Directional Wave Spectra Measured with the Surface Contour Radar

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ABSTRACT

The Surface Contour Radar is a 36-GHz computer-controlled airborne radar which generates a false-color coded elevation map of the sea surface below the aircraft in real-time, and can routinely produce ocean directional wave spectra with post-flight data processing which has much higher angular resolution than pitch-and-roll buoys. When compared with waveriders and the XERB and ENDECO pitch-and-roll buoys, there is good agreement among the nondirectional spectra. There is also good agreement among the angles associated with the $a_1$, $b_1$, and $a_2$, $b_2$ Fourier coefficients of the spreading function for XERB, ENDECO, and the Surface Contour Radar. There are indications that the pitch-and-roll buoys in this study may have calibration problems with the magnitudes of the Fourier coefficients of the spreading function, and that the radar system determines the Fourier coefficients with sufficiently less noise and bias. The high spatial resolution and rapid mapping capability over extensive areas make the Surface Contour Radar ideal for the study of fetch-limited wave spectra, diffraction and refraction wave patterns in coastal areas, and wave phenomena associated with hurricanes and other highly mobile events.

1. Introduction

The Surface Contour Radar (SCR), developed jointly by the NASA GSFC Wallops Flight Facility (WFF) and the Naval Research Laboratory under the NASA Advanced Applications Flight Experiments program, became operational in 1978. It is a computer-controlled 36-GHz bistatic radar which produces a real-time topographical map of the surface beneath the WFF P-3A aircraft on which it is carried. The SCR is one of the most straightforward remote sensing instruments in measurement concept. It provides great ease of data interpretation since it involves a direct range measurement.

The system (Kenney et al., 1979) was designed to measure the directional wave spectrum of the ocean surface. Figure 1 shows the nominal measurement geometry and the horizontal resolutions in terms of the aircraft altitude, $h$. An oscillating mirror scans a 1.42° half-power width pencil-beam laterally to measure the elevations at 51 evenly spaced points on the surface below the aircraft. The nonscanning receiving antenna is a 1.3° × 40° fan beam with the 40° dimension oriented cross-track. The combination of the transmit and receive antennas narrows the along-track interrogated region to a half-power width of 0.96°. At each of the 51 points across the swath the SCR measures the slant range to the surface and corrects in real-time for the off-nadir angle of the beam to produce the elevation of the point in question with respect to the horizontal reference. The elevation is given by

\[ \text{elevation} = h - r \cos \phi_T, \]  
\[ \phi_T^2 = (\phi + \phi_p)^2 + \phi_p^2 \]  

where $h$ is the aircraft altitude, $r$ the radar measured slant range, $\phi_T$ the total off-nadir angle of the beam, $\phi$ the angle of the beam with respect to the perpendicular to the aircraft wings (measured by a shaft angle encoder on the oscillating mirror), $\phi_p$ the roll attitude of the aircraft, and $\phi_p$ is the pitch attitude of the aircraft. Substituting (2) into (1) and expanding in a small angle approximation results in

\[ \text{elevation} \approx h - r[1 - \phi_T^2/2 - \phi_T \phi - (\phi_T^2 + \phi_p^2)/2]. \]  

(3)

The pitch angle appears only as a second-order term in (3), and since its standard deviation is typically 0.5°, it is ignored in the data processing. The roll-angle standard deviation is typically one degree but must be included because it has a first-order effect in (3). The real-time computation of the elevations uses (1) with $\phi_p$ set to zero.

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The standard deviation of aircraft yaw is typically 0.5° and while it does not affect the elevation measurements, it and pitch both affect the position of the illuminated spot on the surface. Pitch and yaw tend to be slowly varying quantities and no attempt to compensate for their variability is included in the data processing. Similarly, any variation in the aircraft ground speed over the 52 seconds it takes to acquire a data set is also ignored. It should also be pointed out that the beam scans in a sinusoidal fashion between approximately ±17° with data being acquired between ±15°. Even though the elevation points are acquired over this truncated sinusoid on the surface, the data are processed as if they were on a rectangular grid. The degrading effects of all of the approximations and neglected quantities are small, as will be demonstrated by the consistency of the SCR spectral data and its agreement with the in situ data.

The elevation measurements are false-color coded and displayed on the SCR color TV monitor so that real-time estimates of significant wave height ($H_S$), taken to be equal to four times the surface height standard deviation, dominant wavelength, and direction of propagation can be made. The real-time display allows optimal selection of aircraft altitude and flight line direction even during a flight over a cloud-covered sea without prior knowledge of the wave conditions. The radar has range resolution cells of 0.15, 0.3, 0.6, and 1.5 m, although the last two have never been used for oceanographic studies. The 15 cm resolution has generally been used, with the 30 cm resolution employed when the $H_S$ reaches 5 m, or when the SCR is operating at 800 m altitude.

The Airborne Oceanographic Lidar (AOL), described by Hoge et al. (1980), is also onboard the WFF aircraft. This laser system can profile the waves at a 400 Hz rate to provide independent corroboration of the elevation data measured by the SCR at the center of its swath. Figure 2 shows comparative data taken at 230 m altitude by the SCR and AOL. The AOL data have been averaged to correspond to the SCR spot size. The agreement is remarkable considering that one system is microwave and the other is optical; they use entirely different ranging techniques; and the AOL is located 10 m aft of the SCR in the aircraft and was looking aft at 15° off-nadir. A relative shift in the time origin of approximately 0.7 s was required for the comparison of profiles between the two instruments.

The 15 cm range quantization of the SCR is apparent in Fig. 2. The noise introduced by this quantization is smaller than the noise in the elevation measurements generated by the fluctuating radar signal when the beam is directed off-nadir (Walsh et al., 1985). The data will demonstrate that reasonable directional wave spectra were obtained even for an $H_S$ as low as 0.9 m.

This paper will deal with variance spectra and, since the SCR measures in the spatial domain rather than the time domain as in the in situ sensors do, the analysis of the system and its data will be made in wavenumber space. The wavenumber values are $2\pi$ divided by the wavelength. The frequency domain will be used for the hurricane directional wave spectrum and when comparing with in situ sensors. Throughout the discussion we will deal with propagation vectors so that the waves will be referenced to the direction towards which they are traveling. The wind is referenced to the direction from which it is blowing.

2. Computation of directional wave spectrum

The oscillating mirror scan rate is variable but is normally set at its maximum to produce approximately 20 raster lines of elevation data per second. For the typical ground speed of 100 m s⁻¹, the aircraft advances 5 m between scan lines. A data span of approximately 52 s is acquired for the development of each directional wave spectrum. The radar records 1024 raster scan lines (containing 51 elevation data points each) while the aircraft travels about 5 km.

For each group of 1024 scan lines imagine that each of the elevation data points occupies a square on a large, rectangular (51 × 1024) checkerboard. Before applying the FFT, all of the data points which occupy red squares are multiplied by −1. Then each of the 51 columns of 1024 points is transformed by a one-dimensional FFT. The pretransform multiplication by −1 shifts zero frequency from one end of the transformed data to the middle so that both positive and negative frequencies are represented (Andrews et al., 1970). Then each of the 1024 rows of
complex Fourier coefficients is itself transformed by a one-dimensional FFT. Since the number of points required by the particular FFT employed is a power of two, 13 zeros are filled in at one end to bring the total to 64. The result of these two sequential one-dimensional FFTs is a two-dimensional FFT with zero frequency in the center. The magnitudes of the complex Fourier coefficients are computed and then squared to obtain the variance spectrum. The along-track dimension of approximately 5 km produces an along-track spectral resolution of $2\pi/5000$ m$^{-1}$ in wavenumber space. Since the swath width is approximately half the altitude, the cross-track spectral resolution would be only about $2\pi/200$ m$^{-1}$ for a 400 m altitude. The fineness of the nominal cross-track spectral resolution would be directly proportional to the aircraft altitude while the along-track resolution would not change with altitude as long as the antenna scan rate and the aircraft ground speed did not vary. Of course one cannot fly higher and higher to improve the cross-track resolution without penalty, because the area illuminated and the noise in the elevation measurements increases with altitude (Walsh et al., 1985).

