Fig. 4) and reproduced by the model (Fig. 12b) in the vicinity of the Palmer Ridge. As the warm plume advepts off of the ridge, its contours become oriented nearly perpendicular to the terrain height contours, resulting in a generation of vorticity.

Finally, to test the accuracy of the vorticity budget calculation, the actual vorticity of the parcel at the time of its maximum value (hour 6) was compared with the time integral of the sum of the individual budget terms; their difference was less than 2%.

6. Nondimensional vorticity equation

For many forecast purposes, it is more important to predict if a gyre will form than it is to predict the maximum vorticity in the flow. Although gyre formation is related to vorticity generation, the two are not synonymous. For example, it is possible to have a low velocity flow with relatively small vorticity and a gyre, or a high velocity flow with relatively large vorticity but no gyre.

A quantity that is more closely related to the presence of a gyre than vorticity is the nondimensional vorticity,
The baroclinic-slope term generates vorticity where contours of $\theta$ intersect contours of $S$.

formed by normalizing the vorticity with the length scale of the gyre $l$ and an appropriate scaling velocity $v$, which removes the velocity and length scale dependence of the gyre. In the case of a gyre forming due to flow over topography, the velocity scale is taken as an “upstream” velocity, unaffected by the topography, and the length scale of the gyre is predetermined by the terrain. A circular gyre of radius $l$ will have a vorticity of $2\hat{v}l$, which can have any value, but a dimensionless vorticity always of two. If instead of a circular gyre we have an incipient gyre that is just beginning to form reverse flow, the dimensionless vorticity will be smaller, of order unity. Therefore, when scaled in this manner a dimensionless vorticity equation is potentially useful in determining whether or not a gyre will form.

An equation for the nondimensionalized vorticity is obtained by scaling the terms of (38) with the following dimensionless quantities denoted by asterisks:

$$V = \hat{v}v^* \quad t = \left(\frac{1}{\hat{v}}\right)t^* \quad \nabla = \left(\frac{1}{l}\right)\nabla^*$$

$$D = \hat{D}D^* \quad \Theta = \hat{\theta}\theta^* \quad s = \hat{s}s^*$$

$$K = KK^* \quad \zeta = \left(\frac{\nabla}{l}\right)\zeta^*$$ (39)

where $\hat{v}$ is a characteristic velocity scale, $\hat{l}$ a horizontal length scale, $\hat{D}$ a mixed-layer depth scale, $\hat{\theta}$ a characteristic horizontal variation of potential temperature, $\hat{s}$ a characteristic horizontal variation of terrain height, and $\hat{K}$, a diffusivity scale.

For the case of a nonentraining mixed layer, the dimensionless vorticity equation becomes

$$\frac{d\zeta^*}{dt^*} = -\left(t^* + \overline{Ro}^{-1}\right)(\nabla^* \cdot v^*)$$

$$- \left(S_d\right)k^* \left[\nabla^* \times \left(\frac{v^*}{D^*}\right)\right]$$

$$+ \left(\frac{B_r\mu}{2}\right)k^* (\nabla^* \theta^* \times \nabla^* D^*)$$

$$+ \left(\overline{Re}^{-1}\right)[K^* \nabla^* s^*]$$ (40)

where

$$\overline{Ro} = \frac{\hat{v}}{l}$$ Rossby number

$$S_d = \frac{C_D\hat{l}}{D}$$ surface drag number

$$B_r = \frac{g}{\theta_0}v^2$$ baroclinicity number

$$\overline{Re} = \frac{\hat{\nu}\hat{l}}{K}$$ Reynolds number

$$\mu = \frac{\hat{D}}{\hat{s}}$$ terrain parameter

Next, we investigate the influence of each of the terms of (40) on the formation of the gyre by systematically varying the dimensionless scaling numbers, or products of scaling numbers, that multiply each of these terms. The scaling numbers for each simulation, listed in Table 3, are derived assuming a horizontal length scale of $\hat{l} = 50$ km, the approximate half-width of the Palmer Ridge; a vertical length scale of $\hat{s} = 500$ m, the maximum amplitude of the Palmer Ridge; a velocity scale $\hat{v}$ equal to the initial wind velocity, which would also be equal to the far upstream velocity; a temperature scale of $\hat{\theta} = \gamma_\theta \hat{s}$, applicable for the initialization scheme which assumes that $\theta$ surfaces are horizontal; and a mixed-layer depth scale $\hat{D}$ equal to the initial depth.

Each simulation is compared with a “standard” simulation (Case 1, Table 3), which has zero surface heat flux and an initial wind speed (the geostrophic wind speed reduced by surface drag) of $\hat{v} = 8.2$ m s$^{-1}$, with an initial southeasterly wind direction from 145°. Both the standard and subsequent simulations do not include shear entrainment, so that in an average sense the mixed-layer depth remains unchanged, preserving the initial depth as an appropriate scaling depth. Except for the lack of shear-induced entrainment, the standard simulation is identical to the simulation shown in Fig. 16, with input parameters given in Table 2. Because the mixed-layer is relatively deep (500 m), neglecting the shear entrainment has almost no effect, and the wind field and vorticity fields at hour 9 for the standard
case are the same as in Fig. 16, with a virtually identical parcel trajectory and vorticity budget. The maximum nondimensional vorticity of the parcel is $\tilde{\omega}_{\text{max}} = 1.0$, reached at hour 6 (Fig. 17).

