ON THE MOVEMENT AND PREDICTION OF TRAVELING PLANETARY-SCALE WAVES
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ABSTRACT

Fluctuations of the planetary-scale waves, represented by spherical harmonics of the 500-mb. geopotential field, are statistically analyzed. A study is made of the prediction of these fluctuations from previous changes and using the non-divergent spherical vorticity equation.

1. INTRODUCTION

When numerical weather predictions were first computed using a planetary grid it was found that the largest or planetary-scale waves were computed to move rapidly westward at roughly 90° of longitude per day (Martin, [11]; Wolff, [13]). This produced a systematic error in the predictions, since the largest-scale waves do not move in this way. Cressman [3] introduced into the vorticity equation a "barotropic divergence" term, scaled so as to have its greatest effect at the largest scales. The magnitude of this term was empirically determined by the requirement that it reduce the mean-square error of the predictions as far as could be done by this means.

Recent observations of the planetary-scale waves at 500 mb. (Deland, [4, 5]; Eliasen and Machenhauer, [7]) have suggested that the fluctuations in position and amplitude of the waves are due to the simultaneous presence of a stationary and a traveling component. In the case of wave number 1, in middle-latitudes in winter, the stationary component has its maximum height over Europe and minimum height over the North Pacific corresponding to the long-term average. The traveling component, of somewhat smaller magnitude than the stationary one, moves westward at speeds variously estimated at 40° to 70° longitude per day.

The previously reported westward wave-speeds are, for the waves of largest scale, significantly less than those corresponding to the non-divergent vorticity equation (Rossby and collaborators [12], Haurwitz [9]). This suggests that a divergence term like that of Rossby and collaborators [12] is necessary for agreement between observed and calculated wave-speeds.

We have investigated the possibility of predicting the fluctuations of the planetary-scale waves, by statistically analyzing the fluctuations of the largest-scale spherical harmonics of the geopotential field. In this paper we present results of correlating the fluctuations (1) with the previous day's change and with fluctuations of other harmonics, and (2) with changes predicted from the barotropic vorticity equation, including non-linear interactions with the other large-scale waves. The first, autocorrelation, analysis was undertaken in response to the subjective observation that there are regularities in the fluctuations which do not fit the simple Rossby-wave model, but which could be useful for prediction. The second procedure provides a partial answer to the question: how well can we predict the fluctuations of the largest-scale waves by means of the (barotropic) vorticity equation?

The geopotential field was analyzed because it was immediately available at the beginning of the investigation and because previous work had already shown that it exhibited the phenomenon of traveling planetary-scale waves. Whether analysis of the stream function derived from the balance equation, as has been done by Eliasen and Machenhauer [7], gives a clearer picture of the phenomenon has yet to be determined. The main fluctuations of geopotential and stream function take place in middle latitudes, while the distribution of the fields among the different degrees of spherical harmonics is strongly affected by the variations in lower latitudes, so the analysis of either height or stream function into spherical harmonics must be considered somewhat "unnatural". It is, however, a convenient, though not precise, way of distinguishing between different latitudinal scales, whichever field is analyzed.

2. DATA

The data consist of surface spherical-harmonic expansions of 500-mb. geopotential over the Northern Hemisphere north of about 20°N., for 00 GMT on each day of January 1–February 28 and July 1–August 31, 1962. The expansions consisted of "odd" (antisymmetric about the equator) harmonics only.
The height field for the Northern Hemisphere at a particular hour is expressed as the sum of the odd harmonics

\[ Z(\lambda, \phi) = \sum_{m=1}^{M} \sum_{n=-m+1}^{m} (Z_{m}^{2} \cos n\lambda + Z_{n}^{2} \sin m\lambda)P_{n}^{m}(\phi) \]

where the sum includes only values of \( n \) for which \( n-m \) is odd. \( \lambda \) is longitude and \( \phi \) is latitude. \( m \) is longitudinal wave-number and \( 2(n-m) \) corresponds roughly to wave-number in the latitudinal direction: the latitudinal scale decreases as \( n \) increases. For a given harmonic \( (m, n) \), \( Z_{m}^{n} \) and \( Z_{n}^{m} \) are constant, and may be referred to as the magnitudes of the “cosine” and “sine” components of the particular harmonic. \( P_{n}^{m}(\phi) \) is the associated Legendre function of the first kind.

