Adaptation of an Isopycnic Coordinate Ocean Model for the Study of Circulation beneath Ice Shelves

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ABSTRACT

Much of the Antarctic coastline comprises large, floating ice shelves, beneath which waters from the open ocean circulate. The interaction of the seawater with the base of these ice shelves has a bearing both on the rate at which Antarctic Bottom Water is formed and on the mass balance of the ice sheet. An isopycnic coordinate ocean general circulation model has been modified so as to allow the incorporation of a floating ice shelf as an upper boundary to the model domain. The modified code admits the introduction of an arbitrary surface pressure field and includes new algorithms for the diagnosis of entrainment into, and detrainment from, the surface mixed layer. Special care is needed in handling the cases where the mixed layer, and isopycnic interior layers, interact with surface and basal topography. The modified model is described in detail and then applied to an idealized ice shelf–ocean geometry. Simple tests with zero surface buoyancy forcing indicate that the introduction of the static surface pressure induces an insignificant motion in the underlying water. With nonzero surface buoyancy forcing the model produces a cyclonic circulation beneath the ice shelf. Outflow along the ice shelf base, driven by melting of the thickest ice, is balanced by deep inflow. The abrupt change in water column thickness at the ice shelf front does not form a barrier to buoyancy-driven circulation across the front.

1. Introduction

Eustacy is affected by a variety of factors including the storage of land surface and ground water, the thermal expansion of the oceans, and the mass balance of glaciers and ice sheets. A report by the Intergovernmental Panel on Climate Change (Houghton et al. 1996) has singled out the ice sheets as representing the greatest uncertainty in accounting for past, and predicting future, changes in sea level. Ice sheet mass balance is a combination of surface accumulation and ablation, calving of icebergs from the fronts of ice shelves and tidewater glaciers, and melting and freezing that occurs at the ice–ocean interface. The latter component of the mass budget has no direct impact on sea level, as the vast majority of the melting ice is already freely floating in the ocean, but the ice shelves may play a role in regulating the discharge from some grounded portions of the ice sheet.

Aside from this indirect impact on global sea level, basal melting also has an important influence on the properties of the waters that circulate beneath an ice shelf. Outside the subice cavity the only heat sink available to the ocean is the atmosphere, so seawater cannot be cooled below the surface freezing point of about \(-1.9^\circ C\). The base of an ice shelf represents a heat sink at depths approaching 2000 m where the freezing point may be as low as \(-3.4^\circ C\) (Millero 1978), so the waters in contact with the ice shelf may be up to 1.5\(^\circ C\) below the freezing point at atmospheric pressure. Such potentially supercooled water is referred to as ice shelf water (ISW). In the Weddell Sea, ISW contributes to the formation of Antarctic Bottom Water (Foldvik and Kvinge 1974), and the exceptionally cold temperatures attained by ISW enhance the density of the product water, through the thermobaric effect. A further consequence of the generation of potentially supercooled water is that if it rises toward the surface it may become supercooled with respect to the in situ freezing point. The result is ice production deep in the water column (Foldvik and Kvinge 1974). Where this happens beneath the ice shelf, the ice accumulates on the base as marine ice (Bombsch and Jenkins 1995), while the freezing of ISW that exits the subice cavity may contribute to the sea-ice budget (Bombsch 1998).

The interaction between ice sheets and oceans is thus an important element of the climate system, and in re-
cent years numerical models have been used to evaluate the key processes operating in the subice cavity (Williams et al. 1998). The approach taken has been to use ocean models of varying sophistication and apply upper boundary conditions derived from a thermodynamic model of the ice shelf–ocean interaction (Holland and Jenkins 1999). The calculated heat and freshwater fluxes represent the only surface forcing on the sub–ice shelf water column, since the overlying ice shelf isolates the ocean from wind, evaporation and precipitation, and the direct effects of atmospheric temperature variations. Dynamic models of the ice shelf itself have not been included to date, and the implicit assumption has been made that any ice shelf thickness changes resulting from melting or freezing are offset exactly by the flow of the ice shelf itself. The disparity of timescales between the slowly evolving ice shelf and the rapidly ventilated waters beneath provides some justification for this modeling approach.

In this paper we focus on the mathematical description of an isopycnic coordinate sub–ice shelf ocean model, and in particular the treatment and parameterization of the mixed layer processes. Observations show the presence of a mixed layer in the water column beneath an ice shelf (Nicholls and Makinson 1998) with water properties relatively well mixed near the ice shelf base and sometimes over a significant fraction of the water column. Section 2 presents the conservation equations for the isopycnic layers of the model and describes how the equations differ for the mixed layer. The necessary modifications to the model required for it to function in the oceanographic regime beneath an ice shelf are then presented. The main issues are that of (i) the incorporation of an arbitrary surface pressure field, (ii) the processes of mixed layer entrainment and detrainment, and (iii) the interaction of layers with topography. We also briefly present some additional changes we have made to the code, associated with internal friction, that are not directly related to the ice shelf problem. Finally the modified model is applied to an idealized ice shelf–ocean geometry and the results are discussed.

2. Model description

As a longer-term science objective a coupled numerical model is being developed suitable for investigation of oceanographic processes in polar regions, referred to as the Polar Ocean Land Atmosphere Ice Regional (POLAIR) modeling system. At present this system consists of a viscous-sublayer parameterization describing the thermodynamic interaction between the ice shelf base and the ocean surface (Holland and Jenkins 1999) coupled to an isopycnic ocean general circulation model (Bleck et al. 1992). That ocean component, the Miami Isopycnic Coordinate Ocean Model (MICOM), already contains an embedded mixed layer model based on a turbulent kinetic energy (TKE) budget. In the next stages of development, a sea-ice, atmospheric boundary layer, and dynamical ice shelf model will be added. We now briefly describe the ocean model and its mixed layer submodel prior to elaborating upon the modifications that are addressed in the sections following.

a. Isopycnic interior model

The primitive equation isopycnic coordinate ocean model used (MICOM) has four prognostic equations: one equation for the horizontal velocity vector, a layer thickness tendency equation, and two conservative equations for the buoyancy related variables. Since not all layers are necessarily constant density (e.g., the mixed layer), we first present the equations for a generalized vertical coordinate $\xi$ (Bleck and Boudra 1981; Bleck 1998):

$$\frac{\partial \mathbf{v}}{\partial t} + \nabla \mathbf{v} \cdot \mathbf{v} + (\xi + f) \hat{k} \times \mathbf{v} = -\alpha_\phi \nabla \phi - \nabla \psi + \frac{\partial \mathbf{C}}{\partial \xi} \frac{\partial \xi}{\partial \xi}$$

$$+ \left( \frac{\partial p}{\partial \xi} \right)^{-1} \nabla \psi \cdot \left( \kappa \frac{\partial \mathbf{C}}{\partial \xi} \right)$$

$$\frac{\partial}{\partial t} \left( \frac{\partial p}{\partial \xi} \right) + \nabla \psi \cdot \left( \frac{\partial p}{\partial \xi} \right) + \frac{\partial}{\partial t} \left( \frac{\partial p}{\partial \xi} \right)$$

$$= \nabla \psi \cdot \left( \kappa \frac{\partial \mathbf{p}}{\partial \xi} \right)$$

$$+ \nabla \psi \cdot \left( \kappa \frac{\partial \mathbf{C}}{\partial \xi} \right) + \mathcal{F}_C.$$

In these relations, $\mathbf{v} = (u, v)$ is the horizontal velocity vector, $p$ the pressure, $C$ represents either one of the model’s dynamically active tracers (i.e., potential temperature $\theta$ or salinity $S$) as well as any passive tracers, $\xi = \partial u / \partial x - \partial v / \partial y$, the relative vorticity, $\alpha_\phi$ the specific volume, $\phi = gz$ the geopotential, $g$ the gravitational acceleration, $f$ the Coriolis parameter (spatially varying), $\hat{k}$ the vertical unit vector, $\kappa$ the isopycnic eddy viscosity, $\kappa_\alpha$ the layer thickness diffusivity, $\kappa_c$ the tracer diffusivity, $\tau$ the wind- and/or bottom-drag-induced shear stress vector, and $\mathcal{F}_C$ represents the sum of all diabatic forcings on the tracer variable $C$. The quantity $(\partial p / \partial \xi) \frac{\partial \xi}{\partial t}$ represents the vertical mass flux through a surface $\xi$. The independent variables $x, y,$ and $t$ have their usual meanings and $\xi$ is now the independent variable in the vertical direction.

The above equations are relationships formulated for variables on $\xi$ surfaces, but the ocean model is formulated for layers (i.e., the material bounded between
two $\xi$ surfaces). A point of nomenclature: in the discrete context the individual layers will be denoted by integer subscript $k$, with the topmost layer assigned index 1 and the bottommost layer assigned index $N$. Furthermore, in general we will use the notation of a superscript symbol ($\bar{\cdot}$) to refer to a quantity at the surface interface of a layer and a superscript symbol ($\hat{\cdot}$) at the basal interface of a layer. To obtain averaged equations for the fluid between some upper $\xi^+$ and lower $\xi^-$ bounding surfaces requires multiplication of each of (2.1) through (2.3) by $\partial p/\partial \xi$, vertical integration of the resulting equations over the coordinate interval $\Delta \xi = \xi^+ - \xi^-$, and division by the pressure jump between the surfaces, $\Delta p = p(\xi^+) - p(\xi^-)$. As a further simplification for use in later sections, we introduce the notation that the layer thickness is exactly defined in terms of the pressure jump as $h = \Delta p$. As the velocity within a layer is vertically constant, it is convenient to also introduce the notation of $\Delta u^+$, $\Delta u^-$ as the velocity jump across the upper $-$ and lower $+$ interfaces of a layer, respectively.

