On Production and Dissipation of Thermal Variance in the Oceans

TERRENCE M. JOYCE

Institut für Meereskunde, Kiel, Germany

8 August 1979 and 27 September 1979

ABSTRACT

An integral relationship is derived expressing the total dissipation of thermal variance by oceanic microstructure in terms of the large-scale forcing at the ocean surface by air/sea heat exchange. The net heat gain by the ocean over warm water and heat loss over cold water is evaluated using zonal averages of annual oceanic heat fluxes and temperatures between 60°N and 60°S. If thermal dissipation occurs in the upper ocean, with a scale depth of 600 m, the average dissipation $\chi$ is estimated to be $10^{-7} \, ^\circ\text{C} \, \text{s}^{-1}$. This value compares favorably with published observations of oceanic microstructure dissipation. The prediction is independent of any dynamical model of turbulent cascade from large to small scales in the ocean.

1. Introduction

Within the last decade, developments in oceanographic instrumentation have greatly extended our ability to measure oceanic temperature variations down to scales of a centimeter. Theoretical models have appeared which have attempted to relate this variability locally to larger scale phenomena and thus assess the importance of the variability. These models have started with some form of the heat equation for the ocean, a simple form of which can be written

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x_1} \left( \kappa_T \frac{\partial T}{\partial x_1} - u_i T \right), \quad i = 1, 2, 3. \quad (1)$$

To obtain the above, a great number of simplifying assumptions have been made: the ocean is assumed to be incompressible, effects of salinity on the
chemical potential of the ocean have been neglected, and internal heat sources such as those due to internal friction have been ignored. Eckart (1962) discusses most of these points and derives a heat equation of more general applicability. When Osborn and Cox (1972) first presented their observations of temperature microstructure, they also provided a model which aided in interpreting the data. Since the essential elements of this model as well as one subsequently suggested by Joyce (1977) do not depend upon terms neglected in (1), this equation will be the starting point for this study.

2. Large-scale forcing of oceanic microstructure

Following techniques developed for the study of turbulence (cf. Monin and Yaglom, 1965) the temperature and velocity fields are split into slowly \( \langle \cdot \rangle \) and rapidly \( \langle \cdot \rangle' \) varying components. The following relations can be easily derived:

\[
\left( \frac{\partial}{\partial t} + \bar{u}_i \frac{\partial}{\partial x_i} \right) \langle T \rangle = - \frac{\partial}{\partial x_i} \langle u_i \langle T \rangle \rangle, \tag{2}
\]

\[
\left( \frac{\partial}{\partial t} + \bar{u}_i \frac{\partial}{\partial x_i} \right) \langle \nabla \cdot (T) \rangle = - \bar{T} \frac{\partial}{\partial x_i} \langle u_i \langle T \rangle \rangle, \tag{3}
\]

\[
\left( \frac{\partial}{\partial t} + \bar{u}_i \frac{\partial}{\partial x_i} \right) \langle \nabla (T)^2 \rangle = -\langle u_i \langle T \rangle \rangle \frac{\partial}{\partial x_i} \langle \nabla \cdot (T) \rangle - \frac{\partial}{\partial x_i} \left[ \frac{\kappa \sigma}{\kappa \sigma} \langle \nabla \cdot (T) \rangle - u_i \langle \nabla (T)^2 \rangle \right], \tag{4}
\]

\[
\chi = 2\kappa \sigma \left( \frac{\partial \langle T \rangle}{\partial x_i} \right)^2.
\]

Further simplifications can be made if one integrates (2)–(4) over the volume of the oceans and uses the fact that \( n_i \langle \bar{u}_i + u_i' \rangle = n_i \kappa \sigma \langle \nabla T' \rangle = 0 \) on all solid boundaries, where \( n_i \) is a normal vector, and that \( \dot{Q}_3 = -\rho c_v \kappa \sigma \langle T \rangle \) is the turbulent flux of heat from the atmosphere into the ocean at the free surface. The assumption of steady state and stationarity will also be made giving the following three relations corresponding to Eqs. (2)–(4):

\[
\frac{1}{\rho C_v} \iint dA \dot{Q}_3 n_i = 0, \tag{5}
\]

\[
\frac{1}{\rho C_v} \iint dA \dot{Q}_3 n_i = - \iiint dV \frac{\partial}{\partial x_i} \langle u_i \langle T \rangle \rangle, \tag{6}
\]

\[
- \iiint dV \frac{\partial}{\partial x_i} \langle u_i \langle T \rangle \rangle = \frac{1}{2} \iiint dV \chi, \tag{7}
\]

