Displacing Potential Vorticity Structures by the Assimilation of Pseudo-Observations

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ABSTRACT

Classic formulations of variational data assimilation in amplitude space are not able to directly handle observations that measure the geographical positions of meteorological features like fronts and vortices. These observations can be derived from satellite images, as is already the case for tropical cyclones. Although some advanced data assimilation algorithms have been specifically designed to tackle the problem, a widespread way of dealing with this information is to use so-called bogussing pseudo-observations: user-specified artificial observations are inserted in a traditional data assimilation scheme. At the midlatitudes, there is a relationship between dry intrusions in water vapor images and upper-level potential vorticity structures. Some prior work has also shown that it was possible to automatically identify dry intrusions with tracking algorithms. The difference of positions between model and image dry intrusions could therefore be used as observations of the misplacement of potential vorticity structures. One strategy to achieve the displacement of potential vorticity anomalies is to sample them, and assimilate the values at displaced locations. The uncertainty of these pseudo-observations is left as a tuning parameter to try to make the displacement both effective and robust. A simple one-dimensional assimilation model is used to study the displacement of curves defined by Gaussian humps. The concept is then illustrated in realistic examples from real synoptic systems, where pseudo-observations of potential vorticity are incorporated in a global variational data assimilation scheme. Overall and despite reasonable optimization, the results contain artifacts. This suggests that the use of pseudo-observations to displace identifiable structures is not an effective strategy.

1. Introduction

Satellite data provide a huge amount of data that are still underused by numerical weather prediction (NWP) systems. There are various explanations for this, including the fact that operational data assimilation schemes are mainly based on quasi-linear frameworks. Most of the data assimilation theory consider observations as “pixels” (e.g., mathematical data vectors where all the structural information is handled through the specification of the background and observation error covariance matrices; Bouttier and Courtier 1999). This approach has overall worked well together with the assimilation of raw satellite data in global models with the four-dimensional variational data assimilation (4D-Var) algorithm (Simmons and Hollingsworth 2002).

At the same time, some important structured information is visible in the images from geostationary satellites. Human vision can detect and track the dynamical evolution of meteorological “coherent structures” such as fronts, clouds, vortices, etc. This information is usually not specifically handled in operational data assimilation schemes (Titaud et al. 2010) despite the fact that coherent structures are often linked with high impact weather. For instance, tropical cyclones (TCs) have major societal impacts. Potential vorticity (PV) structures are also linked with cyclogenesis at the midlatitudes (Hoskins et al. 1985). In fact, exploratory initialization techniques have been specially developed to cope with these two examples: a widespread method is to deduce the position and strength of TCs from microwave and geostationary infrared images (Olander and Velden 2007). Then, a simple model is used to define pseudo-observations of wind along concentric circles (Heming 1994), which are then assimilated. Despite some caveats, the bogussing of TCs has proved beneficial in different NWP models for short-range forecasts (Heming and Radford 1998).

For cyclogenesis and PV, alternative initialization methods may also provide additional information. The prediction of midlatitude cyclones has been shown to be sensitive in a nonlinear way to the initial PV (Beare et al.}
pseudo-observations as for TCs. For instance, Gue\'rin et al. (2006) have assimilated pseudo-observations of PV that are derived from a hand drawing of the tropopause by forecasters. However, it is unclear how this drawn tropopause is sampled by the pseudo-observations and if the correlations of pseudo-observations errors can be safely neglected.

The aim of this paper is to study the displacement of PV structures by the use of pseudo-observations in a classic data assimilation scheme. The strategy is basic: the PV structure is sampled, and then pseudo-observations are assimilated at displaced positions. The uncertainty of these observations is left as a tuning parameter to try to make the displacement both effective and robust, according to some error measure. Section 2 sets up an idealized framework to tackle the problem. A more realistic framework is studied in section 3: the misplacement is automatically deduced from the comparison of the result of the WV tracking algorithm on model and satellite images. Pseudo-observations of PV are then built and incorporated in a data assimilation scheme to try to correct for this misplacement.