3. MOVEMENT OF TRAVELING PLANETARY-SCALE WAVES

In this paper, we are concerned with the behavior of the largest scales only, as given by \( m=1, 2, \) and \( 3 \) and \( n-m=1, 3, \) and \( 5 \).

The two components \( Z_{m}^{2} \) and \( Z_{n}^{m} \) constitute a vector amplitude \( Z_{m}^{2} \), equivalent to scalar amplitude and phase, or the “complex amplitude” if complex exponentials are used. When the vector amplitude \( Z_{m}^{2} \) of a planetary-scale wave is plotted on a polar diagram for successive days as in figures 1 and 2, the plotted point describes a characteristic clockwise circular path. This behavior is sketched in figure 3. The circular motion is interpreted (Deland [4]) as being due to a clockwise rotating vector, representing a westward-traveling component as also suggested by Eliassen and Machenhauer [7].

An interesting feature of the clockwise rotation is that it is often faster, corresponding to faster-moving waves, for smaller fluctuations. Although not apparent in figures 1 and 2 this behavior is evident on many plots of the waves (cf. Deland [4, 5]), especially those for \( Z_{2} \). The relation of speed to amplitude has not yet been quantitatively determined.

The fluctuations of \( Z_{1} \), in figure 2 (for August 1962) appear to be correlated with the simultaneous fluctuations of \( Z_{1} \) plotted in figure 1. The fluctuations of \( Z_{2} \) appear to be the sum of a slow clockwise motion and a faster clockwise motion in phase with the rotation of \( (1, 2) \).

COMPUTATIONS

The rotation of the wave-vectors, corresponding to propagation of the traveling waves, and the relation between the waves of the same longitudinal wave-number \( m \) but different degree \( n \), are both analyzed by computing least-squares vector regressions (Ellison [8], Anderson [11]).

The regressions are of the form

\[ (\Delta X_{i}, \Delta Y_{i}) = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} \begin{pmatrix} \Delta Y_{i} \\ \Delta Y_{i} \end{pmatrix} \]

and

\[ (\Delta X_{i}, \Delta Y_{i}) = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} \begin{pmatrix} \Delta Y_{i} \\ \Delta Y_{i} \end{pmatrix} + \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix} \begin{pmatrix} \Delta Z_{i} \\ \Delta Z_{i} \end{pmatrix} \]
where $c$ and $s$ refer to cosine and sine components and the primes refer to deviations from the sample mean. The multiplication of a vector by a tensor corresponds to ordinary matrix multiplication. The first type of regression has been used recently by Lenhart, Court, and Salmola [10]. Correlation coefficients corresponding to the regressions were computed, and are presented below. For the multiple regression, these are total correlation coefficients, the square root of the fraction of the variance of $\Delta X$ "explained" by regression on both of the other vectors. The square of the correlation coefficient is also tabulated in each case. The following regressions were calculated

$$
\Delta Z_m^{n,J} = [B] \Delta Z_m^{n,J-1}
$$

(1)

$$
\Delta Z_n^{m,J} = [B] \Delta Z_n^{m,J}
$$

(2)

$$
\Delta Z_n^{m,J} = [C] \Delta Z_n^{m,J-1} + [D] \Delta Z_n^{m,J-1}
$$

(3)

$$
\Delta Z_n^{m,J} = [E] \Delta Z_n^{m,J-1} + [F] \Delta Z_n^{m,J-1}
$$

(4)

where $\Delta Z_m^{n,J}$ is the 24-hr. change in $Z_m$ beginning on day $J$ (see fig. 3). The regression and correlation results are presented in tables 1-4. The first, auto-regression, procedure is schematically shown in figure 3.

