A Study of the Bottom Boundary Layer of the Florida Current

GEORGES L. WEATHERLY

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Manuscript received 25 August 1971, in revised form 11 October 1971

ABSTRACT

This is a report of an experiment designed to study the bottom boundary layer of the Florida Current at a representative site in the Straits of Florida. The objectives of the experiment were: 1) to determine the bottom frictional stress \( \tau_b \) and \( \tau_c \) to determine whether the bottom boundary layer is a turbulent Ekman layer; 2) to determine whether the bottom boundary layer is a turbulent Ekman layer; and 3) to determine whether the bottom boundary layer is a turbulent Ekman layer and, if so, to determine the magnitude of the bottom frictional stress. The results of the experiment indicated that the bottom boundary layer is a turbulent Ekman layer and that the magnitude of the bottom frictional stress is given by \( \tau_b = \frac{1}{2} \frac{1}{\rho} \frac{d \bar{u}}{dz} \), where \( \bar{u} \) is the bottom velocity and \( \rho \) is the density of the water.

1. Introduction

The mass transport and velocity structure of the Florida Current from the Straits of Florida to Cape Fear is well known (see Richardson et al., 1969a). While these studies indicate that the average current speed 10–50 m above the bottom is typically 20 cm sec\(^{-1}\), it does not reveal how the velocity is distributed within the bottom boundary layer. This is a report of an experiment in which the velocity structure of the Florida Current in the lowest 30 m was determined at a site in the Straits of Florida. The purpose of the experiment was to determine whether the bottom boundary layer is a turbulent Ekman layer and, if so, to determine the magnitude of the bottom frictional stress. This study was designed to provide a direct measurement of the bottom frictional stress and to determine the magnitude of the bottom frictional stress.

2. Review of Theory

The flow near the ocean floor under the Florida Current is a time-dependent, turbulent Ekman-like boundary layer in which the transfer of momentum to the bottom is accomplished through the frictional force. The flow is characterized by the bottom boundary layer, which is the region where the velocity changes from the free stream to the bottom velocity. The bottom boundary layer is a turbulent Ekman layer and, if so, the magnitude of the bottom frictional stress is given by \( \tau_b = \frac{1}{2} \frac{1}{\rho} \frac{d \bar{u}}{dz} \), where \( \bar{u} \) is the bottom velocity and \( \rho \) is the density of the water.

The theoretical relationship between the boundary layer and the bottom boundary layer is given by

\[ \frac{\partial \bar{u}}{\partial z} = \frac{1}{\rho} \frac{\tau_b}{\rho} \]

where \( \tau_b \) is the bottom frictional stress. This relationship is derived from the Navier-Stokes equations and is valid for a flow over a flat bed.

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A Study of the Bottom Boundary Layer of the Florida Current

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ABSTRACT

This is a report of an experiment designed to study the bottom boundary layer of the Florida Current at a representative site in the Straits of Florida. The objectives of the experiment were to determine the depth of the bottom frictional layer, and 2) to determine whether the bottom boundary layer is a turbulent Ekman layer. A typical value of the bottom stress was found to be \( \approx 0.2 \) dyn cm\(^{-2}\). A mean veering of ~10° is the correct stress was observed in the logarithmic layer. No mean veering was observed above the logarithmic layer; this is believed to be a consequence of the strong modulation of the bottom current by the tidal flow. The implication of \( \tau \approx 0.5 \) dyn cm\(^{-2}\) is considered in a simplified model of the Gulf Stream current system; this analysis suggests that, dynamically, the role of bottom friction is rather small.

1. Introduction

The mean transport and velocity structure of the Florida Current from the Straits of Florida to Cape Fear is well known (see Richardson et al., 1969). While these studies indicate that the average current speed is \( \sim 30 \) m above the bottom is typically 20 cm sec\(^{-1}\), they do not reveal how the velocity is brought to zero at the bottom. This is a report of an experiment in which the velocity structure of the Florida Current in the lowest 30 m was measured for approximately a week at a site in the Straits of Florida. The purpose of the experiment was to see whether the bottom boundary layer of this current is a turbulent Ekman-like layer and to determine the stress exerted on the bottom by the Florida Current.

Section 2 is a brief review of theoretical and experimental studies of stationary planetary boundary layers in which the data will be compared. Section 3 is a detailed description of the experiment and the data. Section 4 discusses in detail the conclusions on the bottom current, stress, and the Ekman veering that were inferred from the data. The results are summarized in Section 5 and two implications of the bottom stress values are considered.

2. Review of theory

The flow near the ocean floor under the Florida Current is a time-dependent, turbulent Ekman-like boundary layer in which the transfer of momentum to the ocean bottom is accommodated. It is an Ekman-like layer because the Florida Current above the bottom is nearly geostrophic, and in this bottom boundary layer the Coriolis force remains dynamically significant. It is time-dependent because tidal modulations of the currents near the bottom are as large as the mean flow. It is turbulent through its entire depth, because the Reynolds number is sufficiently high. 1

The atmospheric ground layer is a turbulent Ekman-like layer and, compared to the oceanic bottom boundary layer, is well studied (see Wimbush and Munk, 1970). Since nearly all of the inquiry into the atmospheric boundary layer has been restricted to stationary conditions (Monin, 1970), little is known of time-dependent planetary boundary layers. The following is a brief description of a stationary planetary boundary layer. Even though no comparable description is available for the time-dependent case, parts of this description are later shown to be relevant to the dynamics of a portion of the flow.

