Numerical Simulations of Gravity Currents in Uniform Shear Flows

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ABSTRACT

The purpose of this study is to examine the effect of wind shear on gravity currents in a neutral atmosphere by using a two-dimensional, nonhydrostatic primitive equation model. The depth of the gravity current is found to be directly related to the sign and the magnitude of the shear; a flow with a positive wind shear produces a gravity current of greater depth than that with a negative shear. As the positive wind shear (i.e., positive $\partial u/\partial z$) increases, the gravity current becomes unstable. For sufficiently large positive shear, the gravity current displays a diffuse structure with two distinct gravity current heads. It is found that enhanced eddy mixing, triggered by the presence of a reversed (rear to front) flow in the prefrontal environment, is the source of this phenomenon. From a vorticity budget analysis, it is found that the rear-to-front flow is less efficient in "ventilating" or "removing" vorticity generated at the leading edge of the gravity current. Therefore, the accumulation of vorticity leads to the development of enhanced eddy circulations that subsequently destroy the gravity current head.

1. Introduction

In this study we use an idealized gravity current propagating in a uniform shear flow to explore the importance of the environmental wind shear in controlling the evolution of the gravity current. We are motivated by the commonly accepted concept that gravity currents (i.e., storm's cold surface outflow) may interact with a low-level wind shear and generate new convection (Rotunno et al. 1988, referred to as RKW hereafter).

In the past, most studies of gravity currents (Simpson 1987; Droegemeier and Wilhelmson 1987; Crook and Miller 1985; Haase and Smith 1989; Bischoff-Gauss and Gross 1989) did not include wind shear. Only a handful of studies treated the effects of environmental shear. These studies include the laboratory experiments by Simpson and Britter (1980); the analytical work by Rottman et al. (1985), Jirka and Arita (1987), Xu (1992, hereafter X92) and Xu and Moncrieff (1994, hereafter XM94); and the time-dependent numerical simulations by RKW and Xu et al. (1995, unpublished manuscript).

Of particular relevance to our study is the work of Rottman et al. and Jirka and Arita. They demonstrated that if the vorticity of the low-level wind is of opposite sign to that of the gravity current head, then the leading edge of the gravity current front becomes more vertical. However, they did not examine how the maximum depth of the gravity current is controlled by the shear flow. Using a steady-state, idealized, two-fluid model, X92 and XM94 examined the quantitative dependence of the gravity current depth on inflow shear. They found that positive wind shear enhances the depth of the gravity current while the negative shear decreases the depth. Reasoning from simple vorticity dynamics, RKW also predicted a more vertical leading edge to the cold pool when there is positive vorticity in the environmental flow. The "optimal" state is reached when the advection of positive vorticity associated with the low-level wind shear balances the generation of negative vorticity at the leading edge of the gravity current. We call this mechanism the "vorticity counteract" theory (Garner and Thorpe 1992) hereafter.

In this paper, we conduct numerical simulations to verify the idealized steady-state solution of X92. We will show that the depth of the gravity current is directly related to the sign and strength of the shear as indicated by X92. However, unlike X92's result, for very strong positive shear, the model produces a gravity current of great complexity. There is a weak gravity current head propagating ahead of the main one. The gravity current behaves very differently from that in the case of moderate positive, uniform, or negative shear. We will show that the mechanism that "ventilates" or "removes" vorticity is very important in determining the structure of the gravity current. Because the efficiency of ventilation depends on the magnitude


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and direction of the flow above the gravity current, positive shear has poor efficiency in advecting the newly generated vorticity away from its source located at the leading edge of the gravity current. The accumulation of vorticity there subsequently leads to instability that diffuses the gravity current. Therefore, the main contribution of this paper is to provide another viewpoint in understanding the dynamical processes that are important in controlling the evolution of the gravity current.

It should be noted that Xu et al. (1995, unpublished manuscript) has also performed a similar time-dependent integration to confirm the gravity current theory of X92 and XM94. Their simulation was able to produce gravity currents of greater depth even when the environmental shear is very strong. Thus, their result is quite different from ours in the case of strong shear. The reason for this disparity is the fact that they treat the flow condition above the cold pool quite differently. Although their simulations were performed with both nearly balanced and unbalanced initial states and the final quasi-steady states were quite independent of the initial settings, we noticed that their flow above the cold pool was mainly front to rear. Consequently, they were able to obtain consistent results when compared with the X92 theory.

