Recurrent Tropical Cyclones: Singular Vector Sensitivity and Downstream Impacts

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ABSTRACT

Singular vectors (SVs) are used to study the sensitivity of 2-day forecasts of recurving tropical cyclones (TCs) in the western North Pacific to changes in the initial state. The SVs are calculated using the tangent and adjoint models of the Navy Operational Global Atmospheric Prediction System (NOGAPS) for 72 forecasts for 18 TCs in the western North Pacific during 2006. In addition to the linear SV calculation, nonlinear perturbation experiments are also performed in order to examine 1) the similarity between nonlinear and linear perturbation growth and 2) the downstream impacts over the North Pacific and North America that result from changes to the 2-day TC forecast. Both nonrecurring and recurring 2-day storm forecasts are sensitive to changes in the initial state in the near-storm environment (in an annulus approximately 500 km from the storm center). During recurvature, sensitivity develops to the northwest of the storm, usually associated with a trough moving in from the west. These upstream sensitivities can occur as far as 4000 km to the northwest of the storm, over the Asian mainland, which has implications for adaptive observations. Nonlinear perturbation experiments indicate that the linear calculations reflect case-to-case variability in actual nonlinear perturbation growth fairly well, especially when the growth is large. The nonlinear perturbations show that for recurving tropical cyclones, small initial perturbations optimized to change the 2-day TC forecast can grow and propagate downstream quickly, reaching North America in 5 days. The fastest 5-day perturbation growth is associated with recurving storm forecasts that occur when the baroclinic instability over the North Pacific is relatively large. These results suggest that nonlinear forecasts perturbed using TC SVs may have utility for predicting the downstream impact of TC forecast errors over the North Pacific and North America.

1. Introduction

The accuracy of tropical cyclone (TC) forecasts is of great concern to both civilian and military interests. The forecast of the TC itself is of obvious concern for regions that may be directly impacted by the storm, but in some cases the storm forecast may significantly affect remote regions as well. It has been shown that when TCs recurve into the midlatitudes, they can have a substantial impact on the midlatitude environment well downstream, and the extratropical transition (ET) of TCs is often associated with a significant degradation of hemispheric forecast skill (Jones et al. 2003). The large sensitivity and complexity of the interaction of the TC and the midlatitude flow as the TC recovers northward (Harr et al. 2000; Klein et al. 2002; Ritchie and Elsberry 2007; McTaggart-Cowan et al. 2003, 2004; Hart et al. 2006; Riemer et al. 2008) means that small uncertainties in the forecast of the storm itself, or the environment, may lead to large forecast uncertainties downstream. Harr et al. (2008) and Anwender et al. (2008) have recently shown that ET events are associated with plumes of forecast uncertainty (represented as large ensemble variance) that spread downstream of the event. They have also shown that the increase in ensemble variance is tied to the ET event itself, rather than the forecast time. Cardinali et al. (2007) show that the impact of SV-based adaptive (targeted) observations over the North Atlantic is almost 4 times higher during extratropical transitions of TCs than the impact averaged for all cases.

In an effort to improve tropical cyclone track forecasts, aircraft adaptive observations have been taken of
the environment around the TC (Aberson 2003; Wu et al. 2005), as well as the storm itself. The premise of adaptive observations is that improving the analysis in sensitive regions will improve the forecast of the feature of interest (Langland 2005). Wu et al. (2007a) have demonstrated positive impact from adaptive observations taken during the Dropsonde Observations for Typhoon Surveillance near the Taiwan Region (DOTSTAR) experiment. The international field program The Hemispheric Observing system Research and Predictability Experiment Pacific Asian Regional Campaign (THORPEX PARC, or T-PARC) will offer an unprecedented opportunity to observe TCs throughout their life cycle, from genesis through extratropical transition, in the western North Pacific. In preparation for the T-PARC adaptive observing component, an intercomparison is under way for different adaptive observing techniques, including singular vectors (SVs; Gelaro et al. 1999), adjoint-derived sensitivity steering vectors (ADSSV; Wu et al. 2007b, 2009b), ensemble deep-layer-mean wind variance (Aberson 2003), and the ensemble transform Kalman filter (ETKF; Majumdar et al. 2002b) for the western North Pacific typhoons of 2006 (Wu et al. 2009a). This western North Pacific intercomparison is similar to one completed for the Atlantic basin (Majumdar et al. 2006; Reynolds et al. 2007), but puts more emphasis on the complicated dynamical systems affecting TC evolution, such as the midlatitude trough and the subtropical jet.

In this study, SVs are used to examine TC sensitivity during recurvature, and the subsequent downstream impacts resulting from the interaction of the TC with the midlatitude environment. The leading SVs represent the fastest-growing perturbations to a particular forecast (in a tangent linear sense), and as such, have been applied in fundamental geophysical fluid dynamic studies (e.g., Farrell 1982, 1988) and predictability studies (e.g., Lorenz 1965; Farrell 1990; Buizza et al. 1993). They have also been used for ensemble design (e.g., Molteni et al. 1996; Puri et al. 2001) and adaptive observing applications (e.g., Langland et al. 1999; Majumdar et al. 2002a). While most of the applications have been for midlatitude system (e.g., Buizza and Montani 1999; Reynolds et al. 2001; Montani and Thorpe 2002), more recently SVs have been applied to tropical cyclones (Barkmeijer et al. 2001; Peng and Reynolds 2005, 2006; Peng et al. 2007). These recent articles have demonstrated how SVs may be used to illustrate the impact of both the remote and near-storm environment on tropical cyclone forecasts. Peng and Reynolds (2006) illustrate how 2-day forecasts of straight-moving storms are most sensitive to the initial state in an annulus around the storm center, while irregular-motion storm forecasts can be more sensitive to remote features in the environmental flow, usually aligned with regions of flow toward the storm.