In the stationary theory, the boundary layer is considered to be divided into two regions, a logarithmic layer overlaying a thicker, turbulent Ekman layer. A third region, the laminar sublayer, may exist immediately above the bottom, provided the bottom is hydrodynamically smooth (to be explained later). It is assumed that the current above the boundary layer is geostrophic and steady, that the bottom is horizontal, that the density variations in the boundary layer are due to temperature only, and that effects of stratification are negligible in the logarithmic layer. External conditions describing the boundary layer are the geostrophic parameter \( f \), the geostrophic velocity \( V_g \), away from the boundary, the average height \( b \) of the bottom roughness elements, a density stratification parameter \( S \) (defined after Eq. 2), the kinematic viscosity \( \nu \), and the fluid density \( \rho \). The appropriate average momentum equations for the boundary layer are:

\[
\begin{align*}
&\frac{f}{\nu} V_g + \frac{f}{\nu} \frac{\partial V_g}{\partial z} - \frac{\partial}{\partial b} (h \rho \nu_0) = 0, \\
&-f \left[ U_0 + \frac{\partial}{\partial z} \left( V_g \right) \right] - \frac{\partial^2}{\partial b^2} (h \rho \nu_0) = 0,
\end{align*}
\]

where \( z \) is the vertical coordinate (positive upwards); \( x \) and \( y \) are the horizontal coordinates; and \( u, U_0 \), and \( P \) are the components of average velocity, geostrophic velocity, and Reynolds stress. The instantaneous deviation from the mean velocity components are denoted by primes (\( u' \) being the \( x \) component, and \( v' \) the \( y \) component). The bottom shear represents some appropriate averaging interval.

Near the bottom the shear stress \( \tau \) is experimentally found to be nearly independent of \( z \) and equal to its value \( \tau_b \) at the bottom. From \( \tau_b \), the friction velocity \( u_0 \), is defined as:

\[
\tau_b = \frac{1}{2} \rho u_0^2.
\]

The friction velocity is the scaling length for the boundary layer (see Blackadar and Tennekes, 1969). The thickness of the nearly constant stress region, defined as that height where \( \tau = 0.8 \tau_b \), is approximately \( 0.2 u_0 / V_g \) (see Monin and Obukhov, 1955).

The viscous sublayer exists, provided that it is less than its thickness \( \delta \). The viscous stresses dominate in this sublayer and the appropriate length scale for \( k \), \( \delta \), \( \nu \), or \( \mu \), is found experimentally that the depth of the viscous sublayer:

\[
\delta = 12 \ln u_0 / \nu.
\]

and that for this sublayer to exist:

\[
\delta / \nu > u_0 / \nu.
\]

The latter expression defines the condition for a hydrodynamically smooth boundary. With a coordinate system aligned so that \( \tau_b \) is parallel to the \( y \) axis, the velocity is given as:

\[
V_g = \tau_b \nabla \psi / \rho.
\]

Overlying the viscous sublayer is the classical logarithmic layer where the velocity is given by:

\[
V(x) = (\tau_b / \rho) \ln (x / \delta),
\]

where \( x/\delta \) is von Kármán's constant and \( \delta \) is the roughness height. For a smooth bottom, \( \delta \) depends on \( h_0 \) and not \( \delta \), and so empirically to be:

\[
\tau_b / \rho = 0.41 \delta.
\]

In hydrodynamically rough flow the logarithmic layer extends into the viscous sublayer, the turbulent wake of the bottom roughness elements having eliminated the latter layer. Consequently, \( \delta \) should no longer depend on \( h_0 \) but on \( \delta \). Empirically it is found that:

\[
\delta_b = 0.36 \delta
\]

and the bottom is rough if:

\[
\tau_b / \rho > 0.2 \nu / \rho \delta_b.
\]

For rough flow, \( \tau_b \) is physically interpreted as that height where, in the valleys between roughness element peaks, the mean velocity is zero.

The Reynolds stress dominates in the logarithmic layer, and in the derivation of Eq. (4) it is assumed that these stresses are nearly constant and equal to \( \tau_b \). Hence, the depth of the logarithmic layer, \( \delta_b \), might be expected to be the thickness of the constant stress layer, \( 0.2 u_0 / V_g \). Observations (see Wimbush and Munk, 1970, Table II) indicate that \( \delta_b \) is actually an order of magnitude larger, i.e.,

\[
\delta_b = 20 \delta / (f V_g).
\]

Overlying the logarithmic layer is a turbulent Ekman layer in which there is a balance between the Coriolis and Reynolds stress terms (see Eq. (1)). The appropriate scaling length for this layer is \( \delta_b / f \), and observations indicate that its depth is:

\[
\delta_b = 0.4 \delta / (f V_g).
\]

(see Monin, 1970). No velocity relation comparable to Eq. (4) is available for the turbulent Ekman layer. However, the similarity arguments of Gill (1968) and Camady (1967) yield useful relations between \( \delta_b / f \), \( \delta_b / \delta \), and the total Ekman veering \( \alpha \) (the angle between \( \tau_b \) and \( V_g \)):

\[
\frac{\tau_b}{V_g} = \frac{\delta_b}{\delta} (\cos \alpha) \int \frac{V_g / \delta}{V_g / \delta} - \cos \alpha \int \frac{V_g / \delta}{\delta}.
\]

(10)

(11)

The values of \( \alpha \) and \( B \) used in this report are \( \alpha = 20 \) and \( B = 3 \), values determined by Dearhoff (1970) in a numerical study of a neutrally stratified planetary boundary layer. Implicit in the arguments leading to Eq. (11) is that no veering occurs in the logarithmic layer (i.e., \( \alpha = \delta_b / \delta \)). Hence, \( \delta_b \) is the total Ekman veering above the logarithmic layer. Actually, in a planetary boundary layer some veering in the logarithmic layer is necessary to account for the observation that these layers do not thicken downstream, and such veering has been observed in laboratory experiments (Charles van Atta, personal communication). Quantitatively, little is known about the total veering in the logarithmic layer under stationary conditions except that it

\[
\text{is less than the total veering.}
\]

\[\text{is less than the total veering.}\]

(2)
is appreciably smaller than the total veering in the turbulent Ekman layer.

The constants $A$ and $B$ in Eqs. (10) and (11) are functions of the stability. For comparison of the results contained in this paper to the observations summarized in Monin (1970), the stability parameter used in that paper is adopted; it is defined as:

$$Sw = \frac{\Delta T}{\Delta T_a}$$

where $\gamma$ is the buoyancy parameter ($\gamma = \frac{\partial}{\partial z}$, where $\alpha$ is the thermal coefficient of expansion and $g$ is the gravitational acceleration) and $\Delta T_a$ is the temperature difference across the planetary boundary layer. Stable conditions, $\Delta T > 0$, apply at the site of the experiment with such a stratification tending to increase the veering. However, for $Sw = 10$, a representative value at the site of the experiment, this increase is rather small, 2°-3°. E.g., for $0.5 < Sw < 10$, the constant $A$ in Eq. (11) and also $B$ in Eq. (11) is nearly constant (see Monin, 1970, Figs. 3 and 4).

