Gulf Stream Kinematics along an Isopycnal Float Trajectory

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ABSTRACT

An isopycnal-following float was deployed near the 400 m depth level in the high speed jet region of the Gulf Stream and tracked for approximately 300 km from the Blake Plateau towards Cape Hatteras during 16-19 May 1983. During its transit through a 400% change in bottom depth, the layer tagged by the float was surveyed with XBTs and hydrostations, and the surface mesoscale structure was determined from NOAA-6, 7 and 8 IR imagery. These data provide the first three-dimensional observations of water parcel motion along a Gulf Stream trajectory in the upper main thermocline. Approaching path segments with anticyclonic curvature, flow in the surrounding Stream volume was divergent and vertically compressed. Approaching path segments with cyclonic curvature, the flow in the surrounding Stream volume was convergent and vertically stretched. The float was upwelled approaching anticyclonic path segments and downwelled approaching cyclonic path segments.

Potential vorticity estimates along the float trajectory agreed within 10%, primarily through compensating changes in curvature vorticity and vortex stretching. Estimates of individual terms in the horizontal divergence from float and hydrographic data indicate that the cross-stream gradient of cross-stream velocity was approximately \(7.0 \pm 1.4 \times 10^{-5} \, \text{s}^{-1}\) and the downstream gradient of downstream velocity was approximately \(2.0 \pm 0.4 \times 10^{-4} \, \text{s}^{-1}\). An independent estimate of the vertical gradient of vertical velocity showed it to have been approximately \(2.5 \times 10^{-4} \, \text{s}^{-1}\). The downstream volume flux divergence in the local volume surrounding the float was approximately \(2 \times 10^{-3} \, \text{cm}^2 \, \text{s}^{-1}\), comparable with previous calculations involving the entire Gulf Stream. A quantitative estimate of the mean horizontal divergence between a track crest and trough based on a wave model of Palmen and Newton using our float, hydrographic data, and satellite data results in a value of \(3.1 \pm 1.9 \times 10^{-4} \, \text{s}^{-1}\), comparable with previous estimates in the Gulf Stream.

1. Introduction

The characterization of the kinematic properties of a three-dimensional water mass trajectory in the Gulf Stream has not previously been attempted. Both upstream of Cape Hatteras and in deeper waters downstream of Cape Hatteras, studies of the mesoscale velocity and density fields of the Gulf Stream have been performed using such methods of observations as moored current meters and AXBT temperature surveys (Bane et al., 1981; Brooks and Bane, 1981, 1983), repeated cross-stream sections (Webster 1961a,b; Levine and Bergin, 1983), IR satellite imagery (Olson et al., 1983; Halliwell and Mooers, 1983), and arrays of moored inverted echo sounders and current meters (Watts and Johns, 1982; Johns and Watts, 1985). These studies have been devoted to obtaining Eulerian descriptions of the two-dimensional horizontal velocity field and its energetics and to mapping of the baroclinic structure and its time evolution. The few Lagrangian descriptions of fluid motion in the Gulf Stream have been mostly limited to the trajectories of isobaric floats (Rossby et al., 1983) or isopycnal trajectories in the deep SOFAR channel (Rossby et al., 1985b).

This work is made possible by the recent development of the isopycnal Swallow Float (Rossby et al., 1985a), which provides a tool to examine the paths of fluid motion in regions with complicated path structures and strong baroclinicity and, for the first time, to directly observe vertical velocity along isopycnal surfaces. Combining these observations with a survey of the nearby Gulf Stream temperature and density field provides data suitable for an examination of pathways for advection and mixing, processes which affect small-scale physical and biological fields.

In the region of study between 31° and 35°N, the Gulf Stream leaves the Blake Plateau south of Cape Hatteras, experiencing a 400% depth change over 100 km. In the area near Cape Hatteras, the meandering of the Gulf Stream reaches a minimum (~15 km standard deviation) (Olson et al., 1983; Halliwell and Mooers, 1983) and coincides with the convergence of bathymetric contours (100–3500 m). In the Cape Hatteras region between 33° and 35°N, a limited number of SOFAR float trajectories (Rossby et al., 1983) do not show float entrainment into the Stream at the 700 m level, whereas south of 33°N there are examples of float entrainment at the 700 m level. These data suggest
that the Gulf Stream region from 31° to 35°N may experience strong localized changes in path curvature and entrainment/detrainment processes.

