Evaluation of a Multigrid Barotropic Tropical Cyclone Track Model

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ABSTRACT

The performance of a multigrid barotropic tropical cyclone track model (MUDBAR) is compared to that of a current operational barotropic model (LBAR). Analysis of track forecast errors for the 2001 Atlantic hurricane season shows that MUDBAR gives accuracy similar to LBAR with substantially lower computational cost. Despite the use of a barotropic model, the MUDBAR forecasts show skill relative to climatology and persistence (CLIPER) out to 5 days.

1. Introduction

Tropical cyclones are complex systems involving dynamics on many scales, complicated physics, and multifarious interactions with ocean, land, and the surrounding environment. To include a domain large enough to predict the storm environment, yet still resolve the fine-scales of motion near the eyewall, tropical cyclone models often employ variable resolution grids (e.g., Kurihara et al. 1998).

Although tropical cyclone modeling involves a wide range of scales, a reasonably accurate track forecast can be obtained without including all of the details of the inner core of the storm. For example, models based on simplified barotropic dynamics can provide useful forecasts of tropical cyclone motion. Since they are computationally cheap to run, the National Hurricane Center’s (NHC) suite of operational guidance products includes barotropic models. Inherently faster than three-dimensional full-physics models, efficient barotropic models open the door to forecasting techniques that utilize very large ensembles.

The first operational barotropic tropical cyclone track models were developed during the late 1950s and early 1960s (Tracy 1966). Due to limited computer power, these models attempted to separate the prediction of the storm environment from the smaller-scale vortex circulation. Because of these limitations, early barotropic models tended to have larger forecast errors than the simpler statistical track models available at that time.

A more successful operational barotropic tropical cyclone model (SANBAR) was developed in the late 1960s (Sanders and Burpee 1968; Sanders et al. 1975, 1980). The SANBAR model was run as part of the NHC operational suite until 1989 when it was replaced by VICBAR. The VICBAR model (DeMaria et al. 1992) introduced an improved discretization scheme, based on B splines and implemented with nesting. The LBAR model1 (Horsfall et al. 1997), with harmonic-sine basis and without nesting, was later developed as a simpler and more portable operational alternative to VICBAR. Some characteristics of these models are given in Table 1.

Recently a new multigrid barotropic model (MUDBAR) has been developed (Fulton 2001). Based on the simple modified barotropic vorticity equation, MUDBAR uses an adaptive multigrid method to refine the

1 Originally the Limited-Area Sine Transform Barotropic model (LASTBAR).
mesh around the moving vortex, with the goal of maximizing accuracy while minimizing computational cost. MUDBAR can also be run in a mode that uses fixed-size movable meshes. This model was developed primarily as a test bed for adaptive multigrid techniques and was previously evaluated using idealized initial conditions. However, MUDBAR’s speed and accuracy make it a useful alternative to existing operational barotropic models. To compare MUDBAR to operational barotropic models, the capability to include initial conditions from real data using a procedure similar to that of the LBAR model was implemented.

The purpose of this paper is to evaluate the accuracy, skill, and efficiency of the MUDBAR model by comparing it to the current operational barotropic model LBAR. Section 2 briefly reviews the LBAR and MUDBAR models and section 3 describes the data used for the comparison. Results for the 2001 Atlantic hurricane season are presented in section 4, and our conclusions are summarized in section 5.

2. Model descriptions

Both LBAR and MUDBAR are developed on a section of the sphere, transforming longitude \( \lambda \) and latitude \( \phi \) to Cartesian coordinates \( x \) and \( y \) via the Mercator projection

\[
x = (\lambda - \lambda_0)a \cos \phi, \\
y = [\tanh^{-1}(\sin \phi) - \tanh^{-1}(\sin \phi_0)]a \cos \phi, \tag{1}
\]

where \( a \) is the radius of the earth. The projection is true at \((\lambda_0, \phi_0)\), where \((x, y) = (0, 0)\), which is taken as the center of the model domain. The models differ in the equations and discretizations used as described below.

a. LBAR

The LBAR model was briefly described in Horsfall et al. (1997). Based on the divergent barotropic (shallow water) equations with a mean fluid depth of 800 m, the model uses a harmonic-sine series representation and a Galerkin projection. Second- and fourth-order diffusion terms are included in the prediction equations. To accommodate nonperiodic boundary conditions, the dependent variables (horizontal wind components and geopotential height) are divided into boundary and interior functions using the technique described by Chen and Kuo (1992). The boundary function is the solution to Laplace’s equation with inhomogeneous boundary conditions; the interior function satisfies homogeneous boundary conditions and thus can be expanded in a double sine series. Boundary conditions for the forecasts are obtained from the Aviation model run (AVN) of the National Centers for Environmental Prediction (NCEP) Medium-Range Forecast Model (MRF).\(^2\) The sine series include 48 terms on a 7200-km square domain. Nonlinear terms are evaluated using the transform method, with 96 transform grid points in \( x \) and \( y \). The model uses centered time differencing with a time step of 180 s. Forward differencing is used for the diffusion terms.