3. The experiment

The site of the experiment, 25°44'N, 70°28'W, was between Miami, Fla., and Bimini, Bahamas, in water 780 m deep (Fig. 1). There, on 22 June 1969, two subsurface, laut-line moorings were set 300 m apart in a NW-SE direction at nearly the same time. One mooring was four Goodyear film current meters, and on the other mooring ten Savonius rotors (Figs. 2 and 3). No part of either mooring extended over 40 m above the bottom. The experiment lasted 155 hr (about 64 days).

d. The current meter mooring

Fig. 2 is a schematic of the current meter mooring. The current meters were at heights of roughly 1, 3, 14, and 30 m above the bottom. The heights above bottom of the mooring of their sensors are given in Table 1. The current meters record velocity data on 16-mm film, and are described by Richardson et al. (1963). On the continuous mode used, sampling every 5 sec, they record ~160 hr of data.

The mooring was anchored with approximately 91 kg of iron weights. Flotation was provided by fifty 15-cm diameter hollow aluminum floats which were clustered about a 2.1 m long, 18-cm diameter aluminum tube. This tube, a modified mako (see Richardson et al., 1969b), provided additional buoyancy and housed a radio transmitter and light used for recovery purposes. The net buoyancy of the mooring minus ballast weights was approximately 54 kg. A timed release was set to drop the ballast weights after 125 hr of data were collected. The mooring line was 1 inch, corrosion-resistant aluminum chain.

b. The Savonius rotor mooring

Fig. 3 is a schematic of the Savonius rotor mooring. The height, above bottom, of each rotor is given in Table 1. Several components of this mooring are identical to those of the current meter array; they are identical to those of the current meter array; they are identical to those of the current meter array.

Each rotor was connected electrically to a centralized recording case which was part of the mooring. Rotor revolutions were recorded by incrementing mechanical counters. Every 84 sec a camera took a picture of a display panel containing the 10 rotor counters, a clock, and a bubble level. The bubble level was used to detect mooring motion and the angle with the vertical of the recording case.

The rotor array records speeds at 10 different levels for 83 hr on one 30.48 m roll of 16 mm film. During the experiment this mooring was recovered after recording 0.24 hr of data. Within 3 hr of its recovery, it was relaunched with fresh film. This was done in order to obtain rotor array records comparable in length to the 155-hr current meter records.

The experiment was designed so that the friction velocity $u_0$ could be determined directly from this mooring's speed data and Eq. (4). The friction velocity $u_0$ could be determined directly from this mooring's speed data and Eq. (4). The friction velocity $u_0$ could be determined directly from this mooring's speed data and Eq. (4). The friction velocity $u_0$ could be determined directly from this mooring's speed data and Eq. (4).

c. Data

Of the four current meters used, three returned with data. Full direction records were obtained from the current meters at 1, 3, and 14 m. A full speed record was obtained from the current meter at 14 m and a partial speed record (the last 68 hr of the experiment) was obtained from the current meter at 5 m. No speed data were collected from the current meter at 1 m, and all data were obtained from the current meter at 30 m. The current meter malfunctions are discussed in Weatherley.
is appreciably smaller than the total veering in the turbulent Kaiman layer.

The constants $A$ and $B$ in Eqs. (10) and (11) are functions of the stability. For comparison of the results contained in this paper to the observations summarized in Monin (1970), the stability parameter used in that paper is adopted; it is defined as

$$S = \frac{\gamma}{g} \left( \frac{W_i}{W} \right),$$

where $\gamma$ is the buoyancy parameter ($\gamma = g\theta$, where $\theta$ is the thermal coefficient of expansion and $g$ is the gravitational acceleration) and $T_s$ is the temperature difference across the planetary boundary layer. Stable conditions, $S > 0$, apply at the site of the experiment with such a stratification tending to decrease the veering $\omega$. However, for $S = 10$, a representative value at the site of the experiment, this increase is rather small, 2°-3°; e.g., for $0.5 \leq S \leq 10$, the constant $A$ in Eq. (11) (and also $B$ in Eq. (11)) is nearly constant (see Monin, 1970, Figs. 3 and 4).

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a. The current meter mooring

Fig. 2 is a schematic of the current meter mooring. The current meters were at heights of roughly 1, 3, 14, and 30 m above the bottom. The heights above bottom of the midpoint of their sensors are given in Table 1. The current meters record velocity data on 16-mm film, and are described by Richardson et al. (1962). On the continuous mode used, sampling every 5 sec, they record ~180 hr of data.

The mooring was anchored with approximately 91 kg of iron weights. Flotation was provided by fifty 15-cm diameter hollow aluminum floats which were clustered about a 2.1 m long, 10-cm diameter aluminum tube. This tube, a modified radiosonde (see Richardson et al., 1969d), provided additional buoyancy and housed a radio transmitter and light used for recovery purposes. The net buoyancy of the mooring minus ballast weights was approximately 54 kg. A timed release was set to drop the ballast weights after 155 hr of data were collected. The mooring line was a 1 inch, corrosion-resistant aluminum chain.

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Each rotor was connected electrically to a centralized recording case which was part of the mooring. Rotor revolutions were recorded by increasing mechanical counters. Every 84 sec a camera took a picture of a display panel containing the 10 rotor counters, a clock, and a bubble level. The bubble level was used to detect mooring motion and the angle with the vertical of the recording case.

The rotor array records speeds at 10 different levels for 83 hr on one 30.48 m roll of 16 mm film. During the experiment this mooring was recovered after recording 82.4 hr of data. Within 3 hr after its recovery, it was relaminated with fresh film. This was done in order to obtain rotor array records comparable in length to the 153.6 hr current meter records.

The experiment was designed so that the friction velocity $u_f$ could be determined directly from this mooring's speed data and Eq. (4). The friction velocity may sometimes be determined from spectra of speed records. In the Appendix it is shown that the speed records were not sampled frequently enough in time to accurately determine $u_f$ values from their spectra.