3. Corrections for aircraft drift angle and ground speed

Long (1979) has done a generalized analysis of platform motion effects on the observation of wave spectra. What we present below is a development of the corrections for the effects of aircraft drift angle (yaw) and ground velocity on the SCR directional wave spectra. In the development we will assume a wave field with wave number $k$ propagating on the sea surface at an angle $\theta$ with respect to the aircraft ground track. The movement of the waves during the measurement period of the SCR produces an encounter spectrum of wave number $k_e$ and direction of propagation $\theta_e$. This is further modified by the aircraft drift angle into the measured spectrum of wave number $k_m$ and direction of propagation $\theta_m$. The corrections are developed and applied sequentially because the drift angle affects only the cross-track spectral component and the ground velocity affects only the along-track component.

a. Drift angle correction

Before Doppler corrections can be applied to the spectrum, the aircraft drift angle (yaw) must be corrected for in order to obtain the proper encounter spectrum. Drift angle is defined to be the amount by which the aircraft ground track differs from its heading. If the aircraft is following its nose the drift angle is zero. If the aircraft heading is to the left of the ground track to compensate for a cross-track component of the wind velocity from the left, the aircraft is drifting to the right relative to its heading and the drift angle is positive. The two-dimensional FFT assumed that the cross track raster scan lines of elevation data were orthogonal to the aircraft ground track. Figure 3 depicts the geometry for a nonzero drift angle. The drift angle will result in the following measured along-track ($k_{ym}$) and cross-track ($k_{xm}$) wavenumber components for encounter spectrum waves of number $k_e$ propagating in the direction $\theta_e$ relative to the aircraft ground track.

$$k_{ym} = k_e \sin(\theta_e + \theta_d)$$  
$$k_{xm} = k_e \cos(\theta_e + \theta_d)$$
\[ \theta_d \sim -2\theta_e, \quad \theta_d \ll 1. \]  

(12)

Another solution is obtained for waves propagating near the cross-track direction (Case B) by substituting \( \theta_e = \pi/2 - \theta_d \) in (11) which results in

\[ \tan \theta_d = -\sin(\pi - 2\theta_d) = \sin(2\theta_d) \]  

(13)

which implies

\[ \theta_d \sim 2\theta_d, \quad \theta_d \ll 1. \]  

(14)

The conditions of (12) and (14) are shown pictorially in Fig. 4. For condition (12) there is no change in magnitude but the \( k_e \) component changes sign. For (14) there is no change in either magnitude or sign.

b. Ground-velocity correction

Since it takes approximately 52 seconds to acquire each set of 1024 scan lines, the data do not represent an instantaneous elevation map of the surface. The waves at the end of the data span would have moved by several wavelengths relative to the positions they were in when the data at the beginning of the span were recorded. If the waves were traveling in the same direction as the aircraft their wavelength would appear longer. Conversely, if they were traveling in the opposite direction as the aircraft their apparent wavelength would be shorter. For waves traveling in any direction not parallel to the aircraft ground track, there would also be a change in the apparent direction of propagation in addition to the wavelength change.

Since the SCR scans its beam cross-track in 0.05 s, it produces essentially an instantaneous picture of the wave structure in the cross-track direction. The measured wave number \( k_m \) can be expressed in terms of the encounter wave number using (4) and (5):

\[ k_m^2 = k_{xm}^2 + k_{ym}^2 = k_e^2[\cos^2 \theta_e + \sin^2(\theta_e + \theta_d)] \]  

(8)

\[ = k_e^2[\cos^2 \theta_e + (\sin \theta_e \cos \theta_d + \cos \theta_e \sin \theta_d)^2]. \]  

(9)

After carrying out the squaring operation on the inner parentheses, substituting \( 1 - \sin^2 \theta_d \) for \( \cos^2 \theta_d \), and \( \sin^2 \theta_e \) for \( 2 \sin \theta_e \cos \theta_d \), then (9) can be rearranged to obtain the encounter wavenumber.

\[ k_e = k_m[1 - \sin \theta_d(\sin \theta_d + \cos \theta_d \sin 2\theta_e)]^{-1/2}. \]  

(10)

If the inner-parentheses sum is zero, then the measured wavenumber will equal the encounter wavenumber even for nonzero drift angle. That condition is met when

\[ \tan \theta_d = -\sin 2\theta_e. \]  

(11)

One solution of (11) is given by (12) for waves propagating near the along-track direction (Case A)
result is that the encounter wavenumber component in the cross-track direction $k_{yx}$ is equal to the actual cross-track wave component $k_x$. The encounter vector wavenumber component in the along-track direction, $k_{yr}$, is computed from the encounter frequency $\omega_e$ and the aircraft ground speed $v_a$ with the sea surface topography assumed to be frozen. The encounter frequency is really a result of the actual wavenumber component $k_y$ and the sum of the aircraft ground speed and the along-track component of the wave phase velocity $v_{py}$.

$$\omega_e = k_{yx} v_a = k_y (v_a + v_{py})$$  \hspace{1cm} (15)

It is possible to rearrange (15) to obtain $\Delta k_y$, the apparent change in the wavenumber caused by the Doppler effect:

$$\Delta k_y = k_{yr} - k_y = k_y v_{py}/v_a.$$  \hspace{1cm} (16)

The wave phase velocity $v_p = (gp/k)^{1/2}$, where $\rho = \tan(kd)$ and $d$ is the water depth, resolves as the reciprocal of the cosine so

$$v_{py} = (gp/k)^{1/2} k/v_y = (gp)^{1/2}/k_{yr}.$$  \hspace{1cm} (17)

Substituting (17) into (16) results in

$$\Delta k_y = (gp/k)^{1/2}/v_a = \omega_d/v_a$$  \hspace{1cm} (18)

which indicates that the change in the vector wavenumber component in the along-track direction is proportional to the ratio of the wave frequency to the aircraft velocity. The change in $k_y$ as a fraction of the total wave number is

$$\Delta k_y/k = (gp/k)^{1/2}/v_a = v_p/v_a.$$  \hspace{1cm} (19)

Even for deep water waves ($\rho = 1$) of 14 s period, the fractional change is only 0.21 for an aircraft velocity of 100 m s$^{-1}$. The change in the magnitude of $k$ will be even less unless $k_y = 0$. Since the changes are small, $\Delta k_y$ is solved for in two steps. First, $k_x$ is computed and used as a first approximation to $k$ in (18) to obtain an estimate of $\Delta k_y$. Then $\Delta k_y$ is used to obtain a better estimate of $k$, which is substituted into (18) to obtain a final estimate of $\Delta k_y$ from which the final value of $k$ is computed. A pictorial representation of the general migration of points in $k$-space is given by Walsh et al. (1981).

4. Elimination of ambiguous lobes in spectra

The technique of producing directional wave spectra with the SCR has some similarities to the airborne application of stereophotography to the observation of ocean waves (Côté et al., 1960; Simpson, 1967; Sugimori, 1975; Holthuijsen, 1983), except that the stereophotographic technique provides the instantaneous topography of the sea surface so there are no Doppler effects present. But far from being a disadvantage, the SCR Doppler effects are not only easily corrected, they provide a means of uniquely determining the direction of propagation of the waves being measured. The technique breaks down if there are waves of the same wavelength and comparable energy propagating in opposite directions, but this is a rare occurrence.

An instantaneous topographic map of ocean waves could represent waves traveling in either of two directions, separated by 180°. When the elevation data are transformed by a two-dimensional FFT, each wave system in the resulting directional wave spectrum has two lobes, the actual spectral component and an identical ambiguous lobe propagating in the opposite direction. As was pointed out by Holthuijsen (1983), it is not possible to discriminate between the two spectral lobes with stereophotography and a priori knowledge must be introduced to reject the ambiguous lobe. While that process might not be too difficult near a shoreline, it could be quite troublesome far out to sea.

The Doppler effects in the SCR spectra are shown schematically in Fig. 5 for a two-component wave system propagating in the left-half plane. The actual spectral lobes for the two flight directions have been shifted into their encounter spectrum positions according to the Doppler effects discussed earlier (with drift angle assumed to be zero to simplify things). The FFT processing causes the ambiguous lobes of the encounter spectrum to occupy symmetrical positions relative to the actual lobes of the encounter spectra. That is, the ambiguous lobe positions are shifted in the opposite direction to that which would have been caused by the Doppler effects had they been real. In applying the Doppler corrections, no a priori knowledge of the actual directions of propagation is assumed. In effect, all of the spectral components are assumed to be real and corrected accordingly. The corrections shift the actual lobes into their proper positions but are in the wrong direction for the ambiguous lobes. The actual spectral lobes become readily apparent by comparing Doppler-corrected spectra obtained on flight lines whose ground tracks differed by 90° or more. The actual spectral lobes will be in the same positions for both data sets but the ambiguous lobes for the different ground tracks will be shifted in position relative to one another.