The initialization procedure for LBAR is the same as for VICBAR (DeMaria et al. 1992). Large-scale wind and height fields are determined by vertically averaging (850–200 hPa) the AVN analysis fields. Heights from the global model are calculated as deviations from the heights in the U.S. standard atmosphere. The tropical cyclone is represented by the sum of an idealized vortex and an initial storm motion vector, both chosen to closely approximate the observed storm. The large-scale and tropical cyclone wind fields are blended as described in section 3 to obtain the initial wind field for the model. The corresponding height field is calculated by solving the nonlinear balance equation, using the analysis heights at the boundaries.

To increase the influence of the boundary conditions, the predictive equations for wind and height contain nudging terms that damp the difference between the LBAR-predicted fields and the corresponding fields from the AVN model run. This nudging is applied in a boundary strip of width \( s_0 = 3000 \) km with amplitude proportional to \((1 - s/s_0)^4\), where \( s \) is the distance to the nearest boundary. The nudging coefficient is chosen so that the difference between the LBAR and AVN model fields has an \( e \)-folding time of 2 h at the boundaries.

b. MUDBAR

The MUDBAR model is based on the modified barotropic vorticity equation

\[
\frac{\partial q}{\partial t} + m^2 \frac{\partial (\psi, q)}{\partial (x, y)} + \beta m \frac{\partial \psi}{\partial x} = 0, \tag{2}
\]

where \( \beta = 2\Omega a^{-1} \cos \phi \) (with \( \Omega \) the rotation rate of the earth) and \( m = \cos \phi_0/cos \phi \) is the map factor. The

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\(^2\) As of 2002, the NCEP’s global model is called the Global Forecasting System. The MRF runs have been replaced by four daily AVN runs.
streamfunction $\psi$ and potential vorticity $q$ are related via the elliptic problem
\[
(m^2\nabla^2 - \gamma^2)\psi = q,
\]
where $\gamma$ is the inverse of the effective Rossby radius of deformation. The model runs reported here use $\gamma = f_0/c_0$, where $f_0$ is the Coriolis parameter at the reference latitude $\phi_0$, and $c_0$ is a specified phase speed, here chosen to be 90 m s$^{-1}$ to match the gravity wave phase speed for the 800-m shallow-water depth in LBAR. Data enter the model through the initial condition (specify $q$) and boundary conditions (specify $\psi$ on the boundary and $q$ on inflow), using the same wind field as described above for LBAR.

The equations are discretized using conservative second-order centered finite differences in space (Arakawa Jacobian) and the fourth-order Runge–Kutta scheme in time, using an adaptive multigrid method that refines the mesh around the moving vortex. The model can be run in a fully adaptive mode with fine-grid patches being created, moved, resized, and destroyed automatically as dictated by the estimated truncation error of the evolving solution (Fulton 2001). For track forecasts, the fully adaptive mode did not offer any efficiency advantages over fixed-size grid configurations, so in this paper we use several nested grid patches of fixed sizes that move with the vortex. Likewise, the model can be run with fourth-order space differencing, but as this did not improve the efficiency of track forecasts (for the same accuracy), here we use only the second-order version.

c. Comparison

The principal differences between LBAR and MUDBAR are the dynamics (divergent vs nondivergent) and discretization (spectral vs adaptive multigrid). Beyond this, we adjusted the parameters of the MUDBAR model to make it as similar to LBAR as possible. The model domain for LBAR is a 7200-km square, but the solution is nudged toward the specified environmental flow in the outer portion of the domain. As described above, the magnitude of the boundary nudging term decreases rapidly with the distance from the boundary. Based upon the behavior of the LBAR nudging term, a domain size of a 6000-km square was chosen for MUDBAR. Choosing this domain for MUDBAR gives a domain size approximately 54° longitude by 49° latitude (depending somewhat on the reference latitude $\phi_0$). Both models are run with a fixed domain centered on the initial vortex position, and both define the storm track using the location of the maximum vorticity.