In section 2, we review the role of wind shear in affecting the behavior of the gravity currents using the steady-state solution as described by X92. Section 3 describes the two-dimensional nonhydrostatic model and initial and boundary conditions. In section 4, we present the results from our numerical simulations. Finally, in section 5, we present a summary and a brief discussion about our findings on the vorticity counteract theory.

2. Review of the steady-state analysis

To understand the role that wind shear plays in propagating gravity currents, we begin by reviewing steady-state solutions for simple two-fluid models. Benjamin (1968) introduced a model where mass, energy, and momentum are conserved at the inflow and outflow boundaries. X92 and XM94 extended Benjamin's theory to include uniform shear as shown by the schematic diagram in Fig. 1. Their results show that for a simple idealized two-fluid, steady-state system the depth of the gravity current is directly proportional to the strength of wind shear (see solid line in Fig. 9).

In addition to these solutions, X92 also reported two different kinds of steady-state solutions. When the dissipation of energy and the generation of negative vorticity, resulting from the mixing of the two fluids along the upper interface through Kelvin–Helmholtz billows, are included in the X92 formula, there is an additional solution that X92 called the “supercritical” gravity current. It is characterized by a well-defined and elevated gravity current head. The width of this head becomes wider as a hydraulic jump propagates downstream from the leading edge of the gravity current. This gravity current is usually associated with the presence of a moderate positive wind shear in the prefrontal environment.

When the wind shear is very strong, X92’s supercritical solution gives way to another regime: a subcritical state that is characterized by a small and narrow gravity current head followed by a shallow cold pool. Without detailed knowledge of the energy dissipation and negative vorticity generation, X92 could speculate only about the existence of the subcritical solution based on the results from numerical simulations and laboratory experiments. The exact reason why this solution developed was unclear to him. In a subsequent study, however, XM94 pointed out that a supercritical gravity current head could collapse if there was no rigid upper boundary to confine the high-speed, front-to-rear flow above the gravity current.

The role of wind shear in determining the structure of the gravity current front can also be examined from a different perspective. At the stagnation point, Benjamin (1968), Rottman et al. (1985), Jirka and Arita (1987), and X92 have shown that the angle between
the gravity current interface and the horizontal surface is always 60° (if there is no surface friction). Rottman et al. (1985) and Jirka and Arita (1987) have given mathematical formulas to show the shape of the gravity current front at locations near the stagnation point. According to their formulas, the slope of the gravity current front is always greater than 60° for positive shear. However, X92 and XM94 showed that the frontal slope decreases less rapidly with height and becomes less than 60° above the stagnation point when the positive shear is moderate. Only when the wind shear becomes sufficiently large as shown by XM94 (see their Figs. 8–9) can the frontal slope increase with height and become greater than 60° above the stagnation point. Therefore, the role of wind shear is twofold. First, the depth of the gravity current is directly proportional to the strength of the positive shear. Second, the slope of the gravity current front is sensitive to the shear strength.

The previous studies that we reviewed were analytical and subject to certain assumptions. In the next section we present a numerical model that will allow us to examine gravity currents in shear flows when a more realistic initial condition is used. We use this numerical model to investigate processes leading to the development of X92’s steady-state solutions. In addition, we confirm that the depth and the slope of the gravity current is indeed directly related to the sign and strength of the shear as indicated by X92 and XM94. Finally, we also conduct sensitivity experiments to find conditions under which X92 and XM94’s solutions are valid.

3. The model and initial conditions

The nonhydrostatic, elastic model used in this study has been described by Chen (1991). It uses finite differences and a time-splitting scheme to solve four prognostic equations on a staggered Arakawa C grid. The model is initialized with a neutral atmosphere that has a potential temperature of 290 K, 514 grid points in the horizontal (x) direction, and 22 points in the vertical (z) direction. It has a rigid lid at the top and a radiative condition on the lateral boundaries. The bottom boundary is free slip. The grid resolution is Δx and Δz = 100 m; the time step is Δt = 1 s. The gravity current is generated by prescribing a 950-m-high and 1.5-km-wide cold air reservoir (cold pool) at the center of the domain. The potential temperature decreases linearly toward the surface with a maximum deficit of 8 K. On average the cold pool is 4 K cooler than the surrounding environment.

We found that the model performs better with this linear profile, especially in the region near the top of the cold pool, than when a uniform or a step function profile is used. When a temperature jump is specified, as used by Bischoff-Gauss and Gross (1989) and Chen et al. (1992), a spurious circulation is often generated at the top of the cold pool because of the sharp discontinuity. This numerical noise becomes very annoying if the temperature difference between the cold pool and the environment is large. Because the gravity current is primarily driven by perturbations from the hydrostatic pressure, the use of the linear profile is equivalent to a uniform profile as long as they have the same average temperature.