### Table 1: Heights above the bottom of the current meters on each mooring.

<table>
<thead>
<tr>
<th>Rotor</th>
<th>Height (m)</th>
<th>Current meter</th>
<th>Rotor height (m)</th>
<th>View height (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.32±0.01</td>
<td>1</td>
<td>1.02±0.03</td>
<td>1.35±0.05</td>
</tr>
<tr>
<td>2</td>
<td>0.64±0.01</td>
<td>2</td>
<td>1.72±0.05</td>
<td>3.04±0.05</td>
</tr>
<tr>
<td>3</td>
<td>1.47±0.01</td>
<td>3</td>
<td>3.13±0.01</td>
<td>4.74±0.05</td>
</tr>
<tr>
<td>4</td>
<td>1.89±0.01</td>
<td>4</td>
<td>4.06±0.06</td>
<td>5.94±0.06</td>
</tr>
<tr>
<td>5</td>
<td>2.92±0.04</td>
<td>5</td>
<td>5.08±0.04</td>
<td>6.98±0.04</td>
</tr>
<tr>
<td>6</td>
<td>3.80±0.06</td>
<td>6</td>
<td>6.06±0.06</td>
<td>7.98±0.06</td>
</tr>
<tr>
<td>7</td>
<td>5.09±0.08</td>
<td>7</td>
<td>7.15±0.08</td>
<td>8.98±0.08</td>
</tr>
<tr>
<td>8</td>
<td>10.03±0.09</td>
<td>8</td>
<td>10.17±0.09</td>
<td>11.98±0.09</td>
</tr>
<tr>
<td>9</td>
<td>20.02±0.06</td>
<td>9</td>
<td>20.17±0.06</td>
<td>21.98±0.06</td>
</tr>
<tr>
<td>10</td>
<td>30.02±0.20</td>
<td>10</td>
<td>30.17±0.20</td>
<td>31.98±0.20</td>
</tr>
</tbody>
</table>

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experienced speeds greater than its threshold value of ~2 cm sec⁻¹. At best, the data recorded by this rotor were qualitative and are not discussed further.

Temperature-salinity profile measurements were made on the first, third and sixth days of the experiment with a free-fall STD. The data collected on the first day are shown in Fig. 4 and demonstrate conditions in the lowest 30 m typical for the duration of the experiment: 1) a nearly constant salinity of 35%, 2) a temperature of about 13°C, and 3) a nearly constant temperature gradient of approximately 1.5×10⁻³°C m⁻¹.

Eight bottom color photographs were taken at the site of the experiment with a freely dropped camera. From these photographs the average height of the bottom roughness elements was inferred to greater than 0.5 cm and less than 2 cm. Shadows of fiducials of known heights were compared to shadows of the bottom roughness elements to obtain this estimate of d. Because the launch coordinates of each mooring and camera drop were known to within several meters (Decca Hi-Fix was used for navigation), all the photographs were obtained within several hundred meters of the moorings. Fig. 3a is a black and white reproduction of one such photograph.

Fig. 3. Reproduction of color photograph made 240 m above the bottom at the site of the experiment. Note circular closed bottom material about camera basket weight. In upper right corner there is indication of surface creep of fine sediment along the bottom.

4. Analysis and results

a. The bottom current

The average current, 14 m above the bottom over the 6 day experiment had a magnitude 10.3 cm sec⁻¹ and a direction of 342.6°+ 2.8°. The velocity data recorded at this level are displayed as a time series in Fig. 6 and as a progressive vector diagram in Fig. 7. Most of the low-frequency large-amplitude fluctuations in these figures are accounted for by the K₁ and O₂ diurnal constituents of the tidal currents.

A recent study by Smith et al. (1969) indicates that the most significant components of the tidal current in the Straits of Florida are the K₁, O₁, M₂, and S₂ constituents, with the diurnal K₁, O₁, constituents having amplitudes approximately twice those of the semidiurnal M₂, S₂ constituents. The predicted K₁, O₁ combined tidal current for the duration of the experiment, calculated using the amplitudes and phases given by Smith et al., accounts for 63.5% of the variance in the speed record collected at 14 m. When the semidiurnal M₂, S₂ constituents are also included, the variance reduction is improved only slightly to 63.6%. The dotted curve in Fig. 8 is that of a mean 10.3 cm sec⁻¹ current modulated.
experienced speeds greater than its threshold value of \(\pm 2 \text{ cm sec}^{-1}\). At best, the data recorded by this rotor were qualitative and are not discussed further.

Temperature salinity profile measurements were made on the first, third and sixth days of the experiment with a free-fall STD. The data collected on the first day are shown in Fig. 4 and demonstrate conditions in the lowest 30 m typical for the duration of the experiment: 1) a nearly constant salinity of 35\%, 2) a temperature of about 7.5\(^\circ\)C, and 3) a nearly constant temperature gradient of approximately 1.5\(\times\)10\(^{-2}\)\(\text{C cm}^{-1}\).

Eight bottom color photographs were taken at the site of the experiment with a freely dropped camera. From these photographs the average height of the bottom roughness elements was inferred to greater than 0.5 cm and less than 2 cm. Shadows of fiducials of known heights were compared to shadows of the bottom roughness elements to obtain this estimate of \(d\). Because the launch coordinates of each mooring and camera drop were known to within several meters (Keka Ha'Puu was used for navigation), all the photographs were obtained within several hundred meters of the moorings. Fig. 5 is a black and white reproduction of one such photograph.

Fig. 3. Savonius rotor array mooring. Refer to Table 1 for specific heights of the rotors above the bottom.

Fig. 4. Salinity and temperature profiles made by a free-fall STD as the site of the experiment on the first day. The STD records its descent and records in and from the bottom, and, because of its response time, double traces sometimes result.

4. Analysis and results

a. The bottom current

The average current 14 m above the bottom over the 64 day experiment had a magnitude 10.5 cm sec\(^{-1}\) and a direction of 342.6\(^\circ\)±2.8\(^\circ\). The velocity data recorded at this height are displayed as a time series in Fig. 6 and as a progressive vector diagram in Fig. 7. Most of the low-frequency large-amplitude fluctuations in these figures are accounted for by the \(K_s\) and \(O_1\) diurnal constituents of the tidal currents.