3. Model initialization and boundary conditions

The 0000 UTC NCEP global model analysis and 5-day forecast fields were collected for most of the 2001 Atlantic hurricane season as part of a separate study of ensemble forecast methods. These same fields were used to compare the LBAR and MUDBAR models. These data were available for all named 2001 Atlantic tropical cyclones except Tropical Storm Allison. Figure 1 shows the best tracks for the storms included in the comparison. Unnamed depressions, subtropical, and extratropical cases were excluded from the verification statistics. The active 2001 season provided a variety of storms across the full range of intensities and latitudes typically experienced in the Atlantic basin, making 2001 a good year in which to conduct a robust model comparison. The forecast sample includes 88 cases with at least a 12-h verification.

It should be pointed out that the global model fields used in this study are not exactly the same as those used in the operational LBAR model. This study uses fields from the control member of the NCEP Global Forecasting System (GFS) ensemble, which in 2001 was a T126 run of the MRF model (truncated to T62 at 84 h and thereafter). The operational LBAR model uses fields from a 6-h-old AVN forecast, which had a T170 truncation in 2001. The LBAR model updates the boundaries at 6-h intervals, while the GFS ensemble control data are available only at 12-h intervals. In addition, there are some differences in the data cutoff times and initialization procedures in the GFS ensemble control and the AVN. Perhaps the most significant difference is that in 2001, the AVN used the vortex relocation scheme of Liu et al. (2000), while the GFS ensemble control did not. By properly relocating the model’s analyzed vortex to the operational position estimate, the relocation scheme significantly reduces binary interactions between the synthetic and analyzed vortices, resulting in substantial reduction in forecast track errors (Liu et al. 2002). To simplify the comparison between LBAR and MUDBAR, the LBAR model was rerun using the same GFS ensemble control fields that were used in MUDBAR. To distinguish the operational LBAR from the reruns, the version that uses the NCEP GFS ensemble control fields will be referred to as NBAR.

The models are initialized using the velocity field
\[
v_0 = (1 - w)v_{anal} + w(v_{vor} + v_{cen}),
\]
where $v_{anal}$ is the analyzed velocity field (from the GFS ensemble control member), $v_{vor}$ is the velocity field of the specified (synthetic) vortex, and $v_{cen}$ is the specified initial storm motion vector. The weighting function $w$ smoothly blends the synthetic vortex into the velocity field; we use $w(r) = \exp[-(r/r_b)^2]$, where $r$ is the distance from the vortex center and $r_b$ is the blending radius (here chosen to be 1000 km). For LBAR this velocity is used directly, with the corresponding height field obtained by solving the nonlinear balance equation. For MUDBAR the initial value of $q$ is $\zeta_0 - \gamma^2 \psi_0$, where $\zeta_0 = k \cdot \nabla \times v_0$ is the initial relative vorticity and $\psi_0$ is obtained by solving $m^2 \nabla^2 \psi_0 = \zeta_0$.

For $v_{vor}$ we use the symmetric vortex with tangential wind given by
Fig. 1. Tracks of the 14 tropical cyclones from the 2001 Atlantic hurricane season included in the model comparison. Numbers in squares identify the storm, and filled and open circles give the storm position at 0000 and 1200 UTC, respectively, on the indicated day. Extratropical and subtropical track segments are not shown.

\[ V(r) = \frac{r V_m}{r_m} \exp \left[ \frac{1}{b} \left( 1 - \left( \frac{r}{r_m} \right)^2 \right) \right], \]  

with maximum wind \( V_m \) at radius \( r_m \), and where \( b \) is a size parameter. This profile has been used by DeMaria (1987), Chan and Williams (1987), Fiorino and Elsberry (1989), and DeMaria et al. (1992). The three parameters of this vortex are chosen to approximately match the intensity and size of the observed storm. The initial storm motion vector \( \mathbf{v}_{\text{cen}} \) is taken from the operational NHC estimate.

For boundary data, the GFS ensemble control forecasts are spatially interpolated and vertically averaged as described above to obtain time-dependent specified geopotential and velocity fields at 12-h intervals out to 120 h. Both models use time-dependent boundary values, interpolating in time as needed. NBAR uses the GFS ensemble control fields for lateral boundary conditions on the velocity and height, and for the nudging term described previously. MUDBAR constructs boundary values of streamfunction by integrating \( \mathbf{v} = \mathbf{m} k \times \nabla \psi \) around the boundary, and uses the corresponding vorticity where there is inflow.

To illustrate the domain, grid sizes, and initial data, Fig. 2 shows a typical initial streamfunction field for MUDBAR. This case is for Hurricane Michelle at 0000
Table 2. Summary of specific model configurations used in error and skill comparisons. All configurations are operational except the first two.