The average speed of propagation of the traveling planetary-scale waves is estimated from the regression coefficients as follows. If the relation between one day's change and the next were a constant rotation, correspond-

$$
\begin{pmatrix}
B_1 & B_2 \\
-B_2 & B_1
\end{pmatrix}
$$

with $B_1 + B_2 = 1$. 

Figure 2.—$Z_4$ for July 1962.

Figure 3.—Sketch of characteristic clockwise rotation and illustration of auto-regression procedure.
With the assumption that the actual relation is the sum of such a rotation plus some additional non-rotational average "distortion", the problem of estimating the speed of propagation is reduced to that of estimating the skew-symmetric part of the regression, i.e., $B_1$ and $B_2$. The skew-symmetric part of the regression tensor is given in terms of the computed components of the tensor by

$$\begin{bmatrix} B_{11} + B_{22} & B_{12} - B_{21} \\ B_{12} - B_{21} & B_{11} + B_{22} \end{bmatrix}$$

The magnitude of the average angle of rotation from one day's change to the next is then given by

$$\Delta \alpha = \arctan \frac{B_{21} - B_{12}}{B_{11} + B_{22}}$$

and its quadrant is easily determined from the skew-symmetric tensor. This angle is tabulated for each of the regressions in tables 1-4, with positive angles corresponding to clockwise rotation.

In view of the "tendency toward zero" of the regression coefficients when the correlation is low, evident in the computed values given in table 1, the question of bias in the estimates of $\Delta \alpha$ arises. It can be shown, using a
simple “rotation plus noise” model, that $\Delta \varphi$ is unbiased to the extent that it is not affected by the addition of random “noise” and/or an additional non-rotational component of the fluctuations.

For the regression of $\Delta Z_2$ on $\Delta Z_{2-1}$ the estimated angle corresponds to the phase speed of the waves; the spatial speed of propagation in degrees of longitude per day is then equal to the phase speed divided by longitudinal wave-number. The speeds of propagation are tabulated in tables 1, 3, and 4, with sign such that westward propagation corresponds to negative speeds, following the usual convention. For the cross-correlations between different degrees, the derived angle $\Delta \varphi$ corresponds to a phase shift between the rotating part of the two harmonics, one being ahead of the other. In this case a positive angle means that the “dependent” vector (on the left hand side of (2)) is on the average rotated clockwise from the “independent” vector (on the right hand side).

### RESULTS

First considering the auto-regressions, $\Delta Z_2$ on $\Delta Z_{2-1}$, it is apparent that the regression coefficients correspond to the simple rotation model to a varying extent. In the winter period only (1,2) and (1,4) and perhaps (2,3) show definite rotation. The rotation is considerably more apparent in the summer months, the regression coefficients for (1,2), (1,4), (2,5), and (3,6) all conforming to the skew-symmetric pattern quite closely, and the other two also doing so fairly well. It is also noticeable that the auto-correlation coefficients are also generally higher in the summer period.

In both periods the behavior of the largest-scale wave (1,2) is closest to the simple traveling wave model. The estimated wave-speeds toward the west are in every case less in the winter than in the summer period, the difference ranging from 10° to 28° longitude per day. In view of the erratic form of some of the regression
tensors, the consistency of the summer-winter speed differences may be partly coincidental.

Let us compare these results with those of Eliasen and Machenhauer [7] (which are for movement of the tendency field, but comparable) and Deland [5]. As shown in Table 1, Eliasen and Machenhauer's results for January 1957 agree quite closely with ours, though, rather surprisingly, better with those for the summer period than those for the winter. Deland's [5] speeds for April 1961 are, on the other hand, all considerably greater algebraically, i.e., less westward than the others. Comparison of figure 1 with figure 1 of Deland [5] shows that the difference is not entirely due to the different methods of analysis: the rotation apparent in figure 1 (of this paper) for August 1962 is clearly more rapid on the average than that shown in the earlier paper for April 1961.

