The Characteristics of Key Analysis Errors. Part III: A Diagnosis of Their Evolution

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ABSTRACT

This paper presents a diagnostic study of the evolution of initial corrections obtained from the key analysis error algorithm that minimizes the short-range (24 h) forecast errors for four specific events poorly forecasted over the eastern part of North America. A potential vorticity (PV) perspective is employed. It is shown that the modification to the low-level structure at the initial time is mainly attributed to the modification of the low-level PV distribution, while changes in the upper-level structure are attributed to the modification of the upper-level PV distribution. The low-level corrections grow mainly through background surface potential temperature advection by the wind corrections attributable to the interior PV corrections. Changes in the diabatic processes and the vertical alignment of low-level PV corrections by differential PV advection also increase the magnitude of the low-level corrections with time. The upper-level corrections grow by advection of background PV from wind corrections. However, the cause of these latter wind corrections responsible for upper-level background PV advection varies from case to case. An investigation of the relative importance of the low-level and of the upper-level initial corrections to produce the final-time corrections also reveals strong variability between cases. Finally, comparison of two cases in which the key analysis errors propagate vertically with two others without significant vertical propagation shows how the relative position of the key analysis errors with respect to the structure of the background flow can influence the evolution of the initial corrections.

1. Introduction

The development of tangent linear model (TLM) and its adjoint in numerical weather prediction makes it possible to calculate corrections to the initial conditions that improve the accuracy of short- to medium-range forecasts (e.g., Klinker et al. 1998; Pu et al. 1997; Gelaro et al. 1998). One such technique implemented at the Canadian Meteorological Centre (CMC; see Laroche et al. 2002) is the key analysis error algorithm (Klinker et al. 1998). Until recently, it was thought that the initial corrections obtained using this method could be representative of some part of the analysis error. However, in the first two parts of this series, Caron et al. (2007a,b, hereafter Part I and Part II) found that both the rotational and the divergent components of the initial corrections for four specific CMC operational analyses are strongly out of balance. Furthermore, the corrected analysis is systematically further away from the observations than the control analysis even when the balanced component of the initial corrections is isolated. Thus the initial corrections from the key analysis error algorithm cannot justifiably be associated with analysis errors, in agreement with the results of Isaksen et al. (2005).

Although the key analysis errors improve the forecast even though they do not bring the initial analysis closer to the observations, it is of interest to understand dynamically/physically how the initial corrections can change significantly the forecast trajectory in a relatively short period of time. While little is known about the answer to this question for key analysis errors, the mechanism responsible for the growth of initial perturbations...
bations in the context of singular vectors\(^1\) (SVs) has been studied quite extensively, mainly from a potential vorticity (PV) perspective employing both simple and primitive equation models. Similar to key analysis errors, the dominant SVs exhibit an alternating positive and negative PV-filament structure with a strong upstream tilt in the vertical. The PV perturbation can amplify and the tilt can become vertically aligned with time as a result of the shear of the background flow and the accompanying differential advection. The upright alignment of the alternating positive and negative PV filaments also amplifies the depth of the wind perturbation and thus the kinetic energy perturbation, in a process termed unshielding (Badger and Hoskins 2001; Montani and Thorpe 2002; Morgan 2001).

In view of our lack of knowledge about key analysis errors, the goal of this paper is to examine how, dynamically/physically, the initial corrections obtained from the key analysis error algorithm modify the short-range forecast and why these corrections grow rapidly in four synoptic-scale events presented in Part I. We will examine especially the relative importance of the upper-level and low-level initial corrections. To our knowledge, this is the first detailed diagnostic study about the evolution of the initial corrections computed from the key analysis error algorithm. We anticipate that the growth mechanism for SVs and key analysis errors share many similarities, because there is a direct relationship between initial corrections from the key analysis error algorithm and the 30 dominant SVs (Gelaro et al. 1998).

Because the initial corrections from the current key analysis error algorithm cannot justifiably be directly associated with analysis errors, the findings on the evolution of the key analysis errors presented in this paper cannot be linked to the dynamics of the analysis errors. However, our results do add relevant dynamical/physical information about the evolution of small- and fast-growing initial perturbations. We also remark that the key analysis error algorithm employed in this paper uses the energy norm at both the initial and final times, but this is not a necessary requirement. The energy norm was adopted because of its simplicity. Other studies have shown that the structure of the key analysis errors is very sensitive to the definition of the initial-time norm (Isaksen et al. 2005; Klinker et al. 1998). However, the diagnosis of the evolution of key analysis errors calculated with other initial-time norms is beyond the scope of this paper.

The organization of this paper is as follows. In the next section, we first describe a representative case occurring on 27 January 2003, presented also in Part I. Section 3 then summarizes the diagnostic approach adopted in this work. Section 4 contains results for the representative case. Three other cases (see also Part I) are presented in section 5. Comparison of the four cases will then be made to assess the variability between the cases. An investigation of the reasons behind some notable differences in the vertical propagation of the initial corrections revealed in each case forms the subject of section 6. Finally, section 7 contains the summary and conclusions.

2. Case study of 27 January 2003

The case to be studied in detail occurred on 27 January 2003. The 24-h forecast of a low pressure system over the Canadian Maritimes using the CMC global forecasting system (Côté et al. 1998; Gauthier et al. 1999) was relatively inaccurate. To perform our analysis, we first interpolated all data (analysis and forecasts), originally on model vertical levels and a global latitude–longitude grid, onto the pressure levels of a regional latitude–longitude grid. The interpolated data were at every 50 hPa, from 1000 to 100 hPa. The regional grid covers North America. It has a horizontal resolution of 0.9°, the same as the global forecast model.