In this paper we combine the three-dimensional path information from a float deployed during the period 16–19 May 1983 with XBT and hydrostation sections and satellite-derived surface temperature maps to seek an understanding of the kinematics of the flow field during a transit from the Blake Plateau near 33°N downstream to Cape Hatteras near 35°N.

The analytical framework for this study is the conservation of potential vorticity (PV). For a layer tagged by an isopycnal-following float the potential vorticity

\[
PV = -\frac{1}{\rho} \left( \left( f + \frac{v}{R} \right) \frac{\partial \rho}{\partial \sigma} - \frac{\partial v}{\partial z} \frac{\partial \rho}{\partial r} \right)
\] (1)

should be conserved along its path from the Blake Plateau to the Cape Hatteras region. Here \( v/R \) is the curvature contribution to the vorticity with downstream velocity (\( v \)) and radius of curvature \( R \), \( \partial v/\partial z \) is the gradient of the downstream velocity in the cross-stream direction, \( \partial \rho/\partial z \) is the density gradient in the layer, \( \partial v/\partial r \) is the vertical gradient of downstream velocity, and \( \partial \rho/\partial r \) is the cross-stream density gradient. A conventional right-handed coordinate system has been used here with the \( r \)-axis oriented cross stream positive to the southeast and the \( s \)-axis oriented downstream.

In a study of potential vorticity along a Gulf Stream Pegasus velocity and CTD section taken during 1982, Johns (1984) calculated the contribution of \( (\partial v/\partial z)(\partial \rho/\partial r) \) to be approximately 20% of the total potential vorticity at the Stream location approximating our float trajectory. Due to a lack of vertical shear measurements, an estimate of the term cannot be included in the present study. Also, since the density difference remains constant over a layer defined by given isopycnals, for the layer the above expression implies conservation of the quantity

\[
\left( f + \frac{v}{R} + \frac{\partial v}{\partial r} \right)H
\] (2)

with \( H \) equal to the layer thickness between selected isopycnals.

Sections of geostrophic current velocity and density are available for Cape Fear and Cape Hatteras, respectively, from Swallow and Worthington (1961). In this region, the orders of magnitude of the various terms in (2) are \( f \sim 10^{-4} \text{s}^{-1} \), \( v/R \sim 10^{-5} \text{s}^{-1} \), and \( \partial v/\partial r \sim 10^{-5} \text{s}^{-1} \), respectively. Actual values of terms cannot be adequately determined from an examination of sections, since the problem is undetermined without path information. Only by using an isopycnal float can the trajectory of a water parcel be tagged and the resulting balance of terms estimated. Thus, within the context of a widening and deepening Gulf Stream core, water parcels may be adjusting their trajectories in three dimensions within the velocity core to conserve potential vorticity.

A quantitative estimate of the mean convergence or divergence associated with particular track segments can be obtained using the gradient current equation. The theoretical framework for this is the frontal model described by Newton (1978) based on the Bjerknes and Holmboe (1944) kinematic description of circulation in atmospheric fronts. The gradient current equation contains the centrifugal (cyclonic motion is positive), Coriolis, and pressure gradient forces, and we assume that the current flows parallel to geopotential contours and the current speed \( v \) is constant along the trajectory.

The equation of motion for gradient flow is described by Newton (1978):

\[
\frac{v^2}{R} = g\frac{\partial Z}{\partial n} - f\nu
\] (3)

where \( R \) is the trajectory radius of curvature, and \( \partial Z/\partial n \) is the geopotential slope on a pressure surface. Solving (3) for the lateral increment between given geopotentials

\[
\frac{\delta n}{f + v/R} = \frac{g\delta Z}{\partial Z/\partial n}
\] (4)

for the case of cyclonic flow, \( R \) is positive and \( \delta n \) is convergent, while for the anticyclonic case \( R \) is negative and \( \delta n \) is divergent. In addition to the simple consequences of gradient flow described above, increased convergence in a trough and increased divergence at a crest can also result from isopycnal tilting due to cross-stream variations of vertical velocity.