In the present study, SVs optimized for 2-day TC forecasts are used to understand how the initial structure of a storm and its environment affect its future state. SVs are calculated for 84 forecasts of 18 TCs in the western North Pacific during 2006. As with the intercomparison mentioned above, this study is also motivated by T-PARC, but focuses specifically on recurving and transitioning tropical cyclones. Also, in this work, in addition to using SVs to study the sensitivity of the 2-day storm forecasts to changes in the initial state, SV perturbations are used to study the downstream impacts of changes to these storm forecasts. This study is complementary to a detailed examination by Chen et al. (2008, manuscript submitted to J. Atmos. Sci.) of the relationship between the sensitivity of all the 2006 TCs and the environmental flow from a dynamical perspective. Chen et al. identify three patterns where the local SV maxima are collocated with weak wind speed regions associated with the confluence between the flow associated with the TC itself and the flow of other synoptic systems, and examine how the frequency of these patterns is dependent on seasonal changes in the environment.

In section 2, we describe how the SVs are constructed and how the nonlinear perturbation experiments are performed. Section 3 describes results, first examining initial-time sensitivity, and then examining evolved perturbations, downstream impacts, and SV-based forecast corrections. Section 4 contains the summary and discussion.

2. Construction of singular vectors

The leading SV represents the fastest-growing perturbation to a given trajectory (e.g., a weather forecast) in a linear sense. Consider a nonlinear model $M$, acting on a state vector $x$, such that $M(x_0) = x$, where the subscript refers to the integration time. Let $x_0$ represent some perturbed initial state, such that $x_0 - x_0 = p_0$ and $M(x_0) - M(x_0) = p_f$. For linear perturbation growth, the initial perturbation can be propagated forward in time using the tangent forward propagator, $L$, representing the model equations of $M$ linearized about the nonlinear trajectory, such that

$$Lp_0 = p_f. \quad (1)$$

Here, $L$ can be represented by its singular values and initial- and final-time SVs as follows:
where $V$ (or $U$) are matrices with columns composed of the initial (final) SVs $v_n$, and $D$ is a diagonal matrix whose elements $d_n$ are the singular values of $L$. The superscript $T$ denotes the transpose and $E$ is the metric that defines how the perturbations are measured. The SVs form an $E$-orthonormal set of vectors at initial and final time. The SVs satisfy the eigenvector equation

$$L E y_n = d_n^2 E v_n,$$

where $y_n = E^{-1/2} v_n$, and $d_n$ and $v_n$ are the $n$th singular value and initial-time SV, respectively. A state vector such as $p_0$ is related to the singular vectors through a transformation using this metric (e.g., $p_0 = E^{-1/2} v_1$). The leading SV maximizes the ratio of the final perturbation energy to the initial perturbation energy:

$$\frac{\langle L p_0; EL p_0 \rangle}{\langle p_0; E p_0 \rangle}$$

where $\langle \rangle$ represents a Euclidean inner product. The second SV maximizes this ratio under the constraint of being orthogonal to the first SV, the third SV maximizes this ratio under the constraint of being orthogonal to the first two SVs, and so on. For complex models, the eigenvector equation may be solved in an iterative fashion using the forward and adjoint propagators linearized about a particular forecast. For further discussion of SV diagnostics and their utility in atmospheric sciences, see Palmer et al. (1998) and references therein.

The SVs are calculated using the tangent and adjoint models of the Navy Operational Global Atmospheric Prediction System (NOGAPS; Hogan and Rosmond 1991; Peng et al. 2004) with a total energy metric at both the initial and final time (Rosmond 1997). For most of the results shown here, the SV calculations do not include moist processes, and contain only simplified versions of surface friction and vertical mixing. However, for some of the cases, the SV calculations include large-scale precipitation, and these are referred to as moist SVs. The SVs are calculated at a reduced resolution of T79L30, although the linearization is based on the trajectory from the full-physics, high-resolution (T239L30) operational NOGAPS forecast. The SV sensitivities are calculated for 2-day forecasts of western North Pacific tropical cyclones from May through December 2006. In total, 72 individual cases for 18 TCs during that time period are considered. Twelve additional cases for storms that are undergoing extratropical transition are also considered. A local projection operator (Buizza 1994) is employed to optimize final-time perturbation energy in a $20^\circ \times 20^\circ$ box enclosing the final-time storm position. In the vertical, the local projection operator extends from the surface to the model level that corresponds to approximately 300 hPa. Evaluation of the final-time SVs ensures that the SVs are focused on the TC and not an unrelated feature inside the local projection operator region. Table 1 provides the storm number, name, date of the initial forecast, and position of the center of the local projection operator (based on the Joint Typhoon Warning Center official 48-h forecast position of the storm). The initial-time SVs indicate regions where the 48-h storm forecasts are most sensitive to changes in the analyses. The vertically integrated energy corresponding to the first three leading SVs is used to construct an SV sensitivity pattern. The SV sensitivity pattern $s$ is a composite of the vertically integrated total energy of the leading SVs, weighted by the singular values as follows:

$$s = \sum_{j=1}^{3} \frac{d_j^2}{d_i^2} a_j(b, i),$$

where $a_j(b, i)$ is the vertically integrated total energy of the $j$th SV at latitude–longitude grid location $b$.