In the following section, we conduct numerical experiments using four different types of flows: uniform (no shear), negative shear, moderate positive shear, and strong positive shear. To study the effect of the shear in a more systematic manner, we employed a shear profile that is uniform (linear shear) in the vertical direction.

4. Numerical results

We began our study by examining several idealized cases using our 2D nonhydrostatic primitive equation model. Although X92 gave an infinite number of solutions for his steady-state supercritical and subcritical gravity currents, he did not prove that his solutions could be realized from reasonable initial conditions. One of our goals is to use the model to identify processes leading to the development of such solutions. In the following, we examine uniform, negative shear, moderate positive shear, and strong positive shear cases.

In the case of uniform flow, the surface inflow speed (relative to the gravity current) is about −7.5 m s⁻¹. For cases of negative shear, moderate positive shear, and strong shear, these speeds vary from −6.5, −7.7, and −9.9 m s⁻¹ at the surface to −11.9, −2.3, and 11.7 m s⁻¹ at the top of the model. The corresponding non-dimensional shear strengths [ad(∂Δρ/ρ) −0.5] are −0.364, 0.364, and 1.455, respectively. To eliminate transient effects, the results shown in Figs. 2–5 are averaged over a period from 40 to 60 simulated minutes.

In general, the result from the simulations confirms the existence of X92’s steady-state solution. Our first two cases consist of a gravity current propagating in a uniform or negative shear flow as shown in Figs. 2a,b. The structure of the gravity current is quite conventional: it has a wedge with a sharp edge pointing toward the oncoming flow. Because no apparent eddy circulation occurs for these two cases, the front-to-rear flow appears to play an important role in stabilizing the gravity current by ventilating vorticity away from its source region that is located at the leading edge of the gravity current. In contrast to these first two cases, when moderate positive shear is introduced (Fig. 2c), the gravity current has greater depth and a steeper slope with a distinct head structure. The snapshot of this case at t = 1.0 h (not shown) shows an undulating fluid interface at the upper part of the gravity current head indicating the presence of strong eddy circulations. This gravity current head is also characterized by the down-
stream (leftward) propagation of a hydraulic jump from the leading edge. In general, the gravity current shown in Fig. 2c resembles the supercritical solution as depicted by X92 in his Fig. 11a. Upon increasing the wind shear (Fig. 2d) further, we found that instead of having a gravity current with much greater depth, the gravity current showed a twin-headed structure. There is a weak gravity current propagating ahead of the main one, which suggests the occurrence of very strong eddy mixing. We will now explore this phenomenon in greater detail.

### a. Vorticity ventilation effect

Because enhanced eddy mixing is very important in determining the gravity current structure in the positive shear cases (see Figs. 2c,d), we computed a vorticity budget to identify the triggering mechanism. In brief, negative vorticity is generated at the leading edge of the gravity current due to the horizontal temperature gradient between the gravity current fluid and the environment. This newly generated vorticity is then advected upward by the environmental flow along the sloping interface of the fluid. Whether the enhanced eddy circulation occurs or not depends on whether the system has an efficient means to ventilate or remove the newly created vorticity. If the ventilation is poor, excess vorticity will accumulate along the fluid’s interface near the gravity current head, eventually leading to the strong mixing. The efficiency of this ventilation mechanism depends upon the environmental flow. For example, the negative shear case shown in Fig. 2b has the most efficient ventilation mechanism, because the flow speed increases with height. Therefore, newly generated vorticity, brought upward by the frontal updraft, can be efficiently advected away from its source.
region by the faster environmental flow at upper levels. On the other hand, the strong positive shear case shown in Fig. 2d has very poor ventilation because of the rapid decrease in the flow speed. Furthermore, there exists a rear-to-front flow at heights above the gravity current. Consequently, the structure of the gravity current head is greatly influenced by the presence of the enhanced eddy mixing when the ventilation of vorticity is poor.

