Scale-Selective Digital-Filtering Initialization

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ABSTRACT

Digital-filtering initialization (DFI) of atmospheric models relies on the fact that the gravity–inertia waves have higher frequencies than the meteorologically relevant rotational modes and assumes that a frequency exists that separates them. This note shows that a Doppler effect of fast-propagating storms may “shift” the frequencies of the small-scale rotational modes into the frequency categories that are deemed to be the ones of the gravity–inertia waves. A forecast is presented in which the impact of this on DFI manifests itself to a substantial extent (i.e., a reduction the depth of the eye of the storm by about 6–7 hPa). As a cure it is proposed to make the filtering scale selective (i.e., filtering the large spatial scales more than the small ones). It is shown that, not only does this leave the storm almost intact, but it also leads to a more balanced initial state. The implementation of such a filter is straightforward in a spectral limited-area model.

1. Introduction

Initial states of atmospheric models are usually unbalanced. When used as initial states of model runs, they will adjust to equilibrium by producing unrealistically high gravity–inertia waves. Initialization is the technique that creates balanced states by removing these waves.

The technique of digital-filtering initialization (DFI) introduced by Lynch (1990) is based on the fact that the gravity–inertia waves lie in a different part of the frequency spectrum than the meteorologically relevant rotational ones. It then assumes that a frequency exists that separates the two. By applying a low-pass filter with this frequency as a cut off, the gravity–inertia waves can be removed. In practice low-pass filters with cutoff periods of roughly a few hours have been found to yield the most satisfactory results. This technique is particularly simple from a practical and a conceptual point of view. This has made it popular in today’s numerical weather prediction (NWP) models [e.g., in the European High-Resolution Limited-Area Model (HIRLAM; Lynch and Huang 1992) and the Aire Limitée Adaptation Dynamique Développement International (ALADIN) model (ALADIN International Team 1997; Lynch et al. 1997)].

High frequencies can also be created by a Doppler effect. Considering this well-known effect in its purest form, an observer standing still with respect to a monochromatic mode with a given wavenumber $k$ that is oscillating with a frequency $\omega$ and moving with a specific velocity $c$, will observe a shift in the frequency as $\omega + ck$. The higher the propagation velocity $c$, the higher the shift in the frequency.

The same effect is also present to some extent in any propagating large-scale rotational wave. An observer standing still with respect to a propagating storm and decomposing an observed time series in a frequency spectrum will find higher frequencies than an observer moving along with it. This may represent frequencies that are not determined by gravitational buoyancy effects but by, for instance, the large-scale dynamical flow. As such the observed frequencies can be higher than what is dictated by the time scales of the underlying physical processes.

This note describes a model run of the ALADIN model of the famous 1999 Lothar storm (Wernli et al. 2002) and shows that this Doppler effect can be detected in the initialized fields by DFI. In particular it...
has been found that applying a standard DFI to this storm at its peak intensity reduces the depth of the eye by about 6–7 hPa. Section 2 presents this case. To investigate the dependency of this effect on the wave-number, a scale-selective filter is then introduced in section 3. It is shown that (i) such a filter can be used to keep the eye of the storm intact, and it is also found that (ii) it also leads to a more balanced state.

2. A Doppler shift

To deal with the loss of information at lateral boundaries of limited-area models due to the temporal interpolation of large-scale data coming from a coupling model (see Termonia 2003), it has been shown by Termonia (2004) that this loss can be monitored in the coupling model by filtering the surface pressure in time. This filtered field can be used to implement coupling strategies that guarantee sufficient data transfer to adequately capture extreme storms at the boundaries. One of the proposed options in that paper is to start or restart the run at a later time, ensuring that the storm is already inside the domain.

To implement these ideas operationally, tests have been carried out where the forecast is started later than the standard initialization time; for example, in the case of the Lothar storm discussed in that paper, the initial state of the model run has been constructed by interpolating the 9-h forecast range of the global Action de Recherche Petite Echelle Grande Echelle (ARPEGE) to the resolution of the ALADIN-France domain, as opposed to taking the interpolated ARPEGE analysis as the initial state for the ALADIN model. The standard 3-h coupling states are then used for imposing the lateral boundary conditions with the Davies flow-relaxation scheme (Davies 1976). This configuration has been the experimental setup for the tests presented in this note.