After these transformations, the horizontal momentum, layer thickness, and buoyancy conservation equations for a layer become

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}^2 + (\xi + f) \hat{k} \times \mathbf{v}$$

$$+ \frac{1}{\Delta p} \left[ \left( \frac{\partial p}{\partial \xi} \right)^+ - \left( \frac{\partial p}{\partial \xi} \right)^- \right] \mathbf{v}^+ - \frac{\partial p}{\partial \xi} \mathbf{v}^-$$

$$= -\alpha_u \nabla p(\xi^+) + \frac{p(\xi^-)}{2} - \nabla \phi(\xi^+) + \frac{\phi(\xi^-)}{2}$$

$$- \frac{g}{\Delta p} (\tau^+ - \tau^-) + \nabla \cdot \left( \kappa_u \Delta_p \nabla \mathbf{v} \right)$$

$$= -\alpha_u \nabla \cdot \left( \kappa_u \nabla p \right)$$

$$\frac{\partial \mathbf{v}}{\partial t} + \nabla (\mathbf{v} \Delta p) + \left[ \left( \frac{\partial p}{\partial \xi} \right)^+ - \left( \frac{\partial p}{\partial \xi} \right)^- \right]$$

$$= \nabla \cdot \left( \kappa_u \Delta_p \nabla \mathbf{v} \right)$$

$$\frac{\partial}{\partial t} (\Delta p \mathbf{v}) + \nabla \cdot (\mathbf{v} \Delta p \mathbf{v}) + \left[ \left( \frac{\partial p}{\partial \xi} \right)^+ + \left( \frac{\partial p}{\partial \xi} \right)^- \right]$$

$$= \nabla \cdot (\kappa_u \Delta_p \nabla \mathbf{v}) + f_c.$$  

(2.4)

(2.5)

(2.6)

The topmost layer is not isopycnic, is referred to as the mixed layer, and has a nonzero $\partial p/\partial \xi \delta \xi \partial \theta|_i$ representing a large flux of mass and properties into and out of an otherwise almost-adiabatic ocean interior. The interior ocean is not exactly adiabatic as there exists subtle diapycnic fluxes $\partial p/\partial \xi \delta \xi \partial \theta|_i$ between neighboring layers due to the background presence of small-scale turbulence. A detailed treatment of these terms is presented elsewhere (McDougall and Dewar 1998) and no modification of that treatment is undertaken in this study.

For all interior model layers (i.e., excluding the mixed layer) we choose $\xi$ to be surfaces of constant potential density $\rho_o$, and for these layers the $\partial p/\partial \xi \delta \xi \partial \theta|_i$ terms vanish if there are no diapycnic exchanges. For future reference we will define this flux of mass between layers as a general diapycnic flux by introducing the notation $w_c^{xy} = (\partial p/\partial \xi \delta \xi \partial \theta|_i)^x$. On isopycnic layers the pressure and geopotential gradient terms in (2.4) can be replaced by the gradient of the Montgomery potential (Montgomery 1937):

$$M = \alpha_o p + gz.$$  

(2.7)

Because the Montgomery potential is vertically invariant throughout the depth of a layer, it does not matter at what level within the layer it is evaluated. For adiabatic isopycnic interior layers Eqs. (2.4)–(2.6) reduce to (Bleck 1998)

$$\frac{\partial \mathbf{v}}{\partial t} + \nabla \mathbf{v}^2 + (\xi + f) \hat{k} \times \mathbf{v}$$

$$= \nabla \cdot \left( \kappa_u \Delta_p \nabla \mathbf{v} \right)$$

$$= -\frac{g}{\Delta p} (\tau^+ - \tau^-) + \nabla \cdot \left( \kappa_u \Delta_p \nabla \mathbf{v} \right)$$

$$= -\frac{g}{\Delta p} (\tau^+ - \tau^-) + \nabla \cdot \left( \kappa_u \Delta_p \nabla \mathbf{v} \right)$$

$$= \nabla \cdot (\kappa_u \Delta_p \nabla \mathbf{v}) + f_c.$$  

(2.9)

(2.10)

As these equations are derived, whether for nonisopycnic or isopycnic layers, under the hydrostatic (and Boussinesq) approximation, the vertical component of the momentum balance is a diagnostic equation, which takes on the form (Bleck 1998)

$$\frac{\partial \mathbf{v}}{\partial t} + \nabla \mathbf{v}^2 + (\xi + f) \hat{k} \times \mathbf{v}$$

$$= \nabla \cdot \left( \kappa_u \Delta_p \nabla \mathbf{v} \right)$$

$$= \nabla \cdot (\kappa_u \Delta_p \nabla \mathbf{v}) + f_c.$$  

(2.11)

b. Mixed layer model

A characteristic problem of pure isopycnic coordinate formulations is an inability to accommodate buoyancy forcing at the ocean surface. The solution adopted in MICOM is to have a variable density mixed layer sitting on top of the pure isopycnic layers that acts as an interface between the adiabatic ocean interior and the buoyancy forcing from the atmosphere. In the present application the surface buoyancy fluxes also arise from thermodynamic interactions at the base of an ice shelf, and so the mixed layer must also exist beneath the ice shelf. While all interior isopycnic layers adhere to the basic conservation equations (2.4)–(2.6), the mixed layer is unique in that its layer thickness is also dependent upon a TKE balance, determined from the input of frictional stress and buoyancy at the surface, and a parameterization of energy dissipation processes (Gaspar 1988). We define the buoyancy in a layer, $b$, as

$$b = g \frac{\rho_o - \rho_s}{\rho_o},$$  

(2.12)

where the potential density is $\rho_o = 1/\alpha_o$ and $\rho_s = 1/\alpha_o$. 

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is a reference density based on a reference specific volume $\alpha_r$. Then, the rate of entrainment of fluid into the mixed layer, $w^*_m$, can be expressed by

$$\frac{w^*_m}{\Delta \Delta b} = \frac{\rho_{o} g m_s u^3_m}{\Delta \rho_i} + \frac{B_M}{2} - \varepsilon_d, \quad (2.13)$$

where $\Delta b$ is the change in mixed layer buoyancy resulting from entrainment, $m_s$ is an empirical constant, $u_m$ is the friction velocity, $\Delta \rho_i$ is the thickness of the mixed layer in pressure units, $B_M$ is the buoyancy flux due to surface forcing, and $\varepsilon_d$ is the parameterized dissipation of TKE (see Gaspar 1988 for further details). The relation between (2.4)–(2.6) and (2.13) is manifested through the common definition of the diapycnic velocity, $w^*_m$, which permits mass exchange between the mixed layer and the isopycnic interior layers by moving the interface representing the mixed layer base.

A source of TKE (e.g., strong shear in the flow between the ocean surface and the base of the ice shelf or alternatively brine rejection associated with marine ice growth) leads in general to a deepening of the mixed layer; the reverse being true for a sink of TKE (e.g., ice growth) leads in general to a deepening of the mixed layer. At times when the mixed layer is deepening, water is entrained from the adiabatic ocean interior into the mixed layer. The exact amount of water entrained is diagnosed by balancing the supply of TKE against the increase in potential energy of the water column resulting from mixing. At times when the mixed layer is shallowing, water is detrained from the mixed layer into the interior isopycnic layer closest in density to that of the detrained water. In this fashion, the waters in the oceanic interior are ventilated by the mixed layer.

c. Equation of state

The equation of state we utilize to compute potential density $\rho_o$, and its inverse the potential specific volume $\alpha_{sp}$ is an approximation of the United Nations Educational, Scientific and Cultural Organization (UNESCO; Millero et al. 1980) equation of state in the form of a polynomial. It also allows for direct analytical “inversion” of the equation of state so that, for instance, potential temperature can be obtained from given values of potential density, salinity, and pressure. The polynomial equation for the density is, expressed in sigma-theta terms,

$$\sigma(\theta, S, \rho_o) = c_{\sigma}^\tau(p_o) + c_{\sigma}^\tau(p_o) \theta + c_{\sigma}^\tau(p_o) S$$

$$+ c_{\sigma}^\tau(p_o) \theta^2 + c_{\sigma}^\tau(p_o) S \theta + c_{\sigma}^\tau(p_o) \theta^3$$

$$+ c_{\sigma}^\tau(p_o) S \theta^2, \quad (2.14)$$

where the empirical coefficients $C_{\tau} \cdot \cdot \cdot C_{\Phi}$ are known functions of ocean reference pressure $p_o$ (Brydon et al. 1999). This equation of state is suitable over the range of temperatures, salinities, and pressures encountered in a sub–ice shelf cavity.

3. Surface pressure

a. Isostatic assumption

A common assumption made in the construction of numerical models of ocean circulation is that the surface pressure either is spatially and temporally constant, or varies at most by a fraction of the overlying atmospheric pressure. The ocean surface then deviates only slightly from a surface of constant geopotential and the deviations are always small in comparison with the depth of the water column. However, a floating ice shelf may exert a pressure approaching 200 atm on the underlying ocean, and the associated surface topography is then of the same order of magnitude as the ocean bottom topography. Computing the oceanic response to changes in pressure of this magnitude would be a challenging problem, but the ice shelf thickness evolves on time-scales that are much longer than those of interest for the dynamic response of the ocean. We can therefore assume that the surface pressure is static and that the surface elevation is always close to that of the isostatically adjusted sea level. The isostatically adjusted sea surface then plays a role identical to that of a constant geopotential sea surface in conventional ocean models.

In the case of MICOM, the isopycnic interior layers are bounded by interfaces that experience a spatially and temporally varying pressure field and may have an entirely arbitrary interfacial topography. The bottom interface of the bottommost layer is unique in that it has a fixed bottom topography and a bottom pressure that undergoes small, temporal variations according to the total weight of fluid in the overlying water column. The surface interface of the mixed layer is also unique in that it has a fixed bottom topography and a bottom pressure that varies at most by a fraction of the overlying atmospheric pressure either is spatially and temporally constant, or varies at most by a fraction of the overlying atmospheric pressure. The ocean surface then deviates only slightly from a surface of constant geopotential and the deviations are always small in comparison with the depth of the water column. However, a floating ice shelf may exert a pressure approaching 200 atm on the underlying ocean, and the associated surface topography is then of the same order of magnitude as the ocean bottom topography. Computing the oceanic response to changes in pressure of this magnitude would be a challenging problem, but the ice shelf thickness evolves on time-scales that are much longer than those of interest for the dynamic response of the ocean. We can therefore assume that the surface pressure is static and that the surface elevation is always close to that of the isostatically adjusted sea level. The isostatically adjusted sea surface then plays a role identical to that of a constant geopotential sea surface in conventional ocean models.