Figure 6 is a superposition of Doppler-corrected spectra from 28 October 1980, for the 34° (solid) and 263° (dashed) ground tracks at 400 m altitude. The local wind was blowing offshore from the SSW at 5 m s$^{-1}$ but the directional wave spectrum consisted of a bimodal system of swell propagating in the left-half plane. Notice that the actual spectra for the two flight directions are essentially identical while the ambiguous lobes are badly mismatched. The ambiguous lobes are not only mismatched but they are shifted out of alignment in the manner indicated by Fig. 5. For example, the “corrected” ambiguous lobes in the 160° direction show little change in wavenumber for
portion of Fig. 7 indicates two flight lines. The first line was flown in the downwind direction, perpendicular to the New Jersey shoreline. The second flight line, flown 2.5 hours after the first, was parallel to the shoreline, starting off the New Jersey coast and ending on the axis of the Delaware Bay. The heavy dots in Fig. 7 indicate the center positions of contiguous sets of 1024 scan lines used to produce the spectra. The radial direction from the center of the mouth of the Delaware Bay to the center of each data set was determined, but only the radials (dotted) for the flight line parallel to the shoreline are shown in the figure.

Figure 8 shows the first six spectra from the outbound leg flown at a 380 m altitude in a format that will be used to give insight into the spectral resolution of the system. The spectra are polar plots in $k$-space with north being indicated by an arrow towards the top of the page and the other radial indicating the direction of the center of the data used in the FFT from the mouth of the Delaware Bay. The origin in $k$-space is in the center of the spectra where the two radials begin. The series of curves making up the spectral presentation are plots of the columns (1024 dimension) of the two-dimensional FFT. The base of each curve indicates the positions in $k$-space of the Doppler-corrected spectral components. The vertical deviation of the curve from its base is proportional to the variance density of the component. Only the

the two flight directions, but the one corresponding to the $34^\circ$ ground track is rotated counterclockwise while the $263^\circ$ ground-track lobe is rotated clockwise. The changes in the ambiguous lobes for the $90^\circ$ wave propagation direction are more apparent in wave number than in direction. The “corrected” lobe for the $34^\circ$ ground track is shifted towards higher wavenumbers while the one for the $263^\circ$ ground track is shifted towards lower wavenumbers. If one looks closely, it can be seen that the $34^\circ$ ground-track lobe is also shifted in the counterclockwise direction.

5. Spectral measurements in vicinity of Delaware Bay

The SCR measured the directional wave spectrum in the vicinity of the Delaware Bay on 5 January 1982. The wind was blowing offshore, nearly parallel to the bay axis, at approximately 17 m s$^{-1}$. The top

FIG. 5. Schematic representation of the Doppler shifts associated with two different aircraft ground tracks (34 and 263°) for a bimodal system of swell propagating in the left half plane and the associated ambiguous lobes in the right half plane. The ×s in the right half-plane indicate the positions which the ambiguous lobes would have occupied had stereophotography been employed so that no Doppler effects were present.

FIG. 6. Overlays for two different ground tracks of the Doppler corrected directional wave spectra for the bimodal system of swell on 28 October 1980. The spectral data on a $10^9 \times 0.01$ m$^{-1}$ wavenumber grid were slightly smoothed by averaging over $3 \times 3$ points with the surrounding eight points each weighted one eighth that of the center point. The solid curves are the average of four spectra and the dashed curves are the average of two, all for 400 m altitude.
Because of the high along-track resolution (0.001 m$^{-1}$), the spectral variance densities were averaged in the along-track direction before plotting using a uniformly weighted, 13-point moving window.

The vertical scale factor used for plotting the variance density was increased linearly with fetch so that any component of constant apparent magnitude in the sequence of six spectra is actually growing linearly with distance from shore. During the 52 s time interval over which data are acquired to generate a spectrum, the aircraft altitude usually has a 2-m standard deviation which contaminates the elevation measurements. This aircraft height variation is removed in post-flight processing by doubly-integrating the output of a vertical motion sensing accelerometer to obtain an independent estimate of the aircraft motion (Walsh et al., 1984). The process is generally quite effective but at times a low-frequency component remains in the data. The spikes near the origin in the cross-track direction (most apparent in spectrum 1 which had the smallest scale factor for spectral variance) are the residue of the aircraft vertical motion not completely removed by the accelerometer.

The only onshore wave system present is the swell near the origin of the spectra. The spectra (not shown) at the last several positions indicated on the downwind flight line of Fig. 7 had the swell traveling towards the north. The spectra show the swell turning progressively towards the northwest, perpendicular to the shoreline, as it nears the coast. It is apparent that the dominant portion of the high wavenumber region of the spectra is made up of waves originating in the Delaware Bay.

Since data in the format of Fig. 8 are difficult to work with quantitatively, the rectangular FFT data are resampled into polar bins of wavenumber versus angle (0.01 m$^{-1}$ x 0.01$^\circ$) and frequency versus angle (0.01 Hz x 0.01$^\circ$). See the appendix for a discussion of the SCR angular resolution, the resampling process, and the number of degrees of freedom contained in the resulting spectra such as Figs. 6, 9 and 32.

Figure 9 shows the spectra from the pass parallel to the New Jersey shoreline on 5 January 1982. Only the right-half plane (0$^\circ$–180$^\circ$) and the wavenumber region from 0.1 to 0.3 m$^{-1}$ is shown. Arrows have been included to indicate the direction from the Delaware Bay, corresponding to the radials in Fig. 7. A number of interesting things are apparent in the sequence. In general, the spectra shift from northeast to southeast, following the radials from the Delaware Bay, indicating that the waves were originating in the bay. However, the two northernmost spectra are actually propagating more northerly than the radials. Since Fig. 7 indicates that these radials graze the shoreline, part of the wave energy may have arrived in that region due to refraction. That might also account for the energy being highest in the first spectrum and then waning over the next three. The
Fig. 8. The first six variance directional wave spectra from the outbound leg in Fig. 7. The spectra are polar plots in $k$-space with north being indicated by the arrow towards the top of the page and the other radial indicating the direction of the data used in the FFT from the mouth of the Delaware Bay.

The first spectrum in Fig. 8 also shows that the waves from the Delaware Bay are propagating in a more northerly direction than the radial from the Delaware Bay.

Spectra 5 through 10 of Fig. 9 have peaks which are south of the radial from the Delaware Bay, which is reasonable since the radials were drawn from the center of the mouth of the bay which has a significant width (Fig. 7). The radial associated with the last spectrum, which is also the most intense, is centered on the spectral peak and Fig. 7 shows that this radial is nearly aligned with the axis of the Delaware Bay.

6. Site of intercomparison with in situ sensors

Comparison of the SCR directional wave spectra with in situ sensors was made during the Atlantic
and related measurements useful for comparing and developing wave measurement systems, evaluating and improving wave models, and providing basic information on wave mechanics in shallow water (IEEE Journal of Oceanic Engineering, Vol. OE-8, No. 4, October 1983).

The ARSLOE study area was centered on the 600 m long instrumented pier and computer facilities of the Field Research Facility (FRF) of the U.S. Army Corps of Engineers, Coastal Engineering Research Center (CERC), located near Duck, North Carolina (bottom of Fig. 7). This location has a relatively smooth bottom topography and a fairly straight coastline without nearby inlets or islands to complicate wave growth and propagation computations.

Figure 10 shows the time history of the wind for the interval of ARSLOE during which SCR flights were made. The wind data were obtained from an anemometer located on top of the FRF building at 21 m above MSL with a 5 m distance between the building top and anemometer. The data shown in Fig. 10 were abstracted from the analog strip recorder at 15 minutes to the hour by David Beesley of NWRI.

Fig. 9. The sequence of directional wave spectra associated with the eleven positions on the flight line parallel to the shoreline in Fig. 7. The spectra are numbered consecutively from north to south, and the direction for the corresponding radial from the Delaware Bay in Fig. 7 is indicated by the arrow in each spectrum.

Remote Sensing Land–Ocean Experiment (ARSLOE) which was a multi-organizational study conducted between 6 October and 30 November 1980. The experiment provided a large number of wind, wave

Fig. 10. Wind speed and the direction from which the wind was blowing during a portion of ARSLOE with vertical dashed lines indicating the times of the SCR flights.
Canada. The times of the three SCR flights considered in this paper are also indicated in Fig. 10.