The gradient motion equation (3) is the starting point for the determination (Palmen and Newton, 1969) of the mean divergence \( (\bar{D}) \) between a track ridge and trough. Here we assume a sinusoidal shape for the contour channel itself and for a streamline in the channel center with amplitude, \( A \), and wavelength, \( L \). The phase speed of wave movement is \( c \), and the particle velocity is \( v \), for a channel centered at a latitude with Coriolis parameter \( f \), where \( \beta = \partial f/\partial y \). Assuming no cross-contour flow, the definition of divergence, \( D = (1/A)\partial A/\partial t \), where \( A \) is the contour channel area, leads to

\[
|\bar{D}| = \frac{16\pi^2 A}{fL^3} \left( \frac{v - c}{\beta L^2} \right).
\] (5)

From (5) and the assumption that the vertical velocity \( w = 0 \) at the sea surface, the vertical velocity at the float depth can be calculated and compared with the observed vertical float motion. According to these models, regions with anticyclonic curvature along the float’s track should be associated with divergent flow, while track regions of cyclonic curvature along the float’s track should be associated with convergent flow. The available AVHRR imagery contains several successive images of the Gulf Stream, which enables us to determine \( A, L \), and \( c \) in the above expression.
2. Methods

The primary instrument used in this study is a Swallow float designed to track isopycnal surfaces that has been described in detail by Rossby et al. (1985a). An isobaric Swallow float was converted into a passive isopycnal follower by the addition of a "compressor," a spring-backed piston in a cylinder designed to give the complete float assembly the compressibility of seawater. Also, the float must have a coefficient of thermal expansion significantly smaller than that of seawater. The float was packaged in a 1.52 m glass tube with a 7.6 cm nominal ID, discussed in detail in Rossby and Dorson (1983).

The compressor consisted of a spring-back piston set in an anodized aluminum cylinder made of tubing with machined end plates. The spring constant and piston diameter were chosen so as to give the instrument the proper compressibility. The float's electronic package sampled temperature at two-minute intervals, averaged four consecutive samples, and stored four hours of continuously updated data, which was transmitted every 15 minutes via the 12 kHz transducer. Float position was determined from LORAN fixes made when the ship was directly overhead, with a typical resolution of about 0.1 microseconds, corresponding to a repeatability of 50 m.

Ballasting to a selected isopycnal surface was accomplished by the addition of lead weights to bring the complete package from a laboratory-determined density to the desired in situ value. The instrument is pressurized to 1500 psi prior to laboratory ballasting to remove bubbles and to insure that all surfaces are properly compressed. The compressibility of the complete float package was determined in the tank by measuring the length of light chain lifted by the instrument in the pressure range from 0 to 800 db. The float used in this experiment had a compressibility approximately 87% that of seawater.

The satellite SST imagery used in this study were derived from AVHRR data collected from the NOAA-6, NOAA-7 and NOAA-8 spacecraft. The data from all three satellites were navigated so that any pixel, the smallest ground element resolved by the sensor, was located in latitude and longitude to within 1.0 km root mean square. Following their navigation, the NOAA-7 passes were atmospherically corrected using spectral channels 4 and 5 with weighting coefficients determined by McClain et al. (1983). No attempt was made to use the visible channels, 1 and 2, or the water vapor channel, 3, for cloud removal in the atmospheric correction process. The atmospherically corrected NOAA-7 images and the uncorrected NOAA-6 and NOAA-8 images were then remapped to a common rectangular coordinate system in latitude and longitude.

3. Results

The float was deployed at the northern end of the Blake Plateau (710 m water depth) near the axis of the high speed jet of the Gulf Stream at a depth of approximately 380 m. Three hydrostations and 55 XBTs were taken along the 300 km float track and ten short (10 to 15 km) XBT sections taken normal to the float track. Figure 1 shows the float path relative to the bathymetry as well as the location of the cross-track XBT sections (heavy lines). At first, the float path is weakly anticyclonic in the transition off the Blake Plateau, increasingly cyclonic when transiting toward the deep abyssal plain, and finally anticyclonic as the Gulf Stream moves shoreward into the shallower Cape Hatteras region. This path traverses three distinct bathymetric regions with steep, gentle and steep values of bottom slope.

In Fig. 2 the acoustically determined float depths are superimposed on the along-track XBT temperature section. The float depth is significantly deeper (by about 70 m) over the cyclonic portion of the track than in the anticyclonic portion of the track corresponding to deepening isotherms in the 8°-18°C temperature range. The fact that the float temperature is not constant reflects the fact that the compressibility was a bit less than desired, as well as the effects of salinity variation on the density. This is shown in this figure by comparing the time series of the acoustically determined float depth with the 15°C isotherm in the transition from the anticyclonic to cyclonic portions of the track. Thus, the float trajectory in this experiment is not perfectly isopycnal, and water parcels are imperfectly tracked.