2. One-dimensional displacement of PV structures

This section assumes that some information regarding the geographical position of a PV anomaly is available from the WV imagery. The methodology is to sample the PV anomaly with pseudo-observations, and then to assimilate them at the displaced locations. The main issues are to understand how many observations are needed, where they should be put and what observation error statistics should be specified. To study the effectiveness of such a strategy, objective measures of performance are required. These measures will be designed to quantify how well the analysis fits the truth (defined as the displaced background). Different shapes will be considered for the PV structure that we aim to displace.

a. The model

The PV anomaly is defined here as a monopolar Gaussian hump of length scale $L_o$ (typically a few hundred kilometers). The $s$ denotes the spatial coordinate that measures the distance along the curve joining the centers of the background and true PV anomalies. The misplacement error will be taken as the quantity $2\delta_s$. The background PV field and the true PV fields are written as

$$x_\delta(s) = e^{-[(s+\delta_s)/L_o]^2},$$

$$x(s) = e^{-[(s-\delta_s)/L_o]^2}.$$  

The two fields have also been normalized by the maximum of the PV anomaly. A general sketch of the problem is shown in Fig. 1a.

All fields are now discretized on a regular grid on a finite domain $[-W, W]$ and written in vector form, in the
framework of data assimilation. Observation errors statistics are taken first to be uncorrelated because this is often assumed in operational data assimilation. Background error covariances are taken as uniform, isotropic, and of Gaussian shape as in Desroziers and Ivanov (2001). Under these hypotheses, the observation error covariance matrix $R$ and the background error covariance matrix $B$ take the following form:

$$R = \sigma_o^2 I,$$

$$B = \sigma_b^2 e^{-\left(\frac{s_i}{L_b}\right)^2} I_{i,j},$$

where $s_{i,j}$ is the distance (measured by $s$) between some points $i, j$, and $L_b$ is length scale of the model for the background error correlations. Here $\sigma_o$ and $\sigma_b$ are the (uniform) observation and background error standard deviations, respectively.

The vector of pseudo-observations $y$ is a sample of the background vector $x_b$ at some positions $s_o$:

$$y = x_b(s_o).$$

These pseudo-observations are displaced at the locations:

$$s_d = s_o + 2\delta_s,$$

where $E_2$ measures the root-mean-square error of the analysis with respect to the pseudotruth. Here $E_a$ specifically measures the amplitude of the anomaly, as defined by its extrema values. Argmax is the argument of the function max, and $E_d$ therefore measures the effective displacement of the anomaly. The perfect value for these metrics is 0 (when $x_a = x_t$), and the value when no displacement has been achieved ($x_a = x_b$) is 1 (except for $E_a$), albeit worse values can be reached for highly distorted analyses.

$$E_2 = \sqrt{\frac{\int_{-W}^{W} (x_{a} - x_t)^2 ds}{\int_{-W}^{W} (x_{b} - x_t)^2 ds}}, \quad E_a = |\max(x_{a}) - \max(x_{t})| + |\min(x_{a}) - \min(x_{t})|,$$

$$E_d = \frac{|\text{Argmax}(x_{a}) - \text{Argmax}(x_{t})|}{|\text{Argmax}(x_{b}) - \text{Argmax}(x_{t})|},$$

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$$s_d = s_o + 2\delta_s,$$

where $H$ is the observation operator, which is in our case the evaluation or linear interpolation of the field at the observation locations $s_d$.

b. Measuring the performance of the displacement

In this linear framework, the analysis minimizes the variance of analysis errors, given that $B$ and $R$ are correctly specified. To evaluate the effectiveness of the displacement, other error measures have to be designed. The three following error metrics are introduced:

$$x_a = x_b + BH^T(\text{HBH}^T + R)^{-1}(y - Hx_b),$$

FIG. 1. General sketch of the one-dimensional data assimilation framework. The background $x_b$ (thin solid line, on the left) is sampled at the positions $s_o$ (gray filled circles). These pseudo-observations are then displaced at the positions $s_d$ (circles). The analysis $x_a$ (solid bold gray line, on the right) is then compared to the pseudotruth $x_t$ (solid bold line, on the right). (a) Configuration at lower levels with two observations of displacement. (b) Configuration at upper levels with three observations. The square marks the local minimum of PV detected along the 1D plot, which is used to defined the tropopause PV gradient height $\Delta PV$ and the position of the first sampling (leftmost gray circle). The second sampling is at the detected maximum of PV, and the third one is $2\delta$ farther.
The last concept to be introduced concerns reference and optimal configurations. The increment produced by data assimilation is the difference between two distant Gaussian humps and has two extrema. This spatial structure requires at least two observations (as background correlations are also Gaussian shaped). In the very special case where PV length scales match the length scales of PV background errors ($L_b \approx L_a$) and where the observations are taken as perfect ($s_o^2 = 0$), there is a configuration that exactly provides a perfect analysis ($E^a = E^d = 0$) with only two observations. It corresponds to

$$s_o = \{ -\delta, \delta \}.$$  \hspace{1cm} (11)

Figure 1a depicts this reference configuration: the maximum of the anomaly is sampled, together with the value of the background field $2\delta$ ahead. Figure 2b shows that in this specific scenario, the reference configuration gives a perfect result, and is therefore a natural choice.