Figure 11 shows three expanded views of the region within the dotted box at the bottom of Fig. 7 with the positions of the SCR ground tracks indicated. Triangular flight patterns were flown on each day with the three ground tracks oriented roughly perpendicular to, parallel to, and at 45° to the crests of the dominant waves. Each leg was approximately two minutes in duration and the aircraft traveled about 10 km, gathering enough data to generate two contiguous FFTs. Three passes around the triangle were made at each altitude. Since one magnetic tape held sixteen minutes of SCR data, there were three legs recorded on the ground tracks perpendicular and parallel to the wave crests and two legs recorded on the ground track oriented at 45°. Mounting a new magnetic tape and modifying the real-time software for a new altitude took about three minutes while the aircraft changed altitude and repositioned to start the next series of triangles.

Table 1 indicates average parameters associated with the various ground tracks. In the text we will be less exact and will generally refer to the data as being acquired at either 200 or 400 m altitude. All the data at a given altitude were acquired within a 25 minute interval and the total data set for the 200 and 400 m altitudes had an elapsed time of about one hour. Some data were lost on account of operator and pilot errors caused by the short duration of the legs or because of hardware problems. For example, on 23 October 1980 the bolts holding the oscillating mirror (46 × 65 cm ellipse) which scans the beam sheared and it fell off just as the first 400 m triangle was completed.

The waves on the three days represented quite diverse circumstances. On 23 October 1980 the waves were wind driven by an onshore wind of 9 m s⁻¹ which had been blowing for about a day. On 28 October 1980 there was an offshore wind of 5 m s⁻¹, but the waves were a bimodal system of swell. On 12 November 1980 the dominant waves were swell propagating towards 210°. But the wind was blowing from the NNW at 9 m s⁻¹, and at higher wavenumbers the waves turned into the downwind direction. This is the only day in which the significant wave height varied on the three legs. It was lower on the 29° legs which were nearer shore and had a significantly smaller fetch owing to the wind direction being nearly parallel to the shoreline. Since they were farther from shore, the 118° and 238° legs would be influenced by waves propagating out of the mouth of Chesapeake Bay.

7. Comparison with in situ sensors

a. Nondimensional spectra

Figure 11 indicates the locations of three wave riders (WN, WE, WS) and the ENDECO (Leblanc and Middleton, 1982) and XERB (Burdette et al., 1979; Lau et al., 1982) pitch-and-roll buoys which were used in comparisons with the SCR. We will first compare the nondirectional spectra and will use the wave-rouder spectra as the standard of comparison. Extensive comparisons were made among the wave riders during ARSLOE (Szabados and Esteva, 1983), and they were the only in situ data available on every SCR flight. The wave-rouder spectra were computed from 17 minute data spans and had approximately 0.01 Hz resolution with 22 degrees of freedom.

Analysis of individual spectra from the three wave riders (WN, WE, WS) and from one which was only 3 km offshore of the FRF pier indicates that there was no significant temporal or spatial variation during the SCR flight on 23 October 1980. Because of the consistency of the wave-rouder spectra, the average of eight spectra, four each from WE and WN, was used as a standard of comparison. The wave-rouder spectra were from the interval 2155 to 2312Z which covers
TABLE 1. Averaged aircraft parameters on the various ground tracks, the number of directional wave spectra averaged, and the significant wave height (SWH).

<table>
<thead>
<tr>
<th>Date</th>
<th>Altitude (m)</th>
<th>Ground track (deg)</th>
<th>Drift angle (deg)</th>
<th>Ground speed (m s⁻¹)</th>
<th>Number of spectra averaged</th>
<th>SWH (m)</th>
<th>Time (GMT)</th>
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<td>23 October 1980</td>
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<td>-7</td>
<td>104</td>
<td>6</td>
<td>1.6</td>
<td>to</td>
</tr>
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<td></td>
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<td>110</td>
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The time period during which the SCR acquired data (Table 1).

The ENDECO pitch-and-roll buoy was colocated with WE 12 km east of the FRF pier and WN was 15 km north of WE. The XERB pitch-and-roll buoy was located 36 km offshore of the FRF pier. Figure 12 shows the comparison of the average wave-rider spectrum with two spectra from each of ENDECO and XERB and three spectra from each of the SCR altitudes. The ENDECO spectra seem to indicate a temporal variation with the spectral peak shifting lower in frequency over the two-hour period between them. But individual spectra from WE for 2115Z and 2315Z indicated no significant variation. Below 0.08 Hz the ENDECO spectra seem to be consistently too high. The XERB anemometer indicated that the wind on 23 October 1980 had been blowing from 23 to 37° at speeds between 9.6 to 11.8 m s⁻¹ for 13 hours preceding the measurements. The XERB spectra do not indicate any trend over the time period the SCR took data, but they are consistently low above 0.15 Hz compared to the wave-rider spectrum.

The SCR spectra are averages for each of the three ground-track directions from the 200 and 400 m altitudes (Fig. 11, Table 1). The three SCR spectra from the 200 m data are all in good agreement with each other and the wave-rider spectrum above the spectral peak frequency. For frequencies at and below the peak, the SCR 45° ground-track data are in the best agreement with the wave-rider spectrum. The 180° ground-track peak spectral density is lower and the leading edge higher than the wave-rider spectrum, and the 315° ground-track peak spectral density is lower still and the leading edge higher still. Since the waves were propagating towards 225°, the best non-directional spectrum was obtained for a ground track parallel to the wave propagation direction, followed by a ground track at 45° to it, and finally at right angles to it. This systematic variation for the long-wavelength waves is caused by the poor cross-track resolution at 200 m altitude and the interpolation procedure in transforming to the polar coordinates (see Appendix). It indicates that the optimum results at low altitude are obtained by aligning the ground track parallel to the propagation direction of the longest waves of interest.

The 400 m altitude data are noisier because they are averages of only one or two FFTs (see Table 1). The low-frequency region shows much less variation with aircraft ground track than the 200 m data because of the wider swath. Although the spectral peak is the same for all three directions, it is reduced in value.

Figure 13 shows nondirectional spectrum comparisons for 12 November 1980. No ENDECO data were available. The XERB anemometer indicated that for 10 hours before the SCR data were acquired, the wind was blowing from 324 to 335° at 15.5 m s⁻¹ diminishing to about 12.6 m s⁻¹, following the same pattern indicated in Fig. 10. We shall see later in the directional analysis that the waves at the spectral peak frequency were swell propagating towards the SSW, but with increasing frequency the waves turned into the downwind direction. The XERB...
spectral data were taken three hours before and three hours after the SCR flew and the wave field definitely changed during that interval. The left side of Fig. 13 shows a comparison of the two XERB spectra with averages of WE, WN, and WS which were closest in time to the XERB observations. Both sets of spectra show that the peak spectral density diminished and shifted higher in frequency over the six hour interval. As was the case in Fig. 12, the XERB spectra in Fig. 13 are below the wave-ridge spectra in the high-frequency region.

The center and right side of Fig. 13 show comparisons of the SCR spectra for data acquired at 200 and 400 m altitudes. The wave-ridge spectra covering the same time interval as the SCR data indicated a spatial variation of the low-frequency portions of the spectra and separate plots for WS and the average of WE and WN are shown in that region. As in the case of Fig. 12, there is excellent agreement with the wave-ridge spectrum (WE, WN) on the low-frequency side of the peak for both 200 and 400 m altitudes for the 30° ground track which was most nearly aligned with the propagation direction of the low-frequency waves. At low frequencies the agreement degrades as the angle between the ground track and the wave propagation direction increases, with the degradation being worse for the lower altitudes. Above 0.07 Hz the 400 m spectrum for the 239° ground track is in good agreement with the spectrum from WS, to which Fig. 11 indicates it was closest.

In the region above the spectral peak frequency, the agreement is excellent for all three flight directions for the 200 m altitude. For the 400 m altitude the SCR data lie somewhat below the wave-ridge spectra. This is an effect of spatial filtering by the larger illuminated area within the beam at the higher alti-
Fig. 13. Comparison of 12 November 1980 nondirectional spectra from XERB, the SCR, and wave riders. The curves labeled WS and AVG OF WE, WN are averages of spectra with starting times of 1715, 1735, 1755 and 1815 GMT.

In both the 200 and 400 m data the ground tracks farther from shore (118° and 239°, see Fig. 11) indicated greater high-frequency energy than the track nearer shore (29°). Since the 118° ground tracks were closer to XERB than the wave riders and the spectral data still agreed with the wave riders, it suggests that XERB may have been measuring somewhat low in the high-frequency region.