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part of the domain it is the ice shelf bottom pressure. We estimate the surface pressure according to

$$p_1 = \frac{gH_i}{\alpha_i},$$  \hspace{1cm} (3.2)

where $\alpha_i$ and $H_i$ are the mean specific volume and thickness of the overlying ice column, respectively. A schematic of the definition of these and other relevant quantities is given in Fig. 1. The expression (3.1) is based on the assumption that the ice shelf cannot support vertical shear stresses between neighboring columns of ice, and is a good approximation over horizontal scales that are large compared with the ice thickness ($\sim 1$ km) as is the case in the present application. The sea level $z^s$ so defined is the equilibrium sea surface position and is not a geopotential surface as is typically the case in ocean modeling. As dynamical motions occur in the ocean, the model sea level, $z_t$, undergoes small height perturbations relative to its reference value $z^s$.

An important consequence of the definition of the sea surface (3.1), whether it be that residing beneath an ice shelf or that occurring over the open ocean, is that the gradient of the Montgomery potential will correctly vanish for the situation where the horizontal pressure force is zero and no flow is expected. Such a situation being when there is no horizontal gradient of the mixed layer specific volume $\alpha$, and the sea surface elevation is equal to its reference value of $z^s$. In that case $\nabla M$ vanishes exactly as seen from combining (3.1) with the definition of the Montgomery potential.

b. Operator splitting

In common with many ocean models, MICOM employs an operator splitting technique to solve the momentum equations in a computationally efficient manner. The solution for the depth-averaged flow is advanced separately from that for the more complex, depth-dependent component. The former is forced by pressure deviations at the seabed, which reflect net convergence or divergence of mass in the overlying water column, while the latter is forced by changes in the vertical density structure, which leave the overall mass of the water column, and hence the pressure at the seabed, unaffected. The details of the operator splitting are described by Bleck and Smith (1990) and Higdon and Bennett (1996), so only the essential modifications will be presented here. The total pressure $p$ is multiplicatively split into “baroclinic” and “barotropic” components according to

$$p = p'(1 + \eta).$$  \hspace{1cm} (3.3)

where $p'$ represents the depth-dependent baroclinic component and the dimensionless variable $\eta$ represents the depth-independent barotropic component as the fractional change in total mass of the water column. At the seabed, the baroclinic component $p'_b$ is time invariant, while spatial gradients in the evolving barotropic component $p'_b, \eta$ provide the main forcing on the depth-averaged flow. In the presence of a static surface pressure, Eq. (3.3) becomes

$$p - p_i = (p' - p_i)(1 + \eta).$$  \hspace{1cm} (3.4)

Note that the surface pressure does not decompose, because it is unaffected by any changes (barotropic or baroclinic) in the ocean. The velocity is split according to

$$\mathbf{v} = \mathbf{v} + \mathbf{v}',$$  \hspace{1cm} (3.5)

where $\mathbf{v}$ is the depth-mean flow and the deviations $\mathbf{v}'$ have the property that they integrate to zero over the total water column.

c. Barotropic component

The prognostic equation for the depth-averaged velocity is written

$$\frac{\partial \mathbf{v}}{\partial t} + f\hat{k} \times \mathbf{v} = -\nabla M + \frac{\partial \mathbf{v}^*}{\partial t},$$  \hspace{1cm} (3.6)

where we have introduced and defined notation for the barotropic component of the Montgomery potential as $\bar{M} = \alpha_i(p'_b - p_i)\eta$. The final term on the right-hand side of (3.6) represents all addition forcing terms in (2.8) that have not been explicitly taken into account in (3.6). Applying the pressure and velocity splitting to the layer thickness tendency equation (2.9), we obtain
\[
\frac{\Delta p'}{\Delta t} + \frac{\partial \eta}{\partial t} + \nabla \cdot (\nabla \Delta p') + \nabla \cdot (\nabla \Delta p') = \nabla \cdot (\kappa \nabla \Delta p'),
\]

where we have used the facts that \( \Delta p = \Delta p'(1 + \eta) \) and that \( \eta \ll 1 \) so that \( 1 + \eta \) may be approximated by 1 where it appears as a factor. Summing this expression over all layers and recalling that \( p_\infty \) and \( p_i \) are constants, not subject to time evolution or diffusion, and that the velocity deviations must integrate to zero, yields a predictive equation for the barotropic pressure component:

\[
\frac{\partial \eta}{\partial t} + \nabla \cdot [\nabla (p_\infty - p_i)] = 0.
\]

\[
\text{d. Baroclinic component}
\]

The corresponding equations for the depth-dependent layer thickness tendencies is obtained by substituting (3.8) into (3.7),

\[
\frac{\partial \Delta p'}{\partial t} + \nabla \cdot (\nabla \Delta p') = \frac{\Delta p'}{p_\infty - p_i} \nabla \cdot [\nabla (p_\infty - p_i)]
\]

and for the depth-dependent layer momentum by subtracting (3.6) from (2.8),

\[
\frac{\partial \nu}{\partial t} + \nabla \nu^2 + (\zeta + f) \hat{k} \times \nu + \zeta \hat{k} \times \nu = -\nabla M' - \frac{g}{\Delta p'} (\tau^+ - \tau^-)
\]

\[
+ \frac{1}{\Delta p'} \nabla \cdot (\kappa \Delta p' \nabla \nu) - \frac{\partial \nu^e}{\partial t}.
\]

The reference to the geopotential function is eliminated by substitution of the definition of the Montgomery potential so as to express \( \nabla M'_E \), only in terms of quantities explicitly calculated by the model. The final expression for the baroclinic component of the mixed layer Montgomery potential is then

\[
\nabla M'_E = \alpha \left( \frac{p_1 + p_2}{2} \right) + \nabla \left( \frac{\phi_1 + \phi_2}{2} \right).
\]

\[
\text{4. Mixed layer processes}
\]

\[
\text{a. Entrainment}
\]

An elegant algorithm for entrainment in an isopycnic layer context, consistent with overall energy conservation in the water column, is presently employed in the MICOM code (Bleck et al. 1989). The TKE produced via the surface stress and buoyancy extraction processes is used to mix denser water from the isopycnic interior into the mixed layer, thereby increasing the mixed layer thickness. The resulting increase in potential energy of the water column exactly matches and thus exhausts the supply of TKE over the course of one model time step. The entrainment algorithm we use is based on identical principles, but looks algebraically different, as we have had to generalize the equations to allow for an arbitrary surface pressure field. The derivation follows closely that of Bleck et al. (1989) and a schematic of the relationship between the various densities and combinations of surface buoyancy fluxes for a typical entrainment scenario is presented in Fig. 2.

A point of nomenclature that needs be mentioned, as it applies both to the entrainment procedure of the present section and detrainment procedure of the next section, is that in the overall context of the model’s mixed layer algorithm there always exist three mixed layer thicknesses: the initial mixed layer thickness \( h_i \), at the beginning of a time step, an intermediate mixed layer thickness \( h_i \), derived from Eq. (2.13) based on the physics of Gaspar (1988) and referred to as the “Gaspar” thickness, and the final mixed layer thickness \( h_f \), at the end of a time step.

We need to evaluate the potential energy (PE) of the water column between the two pressure interfaces representing the sea surface \( p_i \) and the unknown depth to which mixing and entrainment occurs \( p_m \). This will be done for both the mixed and unmixed states, with the aim of finding the value of \( p_m \) for which the change in PE equals the supply of TKE. Introducing the gravitational acceleration as an explicit factor, the PE per unit horizontal area can be expressed in terms of the Montgomery potential:
Fig. 2. Schematic of the entrainment of isopycnic-layer waters into the mixed layer due to either free convection as promoted by an unstable vertical density profile or forced convection due to either surface frictional stress \( u_*^3 \) or an upward-directed surface buoyancy flux \( B_m \) (i.e., buoyancy loss). The leftmost panel shows the initial state of the system with the mixed layer density, temperature, salinity, and thickness denoted \( \sigma_1, \theta_1, S_1, \) and \( h_1 \), respectively. The complementary properties of the isopycnic layer below the mixed layer are indicated by the subscript \( k \). In the presence of an upward surface buoyancy flux \( B_m \), the mixed layer properties are transformed into so-called Gaspar layer properties as shown in the middle panel, and denoted by subscript \( g \). This is an intermediate state of the system in that the buoyancy fluxes are applied only over the initial mixed layer thickness, i.e., \( h_g = h_1 \). This intermediate state includes the possibility that the mixed layer waters have become convectively unstable with respect to the isopycnic layer waters below. The rightmost panel shows the final state after entrainment has occurred with the new, deepened mixed layer overlying the partially depleted isopycnic layer. The final mixed layer properties are denoted with \( m \) subscripts and the final isopycnic layer properties by prime superscripts.

\[
gPE = \int_{p_1}^{p_m} (M - \alpha \rho) \, dp. \quad (4.1)
\]

As the water column in an isopycnic model is layerwise discretized, the vertical integral transforms into a summation. In the initial, unmixed state this summation gives

\[
gPE = \sum_{k=1}^{n-1} \left[ M_k (p_{k+1} - p_k) - \frac{\alpha_k}{2} (p_{k+1}^2 - p_k^2) \right] + \left[ M_n (p_m - p_n) - \frac{\alpha_n}{2} (p_m^2 - p_n^2) \right]. \quad (4.2)
\]

In the final, mixed state there remains only one layer, having a density \( \alpha_m \) equal to the depth-weighted average of all the layers that have been mixed together:

\[
\alpha_m (p_m - p_l) = \sum_{k=1}^{n-1} \left[ \alpha_k (p_{k+1} - p_k) \right] + [\alpha_n (p_m - p_n)]. \quad (4.3)
\]

Note that this expression is actually a statement of volume conservation, which only holds for the case of a linear equation of state. However, the small error we make through the use of this approximation in the diagnosis of the entrainment depth is of little concern, given the uncertainties associated with the parameterizations of TKE production and dissipation we have used. Assuming that volume is conserved during mixed layer entrainment also means that the surface elevation remains unchanged, and this allows us to evaluate the Montgomery potential \( M_m \) of the final mixed layer as follows:

\[
M_m - \alpha_n p_1 = M_1 - \alpha_1 p_1. \quad (4.4)
\]