The distribution of mean sea level pressure in the CMC analysis at 1200 UTC 27 January 2003 (Fig. 1a) shows two low pressure centers near the east coast of North America: a weakening low pressure area over Prince Edward Island (1010 hPa) and a nascent low pressure area well off the coast of Virginia (1012 hPa). The latter is the weather system of interest. At the upper level, the 350-hPa geopotential height (Fig. 1b) indicates a sharp long-wave trough along the east coast, upstream of the low pressure areas. At 1200 UTC 28 January 2003, the central pressure in the nascent cyclone has deepened by 19 hPa over the past 24 h and a well-defined cyclone is now located southeast of Cape Breton (Fig. 2c). During the same period, the upper-level trough at 350 hPa swings eastward without significant change in intensity. It becomes situated over the western edge of the Canadian Maritime Provinces (Fig. 2d).

The 24-h forecast initialized with the 1200 UTC 27 January 2003 CMC analysis shows significant departure from reality (i.e., the reference analysis). The most notable error lies in vicinity of the surface system, the

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\(^1\) SVs represent the fastest-growing linear perturbations that maximize a given norm (e.g., total energy) over a given time interval, regardless of whether the forecast is improved or degraded.
forecast position of which was placed too far north by ~600 km (cf. Figs. 2a and 2c; see also Fig. 12e from Part I of this series for forecast error field). Significant errors also occur at the upper levels. The 350-hPa trough was placed too far to the northeast by ~200 km (cf. Figs. 2b and 2d).

The CMC key analysis error algorithm (see Part I for details) was then used to compute the corrections to the operational analysis that minimized the 24-h forecast errors with respect to the total energy norm over an area delimited by the squares shown in Figs. 2c and 2d. As demonstrated by Part I, the initial key analysis error corrections are unbalanced with respect to the mass and wind in the synoptic-scale CMC three-dimensional variational data assimilation analysis. However, using nonlinear balance PV inversion, it is possible to balance the posteriori the initial corrections while preserving the forecast improvements (Part II). Because the latter set of initial corrections are dynamically consistent, we therefore used the a posteriori balance initial corrections to perturb the operational forecast for this case and the three other cases presented in section 5.

When a forecast is started from the key analysis error corrected analysis, both the surface and upper-level features are significantly closer to the reference analysis after 24 h (cf. Figs. 2e, 2f, 2a, and 2b). The surface cyclone is now located over Cape Breton, much farther south as compared with the operational forecast (Fig. 2c) and is only ~150 km to the north-northwest of the location of the center in the reference analysis (Fig. 2e versus Fig. 2c). In addition to the improvement in the position, the minimum mean sea level pressure is now exactly the same (993 hPa) as in the reference analysis (Fig. 2e versus Fig. 2c). The position of the upper-level trough in the sensitivity forecast is shifted southwestward compared with the operational forecast (Fig. 2f versus Fig. 2b) and coincides with the location in the reference analysis (Fig. 2f versus Fig. 2d).

3. The diagnostic approach

To investigate how the initial corrections from the key analysis error algorithm modify the forecast with time, we adopt, similarly to Part II, the PV perspective that is based on the conservation of PV and its invertibility property.

Ertel’s (1942) PV approximated in isobaric coordinate is defined as

$$q = -g \left[ (\zeta + f) \frac{\partial \theta}{\partial p} - \frac{\partial u \partial \theta}{\partial x} - \frac{\partial u \partial \theta}{\partial y} \right],$$

where $u$ and $v$ represent, respectively, the zonal and the meridional wind components, $\theta$ is the potential temperature, $p$ is pressure, $f$ is the

![Fig. 1. (left) Mean sea level pressure (contour interval 4 hPa) and (right) 350-hPa geopotential height (contour interval 6 dam) valid at 1200 UTC 27 Jan 2003 from the CMC analysis.](image-url)
Fig. 2. (left) Mean sea level pressure (contour interval 4 hPa) and (right) 350-hPa geopotential height (contour interval 6 dam) valid at 1200 UTC 28 Jan 2003. (c), (d) CMC analysis. (a), (b) Global Environmental Multiscale (GEM) model operational and (e), (f) sensitivity forecast initialized at 1200 UTC 27 Jan 2003. The box in (c) and (d) indicates the area for which the 24-h forecast error was minimized.
Coriolis parameter, and \( g \) is the magnitude of the acceleration of gravity. The evolution equation for PV is

\[
\frac{dq}{dt} = S,
\]

(2)

where \( S \) represents the source and sink terms such as latent heating and frictional effects. If \( q \) is divided into the basic state and the perturbation terms (\( q = \overline{q} + q' \)), and if nonlinear terms are neglected, the evolution equation for PV perturbations can be written as

\[
\frac{dq'}{dt} = -\mathbf{V}' \cdot \nabla \overline{q} + S',
\]

(3)

where \( \mathbf{V}' \) and \( \nabla \) represent the three-dimensional wind perturbations and gradient operator, respectively, and \( \frac{dq}{dt} = \frac{d\overline{q}}{dt} + \nabla \cdot \mathbf{V} \). This expression is identical to Eq. (7) in Montani and Thorpe (2002) except that \( S \) is allowed to be influenced by the initial corrections (i.e., \( S' \neq 0 \)).

To invert the PV perturbations, we also need the potential temperature perturbation at the top and bottom boundaries. By applying the same procedure as in (3), the evolution equation for potential temperature perturbation (\( \theta' \)) becomes

\[
\frac{d\theta'}{dt} = -\mathbf{V}' \cdot \nabla \overline{q} + Q',
\]

(4)

where \( Q' \) represent perturbations to diabatic processes such as sensible heat flux and latent heating. Equation (4) will be used to diagnose the growth of the potential temperature corrections at the top and bottom boundaries.

Equations (3) and (4) state that the amplitudes of the PV or potential temperature perturbations can change if there is advection of background PV or background potential temperature by the wind perturbations. Therefore the background PV or background potential temperature can be considered to act as a source. The nonconservative processes also impact the growth of the perturbations.