Figure 1 shows the mean sea level pressure (MSLP) of the ALADIN model for this Lothar storm at 0900 UTC after applying two types of DFI.

Figure 1a shows the output of a standard DFI (the one that is operational in, e.g., ALADIN Belgium) applied on the 0900 UTC ARPEGE forecast after begin interpolated to the grid of the ALADIN-France domain. In this case, the digital filtering has been performed by means of a Dolph–Chebyshev filter (Lynch 1997) with stop-band edge of 3.0 h, a time span of 2.167 h, and a ripple ratio of $r = 0.05$ (see Lynch et al. 1997). It is the one that is currently operational in the ALADIN-France domain, as opposed to taking the interpolated ARPEGE analysis as the initial state for the ALADIN model. The standard 3-h coupling states are then used for imposing the lateral boundary conditions with the Davies flow-relaxation scheme (Davies 1976). This configuration has been the experimental setup for the tests presented in this note.

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![Figure 1](image-url)
ALADIN version running at the Royal Meteorological Institute of Belgium. The minimum MSLP in the eye of the storm is 977.9 hPa.

Figure 1b shows the same setup but after a DFI with a simple filter with Lanczos window with cutoff period $T_c = 3$ h, as described in Lynch and Huang (1992). This will be referred to as DFI$_{3h}$ henceforth. The minimum in the eye of the storm is 976.6 hPa.

All tests presented in this note have been carried out for both the Dolph–Chebyshev and the Lanczos filter. The results were in all cases qualitatively equivalent. For the sake of the presentation, this note will restrict itself to the Lanczos filter henceforth.

The time integrations of the filters in this note were performed according to the so-called diabatic digital-filtering initialization (DDFI) scheme proposed in Huang and Lynch (1993). First an adiabatic backward integration from time step 0 to $-T_{\text{span}}$ is performed and filtered, yielding a filtered state at time step $-(1/2)T_{\text{span}}$. Then the model is integrated with a diabatic forward forecast from $-(1/2)T_{\text{span}}$ to $(1/2)T_{\text{span}}$, yielding the filtered state at time step 0. Here $T_{\text{span}}$ denotes the time span of the filter.

Figure 2a shows the MSLP of a model run at 0900 UTC that is based on the 0000 UTC analysis, without initialization. So this is the storm after having been developing for 9 h. The minimum surface pressure in the eye of the storm is here 971.9 hPa, being 6 hPa deeper than the 0-h forecast range in Fig. 1a.

Figure 2b shows the 0900 UTC interpolated ARPEGE state as in Fig. 1, but without any initialization. This essentially represents the interpolated fields from the large-scale ARPEGE output files. Note the orographically induced waves at the ARPEGE resolution over the mountain regions, in particular the Alps and over the Massif Central. It can be seen how spurious these waves are with respect to the ALADIN-France resolution, by comparing them to the MSLP fields in Fig. 2a. The minimum in the eye of the storm in Fig. 2b is 970.4 hPa.

The two most conspicuous features that can be observed in the different panels in Figs. 1 and 2 are the fact that (i) the DFI correctly filters the spurious waves in Fig. 2b, but that (ii) it also knocks off about 6–7 hPa from the low of the storm.

ALADIN is a spectral LAM having a geometrical structure as proposed by Haugen and Machenhauer (1993). The physical domain is extended with an artificial zone where all the fields are periodically extended to facilitate fast Fourier transforms. That means that the outputs of the model are in fact the spectral coefficients of the fields. These spectral coefficients can be used to study the fields in a scale-selective manner, even though some caution should be taken because the extension zone may introduce some artifacts into the data.