More generally, the analysis error $E$ is a function of the following parameters: the number of observations $N_{obs}$, the position of the sampling pseudo-observations $s_o$, the image-derived misplacement $2\delta$, the error standard deviations $\sigma_o$ and $\sigma_b$, the background error length.
scale $L_b$, and the length scale of the PV anomaly $L_a$. Not all of these parameters have the same status. The shape of the anomaly ($L_a$) and the image-derived misplacement ($2\delta_o$) may have values that vary from case to case. The $2\delta_o$ will be derived from the imagery and will be known. It the following idealized study, it will be set at a 200-km value that will be justified by the real experiments made in section 3. Estimates of $\sigma_b$ can be diagnosed through the method of Andersson et al. (2000), and $L_b$ can be computed using formulas from Pannekoucke et al. (2008). These parameters will be referred to as “external parameters” of the problem. One wishes to minimize $E$ with respect to the “internal parameters” $N_{\text{obs}}$, $s_o$, and $\sigma_o$. The argument of this minimum will be named an optimal configuration $(N_{\text{obs}}, s_o, \sigma_o)$. It depends on the chosen norm $E$ and on the values of the external parameters $\delta_o$, $\sigma_b$, $L_a$, and $L_b$. Because of the low-order model, the minimization can be achieved through a brute exhaustive enumeration. Although it has also proved interesting to derive optimal configurations for different norms, for the sake of simplicity the mean norm $E = \frac{1}{3}(E_2 + E_a + E_d)$ will be used in the rest of the paper. $E_2$, $E_a$, and $E_d$ penalize solutions that have respectively bad shapes, wrong extrema, and wrong displacement. The best results have been obtained with the mean norm that balances all these properties.

c. The case of two pseudo-observations

This section is restricted to the case where two pseudo-observations are assimilated. Figure 2 shows the optimal configurations obtained for different values of the external parameters $L_b$, $L_a$, and some values of the ratio $\sigma_o/\sigma_b$. In this case, only the position of the observations is optimized. The optimal configurations perform as well or better than the configuration of reference, allowing for a better displaced maximum. For instance, when $L_b < L_a$ (Figs. 2a,d,g), the optimal analysis is closer to the truth and offers a better coverage of the displacement. When $L_b > L_a$ (Figs. 2c,f,i), the optimal analysis is less distorted than the referenced one. In particular, the minimum occurring ahead of the anomaly is less pronounced in Fig. 2f in the reference analysis.

This comparison highlights the drawback in determining an optimal configuration: it is sensitive to the external parameters. Even for the same value of $L_a/L_b$, it does depend on the error standard deviations (Figs. 2c,f,i). For a same value of $\sigma_o/\sigma_b$, it may have strong (Figs. 2d,e,f) or weak (Figs. 2g,h,i) sensitivity to the length scales. Keeping in mind a realistic application, this means that one should provide good estimates of these parameters, which is difficult in a real data assimilation scheme. Moreover, the value $L_b$ does not have an obvious interpretation within a 4D-Var framework. The shape of the PV anomaly may also be badly described by a single value $L_a$. Given these constrains, it is important to gain understanding on how much better than the reference configuration the optimal configuration really is.

We choose reasonable ranges for the external parameters: $L_a/L_b$ varies between 0.5 and 2.0, thinking that values below 0.5 would require more observations, and values above refer to the case where the background error model is not suited to the problem of displacement. Values for the parameter $\sigma_o/\sigma_b$ range from 0 to 1.0, as values above are clearly not suited, giving too little confidence to the observations to displace the structure (Figs. 2g,h,i). Figure 3 shows the error obtained for the optimal configurations (Fig. 3a) and for the reference configuration (Fig. 3b), as a function of $\sigma_o/\sigma_b$ and $L_a/L_b$. A comparison between Figs. 3a,b highlights that the configuration of reference is in fact nearly optimal in most cases but not in the region where
Table 1. Different strategies for specifying pseudo-observations of displacement with increasing complexity. The ARPEGE 4D-Var experiments use only the most basic strategy “1.”