The invertibility principle (Hoskins et al. 1985) states that for a given distribution of PV, one can determine all the atmospheric state variables (except humidity) from an appropriate balance relationship between mass and wind. The principle also holds when the PV distribution is written as the sum of discrete PV perturbations. By inverting each PV perturbation to obtain the state variables, one can determine which PV perturbation acts to amplify the initial corrections through (3) and (4). It is also possible to diagnose developments in which the PV perturbations are not growing but only change in relative positions, like for the unshielding of PV perturbations.

We used the piecewise PV inversion technique developed by Davis and Emanuel (1991) based on the Charney (1955) nonlinear balance equation. In our case the basic-state PV is simply the PV distribution from the operational forecast and the total PV perturbation is the initial or evolved PV correction from the key analysis error algorithm (i.e., the PV difference between the sensitivity and the operational forecast, sensitivity minus operational). We adopt here a three-part vertical partitioning of the PV correction: 1) lower-boundary potential temperature corrections (975 hPa) and near-surface (950 hPa) PV corrections (hereafter referred to as LBTC), 2) low-level PV corrections (from 900 to 600 hPa) (hereafter referred to as LPVC), and 3) upper-level PV corrections (from 550 to 150 hPa) and upper-boundary potential temperature corrections (125 hPa) (hereafter referred to as UPVC).

4. Diagnostic results

a. Change to the surface cyclone

The 950-hPa geostrophic vorticity correction at the initial time (Fig. 3a) depicts a region of negative values northeast of the position of the cyclone center in the operational analysis (in terms of maximum geostrophic vorticity at 950 hPa and marked by an X in Fig. 3). A region of positive values lies to the southwest. The amplitudes for both regions are small and they contribute to shift the surface cyclone slightly to the southwest. After 24 h (Fig. 3b), the amplitudes of the negative and positive regions of geostrophic vorticity corrections increase considerably: by about an order of magnitude. This pattern of geostrophic vorticity corrections leads to a significant relocation of the cyclone toward the south near the cyclone center in the reference analysis (see Fig. 2).

To determine the origin of the 950-hPa geostrophic vorticity corrections, we averaged the respective contributions from LBTC, LPVC, and UPVC over each positive and negative area of total geostrophic vorticity corrections delimited by \( |\zeta| \geq 0.5 \times 10^{-5} \) s\(^{-1}\) (\(|\zeta| \geq 1.0 \times 10^{-5} \) s\(^{-1}\)) at 0 h (24 h). The results, shown in Fig. 4, indicate that at the initial time, LPVC and LBTC were the two main contributors for both areas of geostrophic vorticity corrections, whereas UPVC has a marginal impact. After 24 h, LPVC and LBTC are still

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*4 The selection of pressure boundaries and internal limits are linked to vertical interpolation boundaries (1000 and 100 hPa) and 50-hPa vertical intervals described in section 2.*
the dominant terms with LBTC as the main contributor for both positive and negative correction regions signaling a history of considerable temperature advection at the low levels. Note that UPVC is producing significant negative geostrophic vorticity corrections in the positive correction area. Thus upper-level negative PV advection acts to decrease the amplitude of the positive geostrophic vorticity correction. Over the negative areas, UPVC remains a marginal contributor.

The vertical distribution of LPVC (Fig. 5c) shows alternating positive and negative PV correction filaments in the vicinity of the cyclone. The filaments tilt upward against the vertical shear of the background (i.e., operational analysis) flow. Such pattern of PV corrections is similar to those found in studies based on key analysis errors (e.g., Laroche et al. 2002) or SVs (e.g., Montani and Thorpe 2002; Morgan 2001; Hoskins et al. 2000). The positive and negative PV corrections near the center of the cyclone are respectively responsible for the contribution to the positive and negative geostrophic vorticity corrections from LPVC depicted in Figs. 4a and 4c. After 24 h, the amplitude of the LPVC has increased significantly and the PV corrections are now tilted with the vertical shear (Fig. 5d). Consequent to the increase in amplitude of the PV corrections, the associated positive and negative geostrophic vorticity corrections also increase significantly (Figs. 5d versus Fig. 5c). Because LPVC indicates significant growth, it appears that the unshielding of LPVC (i.e., the alignment of like-signed PV correction by differential horizontal advection) is not the primary factor for the increase of the contribution to low-level geostrophic vorticity corrections by LPVC.

The vertical cross section of UPVC shows that positive and negative corrections were located well upstream of the surface cyclone at the initial time (Fig. 5a). The marginal impact of UPVC on the 950-hPa geostrophic vorticity corrections noted in Figs. 4a and 4c is attributed to the large distance between UPVC and the surface cyclone as well as to the confinement of the associated geostrophic vorticity correction to the upper levels. After 24 h, significant growth and changes become noticeable in the distribution of UPVC. A region of strong negative PV corrections develops above and upstream of the surface cyclone. This produces negative geostrophic vorticity corrections within and southwest of the cyclone center, leading to a negative contribution to the positive geostrophic vorticity corrections noted in Fig. 4b, in the verification area selected.

The last component of the piecewise PV inversion, LBTC, is shown in Fig. 6. It can be seen that at the initial time (Fig. 6a), a region of warm correction is located around and west of the cyclone center while a region of cold correction is located due east. These warm and cold correction regions produce respectively regions of positive and negative geostrophic vorticity

![Fig. 3. Geostrophic vorticity corrections at 950 hPa at (a) t = 0 h (contour interval 0.5 × 10⁻⁵ s⁻¹) and (b) t = 24 h (contour interval 2.0 × 10⁻⁵ s⁻¹) produced by the total PV correction. Initial corrections valid at 1200 UTC 27 Jan 2003. Positive (negative) values are in solid (dashed) lines and the zero contour is omitted. The X denotes the center of the low-level system in the operational forecast in terms of maximum geostrophic vorticity at 950 hPa. The long-dashed line shows the location of the cross section in Fig. 5. The H and L indicate, respectively, the maximum and minimum values for the corrections of interest.](image-url)
corrections at low levels (Fig. 5e). At the final time, the warm corrections increase by ~400%, and are now centered to the south of the cyclone center. Cold corrections also increase but only by ~50%. As a consequence, the associated positive and negative geostrophic vorticity corrections increase proportionally (Fig. 5f).