For the run based on the analysis at 0000 UTC presented in Fig. 2a, this yielded the spectral coefficients of
the surface pressure $p_s^{i,n}$, with $n$ being the index of the time step and $i,j$ the indices for the wavenumber of the spectral mode. The wavenumber $\kappa$ is related to the length of the domain $L_x, L_y$ in the $x$ and $y$ direction, respectively, and $i, j$ by

$$\kappa = 2\pi \sqrt{\frac{i^2}{L_x^2} + \frac{j^2}{L_y^2}}, \quad (1)$$

for $i = -i_{\max}, \ldots, i_{\max}$ and $j = -j_{\max}, \ldots, j_{\max}$. The model was run with time step $Dt = 300$ s, resolution $DX = DY = 9.5$ km and $i_{\max} = j_{\max} = 149$. This constitutes $N = 72$ time steps, giving a 6-h span. The values of the spectral coefficients were stored from 0600 to 1200 UTC. This includes the moment when the storm was at its peak intensity.

In this model run, the minimum of the eye of the storm was at 0600 UTC at a grid point located at 49°37′27″N, 1°15′37″E and later at 1200 UTC it was at 50°38′38″N, 9°18′38″E. The average propagation speed calculated from this is 98 km h$^{-1}$.

The spatial spectral decomposition of this integration period is thus readily available. To make the decomposition in the time domain all the spectral coefficients are detrended,

$$P_s^{i,j,n} = P_s^{i,j,0} + \frac{n}{N} (P_s^{i,j,0} - P_s^{i,j,N}), \quad (2)$$

yielding periodic series with period $T = 6$ h. This approach is similar as is done in Errico (1985) for the computation of spatial spectral decompositions. A discrete Fourier transform is then taken from this detrended time series:

$$\hat{P}_s^{i,j} = \sum_{n=0}^{N-1} P_s^{i,j,n} e^{2\pi i mn/N}. \quad (3)$$

The frequency $\omega$ of a mode is related to the period $T$ by

$$\omega = 2\pi/DT = \frac{2\pi}{\Delta t \cdot N}. \quad (4)$$

The fact that the series are not derivable at the end points by the construction in (2) may lead to artifacts in the spectrum on top of the ones originating from the period extension zone in the space domain of the spectrum, but this will not be investigated here.

To represent the spectral coefficients $P_s^{i,j}$ distribution bins are defined as

$$\left[\frac{(k - 1/2)\pi}{149\Delta x}, \frac{(k + 1/2)\pi}{149\Delta x}\right]$$

corresponding to wavenumber $\kappa_k = (k\pi)/(149\Delta x)$ and the following energy norm:

$$\Pi_k(\kappa_k, \omega_k) = \sum_{\kappa(i,j)\in \{k\}} [\hat{P}_s^{i,j}]^2, \quad (4)$$

with the dependence of $\kappa(i,j)$ on $i$ and $j$ taken as in (1) and $\omega_n = (\pi\nu)/(\Delta t N)$, which has been computed.

Figure 3a shows these data for the 0600–1200 UTC period of the Lothar storm forecast. For comparison, exactly the same data is added in Fig. 3b computed for the forecast of 18 December 1999 (this model run has also been considered in Termonia 2004), representing an anticyclonic case.

The line of constant phase speed $\omega/\kappa = 98$ km h$^{-1}$ has been added to the storm case. This line represents the nondispersive part of the wave packet that is propagating at constant speed without deformation. It also represents the Doppler effect described in the introduction. The faster the propagation speed, the more this line will be tilted counterclockwise, thereby pushing the frequencies up, the small scales more than the large scales. Once the propagation speed becomes fast enough, it starts to fill the part of the spectrum that is filtered by the DFI (i.e., the small scales) first.

Note that the artificial “spikes” at a wavenumber of about 0.000 05 m$^{-1}$ corresponding to wavelength 120 km. This is about the width of the periodic extension zone of the model.

A cutoff period of $T = 3$ h corresponds to a frequency $\omega = 0.000 58$ s$^{-1}$. Frequencies higher than this value will be filtered away by the DFI in Fig. 1b, independently from the value of the wavenumber. The hypothesis underlying DFI is that all modes in this belong to the fast, meteorologically irrelevant part of the field.