Following RKW for a Boussinesq system, the vorticity equation can be written as

\[
\frac{\partial \eta}{\partial t} = -\frac{\partial}{\partial x} (u\eta) - \frac{\partial}{\partial z} (w\eta) - \frac{\partial g'}{\partial x} + D_\eta, \tag{1}
\]

where the vorticity \( \eta \) and the buoyancy are defined as

\[
\eta = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}, \tag{2}
\]

\[
g' = g \frac{\Delta \theta}{\theta_0}, \tag{3}
\]

and

\[
D_\eta = \frac{\partial}{\partial x} \left( K_m \frac{\partial}{\partial x} \eta \right) + \frac{\partial}{\partial z} \left( K_m \frac{\partial}{\partial z} \eta \right). \tag{4}
\]

The potential temperature difference between the gravity current and the surrounding neutral atmosphere is denoted by \( \Delta \theta = \theta - \theta_0 \). In this study, we have \( \theta_0 = 290 \) K. The first and second terms on the right side of (1) represent the horizontal and vertical advection, respectively. The third term represents the production of vorticity due to the horizontal gradient of potential temperature at the leading edge of the gravity current. The fourth term represents the subgrid mixing. The eddy mixing coefficient \( K_m \) follows that of Durran and Klemp (1983), which should be referred to for details. In our analysis, we use a frame of reference that has the same speed as the gravity current. Therefore, the flow speed within the gravity current is nearly motionless.

Integrating (1) over the entire domain shown in Fig. 3, we obtain

\[
\frac{\partial}{\partial t} \int L \int_0^R \int_0^H \eta dz dx = \int_0^R (u\eta)_h dz - \int_0^R (u\eta)_r dz + \int_0^H g' dz + \int_L^R \int_0^d D_\eta, \tag{5}
\]

where the subscripts L, R, and d denote the left, right, and top of the integration domain, respectively. The depth of the gravity current is denoted by \( H \). The contributions from the vertical fluxes have disappeared because there is no flow through the boundaries. Also there is no buoyancy contribution at the right side of the domain, because the gravity current is confined to the left side of the domain. If we assume that the flow is steady state and inviscid, the left side of (5) and the subgrid term vanish as well. Therefore, the vorticity generated within the domain due to the buoyancy effect is balanced by the vorticity flux divergence at the left (L) and the right (R) boundary.

For the case of uniform flow, there is no vorticity flux from the R term. Therefore, the flux at the L will balance the buoyancy production term:

\[
0 = \frac{u^2}{2} + \int_0^H g' dz = \frac{C_l^2}{2} \theta_0 \Delta H. \tag{6}
\]

The flow speed \( C_l \) can also be derived from the Bernoulli equation as shown in X92 and it equals

\[
C_l^2 = 2gH \frac{\Delta \rho}{\rho}. \tag{7}
\]

Note that (6) and (7) are identical, because the Boussinesq approximation is already introduced in (1). In addition, the derivation of (6) is identical to that of RKW (8).

Table 1 shows the integrated vorticity budget based on the vorticity distributions shown in Fig. 3. Several terms of the vorticity budget are examined: the flux from the L and R, the buoyancy, the subgrid flux from the upper and bottom boundaries, and the total summations. In general, the buoyancy is the major source term to generate vorticity, while the flux term from the L is the major sink term. The subgrid flux term plays a small role except for the case of strong positive shear.

In Table 1, for uniform flow, the vorticity flux divergence from the L term accounts for 99.39% (with respect to the buoyancy term) of the vorticity loss within the domain, which is consistent with that described by (6). Because of this nearly exact balance of vorticity, it suggests that the system is very efficient in removing the vorticity generated inside the domain. As for the case of negative shear, the vorticity flux convergence is responsible for 64% of the vorticity gain from the R term and a vorticity flux divergence of 168% of vorticity loss from the L term. Again, the vorticity generated at the leading edge of the gravity current has been very efficiently ventilated or depleted from the system.

Turning to moderate positive shear, about 62% of vorticity divergence comes from the R term, and about 13% from the L term. The vorticity gains about 25% in the integration domain. For very strong positive shear, vorticity in the domain increases even more (45%)! Thus, the vorticity is not efficiently ventilated for the case of positive shear. Furthermore, from Table 1 and Fig. 2, we notice that the onset and the degree of mixing is directly proportional to the magnitude of the integrated vorticity retained in the system. This implies that enhanced eddy circulations must be occurring due to the increase of vorticity.

Because of the large residue (negative of \( T \)) given in Table 1, one might question whether cases with moderate or strong shear were in a transient state (though
Fig. 3. As in Fig. 2 except for vorticity. The contour interval is 0.006 s⁻¹.

Table 1. Domain-integrated vorticity budget analysis (m² s⁻²) for cases shown in Fig. 2 averaged over a period from 40 to 60 min.