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Fixed parameters</th>
<th>Optimized parameters</th>
<th>Development</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Positions of the observations $s_o$ (reference)</td>
<td>None</td>
<td>Detection of the maximum of the PV anomaly Link with WV tracking algorithm</td>
</tr>
<tr>
<td></td>
<td>No. of observations (2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correlation of observation errors ($L_o = 0$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ratio $\sigma_o/\sigma_b = 0.1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Positions of the observations $s_o$ (reference)</td>
<td>Ratio $\sigma_o/\sigma_b$</td>
<td>Detection of the maximum of the PV anomaly Link with WV tracking algorithm</td>
</tr>
<tr>
<td></td>
<td>No. of observations (2)</td>
<td></td>
<td>Diagnosing of the shape of the PV anomaly $L_o$</td>
</tr>
<tr>
<td></td>
<td>Correlation of observation errors ($L_o = 0$)</td>
<td></td>
<td>Diagnosing of the background error length scale $L_b$</td>
</tr>
<tr>
<td>3</td>
<td>No. of observations</td>
<td>Ratio $\sigma_o/\sigma_b$</td>
<td>Detection of the maximum of the PV anomaly Link with WV tracking algorithm</td>
</tr>
<tr>
<td></td>
<td>Correlation of observation errors ($L_o = 0$)</td>
<td>Positions of the observation $s_o$</td>
<td>Diagnosing of the shape of the PV anomaly $L_o$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Diagnosing of the background error length scale $L_b$</td>
</tr>
<tr>
<td>4</td>
<td>Ratio $\sigma_o/\sigma_b$</td>
<td>Positions of the observation $s_o$</td>
<td>Detection of the maximum of the PV anomaly Link with WV tracking algorithm</td>
</tr>
<tr>
<td></td>
<td>No. of observations</td>
<td>Correlation of observation errors $L_o$</td>
<td>Diagnosing of the shape of the PV anomaly $L_o$</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Diagnosing of the background error length scale $L_b$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Efficient nonlinear optimization</td>
</tr>
</tbody>
</table>

$L_b > L_o$ and $\sigma_o < 0.1\sigma_b$, where this combination can provide very large errors. In this case, structure functions\(^1\) are too wide, and when a high confidence is given to the pseudo-observations, this gives distorted analyses where both the displacement and the amplitude of the anomaly are overestimated (Fig. 2c). When considering the configuration of reference, we see that there is an optimal choice for the ratio $\sigma_o/\sigma_b$, denoted as $\sigma_o/\sigma_b$, which depends on the value $L_o/L_b$ (and that is plotted as a bold curve in Fig. 3).

So far, it has been shown that within this range of variation of the external parameters, a reference configuration with a lagged value for the ratio $\sigma_o/\sigma_b$ was performing nearly as well as a configuration based on partial optimization (of the position of the observations), for a more reasonable cost, and without exhibiting sensitivity to the external parameters. This strategy is very basic, and is listed in Table 1 together with more complicated strategies associated with none, or partial, or full optimization, and relaxing the correlation between observation errors and/or the number of observations. The more minimization parameters are introduced, the better the results are, but the costlier the implementation will be in a realistic framework. Moreover, if results are highly sensitive to the parameters the methodology is likely to fail in the more complex 4D-Var case. An intuitive approach may consist in using a larger number of pseudo-observations, with the hope to better constrain the analysis. When $\sigma_o = 0$, the analysis fits the observations, such that we are facing a standard interpolation problem (with structure functions as an interpolating basis). Runge’s phenomenon (Runge 1901) shows that it is not always a good idea to increase the number of points without exerting some control over the position of the interpolating points. The question is also whereas pseudo-observations shall be designed to cover an extended area. The purpose is, however, not to glue the analysis to the background over a large region anyway: these pseudo-observations are intended to be used together with other observations that may bring some correction as well, as it is the case for TC initialization.

Figure 4 compares the analyses obtained with the different strategies (e.g., by varying the number of internal parameters over which the optimization is performed). Similar to the preceding results, it has been found that it is difficult to control numerical artifacts far from the pseudo-observations in the case when $L_b > L_o$. When on the contrary $L_b < L_o$, more complete optimization performs better, but the improvement is marginal because strategy 1 already get fair results and moreover there is a sensitivity to the external parameters.

\(^1\) Structure functions, in data assimilation, are lines (or columns) of the B matrix that are described in this section with a Gaussian shape of length scale $L_b$. 