<table>
<thead>
<tr>
<th>Designator</th>
<th>Model</th>
<th>Model type</th>
<th>Domain</th>
<th>Nested?</th>
<th>Boundary data</th>
</tr>
</thead>
<tbody>
<tr>
<td>MBAR</td>
<td>MUDBAR (optimal grid configuration)</td>
<td>Nondivergent barotropic</td>
<td>Limited area</td>
<td>Yes</td>
<td>GFS ensemble (control run)</td>
</tr>
<tr>
<td>NBAR</td>
<td>LBAR reruns</td>
<td>Divergent barotropic</td>
<td>Limited area</td>
<td>No</td>
<td>GFS ensemble (control run)</td>
</tr>
<tr>
<td>LBAR</td>
<td>LBAR</td>
<td>Divergent barotropic</td>
<td>Limited area</td>
<td>No</td>
<td>GFS (AVN run)</td>
</tr>
<tr>
<td>GFDL</td>
<td>GFDL</td>
<td>Baroclinic w/physics</td>
<td>Limited area</td>
<td>Yes</td>
<td>GFS (AVN run)</td>
</tr>
<tr>
<td>NGPS</td>
<td>NOGAPS</td>
<td>Baroclinic w/physics</td>
<td>Global</td>
<td>No</td>
<td>—</td>
</tr>
<tr>
<td>AVN0</td>
<td>GFS (AVN run)</td>
<td>Baroclinic w/physics</td>
<td>Global</td>
<td>No</td>
<td>—</td>
</tr>
<tr>
<td>A98E</td>
<td>—</td>
<td>Statistical-dynamical</td>
<td>—</td>
<td>—</td>
<td>Dynamical predictors from GFS (AVN run)</td>
</tr>
</tbody>
</table>

UTC on 4 November 2001, then a category-four storm (on the Saffir–Simpson hurricane scale) centered just south of the western end of Cuba. The details of the selection of the MUDBAR grids are described in the next section.

4. Results

In this section, a version of MUDBAR will be developed that provides a reasonable balance between computational cost and forecast accuracy. The forecast accuracy will be determined by defining the track error as the great circle distance between the storm position in the model and the best track position from the NHC best track data (Jarvinen et al. 1984). Once the final version of MUDBAR is chosen, the forecast results will be compared with those from the operational LBAR model, and the LBAR model run using the same fields as MUDBAR. The MUDBAR results will also be compared with some of the other operational track models used by NHC. The various models compared in this section are summarized in Table 2.

A statistical $t$-test will be conducted following the method of Franklin and DeMaria (1992) to determine whether model differences are significant. Sample sizes are adjusted to account for serial correlation. The level of statistical significance is taken to be 95%.

a. Optimization of MUDBAR

The MUDBAR model can be run with any number of nested grid patches of virtually any size. The best configuration of MUDBAR for the barotropic track forecast problem is the one that gives the most accurate forecasts for the least computational cost. To determine the best version of MUDBAR, the 2001 forecast cases were run with a single grid, two grids, and three grids. Each of these grid systems were run with a range of horizontal resolutions. The computational cost was measured as the central processing unit (CPU) time required for a single forecast case on a 1-GHz personal computer (PC). The accuracy was measured by the mean track error (at times 12, 24, 36, and 48 h) over all cases for the 2001 season.

Figure 3 shows the accuracy obtained using various numbers and sizes of grids as a function of the cost required. Each single-grid configuration is indicated by an $\times$, each two-grid configuration by a small dot, and each three-grid configuration by a triangle. These results show that mesh refinement is especially advantageous in the short term (e.g., at 12 h), when accurately capturing the initial position and motion is critical; at later times the advantage is reduced, as other sources of error (uncertainties in the environmental flow, lack of baroclinic effects and physics, etc.) begin to dominate. The optimal grid configuration selected for subsequent runs (indicated by the large solid dot) consists of three grids of size $32 \times 32$ grid intervals each, with mesh sizes approximately 174, 87, and 43.5 km. In this and subsequent results we refer to this configuration of the model as MBAR, which is considered the best choice based upon computation cost and forecast accuracy.

b. Comparison with LBAR and NBAR

Figure 4 shows the average track forecast error from the 2001 sample as a function of forecast time for MBAR, NBAR, and LBAR. Also included is the 5-day climatology and persistence (CLIPER) model of Aber- son (1998), since it is often used as a benchmark for the evaluation of forecast skill. If a model has average track forecast errors smaller than CLIPER, it is considered to be skillful. MBAR and LBAR showed statistically significant improvement over CLIPER at all time periods, while NBAR showed significant improvement only out to 48 h.