Time series of float temperature and acoustically determined float depth are shown in Fig. 3. Subsequent to the initial temperature drop due to settling at the equilibrium density surface, the overall shape of the temperature record (solid curve) exhibits a rise of approximately 1°C from 2000 GMT 16 May to 1200 GMT 17 May followed by a comparable drop from 1800 GMT 17 May to 0000 GMT 19 May. The synoptic acoustically determined float-depth time series shows a low frequency deepening of approximately 70 m from 1200 GMT 16 May to 1200 GMT 17 May followed by a shoaling of 70 m in the period 0000 GMT 18 May to 0400 GMT 19 May. Thus, the depth and temperature records are inversely correlated during the experiment, i.e.,, deepening corresponding to warmer temperature and shoaling corresponding to cooler temperatures. These results must be viewed in relation to the float position relative to the Gulf Stream axis, to be discussed later. Examples of an inverse relationship between temperature and depth time series along an isopycnal Swallow float and a RAFOS float path has been previously observed by Rossby et al. (1985a) and Rossby and Dorson (1983), respectively, and are related, in part, to the imperfect isopycnal-following character of the floats, which results in an underestimate of vertical velocity.

As an independent check on the isopycnal-following character of the float, hydrostations were taken at three...
FIG. 1. Float trajectory (heavy curve) superimposed on bathymetric contours for the study area south of Cape Hatteras. XBT sections are indicated by transects perpendicular to the track.

FIG. 2. Acoustically derived float depth (circles) superimposed on the XBT-derived downstream temperature section (°C). Also included is the bottom bathymetry.
locations along the track, and in Table 1 temperature, salinity, and density (σt) data are interpolated to the simultaneous float depth. Based upon these data, the float was determined to have tracked the σt = 26.62 ± 0.05 surface during the course of the observations. The variation in density, corresponding to a depth interval of approximately ±20 m, is consistent with the XBT information.

The satellite images permit us to examine the float track and adjacent Gulf Stream within the context of mesoscale structures simultaneously observed at the sea surface. Figure 4 (upper panel) shows the northern edge of the Gulf Stream prior to the deployment of the float obtained from the NOAA-7 pass for 14 May at 1845 GMT and from the NOAA-6 pass for 1233 GMT 15 May. The northern edge here corresponds to the maximum horizontal gradient of the sea surface temperature (SST) field in the vicinity of the float track. In addition to the individual synoptic northern edges at the two times (solid lines), the figure also includes a rigid displacement of the earlier curve to the north and east (dotted lines).

The local frontal shape at the sea surface has not changed appreciably over this period since the meander simply propagated downstream by 36.9 km, corresponding to a phase speed c = 49.8 km day\(^{-1}\) (57.6 cm s\(^{-1}\)). Figure 4 (lower panel) shows that toward the end of the float deployment the shape of the meander has changed some, but as in the case of the 14 and 15 May data, the 18 May curves change little in shape over 11 hours. The 1938 GMT 18 May curve is closely approximated by the 1812 GMT curve simply displaced 21.9 km to the north and east. This displacement corresponds to a phase speed of 46.2 km day\(^{-1}\) (53.4 cm s\(^{-1}\)) in close agreement with the earlier value. Missing segments in the curves correspond to cloudy regions or to regions in which the northern edge is not distinct in the satellite imagery. Given the near constant phase speed over the period of deployment and the slow change in shape, a simple algorithm was derived to interpolate the northern edge to any time between 1233 GMT 15 May and 1938 GMT 18 May. This algorithm was used to obtain the northern edge curves at 1200 GMT on 16, 17 and 18 May shown in Fig. 5, corresponding to a period of warm filament growth as the meander progresses northeastward. Also in this figure is the float track, the line segment of each day coded in the same fashion as the 1200 GMT northern edge. The float track ranges from about 50 km upstream of the meander trough on 16 May to about 50 km down-

![Graph showing float depth for 16-19 May 1983.](image-url)

**FIG. 3.** Time series of float-transmitted temperature and acoustically derived float depth for 16-19 May 1983.

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**TABLE 1.** Sigma-t at float location and depth.