The PE per unit area of the final mixed layer is simply

\[
gPE_m = M_m (p_m - p_1) - \frac{\alpha_m}{2} (p_m^2 - p_1^2), \quad (4.5)
\]

and substituting from (4.3) and (4.4) above, this can be written

\[
gPE_m = (p_m - p_l) \left\{ M_l - \alpha_l p_1 - \sum_{k=1}^{n-1} \left[ \frac{\alpha_k}{2} (p_{k+1} - p_k) \right] - \frac{\alpha_n}{2} (p_m - p_n) \right\}. \quad (4.6)
\]

Subtracting the expression for the potential energy of the initial state from this and equating the difference to the TKE supplied by surface stress and buoyancy extraction,

\[
g(PE_m - PE) = gTKE, \quad (4.7)
\]

leads us to an expression for the pressure at the base of the mixed layer following entrainment:
where, for convenience, we have introduced the layerwise functions analogous to those of Bleck et al. (1989):

\[
p_m = \frac{g \text{TKE} + p_i (M_1 - \alpha_i p_1 - F_k) + G_i - p_n \left[ M_n - \frac{\alpha_n}{2} (p_1 + p_n) \right]}{(M_1 - \alpha_i p_1 - F_k) - \left[ M_n - \frac{\alpha_n}{2} (p_1 + p_n) \right]},
\]

(4.8)

\[F_k = \sum_{k+1}^{N} \frac{\alpha_k}{2} (p_{k+1} - p_k) \quad \text{and} \quad (4.9)\]

\[G_i = \sum_{k=1}^{i} \left[ M_i (p_{k+1} - p_k) - \frac{\alpha_k}{2} (p_{k+1}^2 - p_k^2) \right]. \quad (4.10)\]

The solution procedure for finding \( p_m \) follows exactly that of Bleck et al. (1989). Trial values are found successively for each layer \( k = 2 \ldots N \) beneath the mixed layer, and the process is continued as long as the trial value is greater than the pressure at the upper surface of layer \( k \), or the seafloor is reached. The correct value of \( p_m \) is the last one that meets this criterion, and once it has been found, updating of layer thicknesses and properties completes the algorithm.

b. Convection

In the MICOM code a separate algorithm deals with the case where buoyancy extraction renders the mixed layer statically unstable with respect to the isopycnic layers beneath. It is assumed that the timescale for convection is small compared to the model time step and thus static instability is removed by an instantaneous rearrangement of the appropriate model layers. Interior layers are incorporated successively into the mixed layer until the density profile is once again statically stable. The process is essentially one of mixed layer entrainment, but since the depth of convective mixing is diagnosed in a manner that is incompatible with the theory outlined in the preceding section, it must be dealt with in a separate routine.

We decided to exploit the similarity between the processes of entrainment and convection and so deal with them both using a single algorithm. In the case of unstable stratification, deepening of the mixed layer lowers the PE of the water column, and the (negative) change in PE between mixed and unmixed states can be found in the manner outlined in the preceding section. The depth of the mixed layer following convection can also be found through the procedure outlined above. In the absence of any TKE supply, the new state will be one with the same PE as the initial state; that is, all the available PE associated with the unstable stratification will have been used for entrainment. With a supply of TKE from surface processes, the mixed layer will deepen further, with the (positive) change in PE between initial and final states equaling the TKE supplied over one time step, as before.

While this procedure is computationally efficient, it suffers one potential disadvantage, in that none of the available PE associated with the unstable water column is dissipated, rather it all contributes to deepening of the mixed layer. However, in a coarse-resolution, hydrostatic model any parameterization of convection is subject to considerable uncertainty. Given that the ultimate aim of a convection routine in such a model is to avoid a situation that cannot be handled by the reduced physics through a somewhat arbitrary reorganization of the vertical structure, there is possibly some advantage in ensuring that this procedure does not represent a sink of energy.

c. Detrainment

In order to maintain the integrity of an isopycnic coordinate system while allowing mass to flux between layers one must ensure that the density of waters received by an isopycnic layer matches its prescribed density. The situation of mixed layer entrainment, discussed in the previous sections, is not of concern in this sense because the mixed layer is nonisopycnic and therefore can receive water of any density. The reverse situation, that is of mixed layer detrainment, is problematic because the density of the waters that are to be detrained into a particular isopycnic layer will not in general match the prescribed density of that layer.

A retreat of the mixed layer occurs whenever the TKE generated by surface stresses is insufficient to maintain the current mixed layer depth in the presence of an input of buoyancy at the surface. In this case the TKE balance of Eq. (2.13) would yield a negative entrainment rate, but since the equation is only valid for zero or positive entrainment, it is used instead to diagnose a new equilibrium depth for the mixed layer. This is the depth for which the supply of TKE is exactly balanced by buoyancy and frictional dissipation, and the entrainment rate is therefore zero. As the mixed layer shoals to this depth and attains a lighter density it leaves beneath it a slab of water of arbitrary density, which we shall refer to as the fossil layer. A schematic of the relationship between the various densities and combinations of surface buoyancy fluxes for a detrainment scenario is presented in Fig. 3.

A variety of solutions to the problem of how to adjoin
FIG. 3. Schematic of the detrainment of mixed-layer waters into an isopycnic layer as forced by a downward-directed surface buoyancy flux (i.e., buoyancy gain). The leftmost panel shows the initial state of the system with the mixed-layer density, temperature, salinity, and thickness denoted $\sigma_1$, $\theta_1$, $S_1$, and $h_1$, respectively. The complementary properties of the isopycnic layer below the mixed layer are indicated by the $k$ subscript. The presence of a downward surface buoyancy flux $B_m$ that is strong enough to suppress the TKE supplied by surface stress $u^*$ leads to the creation of a shallow, intermediate state layer referred to as the Gaspar layer over whose depth scale (i.e., the Monin–Obukov length) are distributed the downward surface buoyancy fluxes, i.e., $h_g$, $h_1$. The relatively shallow Gaspar layer properties are indicated in the middle panel by the subscript $g$. The difference between the shallow, intermediate state Gaspar layer and the deep, initial state mixed layer leads to the existence of a fossil layer in the intermediate state. This fossil layer is split into two sublayers: the upper fossil layer denoted with subscript $uf$, and a lower fossil layer of subscript $lf$. The final state of the system is shown in the rightmost panel in which the upper fossil layer adjoins with the Gaspar layer to produce the final mixed layer; the lower fossil layer adjoins with the isopycnic layer to produce the final state of the isopycnic layer. The final mixed layer properties are denoted with $m$ subscripts and the final isopycnic layer properties by prime superscripts.

For our ice shelf–ocean modeling problem, the simplest approach was to follow the MICOM procedure of splitting the fossil layer, but we found that some modification of the algorithm was still necessary. That used in the original MICOM code (Bleck et al. 1992; Sun 1997) has some shortcomings in that under certain conditions of surface forcing it can either create an artificial temperature extremum or prevent detrainment proceeding at all. The problem arises because there are an infinite number of ways in which the fossil layer can be unmixed, and selecting the most appropriate split for
any situation is not straightforward. While the procedures used in MICOM seem to work very well in the warm water sphere, for which they were originally designed, in our high-latitude polar domains their performance appears to degrade.

If there were an infinite number of isopycnic layers, the detrainment algorithm would be straightforward. We would first diagnose the new thickness of the mixed layer from (2.13), then calculate its new temperature and salinity by applying the surface fluxes to this new layer over one time step. The fossil layer would be subducted directly into the interior, having properties identical to those of the mixed layer prior to receiving buoyancy input. Our detrainment algorithm mimics this behavior as closely as possible, given the finite number of isopycnic layers.

1) MICOM ALGORITHM

The original MICOM algorithm for detraining the fossil layer waters into an isopycnic layer involved solely a temperature-based split of the fossil layer into an upper, warmer layer and a cooler, lower layer (Bleck et al. 1992). The salinity of the upper and lower fossil layers was not split but kept the same as the original fossil layer. The upper fossil layer temperature was assigned precisely that of the Gaspar layer. Using these constraints, coupled with an equation of state, and demanding that the lower fossil layer density exactly match that of the isopycnic layer into which the lower fossil layer waters are received, leads to a polynomial equation in one unknown, the thickness of the lower fossil layer.

It was later recognized that splitting the fossil layer solely on temperature was problematic for polar oceans where the density is largely controlled by salinity variations and not temperature. Accordingly, the splitting of the fossil layer was adapted to involve splitting on both temperature and salinity (Sun 1997). In the instance of a density profile as in Fig. 4a, the approach was to assign the lower fossil layer the temperature and salinity properties of the isopycnic layer. The upper fossil layer temperature was still assigned the Gaspar value as in the earlier algorithm while the upper fossil salinity was now an unknown to be determined. As before, the thickness of the lower fossil layer was solved via a polynomial equation, which once obtained diagnostically led to knowing the salinity of the upper fossil layer.

In the situation that the diagnosed upper fossil layer salinity is less than the Gaspar or greater than the original fossil, as would occur in the situations depicted in Figs. 4b–d, then the upper fossil layer salinity is reassigned to whichever of these constraining values it lies closer to. A new determination of the lower fossil layer
Fig. 5. Schematic based on a temperature–salinity diagram illustrating behavior of present MICOM detrainment scheme for conditions usual to the ice shelf–ocean interaction problem. The thin dashed lines represent the isopycnals of the Gaspar layer $\sigma_g$, the fossil layer $\sigma_f$, and the isopycnal layer $\sigma_k$, which is the recipient of the detrained water. The triangle represents the Gaspar layer, which has properties of $\theta_f$ and $S_f$. The circle positioned with temperature $\theta_i$ and salinity $S_i$ represent the fossil layer properties. A built-in constraint of the present MICOM algorithm is that the upper fossil layer temperature is exactly that of the Gaspar layer. The thick, solid horizontal line of fixed temperature $\theta_{uf}$ describes this constraint. Since the unmixing of the fossil layer must occur along a straight line, the only possible detrainment lines must lie within the conic structure indicated by the shading. This implies the possibility of creating extremely warm temperatures in the lower fossil layer or alternatively not detrainning at all, neither behavior being desirable.

thickness is carried out, but with the lower fossil layer temperature and salinity values no longer constrained to be exactly the same as that of the isopycnic layer. While the temperature and salinity of the lower fossil layer now differ from the isopycnic layer, the density of the lower fossil is still constrained to be that of the isopycnic layer. With these new constraints in place, the thickness of the lower fossil layer is once more solved for and the present MICOM algorithm terminates.