The above results suggest that in order to understand the modifications to the surface cyclone in the present case study, the dynamical/physical processes responsible for the growth of LBTC, LPVC, and UPVC must be addressed.

1) DIAGNOSIS OF THE GROWTH OF LOW-LEVEL PV CORRECTIONS

Inspection of the low-level PV gradient in the operational forecast shows, as typically observed, that the gradient of PV is relatively weak in the proximity of the surface cyclone. Despite considerable wind perturbations, the magnitude of the low-level background PV advection was relatively small (not shown) and cannot explain the significant growth of LPVC. On the other hand, notable differences in precipitation rate were observed between the operational and the sensitivity forecasts in the vicinity of the cyclone (not shown). To determine the effect of latent heat release due to condensation, the diagnostic software package DIONYSOS (Caron et al. 2005) was employed to transform the model precipitation rate into a heating rate, which in turn was distributed in the vertical using the wind, vertical motion, and the temperature values in conjunction with the thermodynamic equation. The resulting distribution of latent heating for each forecast (operational and sensitivity) was obtained and their difference (sensitivity minus operational) was plotted in Fig. 7. The vertical section indicates a region of enhanced condensation behind (i.e., southwestward) the center of the cyclone and a region of reduced condensation lying ahead (i.e., northeastward). The maximal amplitude is located in the midlevels (~650 hPa). Such a pattern of latent heating leads to the creation of positive (negative) PV corrections below the level of maximum positive (negative) latent heat release corrections in agreement with the observed growth of LPVC.

In passing, we mention that the TLM and adjoint models used in the present key analysis error algorithm are dry. The initial corrections are therefore determined by the dry dynamics without knowledge of the
FIG. 5. Vertical cross section of PV correction (black lines) and associated geostrophic vorticity correction (gray lines) at (left) 0 and (right) 24 h taken along the baroclinic shear of the background flow and as shown in Figs. 3 and 6. (a), (b) UPVC; (c), (d) LPVC; and (e), (f) LBTC. The contour interval for PV is 0.05 PVU (0.2 PVU) at 0 h (24 h) and the contour interval for geostrophic vorticity is $0.5 \times 10^{-5}$ s$^{-1}$ (2.0 $\times 10^{-5}$ s$^{-1}$) at 0 h (24 h). Initial corrections valid at 1200 UTC 27 Jan 2003. Positive (negative) values are in solid (dashed) lines and the zero contour is omitted. The X denotes the center of the low-level system in the operational forecast in terms of maximum geostrophic vorticity at 950 hPa.
moist processes. Nevertheless, when these corrections to the dry dynamics are placed in the context of a non-linear forecast model with full physics, they also develop corrections to the moist processes, which in turn increase their growth rate. This phenomenon is similar to that occurring in many extratropical cyclones, which are initiated by dry baroclinic processes but are strengthened by moist processes (e.g., Davis et al. 1993, and references therein).

2) Diagnosis of the Growth of Potential Temperature Corrections in the Lower Boundary

Inspection of the changes in diabatic processes such as sensible heat fluxes reveals no significant contribution to the modification of LBTC with time (not shown). Thus the growth of LBTC is likely caused by the advection of background potential temperature in the lower boundary by the horizontal wind corrections arising from both LPVC and UPVC. Figure 8 shows that the cyclone is located in a strong baroclinic zone with maximum temperature gradient southwest of the center. The low-level winds associated with LPVC create strong warm-air advection to the southwest and weak cold-air advection to the east of the cyclone (Fig. 8a) to intensify the warm and cold LBTC, located initially downstream of the positive and negative LPVC respectively. On the other hand, the low-level anticyclonic flow associated with the developing negative UPVC above and near the cyclone center, and the low-level cyclonic flow induced by the positive UPVC developing to the south (see Fig. 11c), generate warm-air advection to the southwest and cold-air advection east and northwest of the cyclone (Fig. 8b).

The relative contribution of LPVC and UPVC to the growth of LBTC changes with time. In the first 12 h, as LPVC tilts upstream and as UPVC is relatively weak, LBTC grows mainly from the flow induced by LPVC. In the last 12 h, when LPVC becomes vertical or with a downstream tilt and UPVC is large in amplitude, the induced low-level flow from UPVC becomes the primary mechanism for the growth of LBTC (not shown). We remark that although the unshielding of LPVC is not the primary factor for the growth of the low-level geostrophic corrections, the fact that LPVC tilts initially against the shear produces a dynamical structure highly favorable for the generation of significant LBTC with time.

b. Changes to the upper-level trough

We now turn our attention to the modification of the upper-level trough. The distribution of the 350-hPa vorticity corrections at the initial time (Fig. 9a) depicts a region of negative values east of the trough axis
In the operational forecast and two regions of positive values, one to the west and the other to the south of the trough. This pattern of vorticity corrections shifts slightly the trough axis toward the west and increases slightly its southward extension. After 24 h, the magnitudes of the vorticity corrections on the east and west sides of the trough increase dramatically while those to the south grow only moderately (Fig. 9b). This vorticity corrections pattern shifts the upper-level trough southwestward, to the same location as the reference analysis (see Fig. 2).

Similar to the low-level geostrophic corrections, we determined the respective contribution to the 350-hPa vorticity corrections from LBTC, LPVC, and UPVC over each area with positive and negative values of total vorticity corrections delimited by \( |\zeta| = 0.25 \times 10^{-5} \text{ s}^{-1} \) \( (|\zeta| = 1.0 \times 10^{-5} \text{ s}^{-1}) \) at 0 h (24 h). The results, shown in Fig. 10, indicate that both the positive and the negative vorticity corrections at the two times are entirely generated by UPVC. Therefore, to understand the evolution of the upper-level vorticity corrections we must investigate the evolution of the UPVC.