The storm cases are subjectively divided into two groups: 22 cases that are recurving into the midlatitudes (changing direction from westward or west-northwestward to northward or northeastward) and 50 cases that are not recurving. An additional 12 cases for extratropical transition are also considered (see Table 1). For a TC that ultimately recurves into the midlatitudes, the early forecasts of the storm, before recurvature, are put into the nonrecurring group, as the definition applies to the particular 2-day forecast, not the entire TC track. Thus, a storm that ultimately recurves is classified as a nonrecurver for forecasts started more than 2 days prior to recurvature. This is similar to, though not the same as, the groupings used in Peng and Reynolds (2006), who divided the storm forecasts into straight-moving and irregular-motion groups.

Because of the limitations of the tangent linear approximation, SVs are calculated over a 48-h period. Reynolds and Rosmond (2003) showed that for midlatitude baroclinic systems, tangent linear perturbation growth is a reasonable approximation for synoptic scales out to 3 days (although small-scale perturbations become nonlinear much more quickly). Peng and Reynolds (2006) showed that the tangent linear approximation is reasonable when applied to 2-day forecasts of TCs at a T79 resolution (i.e., linear and nonlinear perturbation...
growth are fairly similar). However, here, we are not just interested in the sensitivity of the 2-day forecast to changes in the initial state, but also in the impact these changes will have downstream, beyond 2 days. To study this downstream impact, we examine the difference between full-physics nonlinear control forecasts, and full-physics nonlinear forecasts run from analyses that have been perturbed using the SVs. These differences are referred to as nonlinear perturbations. The initial perturbations are produced by multiplying the initial-time SV state vector by a constant. The same constant is used in each case, so all nonlinear perturbations have the same total energy at initial time. The largest initial-time perturbation values are less than 2 m s$^{-1}$ in

Table 1. List of the 84 TC cases. The “name” column includes the names and the basin annual cyclone number. The northwestern Pacific (WP) and the central Pacific (CP) are listed with the storm number and name. The forecast initial times (“initial” column) are at 0000 UTC. For example, 11 May corresponds to 0000 UTC 11 May 2006. The local projection operator that defines the verification area is a 20° × 20° box centered on the longitude east and latitude north given in the lon and lat columns, respectively. This is determined by the official Joint Typhoon Warning Center (JTWC) 48-h forecast position of the TC for the nonrecurring and recurring cases, and is based on the position of the TC in the NOGAPS forecasts for the extratropical transition cases. The “class” column gives the classification: nonrecurring (nr), recurring (r), or extratropical transition (et).
the wind field and less than 1.5 K in the temperature field. The perturbation total energy is computed over the entire model domain, in order to capture the downstream impacts as well as changes to the storm itself. In the experiments conducted here, forecasts from the control and perturbed initial conditions are run out to 120 h, using the full-physics version of NOGAPS at a T79L30 resolution. These forecasts allow us to 1) compare the nonlinear perturbation growth at 2 days with what is expected from the linear SV calculation, and 2) examine the subsequent impact of these small perturbations, designed to affect the 2-day TC forecast, on the downstream environment out to 5 days.

3. Results

a. Initial-time SV sensitivity patterns

The initial-time SV sensitivity is first examined through composites. Figure 1 shows the storm-centered composites of initial-time SV sensitivity, along with analyzed 500-hPa streamline composites for the 50 nonrecurring cases and the 22 recurring cases. (In the composite and average plots, the sensitivity for individual cases has been normalized by its largest value before the averaging is done.) Consistent with the results of Peng and Reynolds (2006), both composites show a sensitivity maximum in an annulus about 500 km from the storm center. In the recurring composite, the sensitivity pattern extends to the west-northwest (there is just a hint of this extension in the nonrecurring composite). This northwest extension is also apparent in Peng and Reynolds (2006) composites for irregular-motion storms, but the extension is significantly more pronounced here. This difference may have to do with the fact that Peng and Reynolds included all irregular-motion cases in their composite, not just recurring cases. The 500-hPa streamlines indicate that, on average, the anticyclonic circulation to the east of the storm is stronger in the recurring composite, not just recurving cases. The streamlines indicate that, on average, the anticyclonic circulation to the east of the storm is stronger in the recurring composite than in the nonrecurring composite. The streamlines also indicate that the SV sensitivity maximum to the northwest in the recurring composite may be associated with upstream troughs, although the composite averaging has smoothed out any details and the composite trough is weaker than individual systems. The final-time SV structures (not shown) are case dependent, but usually exhibit a dipole pattern around the storm center, indicating that the SVs affect the final-time position of the storm (in both along-track and cross-track directions), although the intensity of the storm is also affected in some cases.