On 28 October 1980, data were not available from either ENDECO or XERB. The wind had been blowing from the SSW for 12 hours at about 5 m s\(^{-1}\). Figure 14 shows a comparison of the SCR nondirectional spectra with the average spectrum for WE, WN, and WS for the same time period. The SCR indicated that the waves were a bimodal system of swell (see Fig. 6), although the wave-riding spectrum...

Fig. 14. Comparison of 28 October 1980 nondirectional spectra from the SCR and wave riders. The wave-riding data is the average of the spectra from three buoys with starting times of 1915, 1935 and 1955 GMT.
indicates three distinct peaks. Figure 11 indicates that the SCR ground tracks were farthest from the wave riders on this day, which might explain why the SCR failed to see the peak in the wave rider spectrum at 0.065 Hz at both altitudes and all three flight directions. It should also be noted that $H_S$ was only 0.9 m and the spectral peak was an order of magnitude lower than either of the two cases already considered. Quantification noise might have played a part in the low-frequency discrepancy.

The SCR data from both altitudes and all three flight directions are consistent with each other. There is general agreement with the buoys in the spectral levels although the SCR definitely reads higher in the 0.20–0.25 Hz region. This was probably caused by the greater distance from shore of the aircraft ground tracks than the wave riders and the spatial variation of the wave field that will be demonstrated later was caused by the offshore 5 m s$^{-1}$ wind.

b. Angular distribution of wave energy

Comparison of the directional wave spectrum of the SCR will be made with the pitch-and-roll buoy data through the use of Fourier coefficients following the development of Longuet-Higgins et al. (1963) and Hasselmann et al. (1980). The directional wave spectrum is represented by a Fourier series,

$$E(f, \theta) = a_0/2 + \sum_{n=1}^{\infty} (a_n \cos n\theta + b_n \sin n\theta)$$ (20)

where

$$a_0 = \pi^{-1} \int_0^{2\pi} E(f, \theta) d\theta$$ (21)

$$a_n = \pi^{-1} \int_0^{2\pi} E(f, \theta) \cos n\theta d\theta$$ (22)

$$b_n = \pi^{-1} \int_0^{2\pi} E(f, \theta) \sin n\theta d\theta$$ (23)

with angles measured clockwise from north. $E(f, \theta)$ can be written as a product of the nondirectional frequency spectrum $E(f)$ and the spreading function $S(f, \theta)$ where

$$E(f, \theta) = E(f)S(f, \theta)$$ (24)

$$\int_0^{2\pi} S(f, \theta) d\theta = 1.$$ (25)

It follows that

$$E(f) = \pi a_0$$ (26)

$$S(f, \theta) = \frac{1}{2\pi} \left[ 1 + 2 \sum_{n=1}^{\infty} \left( \frac{a_n}{a_0} \cos n\theta + \frac{b_n}{a_0} \sin n\theta \right) \right]$$ (27)

$$S(f, \theta) = \frac{1}{2\pi} \left[ 1 + 2 \sum_{n=1}^{\infty} r_n \cos(n\theta - \theta_n) \right]$$ (28)

where

$$r_n = (a_n^2 + b_n^2)^{1/2}/a_0$$ (29)

$$\theta_n = \tan^{-1}(b_n/a_n).$$ (30)

A pitch-and-roll buoy is capable of measuring the first five Fourier coefficients. Since it is difficult to gain insight from the individual coefficients $a_1$, $b_1$, $a_2$ and $b_2$, we will make the SCR-buoy comparisons using $r_1$, $\theta_1$, $r_2$, $\theta_2$ where $r_1$ and $r_2$ are the magnitudes of the fundamental and second harmonic of the spreading function, and $\theta_1$ and $\theta_2$ are the associated angles. The mean direction of propagation is $\theta_1$.

It should be said that Kenneth Steele of the NOAA Data Buoy Office pointed out that the $a_2$ Fourier coefficient in the XERB data was in error because a hardware problem had eliminated the $C_{33}$ term needed in its calculation. It was possible to calculate $C_{33}$ following the approach of Weissman and Johnson (1984) who used the fundamental relation among $C_{11}$, $C_{22}$ and $C_{33}$ (Longuet-Higgins et al., 1963)

$$k^2 C_{11} = C_{22} + C_{33}$$ (31)

to compute $C_{33}$ and then the $a_2$ coefficient.

$$a_2 = C_{22} - C_{33} = 2C_{22} - k^2 C_{11}.$$ (32)

![Figure 15](image-url) FIG. 15. The magnitudes and phase angles of the XERB Fourier coefficients for the two data sets bracketing the SCR data on 12 November 1980.
The procedure just described seemed to work quite well as will be seen in the good agreement between the XERB and the SCR $\theta_2$ values.

Since the generation and dissipation of waves involves integrations over both time and space we should expect that the Fourier coefficients would be smooth functions of frequency, even under varying wind conditions or when sea and swell are both present. This assumption permits us to make some value judgments as to the amount of noise in the Fourier coefficients even when there is not a comparative data set available. Figure 15 shows the two sets of values of $r_1$, $r_2$, $\theta_1$, and $\theta_2$ from XERB on 12 November 1980. The low-frequency wave energy was swell propagating towards the SW whereas with increasing frequency the waves turned into the downwind direction and propagated towards the SSE. The data points for the two observations of $\theta_1$ indicate a definite variation of the wave-field direction of propagation over the six-hour interval between them. However, the variation is small enough that the mean value should serve well to compare with the SCR observations which were made midway between the two XERB observations.

Figure 15 indicates that XERB had a problem with the magnitudes of the Fourier coefficients at the lower frequencies. Both $r_1$ and $r_2$ exceed unity which is not possible since they are normalized parameters. This could have been caused by a problem in the calibration of the buoy sensors with regard to amplitude. Such a problem would not affect the angular coefficients since they are computed from ratios and any calibration error common to both numerator and denominator would have no effect on them.

Figures 16 and 17 show the Fourier coefficients determined from the SCR data at the 200 m and 400 m altitudes, respectively. The solid curves are the averages of the XERB data from Fig. 15. The first thing to notice is the consistency of the SCR data from one altitude to another. Not only is the general behavior of the coefficients virtually the same, even the minor variations from one flight line to another are nearly identical. For example, the amplitude of the $r_1$ coefficient is lowest for the 118° ground track at both altitudes. Also, the mean direction of propagation ($\theta_1$) for the 29° ground track is rotated clockwise relative to the other tracks in the interval 0.09–0.15 Hz for both altitudes. But above 0.2 Hz the 118° ground track $\theta_1$ is rotated clockwise at both altitudes. This kind of consistency with altitude change in the minor variations of the Fourier coefficients from one ground track to another is the same for all three sets of data acquired at ARSLOE. We believe that the
SCR is measuring real spatial variations of the wave field from one ground track to another. We will readdress this issue later in our analysis of the data taken on 28 October 1980.

The agreement between XERB and the SCR in the mean direction of propagation $\theta_1$ is good with the best overall agreement being on the 118° ground track which Fig. 11 indicates was closest to XERB on 12 November 1980. The agreement between XERB and the SCR is also good for $\theta_2$. There is not such good agreement in the case of the $r_1$ and $r_2$ coefficients. Below 0.17 Hz $r_1$ and $r_2$ are consistently too large compared to the SCR. Since the XERB coefficients are certainly in error when they exceed unity, one might suspect that they are running larger than they should be at frequencies just above where they exceed unity.

Figure 18 shows the variation with frequency of $r_1$, $r_2$, $\theta_1$ and $\theta_2$ for the two XERB directional wave spectra on 23 October 1980. The $r_1$ and $r_2$ coefficients show the same characteristic of exceeding unity at lower frequencies that they did on 12 November 1980 (Fig. 15). The mean direction, $\theta_1$, shows no trend with time and indicates that above 0.13 Hz the waves were sea propagating towards the SW which was generated by the NE wind. Below 0.13 Hz there appears to be swell propagating towards the W. Figure 19 shows the coefficients for the two ENDECO directional wave spectra on 23 October. The ENDECO values of $r_1$ and $r_2$ do not exceed unity at low frequencies, but $r_1$ becomes quite small. There seems to be a problem in $\theta_1$ for low frequencies because it sometimes indicates waves propagating away from shore.