The initial PV perturbations at 350 hPa (Fig. 11a) reveal a region of negative corrections (L) and a region of weak positive corrections (H1) on each side of the upper-level trough (Fig. 9a), as well as a stronger posi-

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**Fig. 7.** Vertical cross section of PV correction (black lines, contour interval 0.1 PVU) and changes in condensational latent heat release (gray lines, contour interval 5 \( \times 10^{-5} \text{ K s}^{-1} \)) at 12 h. Initial corrections valid at 1200 UTC 27 Jan 2003. Positive (negative) values are in solid (dashed) lines and the zero contour is omitted. The X denotes the center of the low-level system in the operational forecast in terms of maximum geostrophic vorticity at 950 hPa.

**Fig. 8.** 975-hPa winds (arrow, knots) and potential temperature advection [thick lines, contours are at \(-10, -5, -3, -1, 1, 3, 5, \) and \( 10 \times 10^{-5} \text{ K s}^{-1} \) with positive (negative) values in solid (dashed) lines] attributed to (a) LPVC and (b) UPVC at \( t = 12 \) h. Overlaid on both panels is the potential temperature at 975 hPa from the operational analysis (thin lines, contour interval: 5 K). Initial corrections valid at 1200 UTC 27 Jan 2003. The X denotes the center of the low-level system in the operational forecast in terms of maximum geostrophic vorticity at 950 hPa.
Fig. 10. Contribution of LBTC, LPVC, and UPVC to the 350-hPa vorticity correction for (a), (b) the maximum (H1 in Fig. 9) and (c), (d) the minimum (L in Fig. 9) of vorticity correction at (left) 0 and (right) 24 h. Each contribution represents an area average over each maximum and minimum (see text in section 4b for complete details). Initial corrections valid at 1200 UTC 27 Jan 2003.
tive perturbation area (H2) to the south of the trough. This distribution is very similar to the vorticity corrections displayed in Fig. 9a. The geopotential height corrections due to UPVC (Fig. 11b) are negative south of the trough. The resulting wind corrections produce an eastward flow across the upper-level trough. The accompanying negative and positive background PV advections act to increase the magnitude of the positive (H1) and negative (L) PV perturbation areas. After 24 h, two large PV perturbations (Fig. 11c) appear on each side of the upper-level trough (Fig. 9b). Thus UPVC is responsible for the shift in the location of the upper-level trough in the sensitivity forecast and the negative contribution to the positive low-level geostrophic vorticity corrections.

c. The integrated effect of the upper-level and low-level initial PV corrections

In section 4a, it was shown that UPVC has both a negative (through upper-level negative PV advection) and a positive (through low-level warm-air advection by the horizontal wind corrections from UPVC) impact on the growth of the low-level positive geostrophic vorticity corrections. Because it is likely that the evolution of UPVC is mainly due to the interaction between the basic state and the initial UPVC, a forecast initialized only with the initial corrections attributed to UPVC would allow the determination of the integrated effect of UPVC and the relative importance of the positive and negative impacts on the development of the low-level corrections. A new sensitivity analysis (called PV-Up) was therefore prepared by inverting the initial UPVC and adding the resulting state corrections to the operational analysis. Similarly, a PV-Down sensitivity analysis was prepared by inverting both the initial LBTC and LPVC. For each of the new sensitivity analyses, a 24-h forecast was then performed and the corrections after 24 h were computed (sensitivity minus operational forecast). The PV-Bal (inversion of the full initial PV correction) sensitivity analysis and its forecast would serve as the control.

![Fig. 11. 350-hPa PV perturbations at (a) 0 h (contour interval 0.05 PVU) and (c) 24 h (contour interval 0.4 PVU). (b) 350-hPa background PV (thin lines, contour interval 1 PVU) and geopotential height corrections (thick lines, contour interval 0.5 dam) and wind corrections (arrows, knots) attributed to UPVC at 0 h. Initial corrections valid at 1200 UTC 27 Jan 2003. Positive (negative) values are in solid (dashed) lines. Zero contour is omitted in (a) and (c). The dotted line in (a) and (c) denotes the axis of the upper-level trough in the operational forecast in term of maximum vorticity at 350 hPa. The H and L indicate, respectively, the maximum and minimum values for the corrections of interest.]
The initial and 24-h vorticity corrections for the three experiments are plotted in Fig. 12. A number of conclusions can be drawn by comparing Figs. 12a, 12c, and 12e. First, the 24-h upper-level corrections are mainly associated with the initial UPVC because in the vicinity of the surface cyclone, the upper vorticity corrections in experiment PV-Up are similar to those in experiment PV-Bal. Furthermore, there was very weak upper-level change (Fig. 12c) in the PV-Down experiment. These quantitative results confirm the earlier qualitative results and support the view that the evolution of UPVC is due to the interaction between the initial UPVC and the basic state. Second, a positive low-level vorticity corrections area formed at a similar location in both the PV-Up and PV-Bal experiments. Thus the positive impact by UPVC on the development of low-level positive geostrophic vorticity corrections through low-level warm-air advection exceeds the negative impact from the upper-level negative PV advection. Other results from dry experiments (not shown) confirmed that most of the low-level vorticity corrections formed in PV-Up are caused by the dry dynamics (temperature advection) and not by moist processes.

Finally, we remark that the PV-Up experiment reproduced most of the upper-level vorticity corrections and about half of the low-level vorticity corrections in the control experiment after 24 h. Therefore the initial UPVC is responsible for the majority of the final-time corrections in this case.