When planning for adaptive observing field programs, it is of interest to know not just where the sensitivity is located relative to the storm center, but where the sensitivity is likely to be located geographically. Figure 2a shows the sensitivity averaged over the 22 recurring cases and the averaged analyzed 500-hPa streamlines (note this is not a storm-centered composite). Figure 2b shows the individual locations of the storm centers along with primary sensitivity maxima and secondary sensitivity maxima (when spatially distinct from the primary maxima, and at least 50% of the value of the primary maxima). In many cases, the forecasts of the storms during recurvature will be quite sensitive to the initial state well upstream (as far as 4000 km) over the Asian mainland (in these cases, a secondary maxima is often present in a near-storm location). This suggests that in some cases, adaptive observing by aircraft may not be feasible, and other adaptive observing system components, such as additional or off-time radiosondes, or high-density satellite winds, may be better suited for sampling these sensitive regions. These results are consistent with those found for the Atlantic (Reynolds et al. 2007), where storms with significant nonlocal sensitivity were often sensitive to changes in the initial state to the northwest of the storm, usually over North America.

As averaging of the fields will smooth out the details of the sensitivity patterns and the environmental flow, one TC, Shanshan, will be used as an example to illustrate how storm-relative sensitivity is modulated during the TC life cycle. Shanshan formed on 10 September 2006, reached a minimum pressure of 919 hPa, and, after heading north and west, recurved to the northeast east of Taiwan, undergoing extratropical transition on 19 September. Figure 3 shows the SV sensitivity of TC Shanshan for 2-day forecasts starting at 0000 UTC 13, 14, 15, 16, and 17 September 2006. Early in Shanshan’s life cycle, before recurvature (13 September; Fig. 3a), the sensitivity is concentrated in an annulus around the storm center, much as reflected in the composite (Fig. 1). The sensitivity is strongest to the southeast of the storm, in a region of southerly flow. As the storm starts to recurve (14 and 15 September; Figs. 3b,c), the sensitivity remains large in the region southeast of the storm, but a secondary maxima develops to the northwest, and the 500-hPa streamlines indicate that this region is associated with an approaching trough. Later during recurvature (16 September; Fig. 3d), another region of sensitivity, even farther upstream than the approaching trough, starts to become important. These results are consistent with those of Wu et al. (2009b) who used
ADSSV and potential vorticity diagnostics to study the sensitivity of forecasts of Shanshan from 14, 15, and 16 September. They also find significant sensitivity to the southeast of the storm, with increasing sensitivity to the northwest, associated with the incoming trough, for the later forecasts. The sensitivity during extratropical transition (17 September; Fig. 3e), is very complex, with the strongest sensitivity associated with an upstream ridge.

Examination of other storms, not shown, indicates that the patterns seen for Shanshan before and at the start of recurvature are quite typical. However, the patterns that occur when the storm is forecast to undergo extratropical transition are very complicated and case dependent. Figure 3f shows the final-time (48 h) SV sensitivity that evolves from the initial-time SVs from 17 September, shown in Fig. 3e. The three-lobed structure is more complicated than the storm-centered dipole that is typical of final-time SVs for TCs before extratropical transition (not shown here, examples shown in Peng and Reynolds 2006). The complicated final-time structure indicates that the initial perturbations

**FIG. 1.** Vertically integrated SV sensitivity composited about the storm center (shading, values are nondimensional) for (a) nonrecurring cases and (b) recurring cases. Also shown are the composite analyzed 500-hPa streamlines.

**FIG. 2.** (a) Vertically integrated SV sensitivity averaged for the recurring storms (shading, values are nondimensional) and analyzed average 500-hPa streamlines. Note that this is not a storm-centered composite. (b) The positions of the recurring storms at initial time (TC symbols) and the location of the primary (circles) and secondary (Xs) sensitivity maxima.
that impact the TC during recurvature are also impacting midlatitude features in the vicinity of the storm. A detailed examination of the dynamics involved in the extratropical transition process is beyond the scope of the study. However, the complex nature of the sensitivity patterns shown here is consistent with several previous careful studies on the dynamics of extratropical transitions (e.g., Harr et al. 2000; Klein et al. 2002; Ritchie and
showing that the process is very case dependent and sensitive to the details of the interaction of the storm with the extratropical environment.

For adaptive observing purposes, one may be interested in more details than the vertically averaged total energy sensitivity. For example, the altitude of maximum sensitivity may be of interest. In addition, one may wish to separate total energy into kinetic energy and potential energy components, which will reflect sensitivity to the wind and mass fields, respectively. Figure 4 show the potential and kinetic energy components of the SV sensitivity for TC Shanshan before (13 September) and during (16 September) recurvature. Before recurvature (Figs. 4a,b), the forecast of the storm is sensitive to changes in both the mass and wind field, although sensitivity to the wind field is stronger than to the mass field, particularly to the south of the storm. During recurvature, the remote sensitivity to the northwest is dominated by the potential energy component (Figs. 4c,d). These differences are typical and reflected in the average values of SV energy as a function of pressure level for recurving and nonrecurveing cases shown in Fig. 5. In these plots, the kinetic energy is separated into rotational and divergent components. For the nonrecurveing average (filled symbols), the rotational kinetic energy component is larger than the potential energy below 500 hPa and smaller above 500 hPa (vertically averaged potential energy and kinetic energy contributions are comparable). For the recurving average (open symbols), the potential energy component is larger than the kinetic energy component at most levels, although the kinetic energy component is still significant. The divergent kinetic energy component is relatively small for both the recurving and nonrecurveing cases, though slightly larger for the nonrecurveing cases (perhaps reflecting the relatively lower latitudes at which these cases occur).