<table>
<thead>
<tr>
<th>Station</th>
<th>Float depth (m)</th>
<th>Latitude</th>
<th>Longitude</th>
<th>T (°C)</th>
<th>S (%)</th>
<th>Sigma-t</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>382</td>
<td>33°30.51'</td>
<td>75°57.92'</td>
<td>16.5</td>
<td>36.26</td>
<td>26.67</td>
</tr>
<tr>
<td>5</td>
<td>440</td>
<td>34°01.23'</td>
<td>75°09.75'</td>
<td>16.8</td>
<td>36.28</td>
<td>26.56</td>
</tr>
<tr>
<td>6</td>
<td>395</td>
<td>34°52.72'</td>
<td>74°48.24'</td>
<td>15.8</td>
<td>36.12</td>
<td>26.67</td>
</tr>
</tbody>
</table>

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Fig. 4. Gulf Stream northern edge at 1845 GMT 14 May (from NOAA-7) superimposed on an image at 1233 GMT 15 May (from NOAA-6). The downstream translation of the feature was 36.9 km over this interval (upper panel). The Gulf Stream Northern Edge for 18 May at 0812 GMT is superimposed on an image at 1938 GMT 18 May with a translation of 21.9 km over this period (lower panel).
stream of the meander trough on 18 May. For the approximate 204 km wavelength of the meander this means that the float was in the meander trough during the center portion of the deployment period. At the beginning and end of the track, the float was approximately equidistant between crest and trough.

4. Kinematic properties

a. Potential vorticity

Time series of velocity, curvature and layer thickness can be used to obtain an approximation of the quantity implied by potential vorticity conservation between isopycnals given by (2). We had planned to estimate $\partial u/\partial r$ directly using a second float deployed on the same isopycnal surface, but ballasting difficulties precluded obtaining these data. Therefore, with our dataset we can only examine the quantity

$$f + \frac{v}{R}$$

and attempt to infer the contribution of lateral shear from other data.

Float velocity was calculated at each float position using differenced values of position and time. The velocity time series (Fig. 6) shows speeds between 100 and 145 (±5) cm s$^{-1}$, with the smaller values generally toward the center of the record. The largest velocity values, ($\sim$145 cm s$^{-1}$) near 1600–2200 GMT 16 May, occur during the anticyclonic track section transiting off the Blake Plateau. The lowest velocity values ($\sim$110 cm s$^{-1}$) near 0600–2200 GMT 17 May correspond to the cyclonic track section over the deepest topography along the trajectory. The subsequent generally increasing velocity (110–120 cm s$^{-1}$) occurs during the anticyclonic track portion over the shallower waters off Cape Hatteras.

The float positions determining the trajectory were fitted, using least-square methods, with an eighth degree polynomial $\bar{y}(\bar{x})$ relating latitude and longitude, respectively. The excellent fit accounted for more than 99% of the variation of the data and approximated the curvature quite well. The radius of curvature of the float trajectory, $R$, was determined from the inverse to the curvature, i.e.,

$$R = \frac{1 + (\bar{y}'^2)^{3/2}}{\bar{y}''}$$

where $\bar{y}(\bar{x})$ was determined above. Results were generally on the order of $2 \times 10^3$ km. The standard deviation of observed float positions about the modeled float track was less than ±10 km. The overall structure exhibited anticyclonic (negative) radius of curvature from 1200–2200 GMT 16 May, cyclonic radius of curvature 2200 GMT 16 May–1200 GMT 18 May, and anticyclonic radius of curvature 1700 GMT 18 May–0600 GMT 19 May.

The time series of thermocline layer thickness, defined as the separation between the 8$^\circ$ and 18$^\circ$C surfaces, is shown in Fig. 6. The layer is observed to thicken from minimal values (∼360 m) on 16 May to maxima (∼480 m) near 1800 GMT 17 May with individual values accurate to ±5 m. From this point to the end of the record, values generally decrease toward 350 m layer thickness. On the scale of the sampling period, the $H$ time series is generally out of phase with the $v$ time series.

The $(f + v/R)/H$ time series (Fig. 6) shows very good consistency for most of the record, with values within ±10% of 2.0 × 10$^{-9}$ cm$^{-1}$ s$^{-1}$. For the measurement accuracies in the individual terms $v$, $R$, and $H$, stated above, the accuracy in obtaining (6) is estimated from a first order propagation of errors in the individual terms to be within ±2 × 10$^{-10}$ cm$^{-1}$ s$^{-1}$. Therefore, we assume that the observed variability, of the order of 10%, is comparable to measurement error.