In Figs. 20 and 21 curves of the average Fourier coefficients from XERB and ENDECO are compared with those determined from the SCR data taken at the 200 and 400 m altitudes, respectively. The consistency of the SCR data from one altitude to the next for a given ground track should again be noted. Above 0.12 Hz the agreement among the SCR, XERB, and ENDECO for the mean propagation direction ($\theta_1$) is good. The general amplitude levels of $r_1$ and $r_2$ for the SCR match the levels of those coefficients in the data of 12 November 1980 seen in Figs. 16 and 17. As was the case on 12 November 1980 the XERB $r_1$ coefficient on 23 October 1980 is larger than the SCR coefficient at low frequencies. On the other hand, the ENDECO $r_1$ coefficient is much lower than the SCR coefficient at the lower frequencies in Figs. 20 and 21. But this was the region that ENDECO seemed to read anomalously high in spectral density (Fig. 12). We believe that the SCR coefficients are correct and will demonstrate that with Figs. 22, 23, and 24.
Fig. 20. As in Fig. 18 but of the SCR Fourier coefficients for the three ground tracks at 200 m altitude and the averages of the XERB and ENDECO data from Figs. 18 and 19.

Fig. 22. Amplitude ratios for the ENDECO and XERB data sets from ARSLOE, for data sets from the National Institute of Oceanography pitch-and-roll buoy, and the curve indicating the theoretical ratio for a wave field satisfying (33).

8. Relationships among Fourier coefficients

The generally assumed model for the azimuthal variation of the wave energy (Longuet-Higgins et al., 1963; Cartwright, 1963; Hasselmann et al., 1980) is

\[ A(s) \cos^2\left[ (\theta - \theta_i)/2 \right] \]  \hspace{1cm} (33)

where \( A(s) \) is a normalizing constant to satisfy (25) and \( s \) is not necessarily an integer.

If the azimuthal variation of the wave energy is of the form given in (33), then there is a fixed relationship that must exist between \( r_1 \) and \( r_2 \). Cartwright (1963) used this theoretical relationship to test the consistency of the coefficients obtained from the National Institute
of Oceanography pitch-and-roll buoy. His coefficients are shown at the top of Fig. 22 along with the theoretical relationship indicated by the solid curve. It is seen that the curve and the data follow the same trend, but almost all of the data points lie to the left of the curve. Cartwright suggested that the discrepancies might be due to a bimodality of wind-wave spectra suggested by some results discussed by Longuet-Higgins et al. (1963) but thought it too complicated to treat in his analysis. The bottom of the figure shows the data points from XERB and ENDECO for the ARSLOE data comparisons with the SCR. The ENDECO data points occupy the same position relative to the theoretical curve as the Cartwright data points, although ENDECO certainly has many outliers which could be accounted for by the too low

Fig. 23. Amplitude ratios for the SCR ARSLOE data sets and the theoretical ratio for a wave field satisfying (33).

Fig. 24. Comparison of the (33) spread parameter \( s \), determined from \( r_1 \) and from \( r_2 \), for the SCR ARSLOE data sets and the values from pitch-and-roll data taken during JONSWAP.
$R_1$ values at low frequencies. The XERB data points lie on both sides of the curve, but the mean is still biased to the left. It should be pointed out that fewer points are plotted for XERB than for ENDECO because any points for which either $r_1$ or $r_2$ exceeded unity were not plotted since they were not physically realizable.

Figure 23 shows all of the SCR data points from the three flights during ARSLOE for the 400 (top of figure) and 200 m (bottom) altitudes. Different symbols are used for each day but the same symbol is used for the ground tracks on a given day since analysis of the data showed no variation in the $R_1$ versus $R_2$ characteristic for the various ground tracks. Compared to the in situ data of Fig. 22, the SCR data are an almost perfect fit to the theoretical curve.

In 1973 an extensive set of observations of the directional wave spectrum was made using the University of Hamburg meteorological buoy and an Institute of Ocean Sciences pitch-and-roll buoy during the Joint North Sea Wave Project (JONSWAP). Hasselmann et al. (1980) indicated that well defined steady meteorological conditions were never encountered during the twenty-day period of their study. However, they were able to select a number of cases under fairly steady wind conditions in the absence of swell to study the variation of the spreading function with wave frequency. In analyzing the consistency of the magnitudes of the Fourier coefficients for this data set, Hasselmann et al. used a slightly different approach from that of Cartwright (1963). Instead of developing a scatter plot of $r_2$ versus $r_1$, they plotted $s_2$ (the estimate of $s$ in equation 33 obtained from $r_2$) versus $s_1$, the estimate of $s$ obtained from $r_1$. If the azimuthal energy distribution is of the form of (33), then $s_1$ will be identical to $s_2$ and all the points will fall on a 45° line. The data from Hasselmann et al. are reproduced at the top of Fig. 24. It is apparent that there is a large scatter in the data. For comparison, the SCR data points from ARSLOE have been replotted in the same format in the middle (200 m altitude) and bottom (400 m) of Fig. 24. Once again we see that there is virtually no scatter in the SCR data compared to the buoy data. The slope of the SCR data points appears to be slightly greater than unity and the dotted straight lines in the figure have been least-squares fitted to the data. It is interesting that the right side asymptote of the buoy data also has a slope which exceeds unity, but the scatter in the data was apparently so large that Hasselmann et al. did not consider it noteworthy.

The functional form of the spreading function given in (33) is unimodal and symmetrical and implies that all of the azimuthal angles associated with the Fourier coefficients are aligned in the same direction. But because of the manner in which we have defined the phase angles in (28) and (30) with $\theta_n$ not multiplied by $n$ as Hasselmann et al. (1980) did, $\theta_n$ will be $n$ times larger than $\theta_1$ in the situation just described. Figure 25 demonstrates this effect graphically for $\theta_1$ and $\theta_2$. The SCR data points shown are from the 30° ground track for the 200 m altitude data (Fig. 16). The curve on the left side of the figure is a piecewise linear approximation to the $\theta_1$ data points. The curve on the right side of the figure is the curve on the left multiplied by two. It is apparent from the agreement of this curve with the $\theta_2$ data that the data should be well represented by a unimodal, symmetrical functional form such as given in (33).

Figure 26 shows scatter plots of $\theta_2$ versus twice $\theta_1$ for the SCR data points for all three ARSLOE data sets. There is some scatter but in general the SCR data points fall along the unity slope line which would be the locus of points for wave fields whose azimuthal dependence satisfies (33). It should be pointed out that the cluster of points at the lower right in the 400 m data set are actually not far from the line since they could be equally well represented by slightly negative angles for twice $\theta_1$. Figure 27 shows the same scatter plot for the XERB and ENDECO data sets. The trend of the XERB data points is along the line and they exhibit about the same scatter as the SCR data in Fig. 26. The ENDECO data show more scatter than either XERB or the SCR.

Recall that the three SCR datasets were acquired under quite diverse conditions. The wave field on 23 October 1980 was essentially wind driven with a swell component at low frequencies, on 28 October 1980 it was essentially a bimodal swell, and on 12 November 1980 it was a combination of sea and swell with the wave propagation direction changing significantly as a function of frequency. But on none of the days was there a bimodal distribution of wave energy at a given frequency with sea and swell propagating orthogonal to each other as in the hurricane directional wave spectrum shown in Fig. 32. Figures 23 through 27 seem to indicate that (33) appears to be a good
representation of the azimuthal dependence of a wave field if there is not a bimodal distribution of wave energy at a given wave frequency. The other conclusion is that buoys have trouble measuring the Fourier coefficients of the directional wave spectrum to the accuracy demonstrated by the SCR.

9. Spatial variation of wave field

We will now reexamine our earlier suggestion that the variations in evidence in the SCR Fourier coefficients in Figs. 16, 17, 20 and 21 were real. Figures 28 and 29 show the SCR Fourier coefficients for 28 October 1980 for the 200 and 400 m altitudes,
ground track. This was disturbing at first because the wave field was assumed to be swell and the water depth was approximately 25 m which should have had no effect on the 0.25 Hz waves whose wavelength was 25 m. But, as with the other data sets, there is a great consistency of the $\theta_1$ and $\theta_2$ data between the two altitudes.

This apparent inconsistency was resolved using Figs. 30 and 31. Figure 30 is a plot of the spatial variation of the wave propagation direction at 0.25 Hz. The propagation directions were determined from individual FFTs at each altitude instead of the averages from each ground track direction as was done in Figs. 28 and 29. The symbols indicate the locations of the centers of the 1024 scan line data sets used in each FFT. The radial from each symbol extends in

respectively. This is the day on which the waves were essentially a bimodal system of swell (Fig. 6). Notice that the angles of the Fourier coefficients ($\theta_1, \theta_2$) show less noise at the lower frequencies for the 400 m altitude than the 200 m altitude because of the wider swath. The mean propagation direction ($\theta_1$) for the 200 m altitude data shows very little noise above 0.15 Hz; however, there is a significant divergence with

Fig. 29. As in Fig. 28 but for the three ground tracks at 400 m altitude.