Eq. (5) states that for a steady state, the net flux of heat into the volume must vanish, while (6) obtains from (3). It expresses the fact that, in the presence of external forcing, eddy-like processes must act to stir the mean temperature field removing heat from warm regions and transferring it on average down the mean temperature gradient. In deriving (7) the triple product term \( u_i \langle T' \rangle \) denoting the self-advection of thermal variance into the volume at the free surface has been neglected as well as the molecular heat flux term. Eq. (7) shows that the creation of small-scale variance by stirring must be balanced on average by molecular dissipation.

The local approximation of (7) for each volume element yields both the Osborn and Cox (1972) and Joyce (1977) models depending on the assumptions about the nature of the mean and fluctuating fields, the former applicable in central gyres and the latter to frontal regions. The nature of the cascade from frontal scales to dissipation has already been discussed by Gargett (1978) and need not be repeated here. When (6) and (7) are combined a further result is obtained:

\[
\frac{1}{\rho C_v} \iint dA \dot{Q}_3 n_i = \frac{1}{2} \iiint dV \chi, \tag{8}
\]

which shows the relationship between total dissipation of thermal variance [right-hand side (rhs)] to a surface integral of thermal driving [left-hand side (lhs)] independent of any specific model of turbulent cascade. A similar result has been derived for salinity variance dissipation by Stern (1975, p. 210).

It is worth mentioning at this point that the previous discussion on the cascade to small scales presupposes that the ocean requires dissipation. Reference to (8) will show that the degree to which dissipation is needed depends on the nature of the correlation between heat flux and temperature at the ocean surface.

Only a few scattered measurements of \( \chi \) have been made and so the rhs of (8) is impossible to evaluate. It seems possible to do so for the lhs of (8) and so obtain some average \( \chi \) which can be compared with observations independent of any physical model. To the extent that (8) can be applied locally to each area segment \( dA \) one can also obtain an estimate of the spatial variability of \( \chi \). In what follows, a rough evaluation of the thermal driving is made in which \( Q, T \) are replaced by zonal averages at the ocean surface and heat transfer across the earth/salt boundary is neglected. A more thorough study of the spatial variation of thermal driving could be made but for the present only an order of magnitude estimate is sought.

3. Estimation of thermal driving

In Table 1 the relevant parameters are listed at 10° intervals between 60°N and 60°S. The net heat flux
Table 1. Zonal averages of thermal driving parameters.

<table>
<thead>
<tr>
<th>Latitude</th>
<th>Oceanic area(^1) (x10(^7) km(^2))</th>
<th>Surface temperature(^2,3) (°C)</th>
<th>Heat flux(^4) (W m(^{-2}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>60–50°N</td>
<td>1.14</td>
<td>2.9</td>
<td>6.3</td>
</tr>
<tr>
<td>50–40°N</td>
<td>1.51</td>
<td>9.7</td>
<td>11.0</td>
</tr>
<tr>
<td>40–30°N</td>
<td>2.10</td>
<td>17.0</td>
<td>17.7</td>
</tr>
<tr>
<td>30–20°N</td>
<td>2.54</td>
<td>22.1</td>
<td>23.3</td>
</tr>
<tr>
<td>20–10°N</td>
<td>3.15</td>
<td>24.9</td>
<td>26.3</td>
</tr>
<tr>
<td>10–0°N</td>
<td>3.42</td>
<td>26.4</td>
<td>27.4</td>
</tr>
<tr>
<td>0–10°S</td>
<td>3.42</td>
<td>26.4</td>
<td>26.4</td>
</tr>
<tr>
<td>10–20°S</td>
<td>3.34</td>
<td>24.9</td>
<td>24.9</td>
</tr>
<tr>
<td>20–30°S</td>
<td>3.12</td>
<td>22.1</td>
<td>21.8</td>
</tr>
<tr>
<td>30–40°S</td>
<td>3.29</td>
<td>17.0</td>
<td>16.4</td>
</tr>
<tr>
<td>40–50°S</td>
<td>3.10</td>
<td>9.7</td>
<td>9.8</td>
</tr>
<tr>
<td>50–60°S</td>
<td>2.57</td>
<td>2.9</td>
<td>3.2</td>
</tr>
</tbody>
</table>