5. Additional case studies

Our diagnosis of the evolution of the corrections to the initial state from the key analysis error algorithm for the first case (hereafter referred to as C1) revealed the following characteristics. First, the growth of the initial corrections in the low level is dominated by low-level temperature advection and changes in diabatic processes while upper-level corrections grow by PV advection. The initial upper-level PV corrections are responsible for almost all the upper-level corrections and about half of the corrections in the low levels after 24 h. The impact of the initial low-level PV corrections after 24 h is essentially limited to the low levels. To determine the generality of these characteristics, three other cases also investigated in Part I of this series, designated as C2, C3, and C4 respectively, were investigated (Figs. 13–15). Similar to C1, the operational CMC global forecasting system for these cases showed significant 24-h forecast errors over the eastern part of North America. The forecast for C2, initialized at 1200 UTC 6 January 2003, yielded large errors at both the low and upper levels. There were errors in the location of the low pressure area (Fig. 13a versus Fig. 13c; see also Fig. 12f from Part I of this series for forecast error field) and in the amplitude of the upper-level ridge over the western part of Nova Scotia (Fig. 13b versus Fig. 13d). Cases C3 and C4 are distinct from C1 and C2 in that the forecast errors are mainly confined to the low levels in C3 but to the upper levels in C4. In C3, initialized at 1200 UTC 6 February 2002, there were forecast errors in the location of the low pressure center (Fig. 14a versus Fig. 14c) and in the intensity of the ridge to the northeast of the low. The latter errors reached 12 hPa near Sable Island (see Fig. 13c from Part I). In C4, initialized at 0000 UTC 19 January 2002, forecast errors can be detected in both the location and the intensity of an upper-level trough embedded in a strong westerly flow (Fig. 14b versus Fig. 14d). The intensity errors reached 9 dam over Pennsylvania (see Fig. 13f from Part I). Because the initial corrections from the key analysis error algorithm try to minimize the forecast errors after 24 h, it is not surprising to find the largest 24-h corrections in regions where forecast errors were observed: in both the low and upper levels in C2 (Fig. 12b), in the low levels in C3 (Fig. 17a), and at the jet level in C4 (Fig. 17b). Each of these final-time correction distributions contributes to reduce significantly the 24-h forecast errors (not shown).

We first compare the relative contributions to the 950-hPa geostrophic vorticity from LBTC, LPVC, and UPVC at the initial and final times. Our analysis is restricted to C2 and C3 because no significant low-level corrections developed with time in C4. Figure 16 depicts the results for the positive geostrophic vorticity. At 0 h, LPVC is the only positive contributor in the two cases while LBTC and UPVC have slightly negative or marginal impact (Figs. 16a and 16c). After 24 h, LBTC becomes dominant in C3 (Fig. 16d) but is as large as LPVC in C2 (Fig. 16b), confirming the primary importance of low-level temperature advection in the growth of the low-level corrections. Inspection of the changes in latent heat release (not shown) indicates that diabatic effect considerably influences the growth of LPVC in these two cases. However, the effect of latent heating in C2 and C3 is less than in C1 so that their contributions from LPVC do not amplify as much as C1
with time (see Fig. 4). As for UPVC, its contribution after 24 h is marginal in C3 (Fig. 16d) and negative in C2 (Fig. 16b) and C1 (Fig. 4b) suggesting that upper-level PV advection is not an important factor in the intensification of the low-level corrections. The contributions to the negative areas of 950-hPa geostrophic vorticity show similar results (not shown).

Inspection of the contribution to the upper-level vor-

![Fig. 12](image-url) Vertical cross section, along the section shown in Fig. 3b, of the vorticity corrections (contour interval $2.0 \times 10^{-6}$ s$^{-1}$) obtained after 24 h from various sets of initial corrections. (a) Experiment with the full initial PV correction (PV-Bal), (c) experiment with only the UPVC (PV-Up), and (e) experiment with only the LBTC and LPVC (PV-Down). Initial corrections are valid at 1200 UTC 7 Jan 2003. Positive (negative) values are in solid (dashed) lines and zero contour is omitted. (right) Same as left panels but for initial corrections valid at 1200 UTC 6 Jan 2003 and contour interval $1.5 \times 10^{-6}$ s$^{-1}$. Vertical cross section along the section shown in Figs. 13e and 13f.
Fig. 13. Same as in Fig. 2 but for fields valid at 1200 UTC 7 Jan 2003 and from GEM forecasts initialized at 1200 UTC 6 Jan 2003. The long-dashed line in (c) and (f) shows the location of the cross section in the right panel of Fig. 12.
Fig. 14. Same as in Fig. 2 but for fields valid at 1200 UTC 7 Feb 2002 and from GEM forecasts initialized at 1200 UTC 6 Feb 2002. The long-dashed line in (c) and (f) shows the location of the cross section in the left panel of Fig. 17.
Fig. 15. Same as in Fig. 2 but for fields valid at 0000 UTC 20 Jan 2002 and from GEM forecasts initialized at 0000 UTC 19 Jan 2002. The long-dashed line in (e) and (f) shows the location of the cross section in the right panel of Fig. 17.
ticity corrections in C2 and C4 reveals that the vorticity corrections at the initial and final times are completely attributable to UPVC (not shown), similar to the case of C1. This confirms that upper-level background PV advection controls the evolution of the upper-level corrections.\(^6\)

To evaluate the variability of the sensitivity to the low-level and upper-level initial corrections we performed two additional sensitivity forecast for each additional case. Similar to the case of C1 in section 4c, the initial corrections were obtained from the inversion of the initial UPVC (PV-Up experiment) and from the inversion of both LBTC and LPVC (PV-Down experiment). The results for C2 are dramatically different from C1. Practically all the final-time vorticity corrections at both the low and upper levels in C2 originated from the initial low-level corrections (Figs. 12f and 12b). Whereas the UPVC in C1 grows through background PV advection generated by the initial UPVC itself, we can infer that the upper-level background PV advection in C2 is generated by the initial low-level corrections. Similar to C2, most of the final-time low-level vorticity corrections in C3 originated from the initial low-level corrections (Figs. 17e and 17a). However, unlike C2, the initial low-level corrections in C3 did not propagate to the upper levels with time. Finally, in C4, the upper-level vorticity corrections after 24 h are due to both low- and upper-level initial corrections as both PV-Down (Fig. 17f) and PV-Up (Fig. 17d) experiments show significant contribution to the total vorticity corrections (Fig. 17b). A closer examination of the initial corrections in C4 reveals that most of them are located in the midlevels (around 600 hPa; not shown). Because we adopt 600 hPa as the boundary between the lower and the upper levels, the key corrections in C4 are divided into a lower and an upper part, resulting in the final-time corrections being nearly equally distributed between PV-Up and PV-Down. These results suggest that there are large case-to-case variabilities in the relative importance of low-level and upper-level PV cor-