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>R</th>
<th>B</th>
<th>S</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform flow</td>
<td>0.69 × 10²</td>
<td>0.14 × 10⁻¹</td>
<td>−0.70 × 10²</td>
<td>0.27 × 10⁰</td>
<td>−0.14 × 10⁰</td>
</tr>
<tr>
<td></td>
<td>(−99.39%)</td>
<td>(−0.02%)</td>
<td>(100%)</td>
<td>(−0.39%)</td>
<td>(0.20%)</td>
</tr>
<tr>
<td>Negative shear</td>
<td>0.15 × 10⁰</td>
<td>−0.56 × 10¹</td>
<td>−0.88 × 10²</td>
<td>0.61 × 10¹</td>
<td>0.41 × 10⁰</td>
</tr>
<tr>
<td></td>
<td>(−168.47%)</td>
<td>(−64.51%)</td>
<td>(100%)</td>
<td>(−0.70%)</td>
<td>(--4.66%)</td>
</tr>
<tr>
<td>Moderate positive shear</td>
<td>0.63 × 10³</td>
<td>0.30 × 10⁷</td>
<td>−0.48 × 10²</td>
<td>−0.23 × 10⁰</td>
<td>−0.12 × 10²</td>
</tr>
<tr>
<td></td>
<td>(−13.06%)</td>
<td>(−61.97%)</td>
<td>(100%)</td>
<td>(0.48%)</td>
<td>(25.45%)</td>
</tr>
<tr>
<td>Strong positive shear</td>
<td>0.48 × 10²</td>
<td>−0.14 × 10¹</td>
<td>−0.54 × 10²</td>
<td>−0.42 × 10¹</td>
<td>−0.24 × 10²</td>
</tr>
<tr>
<td></td>
<td>(−89.62%)</td>
<td>(26.78%)</td>
<td>(100%)</td>
<td>(7.83%)</td>
<td>(44.99%)</td>
</tr>
</tbody>
</table>

L: Flux of vorticity from the LB.
R: Flux of vorticity from the RB.
B: Subgrid flux due to buoyancy.
S: Subgrid flux through upper and bottom boundaries.
T = L + R + B + S.
%: Fraction with respect to the buoyancy term (B).
Table 2. Domain-integrated vorticity budget analysis for cases shown in Fig. 2 averaged over a period from 100 to 120 min.

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>R</th>
<th>B</th>
<th>S</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moderate positive shear</td>
<td>$0.40 \times 10^2$</td>
<td>$0.29 \times 10^2$</td>
<td>$-0.66 \times 10^2$</td>
<td>$-0.20 \times 10^6$</td>
<td>$0.33 \times 10^1$</td>
</tr>
<tr>
<td></td>
<td>$(-61.00%)$</td>
<td>$(-44.27%)$</td>
<td>$(100%)$</td>
<td>$(0.31%)$</td>
<td>$(-4.97%)$</td>
</tr>
<tr>
<td>Strong positive shear</td>
<td>$0.73 \times 10^2$</td>
<td>$-0.52 \times 10^2$</td>
<td>$-0.36 \times 10^2$</td>
<td>$-0.57 \times 10^1$</td>
<td>$-0.20 \times 10^2$</td>
</tr>
<tr>
<td></td>
<td>$(-204.42%)$</td>
<td>$(145.08%)$</td>
<td>$(100%)$</td>
<td>$(16.01%)$</td>
<td>$(56.66%)$</td>
</tr>
</tbody>
</table>

quasi-steady). However, when we integrated the model for a longer time (two simulated hours), as shown in Table 2, the magnitude of the residue term for the case of moderate shear was reduced. On the other hand, the residue for the case of strong shear remained quite large, indicating that a significant transient solution still remains. Therefore, it appears that the time interval for a gravity current to reach steady state depends on the strength of the environmental wind shear. Furthermore, we feel that the vorticity ventilation effect has a direct attribution to determine the unsteadiness of the flow.

Although the domain’s vorticity budget analysis, given by Table 1, is very useful in giving us the overall view, the spatial distribution of the vorticity budget can be very helpful in understanding the details. As can be seen in Fig. 4, for the uniform flow case the dominant

Fig. 4. As in Fig. 2 except for the vorticity budget for the uniform flow case, in which (a) is the horizontal advection term, (b) is the vertical advection term, (c) is the buoyancy term, and (d) is the total summation of advection and buoyancy terms. The contour interval is 0.0001 s$^{-2}$. 
terms in the vorticity budget are horizontal and vertical advection (panels a and b). These two terms tend to cancel each other; a negative horizontal advection is usually accompanied by a positive vertical advection. On the other hand, the buoyancy production term (Fig. 4c) and the total summation term (Fig. 4d) are relatively small compared to the advection terms. From Fig. 4 we can see that the vorticity is generated at the leading edge of the gravity current, carried upward by a frontal updraft, and then advected downstream by the environmental flow.