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3. A realistic framework

a. The tracking of dry intrusions and the measurement of the displacement error

A tracking algorithm, based on Morel and Sénési (2002), has been adapted to the specific case of dry intrusions by Michel and Bouttier (2006). It features both an adaptive thresholding technique and a temporal tracking. The algorithm describes the detected dry intrusions in a low-dimensional space of characteristics such as position, contour points, apparent velocity, temperature evolution, and so on. These characteristics at a given time will be called a cell. It can be applied to both satellite images and model images [e.g., output from the radiative transfer model, version 8 (RTTOV 8), Saunders et al. 1999] having background fields as input. A robust temporal link is established between satellite and model cells that describe the same dry intrusion. Michel and Bouttier (2006) and Michel (2010) give a comprehensive description of the algorithm and of its use on WV images.

In this paper, errors in the image placement of WV cells will be directly transposed into errors in the (isobaric) placement of PV structures. Figure 5 shows the graphical summary of the detected cells at 0600 UTC 20 May 2006. In Figs. 5a,b, the cells appear to be displaced from about 300 km along a southwest–northeast direction. This roughly matches the difference of position of other noticeable structures such as the convective area appearing as a light gray shade on the bottom of the images. A similar conclusion can be drawn from the examination of the second case (Figs. 5c,d). This justifies that the center of mass of the cells can be used as measure of local misplacement (at least in image space and when the dimensions of the satellite and model cells are comparable in size). The error made on the measurement of $2\delta_v$ is not statistically taken into account.

We make the further assumption that this measurement...
could be applied at any vertical level where a surrounding PV anomaly can be detected (e.g., assuming a barotropic nature of misplacement error). The reality can be much more complex, with vertically tilted displacement errors. At the present time, WV images hardly help to refine this model at the time being because geostationary imagers only have one or two WV channels with broad contribution functions on the vertical.

b. The characteristics of background error in PV space

Section 1 has studied the effect of the assimilation of pseudo-observations of displacement in a highly simplified one-dimensional framework where the background error standard deviation ($\sigma_b$) and the length scales of the model for background error correlations ($L_b$) were both constant and uniform. For the realistic framework, a former operational version of the French global model, Action de Recherche Petite Echelle Grande Echelle (ARPEGE), is used. The analysis problem is solved using a 4D-Var incremental approach (Courtier et al. 1994). The tangent-linear and adjoint models are adiabatic and of uniform T149 horizontal resolution. Some problems of insufficient convergence were encountered with the operational settings, which were solved using a higher number of iterations (from 40 up to 100). This may be because this framework requires the assimilation process to create strong gradients of PV. To compare results with section 1, it is necessary to understand characteristics of background error in PV space in this realistic framework.

2 ARPEGE is both a global NWP model and a data assimilation system (Courier et al. 1991) with a stretched grid of resolution T358 C2.4 for the forecast, and 46 levels on the vertical (model top at 1 hPa). The stretching factor “C2.4” (Yessad and Bénard 1996) allows the grid spacing to reach 15 km over Europe.
The PV observation operator in ARPEGE is based on a simplified form of Ertel PV with a low Rossby number approximation (Guérin et al. 2006). The linearized PV operator has been introduced in the randomized estimation of $s_b$ in observation space introduced by Andersson et al. (2000). Belo-Pereira and Berre (2006) have introduced an economical estimate to calculate and compare the local correlation length scales. This formula has been applied to random perturbations from the background error covariance matrix from ARPEGE projected in PV space. These estimates therefore give a synthetic view of the geographical variations of the background error statistics in PV space. In a simplified quasigeostrophic framework, Snyder et al. (2003) found out that background error characteristics in PV space include scales comparable to those of the reference flow and PV variance concentrated where the gradient of the reference-flow PV is large. A similar result is shown in Fig. 6, with areas of large $s_b$ along the gradient of the reference flow (cf. Figs. 6a,b). The diagnostic for $L_b$ is
shown in Fig. 7 for both horizontal and vertical length scales. Overall larger values are found in anticyclonic areas (cf. Figs. 6a and 7a), and the length scales are stretched along the reference flow (not shown). Figure 7 also highlights a key feature of geostrophic balanced flows for large-scale variables: as shown by Lindzen and Fox-Rabinovitz (1989), horizontal scale $L$ and vertical scale $Z$ are linked through $L = (N/f_0)Z$ where $N$ is the Brunt–Väisälä frequency and $f_0$ is the Coriolis parameter.