In applying this algorithm to the problem of ice shelf–ocean interaction, situations were found where the above algorithm failed to detrain sufficient waters and the waters that were detrained were found to be unacceptably warmed. As the mixed layer in the sub–ice shelf cavity does not undergo a seasonal cycle, which would cycle the waters everywhere in the cavity through an entrainment and detrainment phase, regions of persistent detrainment can lead to the creation of unrealistically warm water masses. Consider for instance, the scenario of detrainment in a polar ocean illustrated in Fig. 5. The fact that the present MICOM algorithm makes the temperature of the upper fossil layer match that of the Gaspar layer results in an excessively warm temperature for the lower fossil layer. A more natural split of the fossil layer would be along the direction “orthogonal” to the isopycnals as suggested in Fig. 5.

Such a split avoids the generation of extreme temperatures but can only be achieved by giving up the constraint that the upper fossil layer temperature must match that of the Gaspar layer. Imposing such a constraint is reasonable in a tropical ocean where density is dominated by temperature. In a polar ocean, where density is dominated by salinity, a more reasonable constraint is that the upper fossil layer adhere to the salinity of the Gaspar layer and not its temperature. What is needed then is a flexible constraint on the upper fossil layer such that it adheres to the Gaspar temperature in tropical oceans but to the Gaspar salinity in polar oceans with a smooth transition between the two in temperate oceans. Such a flexible constraint is possible by introducing the concept of an orthogonal density split of the fossil layer.

2) MODIFIED ALGORITHM

The main difficulty encountered with the present MICOM algorithm stems from the fact that it treats temperature on a special footing as compared to salinity. We argue that it may be better to base a detrainment algorithm on density, with temperature and salinity each proportionally contributing to the algorithm based on their respective thermal expansion and haline contraction coefficients as relevant weighting.

We remove the above constraint that the upper fossil layer be assigned the Gaspar layer temperature. Instead, we argue that the upper and lower fossil layers can each take on arbitrary temperature and salinity values, provided they are constrained by the overall minimum and maximum temperature and salinities present. These extrema come from the properties of the isopycnic layer, the fossil layer (which has the properties of the old mixed layer), and the Gaspar layer. The present MICOM scheme does not “bound” the upper fossil layer by the new Gaspar temperature, but rather sets it equal to it. We argue that the incoming buoyancy fluxes (both of heat and salt) are distributed over the shallower Gaspar layer and not the deeper, initial mixed layer. In this manner the fossil layer retains the initial mixed layer properties while the Gaspar layer has its temperature and salinity set by the incoming heat and salinity fluxes being distributed over the shorter Monin–Obukov length rather than the initial mixed layer depth. The new mixed layer temperature and salinity achieved are then used in defining the properties of the upper fossil layer.

Removing the constraint that the upper fossil temperature must coincide with that of the Gaspar layer reintroduces an extra degree of freedom into the algorithm with the result being that there are an infinite set of choices for splitting the upper and lower fossil layers. We decide then to adhere to two guiding constraints: (i) the upper and lower fossil temperatures and salinities do not go outside the temperature and salinity extrema that we establish, and (ii) the lower fossil temperature and salinity produce a lower fossil density that exactly matches that of the isopycnic layer. While these constraints are superficially similar to that of the present MICOM algorithm, the main difference lies in the
choice of the temperature extremum. The MICOM algorithm set the upper fossil temperature to that of the Gaspar layer and places no constraint on the lower fossil temperature. Our approach is different in that we do not impose the upper fossil temperature, rather we solve for the upper and lower fossil temperatures subject to the constraint that no new temperature extremum are created.

3) LOWER FOSSIL PROPERTIES

The new constraint we introduce to replace the present MICOM constraint of assigning the upper fossil layer temperature is to stipulate that the fossil layer is split based on an orthogonal density criterion. This constraint says that the unmixing line for temperature and salinity is orthogonal to the isopycnal of the fossil layer (see Fig. 6). Such an orthogonal split has the advantageous property of giving greater weight to the temperature field in tropical oceans while in polar oceans the salinity field controls the splitting of the fossil layer.

To define the direction of such an unmixing line we consider the isopycnals of the temperature–salinity diagram as contours of a scalar field $\sigma$. For the few remaining derivatives in this section we treat both temperature and salinity as being dimensionless, and we do this so that we can compute a “gradient” in temperature–salinity space with temperature and salinity treated as the independent variables of the density function.

Taking the gradient and normalizing by density $\rho$ we arrive at the expression

$$\frac{\nabla \sigma}{\rho} = \beta \hat{i} - \alpha \hat{j}, \quad (4.11)$$

where the $\hat{i}$ and $\hat{j}$ unit vectors are aligned with the salinity and temperature axes, respectively. In the above equation we have made use of the conventional definitions of the thermal expansion and haline contraction coefficients of seawater (albeit dimensionless for present purposes):

$$\alpha = -\frac{1}{\rho} \frac{\partial \sigma}{\partial \theta}, \quad \beta = \frac{1}{\rho} \frac{\partial \sigma}{\partial S}. \quad (4.12)$$

Given that we know the density of the fossil layer (which is just that of the initial mixed layer), the temperature and salinity of the fossil layer (which is just the temperature and salinity of the initial mixed layer), and the density of the isopycnic layer below, we can determine the orthogonal temperature and salinity of the lower fossil layer so that the layer achieves the same density as the isopycnic layer below as

$$(\theta_f, S_f) = \frac{1}{\rho} \frac{\sigma_k - \sigma_s}{\alpha^2 + \beta^2} (-\alpha, \beta) + (\theta_f, S_f). \quad (4.13)$$

The fossil layer, prior to splitting into upper and lower fossil layers, has a thickness $h_f$ that is the difference between the initial mixed layer thickness and the shallower Gaspar layer thickness:

$$h_f = h_1 - h_s. \quad (4.14)$$

The temperature and salinity of the fossil layer prior to its split is the initial mixed layer temperature $\theta_f$ and salinity $S_f$. There are six unknowns to be determined in the problem of splitting the fossil layer into an upper and lower fossil layer, namely the temperature, salinity, and thickness of the upper and lower fossil layers. The orthogonal split has set the temperature and salinity of the lower fossil layer reducing the unknowns to four. The conservation of mass for the splitting of the fossil layer sets the lower fossil thickness:

$$h_{uf} = h_f - h_{uf}. \quad (4.15)$$

This leaves us with three unknowns, namely, the temperature, salinity, and thickness of the upper fossil layer.

4) UPPER FOSSIL PROPERTIES

Two constraints we invoke for the fossil layer are that of overall heat and salt conservation:

$$h_f \theta_f = h_{uf} \theta_{uf} + h_{uf} \theta_{uf}, \quad h_f S_f = h_{uf} S_{uf} + h_{uf} S_{uf}. \quad (4.16)$$

which in effect determines the upper fossil layer temperature and salinity. This situation now leaves us but one unconstrained variable, the thickness of the upper fossil layer. To address this last unknown we look at the properties of the final mixed layer, which results
when we adjoin the Gaspar layer with the upper fossil layer. Invoking heat and salt conservation for this mixing process produces the constraints

\[
h_u \theta_m = h_u \theta_g + h_u \theta_f, \quad h_u S_m = h_u S_g + h_u S_f,
\]

where from mass conservation the thickness of the new mixed layer is simply

\[
h_m = h_f + h_u. \tag{4.18}
\]

It appears that we have gained little as we have added two more equations with two more unknowns, namely, the final temperature \( \theta_m \) and salinity \( S_m \) of the final mixed layer. As we shall see, however, we can in fact ultimately combine these equations to solve for the thickness of the upper fossil layer. This amounts to making decisions about the final values that the mixed layer temperature \( \theta_m \) and salinity \( S_m \) will achieve under various scenarios of either net heat gain or loss and similarly for freshwater gain or loss with the objective of maximizing the amount of detrainment. Specifying both the new mixed layer temperature and salinity now puts us in the situation of having one too many equations in relation to the number of unknowns. This situation means that we will obtain two physically valid solutions for the upper fossil layer thickness and we will take the thicker of these as that will represent the most constrained or conservative solution. We now show these two solutions.

Combining the heat conservation equation \( (4.16) \) for the fossil mixed layer with the heat conservation equation \( (4.17) \) for the final mixed layer leads to an expression for the thickness of the upper fossil layer:

\[
h_u = \frac{h_f (\theta_f - \theta_g) + h_u (\theta_g - \theta_m)}{(\theta_m - \theta_f)). \tag{4.19}
\]

Similarly, combining the salt conservation equations we obtain

\[
h_u = \frac{h_f (S_f - S_g) + h_u (S_g - S_m)}{(S_m - S_f)}. \tag{4.20}
\]

We cannot evaluate these expressions until we come to a decision as to the final mixed layer temperature and salinity. Our objective is to make the upper fossil layer as thin as possible so as to detrain as much water into the isopycnic layer via the lower fossil layer as is possible, without violating the constraints we impose. This objective will be met by maximizing the difference between the orthogonal temperature and salinity and the final mixed layer temperature and salinity, respectively, while adhering to the overall temperature and salinity constraints that ensure we do not create new temperature or salinity extrema. It turns out that this stipulation of maximum difference depends upon, for temperature, whether we are cooling the Gaspar layer or heating it. Analogously for salinity, we need to know whether the Gaspar layer is freshened or salinified. These situations, depicted in Fig. 7, give us a physical interpretation that
guides the assignment of the final mixed layer temperature and salinity. Briefly stated, if the Gaspar layer is heated, then the final mixed layer temperature will attempt to be the Gaspar layer temperature (Fig. 7a), while if the Gaspar layer is cooled, the final mixed layer temperature will be the warmer fossil layer temperature (Fig. 7b). Analogously for salinity, under conditions of freshwater input to the Gaspar layer via buoyancy forcing then the final mixed layer salinity will attempt to lower to that Gaspar value (Fig. 7c), and vice versa for salt input to the Gaspar layer the final mixed layer salinity will be the fresher fossil layer salinity (Fig. 7d). In this manner the maximum amount of detrainment, while not creating new temperature or salinity extremum, is achieved.