A recent study by Smith et al. (1969) indicates that the most significant components of the tidal current in the Straits of Florida are the \(K_s\), \(O_1\), \(M_2\), and \(S_2\) constituents, with the diurnal \(K_s\) and \(O_1\) constituents having amplitudes approximately twice those of the semidiurnal \(M_2\), \(S_2\) constituents. The predicted \(K_s\), \(O_1\) combined tidal current for the duration of the experiment, calculated using the amplitudes and phases given by Smith et al., accounts for 63.4\% of the variance in the speed record collected at 14 m. When the semidiurnal \(M_2\), \(S_2\) constituents are also included, the variance accounted for is improved only slightly to 63.6\%. The dotted curve in Fig. 6 is that of a mean 10.5 cm sec\(^{-1}\) current modified...
by the predicted combined $K_v, Q_h, M_s$, and $S_i$ tidal current.

Sections of two "unsmoothed" current-meter records are shown in Fig. 8. An increase in the noise level in the speed record associated with deceleration is indicated by the expanded time scale (relative to Fig. 6). This phenomenon has been previously observed in decelerating flows in the ocean and in the laboratory (Winship and Munk, 1970). This feature can also be seen in the Savonius rotor-array speed plots shown in Fig. 9. Some of the smaller scale fluctuations in Fig. 9 are noteworthy in that they occur at nearly the same time over vertical distances varying from less than 1 to 20 m.

A. The friction velocity

One parameter which the experiment was designed to determine is the friction velocity. Once it is known, the bottom frictional stress is also known ($\tau_f = \rho_w u_f^2$; see Section 2). Before proceeding to determine $u_f$ directly from the speed data, an estimate of its value was made as described below.

Studies of the atmospheric boundary layer indicate that the ratio of $u_f/V_s$ is a slowly varying function of the surface Rossby number $R_{so} = V_s/(fL_s)$, with $u_f/V_s$ varying from $-0.01$ to $-0.05$ as $R_{so}$ decreases from 10 to 0. Therefore, if $R_{so}$ can be estimated for the site of the experiment, the ratio $u_f/V_s$ (and $u_f$ if $V_s$ is known) can also be estimated. We take $V_s$ as 10.5 cm sec$^{-1}$, the mean speed at 14 m above the bottom. At 25$^\circ$N, $f = 6.3 \times 10^{-4}$ sec$^{-1}$. Typically, for the atmospheric boundary, $u_f \approx 0.05 V_s$, because the land surface is always nearly rough (Monin, 1970). This choice of $u_f$ as a roughness parameter in the Rossby number is fortunate for oceanographers, because the ocean bottom is generally not rough (Winship and Munk, 1970), i.e., $u_f$ for the ocean bottom is typically a function of $u_f$ and not $d$. For definiteness, $u_f = 0$ is used in place of $u_f$ to form a Rossby number for the site of the experiment. Bottom photographs yielded a value of $d$ somewhere between 0.5 and 2 cm. Hence, an estimated range of Rossby number is $0.03 \leq R_{so} \leq 0.002$ (Lettau, 1959). Therefore, a representative value of $u_f$ for the experiment should lie between 0.34 and 0.41 cm sec$^{-1}$. These values, when applied to Eq. (8), imply a logarithmic depth extending 3.5-5.1 m above the bottom. Fits of the rotor array speeds vs $\ln(s)$ (c.f. Fig. 10) imply a logarithmic layer extending typically 4 m above the bottom.

One method of inferring $u_f$ from the speed data is to determine it directly from the straight lines in the log speed profiles shown in Fig. 10, where the slope of each line is $u_f/s$ (the $s$ intercept is $u_f$). In practice, it is tedious to determine $u_f$ in this manner because of the large number of profiles involved. This method is also somewhat subjective because it is not always obvious as to which is the "best" line to draw. Numerical methods were used instead. The log speed profiles indicate that the logarithmic layer extends typically 4 m above the bottom. Four speed records were obtained at heights
by the predicted combined $K_n$, $O_n$, $M_2$, and $S_z$ tidal currents.

Sections of two "unsmoothed" current-meter records are shown in Fig. 8. An increase in the noise level in the speed record associated with deceleration is indicated by the expanded time scale (relative to Fig. 6). This phenomenon has been previously observed in decelerating flows in the ocean and in the laboratory (Winship and Monk, 1970). This feature can also be seen in the Savonius rotor array speed plots shown in Fig. 9. Some of the smaller scale fluctuations in Fig. 9 are noteworthy in that they occur at nearly the same time over vertical distances varying from less than 1 to 30 m.

**b. The friction velocity**

One parameter which the experiment was designed to determine is the friction velocity. Once it is known, the bottom friction stress is also known ($v_f = u_{*b}$, see Section 2). Before proceeding to determine $u_{*b}$ directly from the speed data, an estimate of its value was made as described below.

Studies in the atmospheric boundary layer indicate that the ratio of $u_{*b}/V_{1/2}$ is a slowly varying function of the surface Rossby number $Ro = V_{1/2}/f/s$, with $u_{*b}/V_{1/2}$ varying from $\sim 0.02$ to $\sim 0.05$ as $Ro$ decreases from $\sim 10^{10}$ to $\sim 10^5$. Therefore, if $Ro$ can be estimated for a site of the experiment, the ratio $u_{*b}/V_{1/2}$ (and $u_{*b}$) can also be estimated. We take $V_{1/2}$ as 10.5 cm sec$^{-1}$, the mean speed at 14 m above the bottom. At $25^\circ F$, $f = 63 \times 10^{-5}$ sec$^{-1}$. Typically, for the atmospheric boundary, $u_{*b} = 0.05$, because the land surface is always nearly rough (Manin, 1970). This choice of $u_{*b}$ as a roughness parameter in the Rossby number is unfortunate for oceanographers, because the ocean bottom is generally not rough (Winship and Monk, 1970), i.e., $u_{*b}$ for the ocean bottom is typically a function of $u_{*b}$ and not $a$ [see Eq. (51)]. For definiteness, $u_{*b} = 0.05$ is used in place of $a_b$ to form a Rossby number for the site of the experiment. Bottom photographs yielded a value of $a$ somewhere between 0.5 and 2 cm. Hence, an estimated range of $Ro$ for the experiment is $2.5 \times 10^8 \leq Ro \leq 1.0 \times 10^9$. For this range of the Rossby number, $0.032 \leq u_{*b}/V_{1/2} \leq 0.039$ (Leittau, 1959). Therefore, a representative value of $u_{*b}$ for the experiment should lie between 0.34 and 0.41 cm sec$^{-1}$. These values, when applied to Eq. (8), imply a logarithmic depth extending 1.5-8.1 m above the bottom. Plots of the rotor array speeds vs $u_{*b}$ (c.f. Fig. 10) imply a logarithmic layer extending typically 4 m above the bottom.