Overall these results rather show that the one-dimensional framework from section 1 probably has limited validity because both $\sigma_b$ and $L_b$ are spatially variable for PV. In the following numerical experiments and for the sake of simplicity, a domain- and time-averaged value of the standard deviation will be used:

$$\sigma_{b_{PV}} = 0.810 + 49.7e^{-0.0148\phi},$$  

**FIG. 7.** Diagnosis of (a) horizontal and (b) vertical length scales of background error correlation in PV space at 400 hPa at 0000 UTC 1 Dec 2006 (intervals 50 km and 30 hPa, respectively).
where \( p \) is the pressure in hectopascals and \( \sigma_b \), the averaged background error standard deviation in PVU.\(^3\) Pressure has been chosen as a vertical coordinate because most observations are defined on pressure levels in the ARPEGE data assimilation scheme.

### c. A case study at a horizontal level

We have applied the methodology to the two cases of dry intrusions represented in Fig. 5, for the PV structures that are present at the 350-hPa isobaric level. This level has been chosen as the first level where the average value of PV over the surface of the cells exceeds the value 2.0 PVU used to define the dynamic tropopause (Hoskins et al. 1985). This level proved to be the same for the two cells shown in Fig. 5. The pseudo-observations for displacement are assimilated using the parameter value \( \sigma_o = 0.1\sigma \). Results are shown in Fig. 8. In both cases, the anomaly of PV appears as a localized, compact maximum in the background (Figs. 8a,d), which is correctly spotted by the definition of the pseudo-observation (marked by black dots). The increments have a bipolar structure (Figs. 8c,f). The analyses are effectively displaced, but do not show their maximums exactly at the target location of the pseudo-observations (Figs. 8b,e). The shapes of the PV anomalies in the analyses are affected by the procedure (they seem more compact in the first case, Fig. 8b, and more elongated in the second one, Fig. 8e). The amplitudes are changed, from 4.8 and 5.5 PVU for cells 1 and 2 in the background field to 4.5 and 7.1 PVU in the respective analyses. These changes in the analyses with respect to a truly displaced background have been referred to as “distortion” by Ravela et al. (2007). Figure 8 shows that the analyzed PV anomaly seems both too large in amplitude and projected too far along the displacement axis.\(^4\) The idealized framework with two observations has shown that when \( L_b < L_o \), the analysis was on the contrary not displaced enough and that its amplitude was smaller or equal.

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\(^3\) Following Hoskins et al. (1985), a convenient unit for PV is 1 PVU = \( 10^{-6} \) m\(^2\) s\(^{-1}\) K kg\(^{-1}\).

\(^4\) One reviewer pointed out that this extension is moreover not supported by in situ observations such as soundings.
When the structure functions were too wide \((L_b > L_a)\), the anomaly in the analyses had a pronounced minimum ahead (Figs. 2c,f,i). This behavior is not reproduced exactly here probably because PV fields have a weak positivity constraint that is taken into account in the 4D-Var in the relinearization of the (nonlinear) PV operator. In the 1D framework, it has been found that changing the ratio \(s_o/s_b\) was a good and cheap choice to achieve better results.

Figure 9 shows the analyses obtained for the same two cases with different values of \(s_o/s_b\), which confirms the preceding statement. In particular it is clear that when \(s_o/s_b\) decreases, both the amplitude and the displacement of the anomaly increase, especially for the second cell. The anomalies linked with cells 1 and 2 reach values of 5.0 and 8.1 PVU on the case where \(s_o = 0.01\sigma_b\), and values of 3.9 and 5.7 PVU when \(s_o = 0.2\sigma_b\). This suggests the overall value of the 1D study, and one can reasonably conclude the following:

- the structure functions are respectively slightly larger \((L_b \geq L_a)\) and much larger \((L_b > L_a)\) for the PV anomalies 1 and 2; and
- there is an optimal value for the error standard deviation, which is close to \(s_o = 0.1\sigma_b\) for the first anomaly and \(s_o = 0.2\sigma_b\) for the second one.

The first limitation of this approach is therefore the strong sensitivity of the result to the specification of \(s_o/s_b\). Also, anomalies in the analysis clearly show different spatial structures with respect to the background. This means that structure functions in 4D-Var may not map PV anomalies very well.

d. Displacement of PV structures at upper levels

Overall it has been found that PV anomalies at the tropopause level are reasonably displaced with adequate values of \(s_o\) (Figs. 8b and 9d). When examining the analyses at different levels, it appears that the vertical
correlations in the background error covariance matrix are not able to properly propagate the displacement. In particular, the analyses are also distorted at upper levels (not shown), which implies that there is a need for multiple pseudo-observations through the vertical.