\(^1\) McLellan (1965).  
\(^2\) Dietrich (1950).  
\(^3\) Neumann and Pierson (1966).  
\(^4\) Budyko (1958).

(after Malkus, 1962; Budyko, 1956), mean sea surface temperature (Dietrich, 1950; Neumann and Pierson, 1966) and oceanic surface area (McLellan, 1965; Fig. 1) in Table 1 are also plotted in the figure. While more modern estimates of these quantities exist, for the purpose of this order of magnitude estimate, these older, more readily available data will be used. Reference to the figure will show that since the ocean is heated (cooled) where it is on average warm (cold) a positive feedback exists which must be offset by dissipation. Using the tabulated values the following spatial averages, denoted by \(\overline{\cdot}\), are found:

\[ A = \oint dA = 32.7 \times 10^7 \text{ km}^2, \]
\[ \overline{T} = 19.29^\circ \text{C, 18.79}^\circ \text{C}, \]
\[ \overline{\hat{Q}} = 1.56 \text{ W m}^{-2}, \]
\[ \overline{\hat{Q}}\overline{T} = \frac{1}{A} \oint (\hat{Q} - \overline{\hat{Q}}) \overline{T} dA \]
\[ = 131 \text{ W}^\circ \text{C m}^{-2}, 128 \text{ W}^\circ \text{C m}^{-2}. \]

The Dietrich temperature is symmetric about the equator as the values for the Southern Hemisphere have been used as a reference for Northern Hemisphere temperatures. The data summary of Neumann and Pierson includes Northern Hemisphere measurements and shows the northward displacement of the thermal equator as well as higher surface temperatures primarily due to the influence of the warmer North Atlantic Ocean. The final results are insensitive to the above choice: the Dietrich values are shown second for both \(\overline{T}\) and \(\overline{\hat{Q}}\overline{T}\). Since the area integral does not contain air/sea exchanges in the polar regions it is not surprising that a net heat flux into the oceans remains. In order to apply (8) it is necessary to subtract this "bias" before evaluating the thermal driving. This has been done above and the adjustment found to be of order 10%. If this thermal forcing is now uniformly dissipated within a layer of depth \(H\) then from (8) one finds

\[ \tilde{\chi} H = 2 \frac{\overline{Q}\overline{T}}{\rho C_p}. \]

The above is also equivalent to dissipation exponentially decaying from the free surface with a scale depth \(H\), which could be taken to be the meridionally varying depth of the thermocline. Since the purpose here is to obtain a single estimate of the global mean thermal dissipation, a single value \(H\) of 600 m will be taken characterizing the average thermocline depth. With this \(H\) Eq. (9) gives

\[ \tilde{\chi} \approx 10^{-7} \text{ } \circ \text{C}^2 \text{ s}^{-1}. \]

It must be noted here that the inclusion of zonal variations in the surface temperatures and heat fluxes will probably reduce this estimate rather than increase it since the intense cooling of warm waters advected poleward in western boundary currents and the warming of cold waters in regions of coastal upwelling will partially negate the need for small-scale dissipation.

4. Comparison with observations and discussion

Depending on the degree of isotropy of the measured temperature microstructure the published values can be adjusted up or down by 50%. In Table 2 the observations and sources have been listed together with an indication when stations were taken

Fig. 1. Zonal averages of mean sea surface temperature (upper panel), net heat flux into oceans (center) and oceanic surface area (bottom). Dietrich (—); Neumann and Pierson (○). See text for data sources.
Table 2. Microstructure dissipation measurements.