One method of inferring $u_{*b}$ from the speed data is to determine it directly from the straight lines in the late speed profiles shown in Fig. 10, where the slope of each line is $\mu = k$ (the $z$ intercept is $a_b$). In practice, it is tedious to determine $u_{*b}$ in this manner because of the large number of profiles involved. This method is also somewhat subjective because it is not always obvious as to which is the "best" line to draw. Numerical methods were used instead. The late speed profiles indicate that the logarithmic layer extends typically 4 m above the bottom. Four speed records were obtained at heights
from the Savonius rotor array. A \( u_s \) and \( u_{nf} \) were determined by a least-squares fit of these four speed data to Eq. (4). In order to eliminate significant errors in the speeds, they were smoothed over an hour (more precisely, 55.78 min). The uncertainty in these hourly averaged speeds resulting from time measurements and the quantizing interval was less than \( \pm 0.006 \text{ cm sec}^{-1} \). This averaging interval is also typically greater than \( \delta u_s/\sigma_u \) a time scale for the logarithmic layer, typically about 20 min. A histogram of the \( u_s \) so computed is shown in Fig. 11a. The peak value in this histogram at \( \approx 0.4 \text{ cm sec}^{-1} \) is quite near the predicted range of \( u_s \) values of 0.31-0.42 cm sec\(^{-1}\) found earlier.

A second method of determining the friction velocity is to substitute the appropriate empirical relation for \( u_s \) into Eq. (4) and to solve for \( u_{nf} \). This method is well suited for a smooth bottom, for one need know only that \( \delta u_s/\sigma_u \) and not its exact value. To know \( u_{nf} \) for a rough bottom, \( u_s \) must be known, and to measure \( u_s \) for a deep-sea bottom is not a trivial task.

Eqs. (3) and (7) define the conditions for smooth and rough bottoms, respectively, With \( u_{nf} = 0.4 \text{ cm sec}^{-1} \), the condition for a smooth bottom is \( \delta u_s \leq 0.1 \text{ cm} \), and that for a rough bottom is \( \delta u_s \geq 3 \text{ cm} \). As bottom photographs indicated that 0.5 cm < \( \delta u_s \leq 2 \text{ cm} \), the bottom at the site of the experiment typically was neither smooth nor rough but in between, and neither (5) nor (6) give a correct representation for \( u_s \). Expressions for \( u_{nf} \) are then needed for a smooth-rough transition bottom (\( \delta u_s \approx 2 \text{ cm} \)) and in order to apply the above method of determining \( u_{nf} \). Such expressions for \( u_{nf} \) (from Nikuradse, 1933) are as follows:

\[
\begin{align*}
\delta u_s & = [0.1 4u_{nf}]/u_{nf}, \quad (13) \\
\delta u_s & = 0.48, \quad (14) \\
\delta u_s & = 0.25 [u_{nf}/v]^{0.5}/1.21, \quad (15) \\
3\delta u_s & \leq 7\delta u_s/u_{nf}, \quad (16) \\
7\delta u_s & \leq 14\delta u_s/u_{nf}, \quad (17) \\
14\delta u_s & \leq 90\delta u_s/u_{nf}, \quad (18)
\end{align*}
\]

respectively, where again \( \delta u_s = 0.0 \). Each of the \( u_s \) expressions (Eqs. (5), (6), (13), (14), and (15)) when substituted into Eq. (4), will yield a \( u_{nf} \). In practice, it
Fig. 10. Speed profiles in Site A, averaged from four consecutive ~1-hr (55.78-min) intervals. The $n_a$ shown in Figs. 9 and 11 were also computed from speeds averaged over the same interval. Note that the $n_a$ in Eq. (9), varies considerably in these profiles. The profiles $a$ and $b$ are averaged from two consecutive ~1-hr (55.76-min) intervals. The speeds in profiles $a$ and $b$ were averaged to produce profile $c$, those in profiles $d$ and $e$ were averaged to produce profile $f$. Profile $g$ is an averaged profile for ~4-hr (223.13-min) interval. The speeds in profiles $a$ and $e$ were averaged to produce this profile. Profile $g$ is similar to other ~4-hr smoothed profiles in which no large direction change has occurred in that it has a $n_a$ intercept at $x=0.03$, $u_a=0.01$ cm.

$x<4$ m from the Savonius rotor array. $n_a$ and $n_2$ were determined by a least-squares fit of these four speed data to Eq. (4). In order to eliminate significant errors in the speeds, they were smoothed over an hour (more precisely, 55.76 min). The uncertainty in these hourly averaged speeds resulting from time measurements and the quantizing interval was less than $0.006$ cm sec$^{-1}$. The averaging interval is also typically greater than $0.01/n_a$, a time scale for the logartithmic layer, typically about 20 min. A histogram of the $n_a$ so computed is shown in Fig. 1a. The peak value in this histogram $0.20$ cm sec$^{-1}$ is quite near the predicted range of $n_a$ values of $0.31-0.42$ cm sec$^{-1}$ found earlier.