Considering now the results for the cross-correlations between different degrees, in Table 2, we see that the relationship apparent in figures 1 and 2 shows up as a moderately high correlation of $\Delta Z_i$ with $\Delta Z_i$ in both summer (0.52) and winter (0.53). Correlations of similar magnitude appear for $\Delta Z_i$ on $\Delta Z_i$ (high correlation in winter, low in summer). $\Delta Z_i$ on $\Delta Z_i$ and $\Delta Z_i$ and $\Delta Z_i$ (0.67, in summer, 0.55 in winter). All "adjacent" pairs of harmonics, $\Delta Z_i$ and $\Delta Z_i$, $\Delta Z_i$ and $\Delta Z_i$, etc., are at least moderately well correlated and closely in phase, both in summer and winter. The correlations between $\Delta Z_i$ and $\Delta Z_i$, $\Delta Z_i$ and $\Delta Z_i$, and $\Delta Z_i$, etc., are small, but it is interesting that in the summer the corresponding changes in these pairs are almost opposite in phase. This does not correspond to oppositely moving waves, a relationship that is excluded by the regression computation. The interpretation of this observation is postponed until after the results for the multiple regressions are presented.

The results for the multiple-regression correlations in Table 3 show that the total correlations are in each case higher than for the correlations between pairs of vectors, presumably mainly because the independently varying parts of the vectors on the right-hand side of equation (3) "explain" some of the variation of the dependent vector.
but also in part because of the decreased "degrees of freedom" of the computed coefficients. The interpretation of the partial regressions is complicated by the time lag between the dependent variable and one of the predictors (the predictand itself for previous day). In particular, the opposite phase of $\Delta Z^2_1$ and $\Delta Z^2_3$ etc., previously mentioned, is apparently masked by the fact that here we are regressing $\Delta Z^2_1$ on the part of $\Delta Z^2_1$ that is uncorrelated with $\Delta Z^2_3$ for the previous day.

The calculated wave-speeds present a regular variation with scale, as can be seen from figure 4. The wave-speeds are considerably greater toward the east in winter than in summer, as has already been observed for the autocorrelations. The difference between winter and summer increases with decreasing scale, up to $25^\circ$ to $30^\circ$ of longitude per day for the smallest scales considered here.

According to calculations of Eliasen and Machenhauer [7], the difference between winter and summer speeds cannot be due to the difference in advection by the zonal wind, even allowing for the varying weighting of the latitudinal variation of the zonal wind for different harmonics. They have computed interactions with the zonal harmonics for December 1956–January 1957 (see their table 10) and it is apparent that not only are they small (except for $(1,2)$, $(2,3)$ and $(3,4)$) even in winter but they decrease with decreasing scale. It follows that the difference between the observed wave-speeds in summer and winter is considerably greater, for all but the largest scales, than can be explained by the non-divergent barotropic vorticity equation.

The relations between the different estimates of the speeds of the various waves is schematically represented in figures 5 and 6. Considering $Z^1_3$ in January–February, its actual westward speed from the auto-correlation calculation is $63^\circ$/day. Its fluctuations are correlated ($R=0.53$) with the slower fluctuations of $Z^1_1$; when the speed of $Z^1_3$ is calculated holding $Z^1_1$ fixed (multiple regression), the estimated westward speed increases to $74^\circ$/day. $\Delta Z^1_3$ is weakly correlated with $\Delta Z^1_2$; when this effect is removed (but the effect of $\Delta Z^1_3$ remains), the estimated speed is $69^\circ$ ($>63^\circ$, but $<74^\circ$). And similarly for the other harmonics; note especially the estimates of the speed of $Z^1_2$: $22^\circ$ "actual", $51^\circ$ with $Z^1_2$ fixed but the interaction with the rapidly moving $Z^2_2$ present; $10^\circ$ eastward when the effect of $Z^1_2$ is removed but the effect of the slower $Z^1_1$ is present. The observed speeds show a consistent pattern, in both the winter and summer periods. It is apparent that the speed estimates by simple auto-correlation are affected by the more rapid westward rotation correlated with the motion of the next lower degree. This effect is strongest in summer when the rotation of the lower-degree waves, $Z^1_2$, $Z^2_2$, and $Z^1_1$, is more clearly defined.