Let us examine the vorticity budget from a different perspective. In a vertical profile of the horizontally integrated vorticity budget as shown in Fig. 5, vertical advection plays an important role in redistributing vorticity. In Fig. 5a, for uniform flow, the buoyancy term (B) generates vorticity at low levels, and then the vertical advection (W) redeposits it at the upper levels. The vorticity flux divergence term (L) depletes vorticity by advecting it from the left boundary.

A similar event also occurs for negative shear as shown in Fig. 5b. Most of the vorticity flux (L) is limited to the upper part of the gravity current, and the magnitude of this outward vorticity flux is larger there than in the uniform case because the flow speed at upper levels is larger. Again, the vorticity generation (term B) is located near the surface, and the vertical advection (W) redistributes the vorticity upward. The vorticity increase due to the flux convergence from the R term plays a small role in the vorticity budget analysis.

The vertical profile of the vorticity budget for moderate positive shear, shown in Fig. 5c, is more compli-

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**Fig. 5.** The vertical profile of the horizontal integrated vorticity budget for cases in which the ambient flow is (a) uniform, (b) negative shear, (c) moderate shear, and (d) strong positive shear. The symbols B, L, R, and W denote the contributions from the buoyancy, the flux from the left boundary, the flux from the right boundary, and the vertical advection, respectively.
Fig. 6. The time evolution of a gravity current propagating in a strong shear environment at $t = (a) 1500 \text{ s}, (b) 2100 \text{ s}, (c) 2700 \text{ s},$ and (d) $3300 \text{ s}.$ The contour interval of the potential temperature is $0.5 \text{ K}.$

In summary, our vorticity budget analysis shows that the wind profile above the gravity current is very important in determining its structure of the gravity current. A front-to-rear flow is favorable for stabilizing (little mixing at fluid interface) the gravity current because the flow can carry the newly generated vorticity away from its source region. On the other hand, a rear-to-front (reversed) flow promotes eddy mixing, because there is no efficient way to disperse vorticity. As a consequence, deep gravity currents cannot develop in such a flow.

b. The existence of a subcritical gravity current

In section 4a, our model simulations confirmed the existence of the subcritical solution that is characterized by a substantial drop of the current depth as suggested by X92. The generation of this subcritical so-
olution was found to be very closely related to the inefficient ventilating of vorticity because of the presence of a reversed flow in the prefrontal environment. The vorticity budget analysis discussed in the previous section was vital in understanding the processes that develop such a gravity current.

At first glance, the twin-head structure shown in Fig. 2d for strong positive shear is quite puzzling and unexpected. Because the shear is so strong, the gravity current is not able to maintain its conventional sharp frontal structure. The time evolution of this case is shown in Fig. 6. The event begins with an elevated gravity current propagating in a strong shear environment as shown in Fig. 6a. The width of this gravity current head is quite narrow and the potential temperature gradient at its front is small. Strong eddy mixing is occurring in this gravity current because there is strong shear associated with the updraft and the downdraft overturning flow that exists at the front and back end of the gravity current head. Furthermore, the overturning flow of the downdraft at the back end erodes the gravity current head from the gravity current. Consequently, a second front with a much stronger potential temperature gradient develops about 2 km behind the previous one. This overturning flow is associated with the reversed shear flow that exists in the prefrontal environment.

Ten minutes later (Fig. 6b), the temperature gradient of the original elevated gravity current head becomes weaker, and a second head starts to form resulting in a twin-head structure. The erosion of the first gravity current continued for another 20 min. Finally (Fig. 6d), the first gravity current head vanished, while the second one reached its peak and began to dissipate. A third gravity current, 2 km behind the second one, also began to form. The entire scenario was repeated again with a time interval of 30 min during the first 1.5 h of simulation. For different shear rate, the length of this period is inversely proportional to the strength of the shear.