A second method of determining the friction velocity is to substitute the appropriate empirical relation for $n_a$ into Eq. (4) and to solve for $n_a$. This method is well suited for a smooth bottom, for one need know only that $n_a/n_a$ and not its exact value. To know $n_a$ for a rough bottom, $d$ must be known, and to measure $d$ for a deep-ocean bottom is not a trivial task. Eqs. (3) and (7) define the conditions for smooth and rough bottoms, respectively. With $n_a=0.4$ cm sec$^{-1}$, this condition for a smooth bottom is $d<0.1$ cm, and that for a rough bottom is $d>0.3$ cm. As bottom photographs indicated that 0.5 cm $<d<2$ cm, the bottom at the site of the experiment typically was neither smooth nor rough but in between, and neither (5) nor (6) give a correct representation for $n_a$. Expressions for $n_a$ are then needed for a smooth-rough transition bottom $(3v/n_a)<d<9v/n_a$ in order to apply the above method of determining $n_a$. Such expressions for $n_a$ (from Nikuradse, 1935) are as follows:

$$n_a = \frac{5}{3} (0.17)^{0.5} n_a^3, \quad (13)$$

$$n_a = \frac{5}{3} (0.17), \quad (14)$$

$$n_a = \frac{0.20}{0.20} \left(\frac{d}{30}\right)^{0.5}, \quad (15)$$

for for $3v/n_a < d < 9v/n_a$.

$$0.20 < d < 0.20, \quad (16)$$

$$1.0 < d < 1.0, \quad (17)$$

$$0.20 < d < 9v/n_a, \quad (18)$$

respectively, where again $d_0=3.0$. Each of the $n_a$ expressions [Eqs. (5), (6), (13), (14), and (15)], when substituted into Eq. (4), will yield a $n_a$. In practice, it
was found that only one of the computed $u_v$ values satisfied the appropriate inequality between $d$ and $v$, namely $u_v$ for $3 < u_v$ (see Eq. 6) in $u$ and $v$, respectively. Eq. (6) and (14) result in simple linear equations in $u_v$ and $v$, respectively. Each of the $u_v$ and $v$, respectively, result in transcendental equations in $u_v$ and $v$, respectively. The solutions, $d = 1$ cm, were used to compute $u_v$ and $v$. The values $d = 1$ cm were chosen in the following way. For a smooth-

rough transition bottom, $u_v$ is very nearly a constant, varying approximately between 0.7 and 1.0$^2$ [see Nikuradse (1933, Fig. 17)]. Therefore, the intercept on its speed profiles should be $\approx 0$. Such profiles averaged over several hours consistently had an intercept at $x = 0.05$ cm [Eq. 10], implying that $d = 1$ cm. The bottom photographs indicated that $d$ was closer to 1 cm than to either 0.5 or 2 cm. Fig. 12 shows regression plots of $u_v$, $v$, and $w$, so computed with 0.1 cm intervals, using the same hourly averaged speeds used earlier in the curve-fitting (least-squares fit) method. These plots indicate that a logarithmic layer extends to at least 8 m above the bottom and that if $u_v > 0.2$ cm sec$^{-1}$, it extends to 4 m. The $u_v$ values computed from speed measurements made above the log-year values ($d = 4$ m) shown in Figs. 12 and 13 tend to fall below the 45° line. This is in qualitative agreement with measurements made in the atmospheric boundary layer of the wind and observed in this study, $S > 0$, conditions (see Ninnin and Onokob, 1953). A histogram of the $u_v$ values from the speed record 147 cm above the bottom is shown in Fig. 11b. This histogram, as well as that in Fig. 10a of the "curve-fitting" $u_v$ has a peak value at $-0.4$ cm sec$^{-1}$.

The curve-fitting method enabled $u_v$ values (and $v$ values) to be calculated without having explicit knowledge of $d$. A second method (later referred to as "empirical") made use of empirical relations for $u_v$, $v$, and $w$, respectively, to determine $u_v$ values. $u_v$ values from hourly averaged speeds, it might be expected that the curve-fitting $u_v$ values are consistent with the $u_v$ relations used in the "empirical" method. This did not prove to be the case.

The values of $d$, determined from hourly averaged speeds by the curve-fitting method typically varied by three orders of magnitude in a seemingly erratic manner. However, for averaging intervals $> 1 h$, these values displayed less scatter, and for an interval of $\approx 3 h$ the $u_v$ values were 0.04 to 0.01 cm sec$^{-1}$, to experimental accuracy, were in agreement with the empirical relations for $u_v$. Using an averaging interval of several hours to obtain "better" speed profiles is consistent with earlier speed profile measurements made by Monnot (1947).

### The Ekman layer

Three of the four current meters returned with full direction records; however, only one returned with a complete speed record. In order to compute average current directions from these direction data, we averaged these current directions, some of the Savonius rotor speed records were spliced to some of the current meter records.

The current meter at 14 m is returned with a full speed record. Only the last 68 hr of the 155-hr record from 14 m is used because of error in the average current. An example of the tendency of the curve-fitting $u_v$ values to "summarize" the current of smoothing intervals can be seen in the $u_v$ profiles in Fig. 10. The current meter at 2 m contains speed data. An almost complete speed record was created for this current meter by splicing speed data from Rotor 3 at 2.92 m in the Savonius rotor array. A 4 hr gap remained where the rotor array and current meter speeds did not overlap. This gap was filled by splicing speed records from the current meter at 14 m, the scaling factor being determined from the Savonius rotor array mean speed profile. The current meter at 11 m returned with no speed record. The speed record used to form average directions for this current meter's data was the spliced speed record of current meter 2.

The average directions over the 67 day experiment were found to be $342.6° \pm 2.8°$ at 14 m, $343.1° \pm 2.8°$ at 3 m, and $332.2° \pm 2.8°$ at 1 m. These directions give current climates at 14 m, 1 m, 3 m, and 1 m of

$$
\theta_{14} = -0.5 \pm 0.5°, \quad \theta_{3} = 10.0 \pm 0.5°, \quad \theta_{1} = 10.4 \pm 0.5°,
$$

respectively. Hence, between 1 and 14 m, veering in the correct sense (counterclockwise, looking down) was observed. With $u_v = 0.4$ cm sec$^{-1}$ and $V = 10.5$ cm sec$^{-1}$, the mean speed at 14 m during the experiment, Eq. (11) predicts a total veering throughout the turbulent Ekman layer of

$$
\alpha = 0.8 \pm 2.4° \quad (20)
$$

In (20) the precision results from estimates of the uncertainty in the above $u_v$ and $V$, of $0.05$ and $\pm 1.0$ cm sec$^{-1}$, respectively. Eq. (19), with $u_v = 0.4$ cm sec$^{-1}$, gives an Ekman layer depth $\approx 25$ m. For a planetary boundary layer of depth, nearly all of the veering should occur between 14 and 3 m (Bauerhoff, 1950, Fig. 2). Hence, it might be expected that $\theta_{14}$ is $\theta_{3}$ but, to within experimental accuracy, this is not the case. All of the observed mean veering occurred in the bottom, $\approx 3$ m, i.e., within the Ekman layer.