All results shown so far have been for dry SVs. Dry SVs have been used in previous intercomparison studies (Majumdar et al. 2006; Reynolds et al. 2007; Wu et al. 2009a) and these will be used for T-PARC. However, it is of interest, of course, to see the impact of inclusion of moisture in the SV calculation. Figures 4e,f show the potential and kinetic energy components of the initial-time SV sensitivity for the forecast for TC Shanshan from 16 September when the SV calculation includes large-scale precipitation. As found previously (Peng and Reynolds 2006, Peng et al. 2007), the inclusion of moisture results in similar sensitivity patterns (cf. Figs. 4e,f to 4c,d), but the patterns tend to be weighted more toward the near-storm environment and away from the remote regions. The similarity between the moist and dry SVs away from the immediate storm environment is encouraging in one sense, as the remote regions deemed most sensitive are not significantly changed by the inclusion of moisture. However, the inclusion of moisture in the SV calculation does change the relative magnitudes of the remote and local sensitivity maxima, and therefore may have implications for adaptive observing if, for example, resources must be shared between the two regions.

b. Evolved SV sensitivity perturbations

If SVs are to be used for adaptive observations, then the perturbation growth estimated by the tangent linear calculation should reflect the actual nonlinear perturbation growth, or at least the case-to-case differences in perturbation growth, reasonably well. While the assumption of linear perturbation growth and the neglect of diabatic processes result in SV perturbations that are almost certainly not optimal for the full system, one should none-the-less examine how well perturbation growth in the full nonlinear system matches the expected linear growth from the dry tangent linear system. To study this, small SV-based perturbations (with maximum temperature perturbations less than 1.5 K and maximum velocity perturbations less than 2 m s$^{-1}$) are added to the control analysis. Nonlinear forecasts are then run from the control and perturbed analyses, and the total energy of the difference between the two nonlinear runs (nonlinear perturbation total energy) is compared to the perturbation that is evolved using the tangent linear model (linear perturbation total energy). These calculations have been performed for all 72 TC cases considered thus far, and for an additional 12 cases corresponding to when the TC undergoes extratropical transition in the 2-day forecast. The linear and nonlinear 2-day perturbation total energy (TE) for all of these cases is shown in Fig. 6. The percent difference, \[100 \times (\text{nonlinear TE} - \text{linear TE})/\text{linear TE}\], between the two, is also shown. At least as far as the large peaks are concerned, the two curves follow each other fairly well. In other words, when linear perturbation growth is relatively large, nonlinear perturbation growth is likewise relatively large. The correlation between the two curves is 0.78. Inspection of the percent difference curve shows that there can be significant differences, especially when the linear error growth is relatively small. This may be due to contamination of the signal by small-scale noise in remote regions, an issue discussed in Hodyss and Majumdar (2007). However, overall, the results shown in Fig. 6 indicate that the peaks in linear perturbation growth are well reflected by peaks in nonlinear perturbation growth, and that the nonlinear perturbation growth is relatively small for both the recurving and nonrecurveing cases, though slightly larger for the nonrecurveing cases (perhaps reflecting the relatively lower latitudes at which these cases occur).
growth is usually at least as large as the growth expected from the linear calculation.

The four prominent peaks between case numbers 45 and 70 are all associated with recurving TCs that eventually undergo extratropical transition, suggesting that recurving cases are associated with faster perturbation growth than nonrecurving cases. This turns out to be true on average, but not for all cases. Figure 7 shows the...
average nonlinear perturbation total energy as a function of forecast time, averaged for the 22 recurving and 50 nonrecurring cases (the curve for the 12 extratropical transition cases, not shown, is very similar to the curve for the recurving cases). Here, the nonlinear integrations are extended past the 2-day verification time (to 5 days) in order to examine the impact these changes have on forecast evolution beyond the optimization period. Clearly, on average, the recurving cases are associated with significantly larger perturbation growth over the 5-day period.3 However, of course, averages can mask case-to-case details that may be important.