After 2 h of simulation time, the gravity current evolves into a more steady state with a wavy structure at the top of the fluid interface as shown in Fig. 7. The pattern of these waves resembles lee waves found downstream of a mountain. This gravity current is similar to the subcritical solutions speculated by X92 in that it is characterized by a very shallow depth and a small temperature gradient across the leading edge of the fluid’s interface. Therefore, the significance of this study is that we are able to use numerical model simulations to clearly demonstrate the existence of such a solution.

In his study of a squall-line gust front, Charba (1974) gave a conceptual picture for the structure of an observed gravity current (Fig. 8). His gravity cur-
rent also exhibited a twin-head structure. By examining his case more carefully, we notice that there is a strong mixing zone between the two heads. Because the shear profile is not clearly shown in Charba's paper, it is difficult to determine the importance of the shear in his case. However, from our experiments, the strong mixing zone behind the leading gravity current head can be created only when there is a strong wind shear and, most importantly, when there is a reversed (rear to front) flow above the gravity current. Therefore, we suspect that the wind shear might have played a similar role in Charba's case.

c. The role of the reversed flow and the gravity current slope

In the discussion of the subcritical gravity current, we stressed the importance of the reversed flow above the main body of the gravity current. In fact, no gravity current can exist in a region where a reversed flow prevails. Although the steady-state solution obtained from X92 does not have a constraint based on the reversed flow, we believe that the X92 solution would become unstable in a time-dependent model if the depth of the reversed shear flow is lower than the depth of the gravity current. For example, in Fig. 9, we found that the reversed flow in X92's steady-state solution appears when the nondimensional shear \([ad(gd\Delta \rho / \rho)^{-0.5}]\) is greater than 0.667; a dashed line shows where the nondimensional depth of the reversed flow is less than unity \((Z_d/d < 1)\). Furthermore, this depth is inversely proportional to the strength of the shear and intersects the curve of the nondimensional depth \((H/d)\) at the shear rate of 1.05. Therefore, we suspect that X92's solution is not valid when the shear rate is greater than this value. Indeed, our model result, as shown by the solid--dotted line, exhibits a dramatic change in the gravity current depth when the shear rate is greater than unity. Therefore, the model confirms our speculation that the depth of the gravity current is limited by the depth of the reversed flow.

As has been discussed before, Xu et al. (1995, unpublished manuscript) were able to produce supercritical gravity current head by using their time-dependent model for the case of strong shear. Because of their special procedure to reconstruct flow above the cold pool, we note that their flow direction above the gravity current is actually front to rear, which is just opposite to that of ours. Because the front-to-rear flow can ventilate vorticity very effectively, it is not surprising that a supercritical gravity current head can be obtained in Xu et al.'s simulations.

Because XM94 pointed out the importance of having a rigid wall to maintain a supercritical front-to-rear flow above the gravity current to support a supercritical head structure, their finding can also be related to the vorticity ventilation mechanism discussed in this paper. Without a rigid upper boundary, the front-to-rear flow cannot be channeled to produce a supercritical high speed flow between the gravity current and the upper wall. Therefore, the weakening of the front-to-rear flow is expected. Subsequently, the supercritical gravity current head will collapse due to the accumulation of vorticity.

We also observe another interesting phenomenon. Because of eddy mixing there is often a well-defined head structure at the leading edge of the gravity current. The width of the gravity current head expands downstream as the hydraulic jump at the back end of the gravity current head propagates in the downstream direction. The rate of expansion depends on the strength of the shear. As shown in Fig. 10, the width of the gravity current head is inversely proportional to the strength of the shear at \(t = 2.0\) h and becomes very

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**Figure 9.** The nondimensional plot of depth versus the nondimensional shear \([ad(gd\Delta \rho / \rho)^{-0.5}]\) at \(t = 2.0\) h. The depth of the reversed shear flow \((Z_d/d)\) is represented by the dashed line. The depths of the gravity currents that derived from the X92 idealized two-fluid, steady-state model and calculated from our time-dependent model are represented by solid and solid--dotted lines, respectively.

**Figure 10.** Same as in Fig. 9 except for the nondimensional width of the gravity current head.
small for a flow with a shear rate greater than unity. With increasing positive wind shear and the descending layers of the reversed flow, the eddy mixing becomes increasingly significant. As a result, the wind shear is more effective in eroding the gravity current head. Therefore, the width of the gravity current head becomes narrower as the positive shear of the environmental flow becomes larger.