<table>
<thead>
<tr>
<th>Area</th>
<th>Source</th>
<th>( \chi (^\circ C)^2 ) s(^{-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central Pacific</td>
<td>Gregg \textit{et al.} (1973)</td>
<td>( 4.1 \times 10^{-10} ) (a)</td>
</tr>
<tr>
<td></td>
<td>Gregg and Cox (1972)</td>
<td>( 2.8 \times 10^{-8} ) (b)</td>
</tr>
<tr>
<td></td>
<td>Gregg (1975)</td>
<td>( 5.5 \times 10^{-8} ) (a)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( 1.5 \times 10^{-5} )</td>
</tr>
<tr>
<td>Northeast Pacific</td>
<td>Gargett (1978)</td>
<td>( 6.5 \times 10^{-7} ) (b)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( 1.6 \times 10^{-7} ) (b)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( 1.3 \times 10^{-4} ) (b)</td>
</tr>
<tr>
<td>Gulf Stream</td>
<td>Oakey and Elliot (1975)</td>
<td>( 7.6 \times 10^{-6} ) (b)</td>
</tr>
<tr>
<td></td>
<td>(thermocline)</td>
<td>( 1.1 \times 10^{-6} ) (a)</td>
</tr>
<tr>
<td>Atlantic</td>
<td>Monin (1980)</td>
<td>( 2.9 \times 10^{-6} )</td>
</tr>
<tr>
<td>Northwest Pacific</td>
<td>Monin (1980)</td>
<td>( 3.3 \times 10^{-7} )</td>
</tr>
<tr>
<td>Prediction for upper ocean</td>
<td></td>
<td>( 1 \times 10^{-7} ) (a)</td>
</tr>
<tr>
<td></td>
<td>Average of monotonic</td>
<td>( 5.6 \times 10^{-9} )</td>
</tr>
<tr>
<td></td>
<td>Average of intrusive</td>
<td>( 1.9 \times 10^{-7} )</td>
</tr>
</tbody>
</table>

near water mass transitions and intrusions. The list is by no means complete but is thought to be representative. It must be noted that the unusually low values of \( \chi \) obtained from the central Pacific may have discouraged further sampling in central gyres with the result that investigators have tended to focus near regions of major currents and fronts. No attempt has been made to calculate an average \( \chi \) for the oceanic observations which are not only biased toward active regions but also highly intermittent, as can be seen by the extremely large value of \( \chi \) found by Gregg (1975) from one 10–20 m thick near-surface layer in the California Current. The Soviet data have not been included in the monotonic or intrusive averages since the documentation of the oceanographic environment and measurement techniques was insufficient.

The chief merit of the approach adopted in this paper is also the chief weakness: the results for oceanic dissipation are independent of any dynamics. The largest and smallest scales of variability are linked together with no physics in between. The present state of knowledge about physical processes acting at different scales will be reviewed in a forthcoming book entitled \textit{Turbulence in the Ocean}, edited by J. Woods (1980). It is hoped that a comparison of the distribution of thermal driving and dissipation will prove useful in trying to sort out the relative importance of various processes acting in different places and on different scales. The present calculation is offered as a gauge by which future observations and calculations can be compared independent of any particular physical model for thermal microstructure.

Acknowledgments. The author wishes to thank colleagues at the Institut für Meereskunde in Kiel for providing support and comments for this work while the author was on leave of absence from Woods Hole Oceanographic Institution. Helpful comments on this note were also made by the editor, R. O. Reid. A NATO Air-Sea Interaction Grant SV 19 provided travel funds between the above two institutions. Salary and publication charge support for this research is gratefully acknowledged from the IFM in Kiel and Office of Naval Research Contract N00014-76-C-0197 NR 083-400 to the Woods Hole Oceanographic Institution. This is Contribution No. 4422 of the Woods Hole Oceanographic Institution.

REFERENCES