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\(^6\)This does not necessarily mean that the upper-level background PV advection is controlled by the initial UPVC like in the first case study (C1). PV corrections from other levels can strongly influence the UPVC, and C2 is a good example as will be shown later.
rections of the key analysis errors. There are situations characterized by a large sensitivity to upper-level initial corrections (e.g., C1), but other cases are marked by low-level (e.g., C2 and C3) or midlevel initial corrections (e.g., C4). More case studies are necessary to obtain statistically significant results on the vertical distribution of this sensitivity, but this is outside the scope of this paper.
6. Variability in vertical propagation

We have shown that under some situations the initial corrections do not propagate in the vertical and alter only the forecast in the levels where they are originally found. C2 and C3 represent two opposite scenarios. In C2 the initial LBTC and LPVC induced significant change at both the low and upper levels after 24 h, whereas in C3 the initial LBTC and LPVC only influence the low-level structure at the final time. Here, C1 and C4 also show significantly different behavior. In C1, the initial UPVC strongly influences the forecast at both the upper and lower levels, whereas in C4 the initial UPVC only modifies the upper-level structure with time. This section examines various factors that may explain the different behavior of the initial corrections observed in C2 versus C3 and C1 versus C4.

We first analyze the difference between C2 and C3. Figure 18 shows low-level PV corrections and the upper-level PV distribution in the operational analysis (i.e., the background state) at the initial time for each case. It can be observed that for C2, the two main low-level PV corrections at 800 hPa are located underneath a relatively strong region of background PV gradient at 350 hPa (Fig. 18a), a configuration favorable for LPVC to induce upper-level background PV advection and develop UPVC with time. In comparison, the low-level PV corrections associated with the low pressure system of interest in C3 are located underneath a region of relatively flat gradient of background PV (Fig. 18b) and remain in this unfavorable location with time (not shown). Moreover, the upper-level PV gradient regions in the background state of C3, located upstream and downstream of the low-level PV correction, tend to decrease with time (not shown). Therefore it appears that it is the difference in the location of the low-level corrections with respect to the upper-level background PV gradient regions, which accounts for why the low-level corrections develop significant UPVC with time in C2 but not in C3.

We have shown previously that the interaction between the UPVC and a strong low-level baroclinic zone develops LBTC with time in C1 (see section 4a). Figure 19a shows the upper-level PV corrections at 300 hPa and the potential temperature distribution at 975 hPa at 15 h for C1. It is clear that UPVC is located over a region of strong temperature gradient and explains why UPVC induced significant background potential temperature advection depicted in Fig. 8b. Because the UPVC in C4 does not develop significant LBTC with time, one can presume that it is due to a lack of baroclinicity in the low levels. In the states bordering the Gulf of Mexico, the temperature gradient in C4 (Fig. 19b) is comparable to the gradient observed over the Atlantic Ocean along the east coast in C1. However, in the central part of the United States, where the largest UPVC is located, the low-level baroclinicity is lower, which can partially explain why no significant LBTC develops with time in C4. Another factor that can influence the impact of UPVC on the LBTC is the pen-
etration of the circulation associated with UPVC in the low levels. The penetration depth of a PV anomaly can be approximated by the Rossby penetration depth, adapted from Jones (1995) as

\[ H = \frac{(f + \zeta) L}{N}, \]

where \( L \) represents the radius of the anomaly (taken here as 1000 km) and \( N \) is the Brunt–Väisälä frequency \( [N = (\rho g^2 S_p/R_s T)^{1/2}] \), a measure of the static stability of the atmosphere. The penetration depth is thus proportional to the absolute vorticity of the atmosphere and inversely proportional to the static stability. Figure 20 shows the static stability parameter \( (S_p) \) and the absolute vorticity averaged in the low levels (between the surface and 750 hPa) for the background state in C1 and C4. It can be seen that the atmosphere is much less statically stable in the low levels over the Atlantic Ocean along the east coast in C1 (Fig. 20a) than over the central United States in C4 (Fig. 20b), thereby allowing a greater penetration of the UPVC circulation in the low levels in C1. Also, the mean absolute vorticity in the low levels is greater in C1 (Fig. 20c versus Fig. 20d), which also favors the penetration of the UPVC to the low levels. When \( H \) is computed using (5) with the mean low-level static stability and absolute vorticity, it is not surprising to find greater values in C1 (Fig. 20e) than in C4 (Fig. 20f). As shown by Davis (1992), the penetration of a PV anomaly, when nonlinear balance is assumed, is also proportional to the magnitude of the anomaly. From Fig. 19, it is evident that the magnitude of the UPVC is greater in C1 than in C4. Therefore, it is believed that no significant LBTC (and therefore no significant low-level vorticity corrections) develops in C4 compared with C1 due to a combination of the following factors: a reduced low-level baroclinicity, a larger low-level static stability, a reduced low-level absolute vorticity, and a weaker UPVC.

Our results demonstrate how the relative position of the key analysis errors with respect to features in the background flow and the structure of the background state itself can influence the evolution of the initial corrections. This is consistent with the work of Reynolds et al. (2001) in their study of the relationship between SVs and transient features in the background flow.

7. Summary and conclusions

The goal of this paper is to find how, dynamically/physically, the initial corrections obtained from the key analysis error algorithm modify the forecast and why they grow rapidly by using a PV diagnostic approach. A detailed analysis has been made for a particular case study (C1) and three additional cases (C2, C3, and C4) to determine the generality of the results.