Examination of the 2- and 5-day nonlinear perturbation TE for all of the cases (Fig. 8) indicates a strong correlation (0.86) between 2- and 5-day growth rates, as reflected in the good correspondence between the two curves on a log scale. The correlation between the 5-day nonlinear growth rate and the 2-day linear growth rate is 0.74, indicating that the 2-day linear calculation itself has some utility in predicting the 5-day nonlinear perturbation growth. The 34 recurving and extratropical transition cases are denoted by the filled symbols. Most of the largest perturbation growth cases (6 out of the top 7, 16 out of the top 25) fall into this group, indicating that the largest perturbation growth is often associated with recurving or transitioning storms. However, not all recurving or transitioning cases correspond to larger error growth. The text panels in Fig. 8 identify the storm number [western Pacific (WP##) or central Pacific (CP##)] and date [month and day (MMDD)] corresponding to the largest perturbation growth for the different recurving storm cases. Storms WP02 (Chanchu), CP01 (Ioke), WP14 (Shanshan), WP16 (Yagi), and WP21 (Soulik) have relatively large perturbation growth, while storms WP04 (Ewiniar), WP09 (Maria), and WP11 (Wukong) do not. This may be due, in part, to seasonal and also short-term synoptic-scale variations in the baroclinicity of the midlatitude background flow, which, when strong, may facilitate perturbation energy growth and propagation downstream from the TC disturbance.

3 The growth rate for days 0–2 is 2.39 and 2.06 day$^{-1}$ for the recurving and nonrecurring cases, respectively. For days 2–5, the growth rates are 0.65 and 0.63 day$^{-1}$ for the recurving and nonrecurring cases, respectively.
Figure 9 shows a latitude–time plot of the Eady index of baroclinic instability, $\sigma_E$ (Lindzen and Farrell 1980; Hoskins and Valdes 1990), which gives the expected Eady growth rate maximum, averaged over eastern Asia and the North Pacific (100°E–160°W), from 1 July to 31 October. The Eady index is defined as

$$\sigma_E = 0.31 \frac{f}{N} \frac{dV}{dz}, \tag{5}$$

where $f$ is the Coriolis force, $V$ is the magnitude of the vector wind, and

$$N = \sqrt{\frac{g}{\theta} \frac{d\theta}{dz}} \tag{6}$$

is the Brunt–Väisälä frequency. Here, the Eady index is calculated over the 300–1000-hPa layer. Midlatitude baroclinic instability is relatively weak in July and August during which recurring storms WP04, WP09, and WP11 have relatively small 5-day perturbation growth. Midlatitude baroclinic instability strengthens in September and October, during which recurring storms CP01, WP14, WP16, and WP21 have relatively large 5-day perturbation growth. (Midlatitude baroclinic instability in May, during TC WP02 Chanchu, not shown, is stronger than in July and August.) Figure 10 shows the Eady index of baroclinic instability over the North Pacific for two time periods. The first period, in mid-August, coincides with the recurvature of two storms, Maria (WP09) and Wukong (WP11) that did not exhibit strong 5-day perturbation growth. The second period, in mid-September, coincides with the recurvature of two storms, Shanshan (WP14) and Yagi (WP16) that both had relatively large 5-day perturbation growth. The baroclinic instability over the North Pacific is much stronger for the September average than for the August average. This result is consistent with that of Cardinali et al. (2007) who found that, for verification regions over Europe (as opposed to the TC-based verification regions used here), observations in sensitive regions during extratropical transitions had larger downstream impacts when the downstream flow was more unstable. A case study from Shanshan, discussed below, illustrates how small perturbations to the TC may have a significant impact downstream via energy propagation through the midlatitude baroclinic region. While the TCs enter the midlatitudes in the exit region of the jet in August, they enter the midlatitudes near the entrance region of the jet during September. The larger impact of the September storms is consistent with the results of Klein et al. (2002), who found that the environment is favorable for ET development when upper-level TC outflow enhances the equatorward entrance region of a downstream jet. These results are also consistent with those of Riemer et al. (2008), who studied the downstream impact of TCs during extratropical transitions using a full-physics atmospheric model with idealized initial conditions. They showed that the evolution of the upper-level pattern can be interpreted as a Rossby wave train excited by the interaction of the TC with the midlatitude
jet and its subsequent propagation. Riemer et al. (2008) found slower energy propagation and shorter wavelengths when the midlatitude jet is weaker.

Another way to illustrate similar information is to plot the location of the TCs in the 2-day forecast with circles, and relate the 5-day nonlinear perturbation total energy to the circle size, as shown in Fig. 11. Note that the circle size corresponds to the 5-day nonlinear perturbation total energy evolved from the initial-time SVs, not to storm size or intensity. Storm intensity is actually reflected in the color of the circles. All the largest circles occur north of 23°N, although not all circles north of 23°N are large. Strong storms that do not recurve, but instead make landfall over Asia, have relatively small 5-day perturbation growth. Similar patterns are found for the same type of figure based on 2-day perturbation total energy rather than the 5-day perturbation total energy (not shown).