To show the gravity current front slope near the stagnation point, we replot Fig. 2 in Fig. 11 with a 1:1 aspect ratio. The purpose is to give a correct perspective of the gravity current slope. For no, negative, and moderately positive shear shown in Figs. 11a–c, the maximum slope of the gravity current front is similar to that shown by X92 and XM94. However, the slope is not bounded by 60° for the case of strong positive shear. From Fig. 11d the slope of the gravity current front can be as large as 105°. We suspect that the reason for this steep slope may be strongly associated with the unsteadiness of the flow. Furthermore, the steepness is also explored by XM94, in which they found that only when the inflow becomes sufficiently strong \[ ad(gd\Delta \rho / \rho)^{0.5} > 1.2 \], the frontal slope can increase with height and becomes larger than 60° above the leading edge of the front.

5. Summary and discussion

The purpose of this study is to examine the effects of wind shear on the behavior of a gravity current. We began our study by reconstructing X92's simple two-fluid, steady-state model. We then conducted two-dimensional numerical simulations using a time-dependent, nonhydrostatic, numerical model and studied the evolution of gravity currents propagating in neutrally stratified shear flows. We confirmed that the depth of the gravity current is directly related to the sign and strength of the shear. Our result is in agreement with

![Fig. 11. Same as in Fig. 2 except replotting the gravity current near its stagnation point. The aspect of the plotting scale is near 1:1 in the x and z directions.](image-url)
X92 and X94. However, X92 and X94’s formulas are of only limited validity. When the nondimensional shear rate is greater than 1.05, the gravity current exhibits a totally different behavior that includes a diffuse head structure and shallow depth. This limitation on X92 and X94’s formulas is largely dependent upon the presence of a reversed (rear to front) flow. In this circumstance the onset of eddy mixing is greatly enhanced, resulting in the destruction of the gravity current head. This enhanced mixing is caused by the accumulation of vorticity at the gravity current head, which was not advected away.

In this study we find that the vorticity ventilation effect is very important in determining the structure of the gravity current. The case of uniform or negative shear has good ventilation, while the case of positive shear has a poor one. The efficiency of ventilation depends on the magnitude and the direction of the flow above the gravity current. In general, a front-to-rear flow is the most efficient in advecting the newly generated vorticity away from its source located at the leading edge of the gravity current. In contrast, the weak front-to-rear and rear-to-front flows advect vorticity away least, resulting in an accumulation of vorticity. This subsequently leads to instability that destroys the gravity current head. For moderate positive shear, the flow speed above the gravity current of the front-to-rear flow is not as fast as its counterpart cases of uniform or negative shear. Therefore, the efficiency of ventilating the vorticity is greatly reduced. Consequently, the eddy mixing associated with the accumulation of vorticity is a dominant feature in cases of positive shear.

The role of the environmental shear is twofold. First, at low levels, the wind shear determines the depth and the frontal slope of the gravity current. Second, the wind direction at levels above the gravity current re-shapes the gravity current structure. Our numerical simulations show that a gravity current cannot maintain itself in an environment where there is a reversed flow above the current. However, as evidenced by RKW’s result, it is not necessary to have shear in this reversed flow to enhance the mixing. Therefore, at upper levels, only the wind direction is important to affect the structure of the gravity current.

The concept of the vorticity ventilation effect is, in fact, not new to researchers in fluid mechanics. In their study of vortex sheet evolution, Rottman and Stansby (1993) found that the convergence of vorticity will result in the destabilization of a vortex sheet. This mechanism is essentially the same as the ventilation effect that we have discussed in this paper; the accumulation of vorticity eventually leads to the development of rigorous mixing and eddy circulations!

Another important finding is that the maximum possible angle of 60° applies only to cases with uniform, negative, and moderate positive shear. For strong shear, this angle can be as large as 105°. The reason for this steep frontal slope may be due to the unsteadiness of the solution.

Finally we can explain a point made by RKW. As shown in Fig. 12, RKW suggested that strong lifting is produced if the vorticity generated at the leading edge of a fluid’s interface is balanced by the advection of vorticity from the ambient environment with wind shear. Because the wind shear profile used by RKW is not continuous and the depth of the shear layer equals that of the gravity current, there is no environmental flow above the gravity current. Therefore, there is no mechanism to ventilate the vorticity generated at the leading edge of the gravity current unless that vorticity is balanced by the advection of vorticity (with opposite sign) associated with the environmental shear flow. Only then will the gravity current front steepen without disruption because there is no excess vorticity to induce enhanced mixing.

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Fig. 12. Schematic diagram showing how a buoyant updraft may be influenced by low-level wind shear (after Rottman et al. 1988). The sense of the circulation associated with the wind shear and the cold pool is depicted by arrows.
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