Fig. 30. The locations of the centers of SCR data sets used in determining the directional wave spectra for 200 m (circles) and 400 m (triangles) altitudes and radials indicating the mean direction of propagation of the energy at a wave number of 0.25 m$^{-1}$.

Fig. 31. Cross-sectional cuts through the SCR directional wave spectrum for the 200 m altitude and 160° ground track on 28 October 1980. The vertical scales for the various spectral cuts have been varied to preserve the apparent area under them in the figure. The data are from an average of six spectra and each point is an average over a 0.03 m$^{-1}$ interval centered on the wavenumber indicated. The peak values for the series of curves, from top to bottom are 0.0118, 0.0088, 0.0068, 0.0055, 0.0057, 0.0038, 0.0027, 0.0021, 0.0016, and 0.0011 m$^2$ deg$^{-1}$.
the wave propagation direction. It is apparent that the wave propagation direction at 0.25 Hz swings towards the north as the distance from shore increases. We believe that this consistent spatial variation can be explained by the local wind which was blowing from the SSW at about 5 m s⁻¹.

The averaged data (Figs. 28 and 29) from each of the three ground tracks for both altitudes indicated that the wave propagation direction at 0.15 Hz was to the NNW and it is reasonable to assume that that was also the direction of propagation of the swell for frequencies above 0.15 Hz. Figure 31 shows the variation with wavenumber of the average directional wave spectrum for the 200 m altitude and the 160° ground tracks, which Fig. 11 indicates were the farthest offshore and parallel to the shoreline. The data indicate that as wavenumber increases, the wave energy in the NNW direction diminishes and there is a growth of the wave energy in the NNE direction corresponding to the direction towards which the wind was blowing (reciprocal of the wind direction). What was actually happening in this instance is that the waves were changing from swell to sea as the distance offshore increased, and the SCR was sensitive enough to measure that transition. The spectra used in Fig. 6 were carefully selected to be as close to shore as possible to minimize the effect just discussed, but it still caused some mismatch in the actual spectra in Fig. 6 above 0.2 m⁻¹.

10. Hurricane Debby directional wave spectrum

The SCR can determine even a complex spectrum in great detail. Figure 32 shows a directional wave spectrum generated from data taken at 860 m altitude approximately 240 km west of the eye of Hurricane Debby on 17 September 1982. The spectrum shows the presence of both sea and swell generated by the hurricane. By flying along different ground tracks the ambiguous spectral lobes were rejected and are shown crossed out in the figure. The swell peak spectral density is at 0.09 Hz and has a 25° half-power width which Table A1 indicates is the resolution limit of the SCR at that frequency. The peak spectral density of the sea is at 0.12 Hz with a 35° half-power width which is about three times as wide as the SCR resolution indicated in Table A1.

Figure 33 shows the aircraft position relative to the ground track of the hurricane obtained from the forecasts. Also indicated in the figure is the expected wind at the aircraft position from the hurricane forecast, confirmed by the wind measured at the aircraft altitude.

The spectrum gives an indication of the detail that the SCR can provide to people studying wave growth and dissipation under these highly nonlinear circumstances. The sea and swell spectral lobes both turn through 30° in direction of propagation as frequency increases, the sea in a clockwise direction and the swell in a counterclockwise direction. The extrapolated positions indicated in Fig. 33 for two swell components (290° for the spectral peak and 280°) indicate they were generated at approximately 1000 GMT in regions 100 and 140 km from the eye of the hurricane, respectively. It would have been impossible for a pitch-and-roll buoy to produce this spectral detail.
11. Aircraft operations and data reduction

The WFF P-3 aircraft on which the SCR is carried has an operational flight endurance of approximately six hours. This total time must be divided between the transit (at 150 m s\(^{-1}\) air speed) to the area where data are to be acquired and the data acquisition itself (at 100 m s\(^{-1}\)). Aircraft operations are not inexpensive. The aircraft presently costs $2700/hour to operate, so it would require almost $50,000 just to fly the aircraft from WFF to the West Coast of the United States for an experiment and back again. Once at an experiment site, the aircraft can very quickly acquire data over a large geographical area.

It takes about 30 minutes of computer time to completely process each 52 s SCR data set into a directional wave spectrum, generating either a plot of the form of Figs. 6 or 32, or a false-color coded spectrum displayed on the SCR color TV monitor. The data are generally processed using overnight computer runs at WFF but a sampling of the data can be processed at a remote site to do quick-look analysis during an experiment.

12. Conclusions

The \(\cos^2(\theta/2)\) azimuthal variation of wave energy seems to be an appropriate model when there is not a bimodal distribution of wave energy at a given frequency. The pitch-and-roll buoys used in this study do not measure the Fourier coefficients to the same overall accuracy as the SCR. There is also an indication that the SCR spreading-function measurements may be superior to those of buoys in general. The SCR provides a means to measure sea-surface directional wave spectra easily and directly with high resolution. Such information would be extremely useful in developing and verifying oceanographic models as well as validating indirect remote-sensing oceanographic techniques such as side-looking radars and wave spectrometers (Jackson et al., 1985a,b).

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**APPENDIX**

**SCR and Buoy Angular Resolution and SCR Degrees of Freedom**

1. Angular resolution

After performing the two-dimensional FFT, the SCR along-track spectral bins are averaged in non-overlapping groups of three, resulting in six degrees of freedom with \(k\)-space resolution of roughly 0.0038 m\(^{-1}\) along-track by 0.030 m\(^{-1}\) cross-track for a 400 m altitude. The FFT datapoint spacing after averaging is shown in Fig. A1 where the ordinate is the along-track direction and the abscissa is the cross-track direction. The variance associated with each FFT point indicated is the result of \([\sin(D)/D]^2\) sampling functions in \(k\)-space caused by the rectangular elevation data window, where \(D\) is proportional to the distance from the FFT point in \(k\)-space. In this example, the cross-track direction parameter would be

\[
D = \pi(k_x - k_{x0})/\Delta k_x
\]

(A1)

where \(k_{x0}\) is the \(k\)-space location of the FFT point, \(k_x\) is any cross-track wave number and \(\Delta k_x\) is the cross-track spacing of the FFT points. Two of the cross-track sampling functions are shown at the top of Fig. A1 for adjacent FFT points, and one is shown along the upper left-hand edge of spectrum 1 in Fig. 8. Since the SCR swath width is 0.5236 \(k\), where \(h\) is the aircraft altitude, then

\[
\Delta k_x = 2\pi/(0.5236h) = 12h^{-1}.
\]

(A2)

Because the along-track spectral spacing after averaging is still a factor of eight finer than the cross-track spacing for 400 m altitude data, the angular resolution in the along-track direction is inversely proportional to the wavenumber. It is given by

\[
\Delta \theta_x = \tan^{-1}(\Delta k_x/k_y) = \tan^{-1}(12/hk_y).
\]

(A3)

Table A1 shows the angular resolution determined from (A3) for various combinations of wavenumber and aircraft altitude. The dimensions of the spot illuminated on the sea surface by the half-power beam width are also indicated in Table A1. The high
Table A1. Along-track angular resolution versus wavelength.

<table>
<thead>
<tr>
<th>Wavelength</th>
<th>Wavenumber</th>
<th>Wave frequency</th>
<th>215 m altitude (3.6 × 5.4 m spot)</th>
<th>430 m altitude (7.2 × 10.8 m spot)</th>
<th>860 m altitude (14.4 × 21.6 m spot)</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>0.016</td>
<td>0.062</td>
<td>74.3</td>
<td>60.6</td>
<td>41.1</td>
</tr>
<tr>
<td>300</td>
<td>0.021</td>
<td>0.072</td>
<td>69.4</td>
<td>53.1</td>
<td>33.6</td>
</tr>
<tr>
<td>200</td>
<td>0.031</td>
<td>0.088</td>
<td>60.6</td>
<td>41.6</td>
<td>24.2</td>
</tr>
<tr>
<td>150</td>
<td>0.042</td>
<td>0.102</td>
<td>53.1</td>
<td>33.7</td>
<td>18.4</td>
</tr>
<tr>
<td>100</td>
<td>0.063</td>
<td>0.125</td>
<td>41.6</td>
<td>23.9</td>
<td>12.5</td>
</tr>
<tr>
<td>75</td>
<td>0.084</td>
<td>0.144</td>
<td>33.7</td>
<td>18.4</td>
<td>9.4</td>
</tr>
<tr>
<td>50</td>
<td>0.126</td>
<td>0.177</td>
<td>23.9</td>
<td>12.5</td>
<td>6.3</td>
</tr>
<tr>
<td>25</td>
<td>0.251</td>
<td>0.250</td>
<td>12.5</td>
<td>6.3</td>
<td>3.2</td>
</tr>
<tr>
<td>15</td>
<td>0.419</td>
<td>0.322</td>
<td>7.6</td>
<td>3.8</td>
<td>1.9</td>
</tr>
<tr>
<td>12</td>
<td>0.524</td>
<td>0.361</td>
<td>6.1</td>
<td>3.0</td>
<td>1.5</td>
</tr>
</tbody>
</table>

nominal resolutions at 860 m altitude for 12 and 15 m wavelengths would not be achieved because the footprint is larger than the wavelength and the amplitude would be highly attenuated. The elevation-noise increase with altitude (Walsh et al., 1985) would also corrupt the measurements severely. It can be seen in Fig. A1 that the angular resolution in the cross-track direction would be much higher than in the along-track direction, but at low altitudes it is smeared over a large wavenumber interval compared with the along-track resolution. Information from various ground-track directions could be combined to improve the overall resolution but this additional complexity is not normally justified.