The question can be asked whether the neglected contribution to (2) by lateral shear, $\partial u/\partial r$, might improve the balance shown in Fig. 6. We can infer that the inclusion of lateral shear would decrease the variability by comparison with a near-synoptic Pegasus (an acoustic velocity profiler) velocity section (Halkin and Rossby, 1985). As shown in Fig. 7, during the early part of the record from 1300 GMT 16 May–0600 GMT 17 May the float was inshore by 8 km of the 16$^\circ$C isotherm at 400 m, the marker of the Gulf Stream high speed jet, and shallower than the 400 m datum by 2–17 m (indicated by the dashed line) of the Gulf Stream high speed jet. Subsequently, in the period 0600 GMT 17 May–1700 GMT 18 May the float was generally deeper (up to 40 m) and offshore up to 10 km relative to this datum. Towards the end of the record, from
FIG. 6. Times series of float velocity (v), the trajectory curvature (v/R), the thermocline thickness between the 8°C and 18°C surfaces (H), and (f + v/R)/H.

FIG. 7. Time series of float position relative to the Gulf Stream high speed jet datum indicated by the 16°C isotherm at 400 m. The dashed line indicates float depth above the datum, while the solid line indicates float depths below the datum. In the middle panel the time series of volume flux per unit width (vH) is shown. In the lower panel the layer width is the distance between the 15°C and 16°C isotherms at 400 m.
1700–2200 GMT 18 May the float was once again shallower by 5–10 m and onshore by approximately 10 km. In the Pegasus velocity section, the high speed jet axis shifted southward by approximately 20 km and deeper approximately 100 m in the center of the record and then shifted northward approximately the same distance by the end of the record. The float moved from a region of near-zero lateral shear near the jet axis, to a region of positive shear (of order 10⁻⁹ s⁻¹) in the middle of the track, to a region of near-zero shear toward the end of the track. These qualitative adjustments to the balance expressed in (2) and shown in Fig. 6 would have minimal effect on the computed values in the Blake Plateau and Cape Hatteras regions but would increase values near the maximum cyclonic curvature point near the center of the record. Thus, the effects of relative shear are inferred to decrease variability in (2) with a contribution of order 10⁻¹⁰ cm² s⁻¹.

The estimate of \((f + v/R)/H\) can be combined with a typical value of \(\delta p/\rho\) and compared with the total potential vorticity (1) estimates obtained by Johns (1984). For \((f + v/R)/H \sim 2.0 \times 10⁻⁹ cm⁻¹ s⁻¹\), and \(\delta p/\rho \sim 3 \times 10⁻³\) over the 8°–18°C layer from our hydrostation data, we combine the values to obtain the estimate \(6 \times 10⁻¹² cm⁻¹ s⁻¹\) for the potential vorticity without the contribution of lateral shear and vertical shear of the downstream velocity gradient. For the Gulf Stream location where the 10°C isotherm reaches 400 m, corresponding approximately to our float trajectory, Johns (1984) estimates the total potential vorticity to \(2–3 \times 10⁻¹² cm⁻¹ s⁻¹\). Since the additional terms in the calculation would decrease the total potential vorticity estimate for the anticyclonic portion of the Stream, our estimates taken 100 km from those of Johns (1984) should agree within approximately 50%.

b. Volume flux

The downstream mass flux for a layer of thickness \(H\) can be defined as

\[
T = \int_0^R \rho v H dr
\]  

where the integral is in the cross-stream direction. Foilofonoff and Hall (1983) used Gulf Stream 60 data to examine the Gulf Stream east of Cape Hatteras in terms of a two-layer inertial jet model which conserves potential vorticity. Large-scale downstream divergence of mass was indicated. In particular, for their region of study, each of these fluxes decreased in succeeding downstream sections. A closer comparison to our estimates was done by Richardson et al. (1969), who examined downstream increases in 100 m layers of the Stream between the Florida Straits and Cape Fear sections (\(\sim 400\) km) using transport floats. In the region closest to our observations, between the Jacksonville and Cape Fear sections, they observed a barotropic increase in transport in the 200–700 m depth interval roughly comparable to our study volume. Our study addresses the question whether a volume downstream from these results following an isopycnal path near the Gulf Stream core will experience comparable volume flux divergence in the downstream direction for the mesoscale.

The time series of volume flux per unit width, \((uH)\), which can be computed from \(v\) and \(H\), is shown in Fig. 7. It shows an increase from values on the order 4.0 \(\times 10⁻⁶ cm² s⁻¹\) early in the record near the Blake Plateau to values exceeding 5.2 \(\times 10⁻⁶ cm² s⁻¹\) near the deepest point along the track (\(\sim 3200\) m) where the cyclonic curvature is greatest. Values decrease again to approximately 4.0 \(\times 10⁻⁶ cm² s⁻¹\) in the region near Cape Hatteras. If the Gulf Stream width were constant over this trajectory, these results would indicate a downstream convergence of volume flux in the region between the Blake Plateau and the maximum cyclonic curvature region, and a downstream divergence in the region between the maximum cyclonic curvature region and Cape Hatteras.