Figure 4 shows that the average MBAR errors are comparable to those from NBAR out to about 48 h and are somewhat smaller after that time. The differences between MBAR and NBAR were only statistically sig-
significant at 72 h. This result indicates that MBAR is able to reproduce or slightly improve upon the operational barotropic forecast model for the case when both use the same initial and boundary condition information. The differences in the errors between NBAR and MBAR can be attributed to the differences between the two modeling systems (divergent barotropic versus nondivergent barotropic, treatment of boundary conditions, numerical methods, etc). Figure 4 does not show the computational cost of the runs. Each 5-day NBAR run took an average of 99.2 s, while each MBAR run took only 1.4 s. Thus, the LBAR modeling system could be replaced by a system with about 1/70 the computational cost.

Figure 4 also shows the results for LBAR that were run in real time. The LBAR errors in Fig. 4 are smaller than those for NBAR after 48 h, and comparable to those from MBAR. The differences between LBAR and MBAR were not statistically significant at any time period, but LBAR showed statistically significant improvement over NBAR at 36 h and beyond. Since these forecasts use the exact same modeling system, the differences between LBAR and NBAR must be due to the differences in the initial and boundary conditions. As described previously, LBAR uses a 6-h-old AVN forecast at 6-h intervals for the initial and boundary conditions. Apparently, the use of a 6-h-old model run does not degrade the forecasts. This result is consistent with those shown by Horsfall et al. (1997). The AVN is also run at higher resolution than the GFS ensemble control and makes use of the vortex relocation scheme, which is not included in the fields used to initialize NBAR and MBAR. All of these factors probably contributed to the increased track errors of NBAR compared to LBAR.

c. Model skill

All three barotropic models in Fig. 4 had errors smaller than CLIPER. This result indicates that, despite the use of the very simple barotropic framework, the forecasts were skillful. As a further evaluation of the barotropic modeling system, Fig. 5 compares the skill of LBAR and MBAR to some of the other NHC operational models. In this comparison, skill is evaluated by deter-
mining the error of each model relative to that of the CLIPER model for a homogeneous sample of cases. If the relative error of a particular model is negative (the errors are less than CLIPER), then the model is considered to be skillful. The sample sizes in Fig. 5 are smaller than those in Fig. 4 because not all of the operational models ran at every forecast period; a homogeneous comparison excludes such cases, reducing the sample size.

Figure 5 includes a representative of each basic type of model used at NHC. The A98E model is a statistical-dynamical prediction model that uses geopotential heights from the AVN forecast to modify a CLIPER track forecast. The Geophysical Fluid Dynamics Laboratory (GFDL) model is a limited-area, nested baroclinic model that obtains boundary conditions from the AVN. AVN0 is the track forecast obtained by tracking the representation of the storm in the global AVN forecast fields, and NGPS is a similar forecast determined from the Navy Operational Global Atmospheric Prediction System (NOGAPS). This figure shows that all of the models were skillful out to 120 h, except A98E after 84 h. There was little difference between the skill of the LBAR, MBAR, NGPS, and GFDL models, but the AVN0 model appeared to have the most skill after about 24 h. The AVN0 showed statistically significant improvements over all other models at all forecast times except at 12 and 108 h. At the 72-h forecast time, all models except A98E showed statistically significant improvement over CLIPER. At 120 h, the only models showing improvement over CLIPER were the AVN0, GFDL, LBAR, and MBAR models. This result shows that although the barotropic model is very simple, it can still sometimes produce skillful track forecasts that are competitive with some of the more general models such as the GFDL model and NOGAPS.

5. Conclusions

We conclude that the adaptive multigrid tropical cyclone track model MUDBAR can achieve accuracy similar to the operational shallow-water model LBAR with far less computational work (approximately a factor of 70). Both models show skill (relative to CLIPER) out to 5 days. The fact that MUDBAR can produce an accurate and skillful forecast in almost negligible computer time (about 1.4 s for a 5-day forecast on a modest PC) makes it a promising tool for use in an ensemble forecast scheme employing a large number of members perturbed in a multidimensional parameter phase space. Such a scheme has been developed (Vigh 2002) and is currently being tested.

The great improvement in computational cost of MBAR relative to LBAR illustrates the utility of the multigrid approach in a simple framework. In this barotropic context there is little need for fully adaptive grids (at least for tropical cyclone track forecasting), so the version of MBAR discussed here simply used fixed-size grids. It is left as a topic for future research to determine how much improvement in computational cost might be obtained by applying fully adaptive multigrid methods to a more general primitive equation hurricane modeling system.

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