Forecasts for Shanshan from 13 September (before recurvature) and 16 September (during recurvature) are again used as examples, in this case to contrast the perturbation growth and downstream propagation before and during recurvature. Figure 12 shows the 500-hPa meridional wind component of the nonlinear perturbation at 0, 72, and 120 h forecast times for forecasts started on 13 and 16 September. The control forecast SLP is also shown. For the 13 September case, small perturbations (less than 2.0 \( \text{m s}^{-1} \)) surround the storm at initial time. After 72 h, the perturbations have grown significantly, with maximum values now over 10 \( \text{m s}^{-1} \) (note the change in contour interval). In this case, the perturbations are primarily confined to the storm’s vicinity even after 72 h. At 120 h, there are some perturbations downstream over the North Pacific and North America, although the lack of coherence in the signal may indicate that these perturbations may be due to the growth of small-scale noise in remote regions (Hodyss and Majumdar 2007). For the 16 September case, the initial perturbations are in the vicinity of the storm as well as to the northwest. By 72 h there are significant perturbations (\( > 5 \text{ m s}^{-1} \)) over the North Pacific and by 120 h significant perturbations over the North Pacific (\( > 20 \text{ m s}^{-1} \)) and North America (\( > 10 \text{ m s}^{-1} \)). Time–longitude plots (Fig. 13) of the 200-hPa meridional wind averaged from 30° to 40°N and from 50° to 60°N are useful for illustrating the downstream phase and energy propagation. Downstream phase and energy propagation within 5 days are much stronger for the recurving case (Figs. 13b,d) than for the nonrecurring case (Figs. 13a,c), although downstream propagation might have occurred beyond 120 h in the nonrecurring case. These time–longitude diagrams are qualitatively similar to those shown by Riemer et al. (2008) for their idealized initial condition experiments, despite the fact that the background midlatitude jet in our example is not zonally uniform. These nonlinear diagnostics illustrate how small initial perturbations, constructed to have an optimal impact on the 2-day forecast of the TC, may result in significant perturbations well downstream from the storm itself after 5 days. These results are consistent with those of Harr et al. (2008) and Anwender et al. (2008), who use ensembles to demonstrate the impact of uncertainty in the forecast of the ET of storms on the downstream troughs and ridges. These types of nonlinear SV-based diagnostics may prove useful for adaptive observing guidance when downstream impacts of the TC, as well as the TC forecast itself, are of interest.

c. Pseudoinverse corrections

The results described above demonstrate the significant downstream impact that may occur from SV-based perturbations for recurring storms. As adaptive observing aims to improve forecasts by improving analyses
in sensitive regions, it is also of interest to examine the theoretical impact that reducing the error that projects onto the SVs may have on the forecast. This is accomplished through a series of pseudoinverse calculations, following Gelaro et al. (1998), Buizza and Montani (1999), and Reynolds and Rosmond (2003). Let the control forecast error $e_{t_5} = M(\mathbf{x}_{ctl_0} - \mathbf{x}_{ver_0})$, where $\mathbf{x}_{ctl_0}$ and $\mathbf{x}_{ver_0}$ represent the control analysis and the verifying analysis, respectively. For linear error growth and a perfect model (and ignoring the final-time analysis error), the initial error can be propagated forward in time following (1) ($e_t = \mathbf{L}e_0$). If $\mathbf{L}$ is not singular, then, following (2):

$$e_0 = \mathbf{L}^{-1}e_t = \mathbf{E}^{-1/2}\mathbf{V}\mathbf{D}^{-1}\mathbf{U}^T\mathbf{E}^{1/2}e_t. \quad (7)$$

In large models, it is not feasible to calculate all SVs (indeed, $\mathbf{L}$ is most probably ill conditioned), but one can calculate a pseudoinverse based on a subset (in this case, three) of the leading SVs:

$$e_{SV_0} = \mathbf{E}^{-1/2}\mathbf{V}_3\mathbf{D}_3^{-1}\mathbf{U}_3^T\mathbf{E}^{1/2}e_t. \quad (8)$$

where the subscript 3 denotes a matrix composed of only the first three SVs of $\mathbf{L}$, and the initial- and final-time metric, $\mathbf{E}$, is dry total energy. Thus, $e_{SV_0}$ corresponds to the component of the analysis error that lies in the rapidly growing SV subspace (in a linear and perfect-model context). In vector form, the equation is

$$e_{SV_0} = \mathbf{E}^{-1/2}\sum_{n=1}^{3} v_n d_n^{-1}(u_n\mathbf{E}^{1/2}e_t). \quad (9)$$

This pseudoinverse perturbation is subtracted from the control analysis to form a perturbed (presumably improved) analysis (i.e., $\mathbf{x}_{bet_0} = \mathbf{x}_{ctl_0} - e_{SV_0}$). From this perturbed analysis, a second, presumably better, nonlinear forecast is run [i.e., $M(\mathbf{x}_{bet_0}) = \mathbf{x}_{bet}$], and the corrected nonlinear forecast error is

$$e_{nl_0} = \mathbf{x}_{bet} - \mathbf{x}_{ver}. \quad (10)$$

If the leading SVs dominate the forecast error and perturbation growth is linear, the corrected analysis should produce an improved forecast at the verification time (in this case, 48 h). There is an expectation that an improved forecast at 48 h should lead to improvements at later forecast times (improvements to forecasts beyond the verification times were found for related types of experiments using “key analysis error” corrections by
In addition to the “better” forecasts, presumably “worse” forecasts are run from an analysis to which the SV-based perturbation has been added instead of subtracted (i.e., $x^\text{wor}_0 = x^\text{ctl}_0 + e^{\text{SV}}_0$).