First, we investigate the effect of $\overline{Re}^{-1}$ in the stretching term of (39) by decreasing $\overline{f}$ by a factor of 5 (Case 2, Table 3). Simultaneously, the geostrophic wind is varied by the amount necessary to keep $\nu$ and the initial wind direction identical to the standard case. Through this procedure only the stretching term in (39) is affected by the change in $\overline{f}$. After nine hours of simulation, a gyre has formed (Fig. 19a), which differs from the standard case only in the slightly weaker northerly flow. A maximum dimensionless vorticity of 0.6 is reached by a parcel released at the same time and location as the parcel in the standard simulation. We conclude that the Rossby number (or Coriolis parameter) has only a slight effect on the strength of the gyre, and that even with very small $\overline{Re}^{-1}$ a gyre will form. This is consistent with the vorticity budget of Fig. 17, which shows that up to the time of maximum vorticity (hour 6), the stretching term is a relatively small net source of vorticity.

Second, the effect of the surface drag term is investigated by increasing and decreasing $C_D$ by a factor of 5 while keeping $\overline{\nu}$ and the initial wind direction constant, so that only the dimensionless parameter $\tilde{C}_D$ changes. For the smaller $\tilde{C}_D$ (Case 3, Table 3), the dimensionless vorticity reaches a maximum of 1.1, and the strength of the gyre increases (Fig. 19b). Conversely, for the larger $\tilde{C}_D$ (Case 4, Table 3) the dimensionless vorticity reaches a maximum value of 0.4, and no gyre forms. Again, this is consistent with the standard case vorticity budget, which indicates that the drag term is a net sink of vorticity.

Next, the influence of the baroclinic-depth gradient term is varied by increasing $\tilde{D}$ by a factor of 5 while also increasing $\tilde{C}_D$ by a factor of 5, so that only the terrain parameter $\mu$ changes value (Case 5, Table 3). In this simulation the dimensionless vorticity reaches a maximum value of 0.3 and no gyre forms (Fig. 19c). The effect of a larger $\mu$ is to amplify the baroclinic-depth gradient term, which is seen in Fig. 17 to be a sink of vorticity.

Finally, the effect of baroclinic-slope term on the generation of dimensionless vorticity and the formation of the gyre is investigated by decreasing $\overline{B}$ by a factor of 5, while simultaneously increasing $\overline{\nu}$ by the same amount (Case 6, Table 3). The dimensionless vorticity reaches a maximum value of 0.2 in this simulation, and a gyre does not form (Fig. 19d). By decreasing the amplitude of the baroclinic-slope term, a major source of vorticity (Fig. 17) is diminished, and a gyre does not form.

The sole remaining term in (40) is diffusion. However, because the diffusion is implicit in the model, $\overline{Re}^{-1}$ cannot easily be varied.

The preceding analysis demonstrates a direct cor-
Fig. 19. The simulated wind field at hour 9 for (a) Case 2 with Ro^{-1}/5; (b) Case 3 with S_u/5; (c) Case 5 with μ × 5; (d) Case 6 with B_u/5 and μ × 5.
respondence between the nondimensional scaling numbers that multiply each term of (40) and the formation of the gyre. For forecasters interested in the effects of individual atmospheric variables, we include a sensitivity analysis of the effects of $\theta$, $D$, $\delta$ and $F_0(s)$.

First, by varying $\gamma_0$ we vary $\theta$ to a value of 1.6°C, one-fifth of the original value (Case 7, Table 3). As the prior vorticity analysis would suggest, smaller values of $\theta$ decrease the net effect of the two baroclinic terms, and a gyre does not form (Fig. 20a). For this particular initial atmospheric state (specifically, a depth of 500 m and a wind direction from 145°), we find that for $B_0 \approx 1.0$ a closed gyre will form, for smaller values it will not.

Next, the influence of $\bar{D}$ is investigated by varying the initial boundary layer depth to values between 200 and 2000 m. The effect of $\bar{D}$ is found to be less than that of $\mu$, with a somewhat weaker eddy forming for large $\bar{D}$ (Case 8, Table 3; Fig. 20b). From the scaling definitions we see that a smaller $\bar{D}$ increases $S_\mu$, which amplifies the drag term, while a smaller $\bar{D}$ also decreases $\mu$, which diminishes the baroclinic-depth gradient term. Since both the drag and baroclinic-depth gradient terms are sinks (Fig. 17), compensating effects apparently occur in these two terms, and the net influence of larger $\bar{D}$ is to weakly dampen the eddy.