The above results are consistent with a continuous spectrum of wave-speeds for each harmonic, which seems to be evident in the polar diagrams, as previously mentioned.

It appears that there are traveling waves present which are similar to spherical harmonics of the geopotential field, but not exactly the same. If we refer to the traveling wave that is mostly evident in $\Delta Z^2_1$ as $\phi^1_3$, it appears that $\Delta \phi^1_3$, in summer at least, is made up mainly of $Z^1_1$, some $Z^1_2$, and a little negative $Z^2_1$. Likewise for $\phi^2_3$ and $\phi^1_3$. This raises the question of the form of the traveling waves, and in particular whether the use of spherical harmonics of a stream function derived from the balance equation would result in a better separation of different characteristic motions. This matter will be dealt with in a later article.

It is of interest to relate the mean speeds of the waves to the quasi-geostrophic barotropic vorticity equation, since even the largest-scale waves should be governed by it (Deland [6]). The barotropic vorticity equation can be written
\[
\frac{\partial}{\partial t} (\nabla^2 v) + \nabla \cdot (\nabla^2 v) + \frac{2\Omega}{R^2} \frac{\partial z}{\partial \lambda} = 0 \tag{5}
\]
where $z$ is the geopotential height, $R$ is the radius of the earth, and $\lambda$ is longitude.

If a "barotropic-divergence" term, suggested by Rossby and collaborators [12] and further discussed by Charney
Equation (6) is identical to Eliasen and Machenhauer’s equation (17), except that here we are using the geopotential instead of the stream function. The relation of \( \chi \) to Cressman’s coefficient \( \mu \) is given by

\[
\chi = \frac{f^2 R^2}{g} \mu \approx 7.5 \mu.
\]

With the barotropic-divergence term included, the spherical-harmonic form of the vorticity equation is

\[
\frac{d}{dt} Z_\gamma = -i \, \frac{n_\gamma (n_\gamma + 1)}{n_\gamma (n_\gamma + 1) + \chi} \left\{ \frac{2(\Omega + \omega)}{n_\gamma (n_\gamma + 1) - \omega} \right\} m_\gamma Z_\gamma + \frac{\Omega}{\sigma_\delta} \sum Z_\alpha Z_\beta I_{\gamma \alpha \beta}
\]

where \( \gamma \) identifies a particular harmonic of the geopotential height, of complex amplitude \( Z_\gamma \); \( m_\gamma \) and \( n_\gamma \) are the order (longitudinal wave-number) and degree of the particular harmonic; \( \Omega \) is the rate of rotation of the earth; \( \omega \) is the average zonal wind, corresponding to the amplitude of the \((0,1)\) harmonic, expressed as an angular velocity; \( Z_\alpha \) and \( Z_\beta \) are the complex amplitude of other waves that affect \( Z_\gamma \) through the interaction coefficient \( I_{\gamma \alpha \beta} \); \( \chi \) is the divergence coefficient; and the other symbols have their usual meanings.