A rough independent estimate of local Stream width \((W)\) can be made using the cross-stream XBT sections. In Fig. 7, this estimate has been made using the distance between the 15° and 16°C isotherms interpolated or extrapolated to 400 m level. This distance is shown to decrease from approximately 10.6 km on the Blake Plateau to approximately 2.6 km near the maximum cyclonic curvature region, with values increasing to approximately 9.5 km in the Cape Hatteras region. These correspond to much larger slope variations than for the Stream as a whole. Thus, the “local” Stream width adjusts to decrease the variability observed in the above mass flux per unit width calculation. Nonetheless, the volume flux divergence in the downstream direction

\[
\frac{\partial (uH)}{\partial s}
\]  

associated with the wedge under study near the Gulf Stream core can be estimated from a combination of the above volume flux per unit width and layer width values and found to be approximately 2 \(\times 10⁻³ cm² s⁻¹\). As previously mentioned, we can make a similar estimate using computed mass fluxes in the 200–700 m layer obtained by Richardson et al. (1969) between Jacksonville and Cape Fear. For their approximately 140 km cross sections, the volume flux downstream divergence was also of order 2 \(\times 10⁻³ cm² s⁻¹\). These large downstream changes occurring over a local Stream volume are indicative of strong submesoscale local processes with time scales on the order of several days, which may be the result of perturbation by larger scale processes. There may be significant stirring and mixing processes on these smaller scales of motion.
c. Vertical velocity; divergence

In the region from the Blake Plateau to the location of maximum cyclonic curvature, the volume under study converges and stretches vertically (Figs. 6 and 7). This kinematic adjustment is accompanied by downwelling of water parcels as observed in the float trajectory illustrated in Fig. 8. In the downstream region between the maximum cyclonic curvature region and Cape Hatteras, the Gulf Stream volume under study diverges and contracts vertically. During this period, the float shoals, indicating a region of upwelling. The vertical velocities associated with this motion can be estimated from \( \sim 70 \) m of vertical motion over one day to be approximately \( 8 \times 10^{-2} \) cm s\(^{-1}\). The vertical motion is downward on 16–17 May and upward on 18–19 May.

The mean divergence associated with the down/up-welling can be estimated from (5) using the IR imagery to determine scales of motion of the wave field. Averaging the two phase-speed estimates previously determined from IR image pairs (Fig. 4), which bracket the tracking interval, we determine a representative meander phase speed to be approximately 48.0 km day\(^{-1}\) (55.5 cm s\(^{-1}\)). This value is comparable with estimates of propagation speeds made by others, such as Bane et al. (1981), who found an average phase speed of 39 cm s\(^{-1}\) for meanders they observed in the Onslow Bay region. This value is approximately half the float velocity shown in Fig. 6.

The meander amplitude and wavelength are shown in Figs. 4 and 5, respectively. The meander amplitude (half the crest to trough distance) is estimated to be approximately 31.5 ± 3.2 km; the wavelength is estimated to be approximately 204 ± 20 km; and from float data a typical velocity is taken to be 100 (±10) cm s\(^{-1}\). Substituting these values into (5) to calculate the mean divergence we obtain

\[
|\vec{D}| = 3.1 (±1.9) \times 10^{-6} \text{ s}^{-1},
\]

where the error is determined from a first-order propagation of the errors in the individual parameters in \( |\vec{D}| \) presented above.

This value is approximately 4% of the vorticity term \( v/R \) previously discussed. This estimate is comparable to upwelling divergence estimates in the Gulf Stream by Chew (1974) of \( 6 \times 10^{-6} \) and by Chew et al. (1985) of \( 7 \times 10^{-6} \) s\(^{-1}\) based on isotherm shoaling rates. In addition, their results are in qualitative agreement with our observations, associating convergence (divergence) with offshore (onshore) flow and downwelling (upwelling) in a meander.

It is useful to examine the individual terms in the mean horizontal divergence

\[
|\vec{D}| = (\partial u / \partial r) + (\partial v / \partial s)
\]

where \( u \) is the cross-stream velocity, to see where the major contribution is located. The mean cross-stream velocity gradient between a crest and a trough, \( (\partial u / \partial r) \), can be estimated from the strain rate of the study volume. In particular, the width constriction \( \Delta W = -8.0 (±0.8) \times 10^2 \) cm occurred over approximately 1.5 days \( (1.7 \times 10^5 \) s) over a mean layer width, \( W \), of 6.6 \((±0.6) \times 10^3 \) cm. Combining these values,

\[
(\partial u / \partial r) = \frac{\Delta W}{W \Delta t} = 7 (±1.4) \times 10^{-6} \text{ s}^{-1},
\]

where the error was determined by propagating errors to first order in the individual terms.