Third, the effect of wind speed is determined by varying the initial wind speed $\bar{v}$ from 3 to 16 m s$^{-1}$. Other conditions remaining the same, we find that for $\bar{v}$ less than $\sim 12$ m s$^{-1}$ the gyre forms, for larger $\bar{v}$ it does not (Case 9, Table 3; Fig. 20c). The effect of larger $\bar{v}$ is to decrease $\alpha^{-1}$ (diminish the stretching term, a small net source), decrease $B_0$ (diminish the baroclinic terms, a large net source) and decrease $\rho^{-1}$ (diminish the diffusion term, a small sink). The first two of these effects act to reduce the generation of positive dimensionless vorticity, which accounts for the disappearance of the gyre for very large ambient wind speeds.

Finally, the influence of a positive surface heat flux is found by setting $F_0(s)$ to a constant value ranging from 0.1°C to 0.3°C m s$^{-1}$ (100 to 300 W m$^{-2}$) throughout the simulation. The effect of increasing the surface heat flux is to decrease the strength of the gyre, so that with $F_0(s) = 0.3°C$ m s$^{-1}$ only a narrow horseshoe-shaped eddy exists near the foothills (Case 10, Table 3; Fig. 20d). The weaker gyre is apparently due in part to the entrainment of air with zero vorticity from above the mixed layer and also to larger PBL depths resulting from entrainment.

7. Summary and discussion

The mesoscale Denver Cyclone of 1 August 1985 formed within a synoptic scale southeasterly flow characterized by initial strong stability in the lowest kilometer. A narrow zone of strong convergence separated the northerly flow close to the mountains from southeasterly flow farther east. In its mature state the cyclone was associated with a plume of warmer potential temperature air that had advected northward from the Palmer Ridge.

The above features are common to most, if not all, Denver Cyclones. One difference between the present case and many other observed Denver Cyclones is that the northerlies began to form $\approx 70$ km north of Denver, near the lowest terrain elevations between the Palmer and Cheyenne ridges. Limited observations suggest that cyclones that form during midday tend to originate farther south, near the base of the Palmer Ridge, whereas cyclones that form during the early morning hours tend to originate closer to the Cheyenne Ridge. A possible explanation for this is that nocturnal drainage flow on the slopes of the Cheyenne Ridge augments the initial development of northerlies in the morning hours, as does thermally driven upslope flow on the slope of the Palmer Ridge during midday.

A simple two-dimensional mixed-layer model has demonstrated that mixed-layer dynamics are sufficient to produce the major features of the Denver Cyclone. In particular, the model has successfully simulated the development of the wind gyre and its associated narrow zone of convergence. In addition, the model has accurately simulated the development of the observed warm plume of potential temperature, which advects off the Palmer Ridge, and has shown moderate skill at predicting mixed-layer depths.

An analysis of the mixed-layer vorticity budget indicates that baroclinicity is of paramount importance in the formation of the Denver Cyclone. Baroclinicity exerts its influence through its effect on the stress divergence profile, which is directly proportional to the baroclinicity. The stress divergence profile, when present over sloping topography, creates a torque throughout the depth of the mixed layer; and this torque generates vorticity. Model simulations used in conjunction with a dimensionless form of the mixed-layer vorticity equation demonstrate the effects of terrain parameter and Rossby, surface drag and baroclinicity numbers through their dependency on Coriolis parameter, wind speed, mixed-layer depth, baroclinicity and surface drag.

Although the mixed-layer model is successful in simulating the observed wind and temperature fields of the Denver Cyclone, it has several limitations. Foremost is the fact that the model parameterizes the effect of the free atmosphere on the mixed layer. Also, three-dimensional aspects of the flow, such as vertical shears, cannot be predicted by a mixed-layer model. The fact that the Denver Cyclone of 1 August 1985 first developed during the early morning hours, when the mixed-layer assumption is less likely to be valid, leaves open the possibility that some additional mechanism, which lies outside the scope of mixed-layer dynamics, is also capable of creating a Denver Cyclone. Alternatively, the ability of the mixed-layer model to accurately simulate the development of the Denver Cyclone during the stably stratified early morning hours may imply
Fig. 20. The simulated wind field at hour 9 for (a) Case 7, with $\theta/5$; (b) Case 8, with $\theta \times 4$; (c) Case 9, with $\theta \times 2$; (d) Case 10, with $F_0(s) = 0.3^\circ \text{C m s}^{-1}$. 
that even if the boundary layer is not well-mixed, physical effects neglected by the mixed-layer assumption are relatively unimportant.

Because of the limitations of the mixed-layer model, work in progress includes use of a three-dimensional model to simulate the Denver Cyclone over a complete diurnal cycle. An analysis of field studies is also in progress, investigating the variability of the mixed-layer depth, flow variability above the mixed layer, and the detailed structure of the convergence zone associated with the Denver Cyclone.

Acknowledgments. The authors wish to thank Dan Wolfe, Chuck Wade and Wendy Schreiber for their help in making the rawinsonde soundings, to Boba Stankov for providing the filtered profiler data, and to Dave Dempsey and Rich Rotunno for their helpful discussions on mixed-layer models.

REFERENCES