It is seen that the theoretical effect of the barotropic-divergence term is to reduce all wave-speeds (neglecting interactions with other waves) in the proportion

\[
\frac{n_\gamma (n_\gamma + 1)}{n_\gamma (n_\gamma + 1) + \chi} \approx \frac{1}{n_\gamma (n_\gamma + 1) + \chi}.
\]

We can thus estimate magnitudes of the coefficient \( \chi \) that would correspond to the observed speeds. The mean Rossby-Haurwitz wave-speeds are calculated from the formula

\[
\omega_{RS-H} = \frac{2(\Omega + \omega)}{n_\gamma (n_\gamma + 1) - \omega}.
\]

Thus the value of \( \chi \) corresponding to the observed wave-speeds is given by

\[
\chi = n_\gamma (n_\gamma + 1) \left( \omega_{RS-H} - \omega_{obs} \right) / \omega_{obs}.
\]

The observed speeds (from table 1), Rossby-Haurwitz speeds, and the corresponding values of \( \chi \) are listed in table 1.

It is immediately obvious that the values for \( \chi \) are very variable. They are, however, mostly much less than 30, corresponding to Cressman’s value of 4 for his factor \( \mu \).

In general, though with conspicuous exceptions, they can be considered roughly equal to the value of 5 deduced by Rossby and collaborators [12] as appropriate to their free-surface model of the atmosphere. The results do not fit the free-surface model in two ways, however; firstly there is a consistent increase (for the well-defined values) of \( \chi \).
with decreasing scale, and secondly, in some cases the waves move toward the east although the Rossby-Haurwitz wave-speed is westward. The values of \( \chi \) are larger in winter or negative, which is consistent with the observation that the summer-winter differences in wave-speed are greater than can be explained by the difference in zonal wind.

The discrepancy of sign of observed and Rossby-Haurwitz speeds is immediately removed if we re-define the barotropic divergence term in equation (6) so that the local derivative \( \partial \varepsilon / \partial t \) refers to a coordinate system moving with the mean zonal wind. This is a straightforward procedure.

4. PREDICTION OF TRAVELING PLANETARY-SCALE WAVES

PREDICTION BASED ON PREVIOUS CHANGES

We have already calculated the fractional reduction in variance of each day's change that can be obtained by estimating it by the linear regression on the previous day's change. It is equal to the square of the vector correlation coefficient, and is tabulated for each wave as \( R^2 \) in table 1. These values correspond to a skill score for prediction, since they give the fractional reduction of mean square error of the geopotential height compared to holding the wave constant, exact prediction of the particular wave corresponding to 100 percent. Similarly for prediction based on the previous day's change and that of another "adjacent" wave; the corresponding ratios are given in table 4. It should be mentioned that these ratios are necessarily higher than would be achieved if the regression coefficients were used with an independent sample as in a realistic prediction situation. How much higher depends on the stability of the regression coefficients which needs to be determined from a larger sample than we have considered here.

As would be expected, the ratios in table 4 are larger than those in table 1, but they are in no case much higher; the addition of another wave as a predictor does not produce any significant improvement.

The results must be considered somewhat disappointing as regards practical prediction of these waves. Only \( Z_2 \)'s behavior in summer is regular enough for more than 50 percent of its variance to be accounted for. In summer, results for many of the other harmonics fall in the 20–35 percent range, which may or may not be useful. In winter only the largest-scale wave has more than 30 percent of its variance explained by the regression.

PREDICTION USING THE BAROTROPIC VORTICITY EQUATION

We have computed the vector regression of the observed changes on "predicted" changes calculated from the spherical vorticity equation (7) with values of \( \chi \) ranging from 0 to 24. The predicted changes were calculated for simple Rossby-Haurwitz waves and using the non-linear vorticity equation including interactions with wave-numbers up to longitudinal wave-number 6. The restriction to long-wave interactions was mainly because of limitations on computer storage, but also because the interactions with the other long waves are the most important ones for the planetary-scale waves (see Eliasen and Machenhauer [7]). The results of these regressions are given in table 5.