From an examination of the contribution to low-level
Fig. 20. (top) Stability parameter ($S_p$, contour interval $2 \times 10^{-4}$ K Pa$^{-1}$) average between the surface and 750 hPa.
(middle) Absolute vorticity average between the surface and 750 hPa (contour interval $4 \times 10^{-5}$ s$^{-1}$). (bottom) Rossby vertical penetration depth [as in Eq. (5), contour interval 5 km] computed from the mean stability and the mean absolute vorticity. (left) At $t = 15$ h for initial corrections valid at 1200 UTC 27 Jan 2003. (right) At $t = 15$ h for initial corrections valid at 0000 UTC 19 Jan 2002.
and upper-level geostrophic vorticity corrections due to different PV corrections (LBTC, LPVC, and UPVC), it has been shown that at the initial time the low-level geostrophic vorticity corrections are completely attributable to UPVC. The low-level geostrophic vorticity corrections grow mainly through background surface potential temperature advection by the winds induced by LPVC and/or UPVC. The contribution from LPVC also increases with time due to both the creation of low-level PV corrections from latent heat release and the superposition of LPVC (unshielding). As a result, the low-level geostrophic vorticity corrections are attributable to LBTC and LPVC at the final time (24 h). Interestingly, in the cases studied, UPVC generates little direct contribution on the low-level corrections at both the initial and final times. This result can be partially explained by the importance of the diabatic processes in the evolution of the key analysis errors, which favor direct positive contribution from LPVC and negative contribution from UPVC (volume-integrated PV conservation). Nevertheless, dry dynamical processes play a substantial role because even without diabatic processes (i.e., in dry run experiments), the direct contribution from UPVC on the low-level corrections remains small or negative (not shown). In the upper levels, the vorticity corrections are completely attributable to UPVC not only at the initial time but throughout the model integration of 24 h. The evolution of UPVC is dictated by the advection of background PV advections by the wind corrections. Significant variability was found in the origin of the upper-level wind corrections that trigger background PV advection. In C1, most of the evolution of UPVC is controlled by the initial UPVC. In C2, the growth of UPVC is initiated by the low-level initial corrections. The latter results were obtained by perturbing the operational analysis using only the initial state corrections attributed to UPVC or only to LBTC and LPVC combined.

Studies of the sensitivity of the final-time corrections to the initial low and upper-level corrections revealed some significant case-to-case variability in the relative importance of the initial low-level and upper-level PV corrections. In C1 most of the final-time corrections can be attributable to initial changes in the upper-level PV fields whereas final-time corrections in C2 and C3 are associated with modification of the low-level PV and boundary temperature fields. In C4, it appears that the modification of the PV field in midlevels can explain most of the forecast corrections after 24 h.

The initial corrections have generally been found in a strong baroclinic environment, characterized in a PV perspective by intense gradient of potential temperature at the surface and intense gradient of PV at the upper levels, in agreement with the results of Reynolds et al. (2001) with respect to SVs. It is also consistent with earlier results that showed a clear correspondence between the localization of the energy-norm-based key analysis errors (Klinker et al. 1998; Isaksen et al. 2005) as well as the energy-norm-based SVs (Buizza and Palmer 1995) and the maximum value of the Eady index, a measure of the baroclinicity of the background flow. However, in some cases, the key analysis errors have been found in regions not favorable for their growth. In C3, the low-level corrections are located beneath a region of relatively flat upper-level PV gradient that prevents the upward propagation of the low-level key analysis errors that was observed in C2. In C4, the structure of the low-level background state has been found not favorable for the downward propagation of upper-level key analysis errors in contrast to the situation in C1.

The growth mechanisms for the key analysis errors found in this work agree reasonably well to those diagnosed by Morgan (2001) for SVs. This is not surprising because it is known that key analysis errors and the dominant SVs share many similarities (Gelaro et al. 1998). The only difference between the results of Morgan (2001) and ours concerns the importance of the unshielding process in the growth of low-level corrections, which was found to be of lesser importance in our work. We have also highlighted the role of diabatic processes in the amplification of LPVC. Nevertheless, PV corrections that are tilted upshear in the low levels produce a dynamical structure favorable for creating significant LBTC with time to develop low-level corrections.

It is interesting to do some comparison between the results obtained here and PV-based studies of rapid development of synoptic perturbations (e.g., Huo et al. 1999; Balasubramanian and Yau 1994; Davis and Emanuel 1991), where PV perturbations represent transient features in the background flow and are usually obtained by subtracting a time-mean PV from instantaneous PV fields. As for the evolution of key analysis errors, the development of significant lower boundary temperature perturbations and low-level diabatic PV perturbations are associated with the rapid growth of low-level synoptic perturbations. A major difference is that in synoptic events the upper-level PV perturbation usually has a significant direct contribution on the low-level perturbations. Interestingly, even if the key analysis error UPVC does not contribute directly to the growth of the low-level corrections, they do contribute to amplify the low-level corrections indi-
rectly through their associated low-level horizontal wind corrections, which act to amplify the LBTC. Such interaction between the upper-level perturbation and the surface baroclinicity is also typically observed in growing synoptic perturbations. Further investigation would be necessary to better analyze the similarities and the differences between the evolution of key analysis error perturbations and growing synoptic perturbations, but this is outside the scope of the present paper.

Because the initial corrections from the current key analysis error algorithm cannot justifiably be directly associated with analysis errors, the findings on the evolution of the key analysis errors presented in this paper cannot be linked to the dynamics of the analysis errors. However, our results do add relevant dynamical/physical information about the evolution of small- and fast-growing initial perturbations. Finally, Klinker et al. (1998) and Isaksen et al. (2005) have shown that the structure of the key analysis errors is very sensitive to the initial-time norm. It would be of interest to investigate how the dynamical/physical processes involved in the evolution of the key analysis errors are modified when other norms, other than the energy norm, are used.

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