The four scenarios outlined above give four different physically meaningful solutions to the upper fossil layer thickness solutions (4.19) and (4.20). Under the case of heat input (Fig. 7a) at the ocean surface the upper layer fossil thickness is

\[ h_{uf} = h_f \frac{\theta_f - \theta_{uf}}{\theta_f - \theta_{uf}}, \]  

(4.21)

which is a well-behaved solution as we can formally guarantee the desired bounded property of \( 0 \leq h_{uf} \leq h_f \). Under the case of heat extraction (Fig. 7b) the solution is

\[ h_{uf} = h_f - h_f \frac{\theta_f - \theta_{uf}}{\theta_f - \theta_{uf}}, \]  

(4.22)

which becomes a well-behaved solution as we restrict the permissible solutions to the desired range of \( 0 \leq h_{uf} \leq h_f \). Under the case of freshwater input (Fig. 7c) the solution is analogous to (4.21) with salinity substituted for temperature. Under the case of salt input (Fig. 7d) the solution is analogous to (4.22) again with salinity substituted for temperature.

Of the two temperature-based upper fossil layer thickness solutions (4.21) and (4.22), only one can be realized at any given instant in time as the Gaspar layer is either being heated or cooled but not both simultaneously. Similarly, only one salinity-based thickness solution is physically relevant at a given instant. This finally leaves us with one temperature-based upper fossil layer thickness estimate and one salinity-based one. Again, we choose the larger upper-fossil thickness as the one to represent the most constrained solution, whether guided by temperature or salinity extremum considerations. This then is the solution of the upper fossil layer thickness. All other dependent quantities of the upper and lower fossil layers are now determined and the algorithm is complete.

The advantage in this approach is clear in that we can always detrain while not altering the isopycnic density of our interior layers, and that we treat tropical water masses and polar water masses on an equal footing with a smooth transition between the two types because our split is on Gaspar density and not on Gaspar temperature.

d. Surface buoyancy fluxes

Because the base of the ice shelf is a thermodynamically active boundary, the waters directly in contact with it will be in a continual state of adjustment toward conditions of thermodynamic equilibrium. We use this fact to diagnose the surface heat and freshwater fluxes that are applied to the mixed layer beneath the ice shelf. The procedure is described in detail by Holland and Jenkins (1999). We use their three-equation formulation, which explicitly accounts for the differing diffusivities of heat and salt in the inner part of the ice–ocean boundary layer, the molecular sublayer. Heat conduction into the ice shelf is parameterized in a way that accounts approximately for both vertical diffusion and constant vertical advection within the ice. The melting and freezing that occur at the ice base are the physical processes responsible for this vertical advection.

Throughout the model domain, whenever we compute the response of the mixed layer to surface forcing, we always convert from potential to in situ temperature before undertaking the calculation. The altered in situ temperature is then converted back to potential temperature for use in all other model calculations, for example, pressure gradients or mixed layer entrainment/detrainment. This back and forth transformation between potential and in situ temperature, which is carried out in double-precision arithmetic, ensures the correct application of surface heat fluxes, regardless of the surface pressure, and does not result in any net drift in temperature.

e. Thermodynamic initialization

The waters throughout the cavity are first assigned properties through lateral extrapolation of values found at the ice front. If the assigned properties of the mixed layer are far removed from a condition of thermodynamic equilibrium with the ice, there will be an initial spike of high melting, which could potentially cause a numerical instability. Therefore, before taking the first model time step, we relax the temperature and salinity of the mixed layer beneath the ice shelf to a state of thermodynamic equilibrium.

We first calculate the in situ freezing point \( T_f \) based on the initial assigned temperature \( T_o \) and salinity \( S_o \) and the pressure at the ice shelf base \( p_1 \).

\[ T_f = \lambda_1 S_o + \lambda_2 + \lambda_3 p_1, \]  

(4.23)

where \( \lambda_1, \ldots, \lambda_3 \) are empirical constants (Millero 1978). The adjustments we make mimic the effects of melting or freezing on the mixed layer properties, so the temperature and salinity are constrained to evolve along a straight line in \( T/S \) space (Gade 1979) given approximately by
where \( L_f \) is the latent heat of fusion and \( c_p \) is the specific heat capacity of seawater at constant pressure. The salinity of the mixed layer waters, once they have reached the freezing point, can then be expressed in terms of the initially assigned conditions:

\[
S_f = S_a \left[ 1 - \frac{c_p}{L_f} (T_a - T_f) \right].
\]

As a final adjustment, the mixed layer temperature is recomputed with the salinity from (4.25) using (4.23). We emphasize again that the above expressions are all in terms of the in situ temperature, \( T \), rather than the potential temperature, \( \theta \).

5. Interactions with topography

a. Layer discontinuity

As a result of the horizontal discretization, surface and bottom topography acquires a stepped structure within MICOM. This raises the possibility of a layer becoming discontinuous, if it is thinner than a particular topographic step is high. In our sub–ice shelf application, this is a particularly strong possibility, because both sea surface and seabed have topography. In particular, the vertical front of the ice shelf introduces a surface step of up to several hundred meters, making it likely that the mixed layer is one of the layers containing a discontinuity. A particular example of this was earlier illustrated in Fig. 1 in the case of the mixed layer.

1) Advection

In principle, there is no difficulty in estimating the mass flow that should pass between grid points that are separated by a topographic discontinuity. The calculation of the Montgomery potential gradient is unaffected by such a discontinuity and will correctly drive flow in the direction that is energetically favorable, that is, dense layers will be forced down bottom slopes, light layers up surface slopes. However, this does not in itself ensure that mass and tracers can be advected across a discontinuity.

The cause of the difficulty arises from the fact that MICOM is discretized on an Arakawa C grid (Arakawa and Lamb 1977) in which the thickness and other scalar properties of layers are defined at grid points horizontally staggered with respect to the points at which the velocity components are defined. A similar problem would also arise if the discretization was performed on the Arakawa B grid. In order to compute the flux of mass or tracer, using (2.5) or (2.6), from one scalar point to a neighboring one, a layer thickness must be assigned to the velocity point that lies between the two neighboring scalar points. The manner in which this assignment is made in the original MICOM code and in our modified code is illustrated in Fig. 8.

Consider the specific example of a two-layer fluid overlying a continental shelf and adjacent deep ocean. The surface of the top layer has a pressure denoted \( p_1 \) and the bottom of the bottom layer has a pressure \( p_2 \), while the pressure along the mutual interface between the two layers is denoted \( p_3 \). We horizontally discretize this fluid environment by considering two “scalar” columns, labeled column \( a \) and \( c \), with column \( a \) being representative of the shallow layer of fluid on the continental shelf and \( c \) representing the deep-ocean column. A scalar column here refers to a column that contains vertically coaligned scalar variables of the Arakawa C grid; a “vector” column refers to vertically coaligned velocity points. Vector columns occur halfway between neighboring scalar columns. In Fig. 8, halfway between the columns \( a \) and \( c \) we have such a vector column, labeled \( b \). The assignment of layer thicknesses at the vector column requires that we establish the pressure \( p_2 \) at the position of the vector column. Referring to Fig. 8, we use \( X \) to mark where this pressure is assigned to occur for the original MICOM code and our modified code. In the original MICOM code, \( p_2 \) coincides exactly with \( p_3 \) at a vector column. The thickness of the bottom layer at the vector column is then zero; that is, the thickness is evaluated as \( p_1 - p_2 = 0 \). In the modified code, \( p_2 \) is assigned to occur higher up in the vector column \( b \) such that \( p_1 - p_2 \neq 0 \), and thus a nonzero thickness of the bottom layer at the vector column is prescribed.

To understand better how this problem can at all occur, it is first necessary to realize that the vector column \( b \) has a total column thickness defined by the difference of the deeper of the neighboring surface elevations at columns \( a \) and \( c \) and the shallower of the two seabed elevations at \( a \) and \( c \). Given such a thickness of the vector column \( b \) there still remains a degree of freedom in assigning the thickness of the upper and lower layer within this vector column. It is that assignment that is the key difference between the original MICOM code and our modified code.

The algorithm used in the original MICOM code assigns zero layer thickness to the bottom layer at the vector column \( b \). This is so because it uses the maximum “seafloor depth” to evaluate the layer thickness at vector points. Visually one can see this by inspecting the lower-left panel of Fig. 8 and noting that the original MICOM assigns the position of \( X \), which represents the value of \( p_2 \) at vector column \( b \), as the average height between \( p_2 \) at column \( a \) and the \( p_2 \) at column \( c \). Mass flux in the bottom layer from column \( a \) to column \( c \) would then be forced to zero because the bottom layer has zero thickness at the vector column. This would lead to the entrapment of dense water on the continental shelf. In the ice shelf–ocean context, the upside-down version of this problem would occur and buoyant water would be trapped beneath the deeper parts of the ice shelf base with no possibility of floating upward to lower elevations.
We introduce the term shelf horizon depth to mean the shallower of the bottom elevation of the two columns a and c, which in this instance occurs at column a. We assign layer thicknesses at vector column b by using the shelf horizon depth concept to produce a nonzero thickness for the bottommost layer, in a physical situation where that is an appropriate assignment. This can be seen visually in the lower-right panel of Fig. 8 where the value of \( p_3 \) and \( p_3 \) in column c is not allowed to fall below the shelf horizon depth. Our algorithm works to assign the position of the X higher up in the vector column b than in the case of the original MICOM code. Our approach is, however, similar to the original MICOM scheme in that is also ensures that the sum of all the layer thicknesses does not exceed the total water column thickness at the vector column. The distinguishing feature is that any layer with nonzero thickness in either of the neighboring scalar columns a or c is now assigned a nonzero thickness at the vector column b. The result is that gravity currents flow freely both for the case of a dense plume descending a continental slope and that of a buoyant plume ascending the base of an ice shelf.

2) DIFFUSION

While advection is allowed to proceed across a layer discontinuity, diffusive fluxes are modified according to a horizontal “line-of-sight” criterion. If a given layer abuts other isopycnic layers, the diffusivities and viscosity are simply set to zero, whereas if it abuts a solid wall (either ice shelf or seabed), scalar diffusivities are set to zero, while the standard viscosity is used. In this latter case velocity boundary conditions are applied consistent with either free slip or no slip at the solid wall.