Pseudoinverse calculations are performed for 13 cases corresponding to three recurving storms that were shown to have significant downstream impacts in section 3b. These include case numbers 49–53, 55–59, and 63–65 (Shanshan, Yagi, and Soulik, respectively). As was the case with the nonlinear perturbation experiments, the nonlinear forecasts in the pseudoinverse calculation are performed using a reduced-resolution (T79L30) full-physics...
version of NOGAPS. Because the impact region of these perturbations can be quite extensive by 5 days in some cases, we calculate the difference in forecast errors (using the total energy metric) over the entire Northern Hemisphere midlatitudes (20°–70°N). As the initial perturbations are very localized as compared with the verification region, one would not necessarily expect a large reduction in error over the large domain. Nevertheless, based on previous results showing that extratropical transitions can have a significant impact on hemispheric forecast skill (i.e., Jones et al. 2003), one would hope to see some impact reflected in the hemispheric skill scores.

Figure 14 shows the average percent reduction in the Northern Hemisphere forecast error TE as a function of forecast time due to the SV-based correction: 100 × [TE(control error) − TE(corrected error)]/TE(control error). The average percent reduction for the corrected (better) cases is 2.8 at 2 days and reaches a maximum of 4.8 at 4 days. There is significant case-to-case variability, with the maximum percent correction reaching 16.8% at 4 days, and in one case the “improved” forecast was actually worse by 7.2% after 5 days. Also included are average results for the worse forecasts (to which the SV-based corrections were added rather than subtracted). As expected, these perturbations result in degraded forecasts (negative percentage reductions correspond to increases in forecast error), although it is not immediately apparent why the average increases in forecast error would be greater than the corresponding reductions for the better cases. Examination of this limited number of cases did not show a clear relationship between the percent correction and recurvature or downstream impact. This is not unexpected, as the details of any particular pseudoinverse correction will depend upon the appropriateness of the linearity assumption, and the specific projection of the error onto the leading three SVs for the particular case. Nevertheless, it is encouraging to see average positive impact from the pseudoinverse corrections even when measured on a hemispheric scale.

4. Summary and discussion

The sensitivity of recurving western North Pacific TC forecasts to changes in the initial state and subsequent
downstream impacts have been investigated using SVs. Both recurving and nonrecurving TC forecasts are sensitive to changes in the initial state in an annulus about the storm center, but during recurvature TCs also show significant sensitivity to the northwest. A similar northwest extension of the sensitivity is found for the Atlantic TCs with remote sensitivity maxima (Reynolds et al. 2007), and, to a lesser extent, for the nonstraight-moving storms considered by Peng and Reynolds (2006). For the Atlantic storms, this resulted in significant sensitivity over North America. In this case, for the western North Pacific, this results in significant sensitivity over the Asian mainland, sometimes as far as 4000 km to the northwest of the storm itself. This has implications for adaptive observing, as it may not be feasible to target these upstream sensitive regions with aircraft, and other observing system components, such as high-density satellite winds, or off-time radiosondes, may be more suitable. Partitioning the SV total energy sensitivity into kinetic energy and potential energy components indicates that for nonrecurving cases, the kinetic energy in the vicinity of the storm is the dominant component, although potential energy sensitivity is also significant. For recurving cases, the sensitivity to the northwest of the storm is dominated by the potential energy component of the total energy.

The downstream impacts of the TCs have been investigated through the comparison of control nonlinear forecasts and nonlinear forecasts run from analyses to which SV-based perturbations have been added. These SVs are optimized to have an impact on the 2-day forecast of the storm, not for downstream impact. Nevertheless, these small perturbations may continue to grow and propagate downstream quickly beyond the 2-day optimization period, and may have a significant impact on the flow over the North Pacific and North America within 5 days. There is very significant case-to-case variability in the 5-day nonlinear perturbation growth, and the largest perturbation growth occurs for recurving cases. These results are consistent with plumes of forecast uncertainty spreading downstream from ET events found by Harr et al. (2008) and Anwender et al. (2008). However, not all recurring cases have large 5-day perturbation growth. In general, smaller downstream perturbation growth is found for recurring cases that occur in July and August, when the baroclinicity of the midlatitude environment is weaker than in September and October. The SV-based pseudoinverse corrections also exhibited significant case-to-case variability, reaching a maximum percent reduction in the hemispheric forecast error in one case of 16.8% after 4 days. As expected from previous studies detailing the sensitive and complex interaction between a TC undergoing extratropical transition and the midlatitude jet (e.g., Harr et al. 2000; Klein et al. 2002; Ritchie and Elsberry 2007; McTaggart-Cowan et al. 2003, 2004; Hart et al. 2006; Riemer et al. 2008), it is not surprising that TC sensitivity and downstream impact are very case dependent. This case dependence suggests that perturbed forecasts of this type may be useful in adaptive observing campaigns where the concern is not only the direct impact of the TC itself, but also the indirect impact of the TC on the downstreamflow as it recurves and undergoes extratropical transition. (The relatively high correlation between the 5-day perturbation growth rates and the 2-day perturbation growth rates suggest that the latter may provide some information on the expected magnitude of the 5-day impacts.) These types of storm-specific perturbations may provide information on potential downstream impacts of recurving storms that is supplemental and complementary to the current global ensemble systems. The T-PARC field campaign will provide an excellent opportunity to study these interactions and test adaptive observing capabilities.

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