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| Table 5.—Correlations of changes with those predicted by spherical vorticity equation, for different values of divergence factor \( \chi \). \( \Delta \theta \) is angle corresponding to regression tensor. |
|---|---|---|---|---|---|
| | January-February | | July-August | |
| | | | | | |
| | | | | | |
| \( Z_2 \) | \( R \) | \( R^2 \) | \( \Delta \theta \) | \( R \) | \( R^2 \) | \( \Delta \theta \) |
| 0 | 0.56 0.34 -23° | 0.55 0.33 -26° | 0.78 0.61 +19° | 0.77 0.59 +16° |
| 6 | 0.58 0.35 -9° | 0.55 0.30 -3° | 0.78 0.61 -10° | 0.78 0.59 -11° |
| 18 | 0.56 0.34 -25° | 0.55 0.30 -15° | 0.78 0.61 -24° | 0.78 0.59 -26° |
| \( Z_4 \) | 0 | 0.54 0.29 -11° | 0.56 0.31 -6° | 0.46 0.21 -11° | 0.42 0.18 -24° |
| 6 | 0.54 0.29 -14° | 0.56 0.31 -6° | 0.46 0.21 -21° | 0.42 0.18 -27° |
| \( Z_6 \) | 0 | 0.45 0.20 -15° | 0.42 0.18 -18° | 0.66 0.44 -15° | 0.52 0.27 15° |
| 6 | 0.45 0.20 -38° | 0.42 0.18 -38° | 0.66 0.44 -4° | 0.52 0.27 -3° |
| \( Z_8 \) | 0 | 0.35 0.12 -68° | 0.30 0.09 -33° | 0 | 0.52 0.27 0° |
| 12 | 0.35 0.12 -94° | 0.30 0.09 -64° | 0.50 0.25 4° | 0.50 0.25 -7° |
| \( Z_9 \) | 0 | 0.30 0.15 -38° | 0.31 0.10 -33° | 0 | 0.66 0.44 -10° | 0.66 0.44 -6° |
| 6 | 0.30 0.15 -60° | 0.31 0.10 -61° | 0.66 0.44 -5° | 0.66 0.44 -8° |
| \( Z_{10} \) | 0 | 0.49 0.24 -74° | 0.26 0.07 -128° | 0 | 0.32 0.16 -15° | 0.32 0.16 -15° |
| 12 | 0.49 0.24 -77° | 0.26 0.07 -127° | 0.32 0.16 -19° | 0.32 0.16 -19° |

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The squared correlation coefficients again represent the ratio of explained to total variance for each harmonic. The ratios are similar to those obtained for auto-regression of daily changes in the previous section. Again only for the largest scales, in summer in particular, can the correlations be considered large enough to be useful. The non-linear prediction does not yield any apparent improvement over the simple Rossby-Haurwitz wave model.

If the regression angles $\Delta \phi$ for different $x$'s are compared, it is seen that the best fit between predicted and actual changes occurs for smaller values of $x$ than those deduced from the auto-regressions. In fact for all but $Z_2$ the best fit would be obtained for negative values of $x$, which we did not use in the computations. This is because in computing the predicted change from the vorticity equation the changes are proportional to the deviation from the mean. Large deviations from the mean, which are given most weight in the regression, are most often followed by changes toward the mean. This tendency toward the mean superimposed on the rotation increases (negatively) the apparent angle of rotation deduced from the regression.

All the vector correlation coefficients obtained using the barotropic vorticity equation include the regression toward the mean, which is relevant for prediction but not to analysis of wave motions.

### 5. CONCLUDING REMARKS

The principal results of the paper are the details of the behavior of the planetary-scale traveling waves. Some general summarizing comments are, however, appropriate. The waves move approximately in accordance with the free-surface model of Rossby and collaborators [12], but with a wide range of varying speeds, as of course do the synoptic-scale waves. The different (latitudinal) harmonics of the same longitudinal wave-number are correlated with each other, but have different characteristic speeds of their own. The variations of movement of each wave about its mean speed are not explainable in terms of interactions, through the non-linear terms in the barotropic, vorticity equation, with other waves.

The fluctuations of the waves are so irregular that prediction of them to a useful accuracy does not seem to be practical by the methods attempted in this paper, except for the largest-scale wave $Z_1$.

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