If a layer is not completely disjointed by topography, but a partial overlap exists between neighboring scalar grid points, normal diffusive exchange is permitted but fractionally weighted according to the proportion of the layer that is in contact with the solid wall. In this manner there is a smooth transition of the diffusive fluxes if the discontinuity in a layer should gradually appear or disappear.

b. Layer thickness diffusion

As the horizontal resolution of present-day ocean general circulation models is often inadequate to resolve
the detailed structure of baroclinic eddies, a parameterization of their impact is utilized. In the isopycnic framework this is layer thickness diffusion, and is represented by the term on the right-hand side of (2.5). This term acts to flatten our isopycnals, so as to make them coincide with geopotential surfaces, and mimics the way mesoscale eddies remove potential energy from the ocean interior. In the present MICOM code, the flattening of the isopycnals is prohibited once this process would force layer interfaces to go deeper than the bedrock. In the situation of an imposed surface topography, the algorithm is modified so as to also prevent isopycnals passing upward shallower than the base of the ice shelf.

6. Interfacial friction

The horizontal momentum equations (2.8) include a stress term \( \tau_i^k \) that describes the vertical flux of horizontal momentum between neighboring layers. These viscous stresses can occur on the interface between any two adjacent layers or between a layer and an adjacent solid boundary. Such interfacial friction is commonly parameterized in ocean models using a quadratic drag law of the form

\[
\tau_i^k = \rho_c c_d |\Delta v_i^k| \Delta v_i^k, \quad (6.1)
\]

where \( c_d \) is a nondimensional and empirical drag coefficient, typically of order 10\(^{-3} \). The velocity jump across the lower + and upper − interfaces bounding layer \( k \) are defined

\[
\Delta v_i^+ = v_{k+1} - v_k \quad \text{and} \quad \Delta v_i^- = v_k - v_{k-1}. \quad (6.2)
\]

In MICOM currently only bottom friction is explicitly included. The velocity at the seabed is assumed to be zero, so the velocity jump appearing in the drag law (6.1) is simply the layer velocity. For simplicity, the drag law is linearized through the use of an effective drag coefficient \( c'_{d} \) based on the velocities from the previous time step, denoted by the “old” superscript:

\[
c'_{d} = \rho_c c_d |\Delta v_i^k|^{old}. \quad (6.3)
\]

We note that this linearization renders the computation of interfacial stress susceptible to numerical instability under certain flow conditions, but we do not offer an alternative. The next step in the MICOM algorithm is to define a viscous thickness scale \((\Delta p)_v\), typically equivalent to 10 m of water, over which the frictional stresses are actually applied. Any fluid, of whatever density, found within this distance of the seafloor is subjected to drag. The viscous stress applied to each isopycnic layer is linearly weighted by the fraction of that layer residing within the bottom \((\Delta p)_v\) of the water column. Although the velocity vector of any layer that is subject to bottom drag will turn away from that of a purely geostrophic flow, the above formulation cannot provide a general parameterization of the bottom Ekman layer, (Ekman 1905), even if there is sufficient model resolution near the seabed.

With the addition of static sea surface topography, surface friction could be introduced in a manner analogous to that described above for bottom friction. The algorithm would be more straightforward, because the mixed layer, aside from being the only nonisopycnic layer, is distinct from the other layers in that it is the only one that has a minimum specified thickness (typically equivalent to 10 m of water). As a consequence, only the mixed layer would experience the surface stress generated at the ice shelf base. For the sake of completeness, we mention that in the open ocean, away from the ice shelf base, the mixed layer experiences a surface stress due to the presence of sea ice and the atmospheric winds.

Aside from surface and bottom stresses, we also wish to consider layer-to-layer interfacial stresses in the ocean interior that arise both from small-scale mixing processes and as a result of the breaking of the internal wave field. At the interface between any two adjacent layers we impose the quadratic drag law, but with a drag coefficient that is much reduced compared with that used for the seabed or surface friction. Our general formulation of the viscous stress is then Eq. (6.1) applied to every layer interface, but with the constant interface drag coefficient, \( c_{d} \), replaced by a variable drag coefficient, \( c'_{d} \):

\[
c'_{d} = c_d \left[ e^{-|p_k-p_h|/(\Delta p_v)} + e^{-|p_h-p_v|/(\Delta p_v)} \right] + 0.001. \quad (6.4)
\]

This formula has the desired effect of making the effective drag coefficient, \( c'_{d} \), equal to the standard value, \( c_{d} \), near the ocean bottom, \( p = p_h \), and surface, \( p = p_v \), and allowing it to decay gradually to a value two orders of magnitude smaller far away from the boundaries.

For an \( N \)-layer ocean model, there are \( N \) horizontal velocities. Our algorithm for the application of interfacial stresses is to simultaneously solve (6.1) for all the layer velocities. Applying the boundary conditions of zero velocity at the seabed and ice shelf base, we arrive at a system of \( N \) nonlinear equations. We linearize the drag law by adopting an effective drag coefficient based on velocities at the previous time step, in analogy with (6.3). As the net viscous stress acting on any given layer is a function of only the velocities of that layer and of the layers directly above and below it, we then arrive at a system of \( N \) linear equations in tridiagonal form. These are conveniently solved using a standard implicit technique, such as Gaussian elimination. It is also noted that this formulation is capable of producing an “Ekman spiral,” given sufficient resolution within the physical boundary layer.

7. Model application

a. Configuration

We have applied the model to a 10\(^{\circ} \) long × 10\(^{\circ} \) lat × 1000-m-deep box with an idealized ice shelf floating
in the southern half of the domain (see Fig. 9). The central latitude, which coincides with the ice shelf front, is 75°S, so the horizontal dimensions of the domain are approximately 250 km in width east–west by 1000 km in length north–south. In thickness, the ice shelf itself is wedge shaped with the thickness increasing linearly from 200 m at the front to 700 m at the southern boundary of the domain and being zonally constant. The grid is isotropic with the grid cells having dimensions equivalent to 0.5° of longitude. This gives a horizontal resolution that ranges from approximately 10 km in the southern part of the domain and nearly 20 km in the northern. We divide the water column north of the ice shelf front into 11 layers (10 isopycnic layers plus the mixed layer) of equal thickness and extend the layer interfaces horizontally below the ice shelf. The potential temperature in all the layers is initially $-1.8{^\circ}C$, while the salinity increases in equal steps from 34.4 psu in the open ocean mixed layer to 34.8 psu in the bottom layer. Beneath the ice shelf the mixed layer first takes on the properties of the isopycnic layer it displaces, and is subsequently relaxed to the freezing point, as described in section 4e. Restoring conditions are used for the temperature and salinity along the northern wall so as to allow the model to evolve toward a steady state. Without such restoring, the only thermodynamic forcing would be that occurring at the ice shelf base and this in itself would lead to a gradual cooling of all of the waters in the domain.

b. Initial tests

The model setup described above, but with no thermodynamic initialization of the sub–ice shelf mixed layer or any subsequent thermodynamic interaction between ice and ocean, has an exact solution of zero motion for all time. Given that we know of no analytical solutions for buoyancy-driven circulation within a cavity, reproduction of such a motionless state provides us with our one chance of verifying at least some of our model code against a known solution. We show here the results of two such tests, one in which the ocean was completely uniform, with a salinity of 34.8 psu in all layers, and one in which the ocean was stratified in the manner described above.

Depth-averaged velocities at the end of a 20-yr simulation of the homogeneous ocean are shown in Fig. 10a. Peak values of the order $10^{-6}$ cm s$^{-1}$ occur along the first row of grid points below the ice shelf, while the domain average is four orders of magnitude smaller (Fig. 10b). There is a linear growth in the magnitude

![Fig. 9. Idealized box domain with a floating ice shelf in the southern half of the domain (north is to the right). The ice shelf front has a uniform thickness of 200 m while at the grounding line (i.e., where the ice shelf meets the bottom topography) the thickness extends to 700 m. There is a uniform slope between the ice shelf front and the grounding line. The model domain extends south–north over 10° of latitude, or equivalently, about 1000 km; the domain extends west–east over 10° of longitude, or equivalently, about 250 km. The maximum depth of 1000 m of the water column occurs in the open-ocean part of the domain; the minimum depth of 300 m occurs beneath the ice shelf along the grounding line.](image)

![Fig. 10. Homogeneous ocean with no ice shelf base thermodynamical forcing. (a) Depth-averaged flow velocities (cm s$^{-1}$) at the end of the 20-yr model simulation. The position of the ice shelf front is at 75°S. (b) Time series over the last 10 yr of the model integration of the domain integrated zonal and meridional depth-averaged flows.](image)
of the zonal component with time, but for any conceivable integration time this noise will remain inconsequential.

Analogous results for the stratified case are shown in Figs. 11a and 11b. In this case peak velocities of the order 1 mm s\(^{-1}\) occur along three bands in the most southerly part of the cavity. Once again domain-averaged values are four orders of magnitude smaller. While there is no growth in magnitude over the final 10 yr of the integration, and the general noise level is insignificant, the same cannot be said of the peak values. However, these velocities have a physical origin. They reflect the fact that within the mixed layer, isopycnic surfaces are no longer horizontal, but are effectively vertical. In our discretized space this manifests itself as a sharp density front in the mixed layer wherever its base intersects an isopycnic layer interface. Discretization also leads to steps in the surface topography, and where these cause discontinuities in the mixed layer the front can be preserved. The density gradient within the mixed layer then exactly balances the gradient in Montgomery potential, and the motion remains insignificant. As the horizontal grid resolution increases toward the south, the vertical steps in the topography become smaller. South of 77.2\(^{\circ}\)S the surface steps are less than 10 m, so the mixed layer is continuous in this part of the domain, and the density fronts within it cannot be maintained against diffusion. The exact balance of forces is disrupted and zonal bands of motion develop at the points where isopycnic layer interfaces intersect the mixed layer.