For the case of the Lothar storm in Fig. 3a, it can be seen that much more is lying above this cutoff of the filter and also, perhaps more importantly, that it is mostly concentrated around the line of the constant phase speed.

For the sake of completeness, the two runs in Fig. 3 were also carried out after initializing their analysis by means of a DFI with cutoff periods of 15 min, 1 h, and 3 h. No substantial differences with the data in Fig. 3 could be found.

This experiment clarifies the mechanism behind of the shortcomings of the DFI in Fig. 1. Indeed, this change of the rotational field is felt in the small scales. It suggests that this Doppler effect is, besides gravity–inertia modes, an additional source of high frequencies. However, it is not possible to distinguish these two in Fig. 3. In the next section, it will be shown that the filtering may be improved by considering filters that are scale selective.
3. Scale-selective DFI

The idea behind a scale-selective digital-filtering initialization (SSDFI) is straightforward. Denoting the filtering of a signal $f(t)$ by $H(\omega)F(\omega)$, with $F(\omega)$ the Fourier transform of $f$, a low-pass filter is given by

$$H(\omega) = \begin{cases} 1 & \text{if } |\omega| \leq \omega_c \\ 0 & \text{if } |\omega| > \omega_c. \end{cases}$$

(5)

Such a filter can be made scale selective by making the cutoff frequencies dependent on the wavenumber [i.e., using $\omega_c(\kappa)$]. This note considers

![Fig. 3. Energy norm per wavenumber and frequency of the (a) Lothar storm case on 26 Dec 1999 and (b) a run without the storm on 18 Dec 1999, between 0600 and 1200 UTC. Thick lines represent the 1-, 10-, 100-, and 1000-Pa contours, the highest contour level in (a) being 1000 Pa. The solid lines represent the (a) $\omega_\kappa = 98 \text{ km h}^{-1}$ phase speeds, corresponding to the propagation speed of the Lothar storm and (b) the 3-h cutoff period at a frequency of 0.000 58 s$^{-1}$.](image)
where \( \omega_c^0 \) is the cutoff frequency of the constant mode and \( c \) determines the slope of the curve. The corresponding cutoff period will be denoted by \( T_0^c \).

Note that, in the case of the Lothar storm, choosing \( c = 98 \text{ km h}^{-1} \) will guarantee that the nondispersive part of the storm is not filtered.

A discretized filter is specified by the coefficients \( h_n \) defined by

\[
y_0 = \sum_{n=-N}^{N} h_n x_n, \tag{7}
\]

where \( x_n \) is the incoming signal and \( y_0 \) the filtered value at \( t = 0 \), and where the subscript \( n \) denotes the time steps.

As mentioned before, the presentation in this note is restricted to the simple filter with the Lanczos window. The coefficients of this filter have the same form as in Lynch and Huang (1992):

\[
h_n(\kappa) = \sin \frac{n\pi}{N+1} \sin \omega_c(\kappa) \Delta t, \tag{8}
\]

but here \( \omega_c \) depends on \( \kappa \):

\[
\omega_c(\kappa) = \omega_c^0 + c \kappa, \tag{6}
\]

\[
\omega_c(\kappa) = \begin{cases} \omega_c^0 + \frac{\kappa}{\kappa_c} \left( \frac{\pi}{\Delta t} - \omega_c^0 \right) & \text{if } \kappa \leq \kappa_c, \\ \frac{\pi}{\Delta t} & \text{if } \kappa > \kappa_c, \end{cases} \tag{9}
\]

which is the form of (6), but for all wavenumbers \( \kappa \) exceeding the critical wavenumber \( \kappa_c \) the filter leaves the modes intact [i.e., \( h_n(\kappa \geq \kappa_c) = 1 \) and \( h_{n=0}(\kappa \geq \kappa_c) = 0 \) exactly]. The critical length corresponding to \( \kappa_c \) will be denoted by \( L_c = 2\pi/\kappa_c \).