To put the resolutions of Table A1 in perspective, it should be kept in mind that the highest resolution attainable with the two harmonics measurable with a pitch-and-roll buoy is 72° (Panicker, 1974). When a nonnegative smoothing function is used (Longuet-Higgins et al., 1963) to weight the Fourier series coefficients to remove the negative side lobes, the resolution degrades to 135°.

The significance of the pitch-and-roll buoy angular resolution needs some discussion. Suppose that the only waves on the sea surface are a long-crested system propagating towards the north. The data out of the buoy would indicate that it was only pitching back and forth in the north–south plane as the waves passed by and not rolling in the east–west plane. Obviously the buoy can tell that this is a long-crested wave system with a very narrow spreading function. How is that determination reconciled with the very poor angular resolution just cited for the buoys?

In resolving this seeming paradox we will examine the buoy Fourier coefficients for three wave systems with very different spreading functions: narrow, broad, and bimodal. To simplify the mathematics we will assume that the spreading functions are rectangular and symmetrical about north. Under these conditions $b_0$ and $b_2$ will be zero and $r_1$ and $r_2$ will equal $a_1$ and $a_2$, respectively, then using (29), (21), and (22) it can be shown that

$$r_1 = \int_0^{2\pi} S(f, \theta) \cos \theta d\theta \quad \text{(A4)}$$

$$r_2 = \int_0^{2\pi} S(f, \theta) \cos(2\theta) d\theta \quad \text{(A5)}$$

If $\theta \ll 1$, then

$$\cos \theta \approx 1 - \theta^2 / 2 \quad \text{(A6)}$$

$$r_1 \approx 1 - 0.5 \int_0^{2\pi} S(f, \theta) \theta^2 d\theta = 1 - \sigma^2 / 2 \quad \text{(A7)}$$

$$\sigma \approx (2 - 2r_1)^{1/2} \quad \text{(A8)}$$

where $\sigma$ is the rms spread angle. Cartwright (1963) and Hasselmann et al. (1980) have used (A8) to measure the width of the spreading function.

Figure A2 shows the three spreading functions used to demonstrate the angular resolution characteristics of pitch-and-roll buoys. The top of the figure indicates a spreading function of 15° width which could be thought of as swell. The middle of the figure indicates a spreading function of 120° width, and the bottom shows a bimodal spreading function whose lobes are each 15° wide which could represent two swell systems propagating at 67° to each other. Also shown are the buoy-determined spreading functions developed from $r_1$ and $r_2$ which were computed from (A4) and (A5). It is apparent that except for Case 2 in Fig. A2, the buoy does not have the resolution to do a reasonable job in plotting the azimuthal variation of wave energy. The buoy spreading function also goes negative (most notable in the top case), and the weighting factors to make it non-negative broaden the spreading function even more.

Table A2 lists the buoy values for $r_1$, $r_2$, and the rms spread angle determined from (A8) and the actual spread angle. In the first case the buoy was able to determine the correct width of the spreading function to within a small fraction of a percent. Therefore, the broad angular resolutions cited above do not mean that the buoys are incapable of determining that the spreading function of a wave field is
generated between each pair of spectral points in the cross-track direction of the encounter spectrum. The variance value assigned to each point is obtained by linear interpolation of the values of the adjacent points in the encounter spectrum. (In Fig. 8 this would result in three additional curves between each of the existing curves.) The variance associated with each of the spectral points is then added to the value of the variance in the polar bin (10$^\circ \times 0.01$ m$^{-1}$ or 10$^\circ \times 0.01$ Hz) in which the point falls. This technique allows transformation to frequency space without use of the Jacobian. As a final step, the variance in each bin is multiplied by a constant to make the total variance of the spectrum after the transformation equal to the variance before the transformation.

Figure A3 shows the mapping of spectral points from the encounter spectrum into polar bins in wavenumber space and frequency space. Each dot represents a nonoverlapping average of three along-track spectral bins so they are independent and have six degrees of freedom. The number of degrees of freedom of a spectral estimate in the polar bins (10$^\circ \times 0.01$ m$^{-1}$ or 10$^\circ \times 0.01$ Hz) depends on the number of points that fall in the bin. For example, at 0.10 m$^{-1}$ there would be 18 degrees of freedom for the spectral estimate in the along-track direction (three points in bin) and 30 degrees of freedom for the cross-track spectral estimate (five points). For 0.10 Hz there would only be 18 degrees of freedom (three points) for the cross-track spectral estimate and 12 degrees of freedom (two points) for the along-track estimate. The degrees of freedom increase as either wave number or frequency increases.

The dots in Fig. A3 represent the conditions under which the data in Hurricane Debby were acquired (Fig. 32) except that corrections for Doppler and drift angle have not been included. When they are, the mapping is no longer symmetrical in all four quadrants because the points shift slightly in the along-track direction and rotate. The point at zero frequency and the first three points in the along-track direction are circled to indicate that they are arbitrarily set to zero since they are frequently contaminated by a residue of the aircraft vertical motion.

The dots in Fig. A3 (except for the ones in the along-track direction) can be viewed as making up a

2. Degrees of freedom in SCR spectral estimates

Because of the wide spacing of points in the cross-track direction (Fig. A1), three additional points are very narrow. The buoy also came to within a few percent of the rms spread angles for Cases 2 and 3, so (A8) provides a good measure of the rms spreading angle even when the angles involved are not much less than unity. However, the buoy saw little difference between the last two cases because of its poor resolution. On the other hand, the SCR would have provided a good reproduction of the spreading functions of Cases 1 and 3 if Table 1 indicated that its resolution was sufficient. For example, $k = 0.063$ for 860 m altitude, or $k = 0.126$ for 430 m, or $k = 0.251$ for 215 m. If the SCR resolution were less, the spreading function reproduction would degrade accordingly, but it would still give a better picture of what was going on than the buoy as long as the SCR resolution were higher than that of the buoy.

<table>
<thead>
<tr>
<th>Case</th>
<th>$r_1$</th>
<th>$r_2$</th>
<th>$\sigma$</th>
<th>Actual $\sigma$</th>
<th>Error in buoy measurements of $\sigma$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.997</td>
<td>0.989</td>
<td>0.076</td>
<td>0.076</td>
<td>0.02 low</td>
</tr>
<tr>
<td>2</td>
<td>0.827</td>
<td>0.413</td>
<td>0.588</td>
<td>0.605</td>
<td>2.8 low</td>
</tr>
<tr>
<td>3</td>
<td>0.823</td>
<td>0.358</td>
<td>0.595</td>
<td>0.605</td>
<td>1.7 low</td>
</tr>
</tbody>
</table>
series of curves which are orthogonal to the abscissa in the cross-track direction and turn with increasing abscissa value towards being parallel to the abscissa. These curves indicate how the curves making up Fig. 8 map into the polar presentation. The interpolation process described at the beginning of this section would generate three additional curves between each pair of existing curves in Fig. A3 and no bin would be void of data points. However, Fig. A3 represents an aircraft altitude of 860 m. If the altitude had been 215 m, there would have been only one-fourth as many curves although the spacing of the points along the curves would have been the same. One can view Fig. A3 as the density of points for 215 m altitude data after the interpolation process. It is apparent that some bins at lower abscissa values will be void of points. For 430 m altitude data, after the interpolation process there would be twice as many curves as indicated in Fig. A3.

REFERENCES