The above results illustrate that genuine noise in the depth-averaged velocity fields is stable at the level of 10\(^{-5}\) cm s\(^{-1}\) or less. This gives us confidence that we have implemented the surface pressure field and associated model modifications in a manner that has not caused any spurious forcing on the ocean.

c. Full model

When the stratified domain is forced by the thermodynamic interaction between the mixed layer and the ice, a circulation pattern emerges that, after 20 yr, leads to net melting of 0.5 cm yr\(^{-1}\) averaged over the ice shelf base. There is spatial variability (Fig. 12) with melting, at peak rates of around 10 cm yr\(^{-1}\), concentrated at the southern boundary and broad areas of more gentle melting and freezing to the north. The zone of freezing occupies the central and western portion of the ice shelf base with peak rates of around 0.5 cm yr\(^{-1}\) occurring in the northwestern corner. Overall, the thermodynamic interaction with the ice shelf supplies a surface freshwater flux of 22 m\(^3\) s\(^{-1}\), and the resulting thermohaline circulation drives a transport of 0.06 Sv into and out of the sub–ice shelf cavity. The circulation beneath the ice shelf is cyclonic, with inflow concentrated in the east
and outflow in the west. A vertical section of meridional velocity across the ice front (Fig. 13a) shows a level of zero motion sloping down from east to west, consistent with a picture of deep inflow, upwelling within the cavity and outflow along the ice shelf base. Peak velocities reach only a few millimeters per second. Near the ice shelf base the outflowing water has a temperature of around $2^\circ C$ (Fig. 13b) and is, thus, classed as ISW. Deeper in the water column, much of the outflow has been unaffected by the ice shelf and has retained its initial temperature.

The cyclonic gyre beneath the ice shelf shows up in the depth-mean flow (Fig. 14a). Viewed in this way, the gyre is intensified to the west and is centered just south of the ice shelf front. Although much of the depth-mean flow appears to be steered along the ice front, the step change in water column thickness does not form a complete barrier to the circulation (Fig. 14b). There are broad regions at the east and west where the velocity vectors have a significant component across the ice shelf front. This is not surprising considering that the only forcing on the circulation is the tilting of isopycnic surfaces by the freshwater flux at the ice shelf base. The forcing on an individual model layer is largely independent of that on the other layers, and a nonzero depth-mean flow emerges only because of the influence of friction and the presence of the surface topography itself. This point is well illustrated by the flow of the mixed layer (Fig. 15a), which shows no hint of the gyre, but instead illustrates behavior we would associate with a buoyant plume ascending the ice shelf base. Because the mixed layer remains thin over much of the cavity (Fig. 15b), the dominant balance of forces is between buoyancy and friction, and deflection by the Coriolis acceleration is relatively minor. The mixed layer flow is largely sustained by the entrainment of fluid along the southern boundary, where there is strong horizontal divergence, and the same occurs to a lesser extent along the eastern boundary. This pattern of entrainment gives rise to the regions of strong melting described above. As the waters of the mixed layer ascend the ice shelf
The sub-ice shelf circulation presented above is broadly similar to that described in earlier studies. The concept of a buoyant flow of ISW ascending the ice shelf base after being initiated by mixing and melting in the deepest parts of the cavity, much like the behavior of our mixed layer, was first introduced by Robin (1979). The idea was subsequently developed by MacAyeal (1984; 1985b) and formed the basis of the one-dimensional models of Jenkins (1991) and Jenkins and Bomboisch (1995). Helmer and Olbers (1989) found similar behavior in a two-dimensional, vertical plane model. With the advent of three-dimensional modeling, attention shifted away from the overturning circulation. Determann and Gerdes (1994) and Grosfeld et al. (1997) concluded that the circulation was dominated by a strong depth-averaged horizontal flow. In applications to idealized domains, they found that the depth-integrated transport formed a cyclonic gyre in the cavity, qualitatively similar to the gyre indicated by the depth-averaged flow vectors in Fig. 14a. However, the results we have presented here are quantitatively quite different to those of all the earlier studies. The rates of melting and freezing and the strength of both the horizontal and overturning circulations are all an order of magnitude smaller in our model results.

The likely cause of at least some of these differences is the differing parameterizations of vertical diffusion. In a model where melting at the ice–ocean interface is the only source of forcing, the transport of heat to the ice shelf base is of paramount importance. If vertical heat transfer is easy, melting will be rapid and the circulation will receive strong forcing. However, there is a natural brake on this system in that strong melting suppresses vertical mixing. This brake is incorporated into our model through the use of the TKE budget of the mixed layer to diagnose entrainment rates. In contrast, the three-dimensional models of Determann and Gerdes (1994) and Grosfeld et al. (1997) and the two-dimensional model of Helmer and Olbers (1989) used constant eddy diffusivities to estimate vertical fluxes, and the one-dimensional models of MacAyeal (1985b), Jenkins (1991), and Jenkins and Bomboisch (1995) all used a simple entrainment scheme that did not consider the surface freshwater flux. The impact of the negative feedback of melting on vertical heat transfer is evident in Fig. 12. Only in the vicinity of the grounding line, where entrainment of warm water is driven by horizontal divergence in the mixed layer flow, can melting in excess of 1 cm yr$^{-1}$ be sustained. Introducing an additional source of energy for mixing, such as strong tides, to the model would not necessarily lead to greater or more widespread melting. While a greater supply of TKE should increase the overall depth of the mixed layer, continuous entrainment would still only occur at those locations where it is required to maintain the thickness against thinning driven by horizontal divergence of the mixed layer flow.

Another difference between our model and the three-dimensional models of Determann and Gerdes (1994) and Grosfeld et al. (1997) is the manner in which heat and salt transfer at the ice shelf base is parameterized. The various formulations of this process that have been used in models of sub-ice shelf circulation are discussed by Holland and Jenkins (1999). That used in our model makes the rate of heat and salt transfer dependent on the friction velocity, while that used by both Determann and Gerdes (1994) and Grosfeld et al. (1997) effectively assumes a constant friction velocity. The two formulations would give equal melt rates only if the mixed layer velocity were about 35 cm s$^{-1}$ (Holland and Jenkins 1999). Our weaker flow thus gives much weaker forcing, which once again contributes to the weaker flow.
The results we have presented here differ from those of earlier three-dimensional modeling studies in another important respect. The major conclusion of both Determann and Gerdes (1994) and Grosfeld et al. (1997) was that the circulation beneath the ice shelf remains independent of that in the open ocean unless \( f/H \) contours cross the ice shelf front. That conclusion is not supported by our results. Our model forcing is confined beneath the ice shelf, but the resulting circulation extends over the entire domain and drives exchange between the cavity in the south and the open ocean to the north.

8. Closing remarks

The study of ocean circulation beneath ice shelves has, of necessity, progressed largely without the aid of the direct observational evidence normally taken for granted in oceanography. There exists a growing database of ship-based observations made along ice fronts, but these features themselves exert a strong influence on the water column. They guide a vigorous coastal current, are the sites of wind-forced upwelling and downwelling (van Heijst 1987) and possibly also of thermohaline convection (Foldvik and Kvinge 1974), and represent sharp changes in water column thickness that induce rectified tidal flows (MacAyeal 1985a; Makinson and Nicholls 1999) and may act as a barrier to some of the flow in the open ocean, such as that forced by the wind (Grosfeld et al. 1997). Using ship-based observations to study sub-ice shelf processes is like trying to understand continental shelf processes based only on data collected along the shelf break. While gross budgets and water mass transformations can be inferred, only theory, supported by a few isolated observations (Jacobs et al. 1979; Nicholls and Makinson 1998), can guide us to the important mechanisms that effect those changes. Yet here we encounter another difficulty. The most tractable theoretical problems in oceanography are those in which the ocean can be assumed to be homogeneous. Although the ocean beneath an ice shelf may appear to be weakly stratified, spatial variations in the stratification are what drives the flow. The assumption of a homogeneous water column yields the null result discussed in section 7b.

Starting with the work of Determann and Gerdes (1994), recent efforts to understand processes beneath ice shelves have involved the application of suitably modified ocean general circulation models to the problem. The model presented in this paper is the first to employ isopycnic coordinates. Such a choice of vertical coordinate is natural for the density-driven flows of interest in this study. It ensures optimum resolution of the water column and allows the introduction of entirely arbitrary topography while avoiding the problems encountered in geopotential coordinates with up- or down-slope flows or the difficulties encountered in terrain-following coordinates with estimating pressure gradients. Precise control over diapycnic diffusion and the exact conservation of potential vorticity are also more easily implemented in isopycnic coordinates. The one drawback is the requirement for a nonisopycnic layer, which can admit buoyancy forcing, at the top of the water column. Here some of the advantages outlined above are lost, and the use of an embedded mixed layer model adds complexity, particularly when dealing with the surface topography imposed by an ice shelf. In the early parts of this paper we presented a strategy to overcome these complications.

The idealized runs we presented in section 7 have most in common with the simulations discussed by Grosfeld et al. (1997). Their setup differed slightly in that they imposed a wind field on the open ocean, and their initial conditions were an unstratified ocean with a temperature slightly colder than the one we have used. There are qualitative similarities between the results, namely, the cyclonic flow beneath the ice shelf and the general pattern of melting and freezing, but also distinct differences, particularly in the strength of the circulation. A cursory comparison with observation may suggest that the stronger circulation is closer to reality, but in nature there is considerable extra forcing on the system. For example, the growth and decay of sea ice drives a seasonal cycle of mixing and restratification in the water column to the north of the ice front, and the impact of this on flow beneath an ice shelf has been documented (Nicholls and Makinson 1998). Any process that excites motion in the water could promote more rapid vertical heat transfer, which would provide further forcing on the flow. Thus, the weak response to our idealized forcing may be correct.

If the differences between our results and those of the earlier three-dimensional modeling studies lie in the different parameterizations of vertical heat transfer, we may conclude that a good representation of this process is essential for a realistic simulation of sub-ice shelf circulation. Our other principal conclusion is that the ice front presents less of a dynamic barrier to the buoyancy-driven circulation than has been suggested by the earlier studies. These differences highlight the need for thorough testing and intercomparison of a range of numerical models before more definitive statements can be made about the key processes controlling the circulation beneath ice shelves. We hope that the model we have presented here, both in its current form and with future developments, will prove a useful tool in advancing our currently limited understanding of these processes.

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