The same basic strategy (Table 1) seems however to fail at the upper levels. Figure 10 shows, on the same two cases as before, the effect of assimilating two pseudo-observations at the 250-hPa level. To more directly compare the experiments with the 1D framework, the graphics are longitudinal and latitudinal 1D plots along the direction of the two pseudo-observations. The PV anomaly in the analysis is in both cases slightly overestimated. A more serious caveat is that the PV tight gradient on the left of each curve is not displaced. This behavior does not exist in the simple one-dimensional study. The geometry is indeed a bit different, as the 250-hPa level is above the dynamical tropopause. In this lower stratospheric region, there is a zone of abrupt change of PV that is located at 45°N for the first cell (Fig. 10a) and at 59°N for the second cell (Fig. 10b). This zone will be referred to as a (lower stratospheric) “PV gradient.” It introduces an asymmetry in the background fields, which cannot be reasonably described by a symmetric Gaussian hump as previously. The idea is to extend the one-dimensional framework to tackle the displacement of this PV gradient, by trying to keep the same level of simplicity. A new configuration of reference with three pseudo-observations in this upper-level configuration is depicted in Fig. 1b. The detection of the PV gradient (in addition to the local maximum of PV) is also introduced in the algorithm that generates the pseudo-observations.

The 1D plots of the analysis are shown in Figs. 10c,d, and can be compared to the case where only two

![Graphs showing potential vorticity](image)
pseudo-observations are assimilated. The displacement is better realized with lower values of $E_d$. The assimilation of three, rather than two, pseudo-observations does not lead to a degradation of the amplitude of the anomaly, with a slight overestimation in both cases ($E_a > 0$). Overall, the configuration with three observations seems better suited than the one with two observations because distortions of the PV fields are reduced on these two cases.

e. Full displacement of PV fields

Based on the previous results, it was felt that a larger sample of cases should be studied in a more systematic way and also should deal with the full vertical structure of PV fields. For this purpose, pseudo-observations have been generated every 50 hPa and where a PV anomaly is detected. At the lower levels, the algorithm uses the configuration of reference with two pseudo-observations built to displace a Gaussian hump. At the upper levels, the algorithm uses the configuration with three pseudo-observations built to displace both the PV gradient and the Gaussian hump, all under strategy 1. The choice for the vertical spacing (50 hPa) does not prevent for the occurrence of vertical correlations between the observations. We have found that with the preceding choice ($\sigma_o = 0.1\sigma_b$) pseudo-observations were having too much confidence in the analysis. To approximately counteract this effect, the empirical choice was made to double the error standard deviation ($\sigma_o = 0.2\sigma_b$).

The same preceding metrics are used to measure the relative error made between the effective and imposed displacements $E_d$ and the relative error $E_a$ made on the amplitude of the analyzed anomaly. We then chose 20 dates of special interest, where there has been an alert raised by forecasters on the divergence of several operational model for medium-range forecasts (48–60 h). They all concern the development of a low over Europe. The full algorithm involves the following steps:

- running the tracking algorithm on WV satellite images;
- running the tracking algorithm on WV ARPEGE background-derived images;
- linking the model and the image WV cells;
- measuring a displacement error;
- linking the WV model cell with a background PV anomaly;
- generating the pseudo-observations of displacement; and
- assimilating the pseudo-observations of displacement.

It has been found that $2\delta_b$ should be greater than the grid size of the minimization (T149) for the procedure to work. Therefore, a threshold condition on $2\delta_b$ has also been introduced to prevent otherwise inappropriate analyses increments. Among the 20 cases, 13 were shown to lead to an effective generation of observations. On some cases however, several dry intrusions were spotted, leading to a total of 20 PV anomalies.

The distribution of the amplitude and displacement errors measured a posteriori is shown in Fig. 11 for each vertical level. Overall, the assimilation of pseudo-observation induces a mean amplitude error of $-1.8\%$, showing that there little bias (e.g., the procedure does not systematically deepen or weaken the PV anomalies). Albeit the dispersion of $E_a$ reaches an 18.9\% value, meaning that the amplitude of the PV may be largely affected. Looking at the displacement (Fig. 11b), the PV structures seem to be effectively displaced, but by a distance on average 15.8\% lower than the imposed displacement. Standard deviation for $E_d$ reaches 47.6\%, which means that an anomaly typically encounters a displacement between 150 and 250 km when the prescribed one is 200 km.

Moreover, there is an undesirable dependence of the errors on the vertical level. Both $E_d$ and $E_a$ exhibit the same behavior of slightly decreasing when the pressure drops. The relationship between $E_d$ and $E_a$ is expected by the one-dimensional framework when $L_b > L_a$. This slope could therefore be interpreted as $\sigma_o$ being too large (too small) at the upper (lower) levels. This is possible as the approximation $\sigma_b$ comes from a spatial mean that is not restricted to the special case of dry intrusions.