The mean downstream velocity gradient between a wave crest and trough, \( (\partial v / \partial s) \), can be estimated from the change in float velocity in the first half of the data (Fig. 4). For \( \Delta v = 20 (±2) \) cm s\(^{-1}\) over a downstream distance 100 ± 10 km, we obtain \( (\partial v / \partial s) \sim 2.0 (±0.4) \times 10^{-6} \) s\(^{-1}\). These values compare reasonably, within an order of magnitude, with the estimate of \( |\vec{D}| \) obtained above. It should be noted that the major contribution to the mass flux divergence will be the cross-stream \( (\partial u / \partial r) \) term, since the cross-sectional area over which it operates will be much larger.
Finally, an additional order of magnitude comparison with the mean divergence $|\bar{D}|$ can be made based on the continuity equation, i.e.,
\[
\frac{\partial w}{\partial z} = -\left( \frac{\partial u}{\partial r} + \frac{\partial v}{\partial s} \right)
\]
and the assumption that the vertical velocity gradient scales like the first baroclinic mode
\[
\frac{\partial w}{\partial z} \sim w/H.
\]
For the previously determined $w \sim 10^{-1} \text{ cm s}^{-1}$ over the layer depth $H = 4.0 (\pm 0.7) \times 10^{-4} \text{ cm,} \ \frac{\partial w}{\partial z} \sim 2.5 \times 10^{-6} \text{ s}^{-1}$, the same order of magnitude as the above $|\bar{D}|$ estimate.

5. Summary and conclusions

The Gulf Stream volume defined by the trajectory of an isopycnal float and cross-stream temperature surveys was examined for kinematic adjustment in a region with curvature reversals associated with meanders. To within measurement error of $\pm 10\%$ potential vorticity was conserved over a 300 km path along which the depth increased over 400%. Local departures in potential vorticity are reduced by inclusion of the inferred lateral shear term and, presumably, also by unresolved smaller scale lateral shear. The main adjustment takes place between curvature vorticity and vortex stretching. While the path of the trajectory is consistent with what we might expect from vortex stretching over the changing bottom depth, the present observations are insufficient for a detailed examination of the role of the bathymetry. The downstream evolution of the geometry of our study volume and its velocity can be described within the context of Palmen and Newton's (1969) model of streaming through a meandering front. Approaching regions with anticyclonic curvature (crests), flow in the study volume was horizontally divergent and vertically compressed, while for regions of cyclonic curvature (troughs) the flow in the study volume was horizontally convergent and vertically stretched. Upwelling occurs approaching regions of anticyclonic curvature, while downwelling occurs approaching regions of cyclonic curvature.

Vertical velocities of approximately $10^{-1} \text{ cm s}^{-1}$ were observed. These values are underestimates of water parcel vertical velocity due to the imperfect isopycnal character of the float. Horizontal divergence was estimated by two methods. The first, using Palmen and Newton's (1969) model for mean divergence between a meander crest and trough, yielded values of about $3.1 (\pm 1.9) \times 10^{-6} \text{ s}^{-1}$, about 4% of the Coriolis parameter. This is much larger than one would expect for open ocean regions. For example, Leetmaa (1977) estimated a horizontal divergence of order $10^{-8} \text{ s}^{-1}$ over a 100 km region for a MODE eddy. Estimates of the downstream gradient $\partial u/\partial r$ are not inconsistent with this result. The cross-stream gradient $\partial u/\partial s$ is larger in magnitude, suggesting large cross-stream fluxes are possible at meander scales. Also, very large volume flux divergences in the downstream direction were observed along the float path. The second, independent estimate of horizontal divergence is based on the assumption that on the scale of the meandering field the vertical motions are constrained by the structure of the first baroclinic mode so that $\partial w/\partial z$ can be approximated by $w/H$. This divergence is the same order of magnitude as the first estimate. The large value of vertical velocity, horizontal divergence, and downstream volume flux divergence observed on meander scales in the Gulf Stream, with wavelengths of order 200 km and amplitudes of order 30 km, suggests that on these scales significant stirring and mixing processes are taking place which may have important consequences for physical and biological tracer distributions throughout the frontal zone.

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