The transfer function of the discrete filter can be calculated from the coefficients by

\[
H(\omega, \kappa) = h_0(\kappa) + 2 \sum_{n=1}^{N} h_n(\kappa) \cos n \omega \Delta t \tag{10}
\]

(see e.g., Lynch and Huang 1992). This is shown in Fig. 4 for the cutoff frequency \( \omega_c^0 = 2\pi/1.5h \) and \( L_c = 100 \) km. The contour lines of 1.0 for wavenumbers smaller than \( \kappa_c = 2\pi \times 10^{-5} \text{ m}^{-1} \) illustrate the Gibbs effect of the truncation of the filter. Note that there are no Gibbs waves for wavenumbers higher than \( \kappa_c \) even if the Eq. (10) is applied.

In the ALADIN model the DFI is performed on
spectral coefficients of the fields. An adaptation to have filter coefficients as in (8) for each component with wavenumber indices \( i, j \) in (1), is therefore relatively easy. Scale-selective filters of this type have thus been implemented in the source code of the ALADIN model.

Extensive testing has been carried out with such an SSDFI with different \( \kappa_c \) and different \( T^0_c \) for the Lothar case described above. Figure 5 shows two tests with \( L_c = 100 \text{ km} \) and with \( T^0_c = 3 \text{ h} \) and \( T^0_c = 1.5 \text{ h} \), respectively denoted by SSDFI\(_{3h}\) and SSDFI\(_{1.5h}\), the second one corresponding to the transfer function in Fig. 4.

The balance of the initial state can be quantified by the tendency of the surface pressure or the balance ratio of the form:

\[
Br = 100 \frac{\sum_I \sum_L | \mathbf{V} \cdot \Delta p_L \mathbf{V}_{II} |}{\sum_I \sum_L | \mathbf{V} \cdot \Delta p_L \mathbf{V}_{II} |},
\]

introduced in Lynch and Huang (1992), where \( \mathbf{V} \) is the horizontal wind vector labeled with gridpoint indices \( I, J, \) and \( L \) for the vertical sigma level. Here \( \Delta p_L \) is the pressure difference between the two half-levels above and below level \( L \). The smaller \( Br \) is, the more balanced are the fields. Figure 6 shows \( Br \) of the Lothar storm at 0900 UTC in three cases: the uninitialized one, one initialized with a DFI with cutoff period \( T^0_c = 3 \text{ h} \) (DFI\(_{3h}\)), and one with SSDFI\(_{1.5h}\).

As can be seen from Fig. 4, SSDFI actual filters a smaller part of the frequency–wavenumber spectrum than a full DFI. However, from Fig. 6 it can be seen that the model states are more balanced after SSDFI than after the full DFI. So surprisingly, filtering less leads to more balanced states. This is also confirmed by a test with a filter with \( T^0_c = 3 \text{ h} \) (not shown) corresponding to less filtering than the one for \( T^0_c = 1.5 \text{ h} \). It yielded almost the same \( Br \) but slightly larger, contrary to what one might expect in the first place.

Another surprise is that after about half an hour the forecast range DFI is actually worse than not filtering at all. This phenomenon can be explained by the fact that the full DFI eliminates those modes of the slow rotational signal that have been Doppler shifted to frequencies higher than the cutoff frequency. The remaining slow modes then have to adjust to a new balanced state. This perturbation is, compared to the magnitude of the signal itself, smaller than the perturbation of the fast gravity–inertia waves. So the effect is small, but its adjustment takes place on a longer time scale. Similarly \( \sum_I \partial p_c / \partial t | \) has been computed (not shown here) and this led to the same conclusions.

From Fig. 5 it can be seen that taking a cutoff period of \( 3 \text{ h} \) as was done in the test in Fig. 1b, deepens the low by 3.1 hPa, which is half the difference with respect to the MSLP in the coupling model ARPEGE in Fig. 2b. Using a smaller cutoff period of \( T^0_c = 1.5 \text{ h} \) makes the storm deeper (970.9 hPa) as expected, even to the extent that it becomes comparable to the one of the reference in Fig. 2b (970.4 hPa).