Despite some similarities, the realistic framework is different enough from the 1D framework because of spatially variable $\sigma_b$ and $L_b$, and interaction of the pseudo-observations because of the vertical correlations. The specific study of the two cases shown in this paper shows that the analyses are severely distorted when pseudo-observations are introduced on the vertical (Michel 2008, see chapter 4), thus suggesting that the interaction of the pseudo-observations is problematic because of the vertical correlations. Some other attempts with more pseudo-observations have not yielded better results up to now, in particular introducing numerical artifacts far from the pseudo-observations. A last try consisted in using time-distributed pseudo-observations every hour in the 4D-Var. This provided worse results especially when the displacement was changing over time, with second local minimums of PV created in some cases (not shown). This study therefore supports the idea that distortions

5 In the other seven cases, it either happened that the tracking algorithm did not retain the satellite or the model cell as valid, or did not link them because other (model or satellite) cells were present in the surrounding; thus, the algorithm has rejected the linking and therefore the generation of pseudo-observations.
are difficult to avoid and that a pseudo-observation approach is inappropriate for the displacement problem.

4. Conclusions

The classic formulation of data assimilation does not allow to directly incorporate observations that measure the position of a meteorological feature. Among the possible alternative methods of initialization are the ensemble Kalman filter (Chen and Snyder 2007), the use of an alternative model error in position space (Lawson and Hansen 2005; Ravela et al. 2007; Michel 2010), or the assimilation of pseudo-observations (Heming 1994). The long-term goal of this work was to automatically specify pseudo-observations from the automatic comparison of WV images made possible by the tracking algorithm designed by Michel and Bouttier (2006). A case study where pseudo-observations are built to strengthen a PV anomaly is described by Michel (2010). This paper rather studies the limitations of a strategy that assimilates pseudo-observations with the aim of displacing a PV structure.

The problem is cast in a one-dimensional idealized framework. The PV anomaly is described by a Gaussian hump. Very simple models are used for the background and the observation error covariances matrices. Once some error measure is defined, it is possible to try to optimize the parameters of the problem (the number, the position of the observations, and their error standard deviation) to reduce the error of the analyses. A configuration of reference is shown to have acceptable performance with a special value of the ratio $\sigma_o/\sigma_b$ and when the structure functions in the variational data assimilation scheme roughly matches the shape of the PV structures.

In the realistic framework, an automatic linking is introduced between the cells that are detected as dry intrusions by the tracking algorithm on model and satellite WV images. A translation operator links the detected cells with the closest PV anomaly in the background. The measured displacement is deduced from the vector difference of the center of mass of the cells, and is applied to PV background fields. To gain some understanding on the structure of background errors in PV space, two randomized diagnostics are introduced using the linearized PV observation operator. Estimates of background error standard deviations exhibit strong values along the higher values of the gradient of the background PV field, consistent with the findings of Snyder et al. (2003). Estimates of background error correlation length scale also exhibit strong flow dependency,
with shorter (larger) values in (anti)cyclonic areas. This
turns out to be significantly different from the simpli-
fying assumptions made in the one-dimensional frame-
work. Experiments at the tropopause level over two
cases of PV anomalies show however that the behavior of the
procedure shares reasonable similarity with the
one-dimensional framework. To prevent analyses to be
distorted at upper levels, the method is extended with
three observations.

Then, a more extensive set of 13 cases is used to
smoothly displace full three-dimensional PV structures.
It is shown that overall the procedure is not working as
well as in the two former single-level examples. The
procedure introduces a random error on the amplitude
of the PV anomaly that is of zero mean but of standard
deviation reaching 20%. There is an uncertainty on the
effective displacement as well, which reaches 50% of the
imposed displacement. The naive strategy adopted on
the vertical is probably one of the reasons why the full
displacement does not work well, although we do not
plan to further test this conjecture.

A longer-term goal of this study was to understand if
WV could be used as a proxy to measure misplacement
in PV fields. However, it is probably inappropriate to
use of pseudo-observations in traditional data assimila-
tion schemes with the aim of displacing PV structures.
Even if reasonable strategies can be designed, they have
not yet prevented numerical artifacts from appears in
the analyses. It is suggested that the use of alternative
and better suited methods to displace features in mete-
orological fields should be considered before ongoing
further experimentation on the PV–WV relationship.

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