FIG. 5. MSLP of the ALADIN model at 0900 UTC 26 Dec 1999 after (a) SSDFI\(_{3h}\) (at a low of 973.5 hPa) and (b) SSDFI\(_{1.5h}\) (at 970.9 hPa). The filter is a simple digital filter with a Lanczos window. The contour intervals are 2 hPa.
The improvements in the initialization of this storm forecast carry over later in the forecast. Figure 7 illustrates this. It shows the forecast at the 6-h forecast range based on no initialization (figuring as a reference) and one initialized with DFI_{3h}, SSDFI_{3h}, and SSDFI_{1.5h}. It can also be seen here that SSDFI_{3h} reduces the difference between no initialization and DFI_{3h} by half and also that SSDFI_{1.5h} (972.5 hPa) yields practically the same result as the reference (972.4 hPa).

### 4. Discussion and conclusions

In this note, it is shown that the hypothesis that the fast gravity–inertia waves can be distinguished from the slow rotational ones by their time frequencies can be complicated by a Doppler effect. It has been explicitly shown that if the propagation velocity of a storm becomes fast enough this leads to frequencies coming from the meteorologically relevant signal being shifted into that part of the frequency spectrum that has been deemed to be exclusively the domain of the gravity waves.

It has been shown that a scale-selective digital-filtering initialization can cure this if it leaves this Doppler part of the frequency–wavenumber spectrum intact.

In the case of a forecast of the famous 1999 Lothar storm, this allows an initial state where the low of the storm is practically as deep as the one found in the coupling model. Contrary to what one might expect from intuition, this also leads to a slightly more balanced state than for the standard DFI. An explanation has been put forth: the state is more balanced in the case of SSDFI since in the case of DFI the slow modes are inadvertently perturbed and adjust to balance. This adjustment itself is also slow.

The results in this note are restricted to the case of an extreme storm. Some tests have been carried out with a forecast on 18 December 1999 in which case it was concluded that SSDFI also performed better. It has, however, not been rigorously tested whether SSDFI is better on average, the storm being an extremely rare case but nevertheless a most crucial one. In the procedure proposed by Termonia (2004) storms are detected in the coupling model. This would allow for a coupling frequency where the parameters of the filter can be adapted accordingly.

The Doppler shift of the frequencies grows linearly with the wavenumber. So for any nonzero large-scale flow, this Doppler effect will always shift some small scales into the filtered frequencies, provided that they are sufficiently small. Identifying, in an operational forecast suite, whether these lie below the resolution of the models and getting an estimate of the overall average loss of the meteorologically relevant signal, represents a challenging research question. This effect may become even more relevant when increasing the resolution in operational models. Indeed, if higher wave-
numbers $k$ are introduced, a given Doppler shift $c_k$ will already be felt at smaller propagation speeds $c$. For instance, at 1-km resolution, frequency shifts of the same magnitude as the ones that have been encountered in a 10-km resolution model, may already occur at large-scale propagation speeds that are 10 times smaller. Given the present analysis of the Lothar storm, this suggests that a standard DFI may filter relevant kilometer-scale features at large-scale flows of about 10 km $h^{-1}$. A systematic study, which lies beyond the scope of this note, is needed.

The implementation of a scale-selective DFI in a spectral model turns out to be quite straightforward. Considering the fact that fast Fourier transforms are computationally very cheap, one could consider applying some transforms in gridpoint models just for the sake of having SSDFI. However, special care should be taken to add a geometrical structure to the domain to

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**FIG. 7.** MSLP of the model run at 1500 UTC 26 Dec 1999, based on the 0900 UTC initial state with (a) no initialization (972.4 hPa), (b) DFI$_{3h}$ (974.5 hPa), (c) SSDFI$_{3h}$ (973.1 hPa), and (d) SSDFI$_{1.5h}$ (972.5 hPa).
make the fields periodic before the Fourier transforms while creating a minimum of artifacts in the physical part of the domain. An elegant and accurate solution has been given by Boyd (2005). This could be done in the filtering part of the model code only.

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