The smallest estimate of $\alpha$ in Eq. (20) of 7.8° is near the accuracy of $\pm 0.5°$, at which veering can be estimated from the current meter records. It could be argued that it is not surprising that near the veering was observed above the logarithmic layer. However, the fact that no such mean veering could be a consequence of the current's unsteadiness. There is some indication that this may be the case.

In Fig. 8 are shown sections of the "unsmoothed" (1 min averaged) direction records from 1 and 3 m. Typically, in the logarithmic layer (as illustrated in Fig. 8), veering developed rather quickly and persisted after the current direction had steadied and become northward. The veering at such times was not constant in magnitude but varied proportionally with the current speed.

In Fig. 13a is a time series plot of 50-min averaged values of $\theta_{14}$, the veering in the logarithmic layer. For reference, the velocity data similarly averaged from 14 m above the bottom are shown in Fig. 13c and d. During the periods of steady northward flow, the maximum value of $\theta_{14}$ was achieved when the current speed also reached its maximum value, this occurring time $t = 84$ hr. At this time $\theta_{14}$, the veering between 1 and 14 m (see Fig. 13b), was larger than $\theta_{14}$, suggesting that the Ekman layer was then extending to at least 14 m above the bottom.

In Figs. 13a and b the veering is generally well behaved and in the correct sense, provided that $V (14 m) \geq 10$ cm sec$^{-1}$ and the current direction was steady and northward. At these times, $\theta_{14}$ and $\theta_{14}$ (see Figs. 13c and d) varied directly with $V (14 m)$, in apparent disagreement with Eqs. (10) and (11). A consequence of Eqs. (10) and (11) is that the total Ekman veering $\alpha$ is
FIG. 11. Histogram of $u_w$ determined by fitting Eq. (4) to the rotor array data for $r < 6$ m, $v$, and that of $u_w$ at $1.47$ m displayed in Fig. 12, b.

was found to be only one of the computed $u_w$ values satisfied the appropriate inequality between $d$ and $v, n_e$ [Eqs. (3), (7), (16), (17) and (18)], respectively.

Eqs. (6) and (14) result in simple linear equations in $u_w$. Eqs. (5), (13) and (14) result in transcendental equations that easily solved numerically using the Newton-Raphson method. These equations, along with $d = 1$ cm, were used to compute values of $u_w$. The value of $d = 1$ cm was chosen in the following way. For a smooth

rough transition bottom and $u_w$ is very nearly a constant, varying approximately by $0.7-1.0$. $u_w$ [Nikuradse (1933, Fig. 11)]. Therefore, the intercept $\alpha$ for speed profiles should be $\sim 0.30$. Such profiles averaged over several hours consistently had an intercept at $z = 0.03$ cm [e.g., Fig. 10g], implying that $d = 1$ cm. The bottom photographs indicated that $d$ was closer to 2 cm than either 0.5 or 2 cm. Fig. 12 shows regression plots of $u_w$ values so computed with $d = 1$ cm, using the same hourly averaged speeds used earlier in the curve-fitting (Eq. 4a) and were used in the curve-fitting (Eq. 4a). These plots indicate that a logarithmic layer extends to at least 3 m above the bottom and that if $u_w > 0.2$ cm sec$^{-1}$, it extends to 4 m. The $u_w$ values computed from speed measurements made above the logarithmic layer ($> 4$ m) shown in Figs. 12c and 12d tend to fall below the 45° line. This is in qualitative agreement with measurements made in the atmospheric boundary layer under stability, $S > 0$, conditions (see Monin and Obukhov, 1953). A histogram of the $u_w$ values from the speed recorded at 4 m above the bottom is shown in Fig. 11b. This histogram, as well as that in Fig. 10a, of the "curve-fitting" $u_w$ has a peak value at $\sim 0.4$ cm sec$^{-1}$.

The curve-fitting method enabled $u_w$ values (and $v_w$ values) to be calculated without having explicit knowledge of the origin of $d$. A similar method referred to as "empirical" [see also Appendix A] was used in the "empirical" method. The former did not prove to be successful.

The values of $u_w$ determined from hourly averaged speeds by the curve-fitting method typically varied by three orders of magnitude in a seemingly erratic manner. However, for averaging intervals $> 1$ hr, these $u_w$ values displayed less scatter, and for an interval of $\sim 3$ hr the $u_w$ values were $0.01-0.05$ cm sec$^{-1}$ and, to experimental accuracy, were in agreement with the empirical relations for $u_w$. Using an averaging interval of several hours to obtain "better" speed profiles is consistent with earlier speed profile measurements made by Mosesy (1947).

c. The Ekman veering

Three of the four current meters returned with full direction records; however, only one returned with a complete speed record. In order to compute average current directions from these data, and hence average veering, some of the Savonius rotor speed records were spliced to some of the current meter records.

The current meter at 14 m returned with a full speed record. Only the last hour of the 155-hr record from current meter 2 at 3 m contained speed data. An almost complete speed record was created for this current meter by interpolating speed data from Rotor 5 (at 2.92 m) in the Savonius rotor array. A 4-hr gap remained where the rotor array and current meter speeds did not overlap. This gap was filled by interpolating scaled speeds from the current meter at 14 m, the scaling factor being determined from the Savonius rotor array mean speed profile. The lowest current meter at 1 m returned with no speed record. The speed record used to form average directions for the current meter data was the scaled speed record of current meter 2.

The average directions over the 63 day experiment were found to be $342.5^\circ, 2.8^\circ$ at 14 m, $342.1^\circ, 2.8^\circ$ at 3 m, and $332.2^\circ, 2.8^\circ$ at 1 m. These directions give veering between 3 and 14 m, 1 m, and 3 and 14 m of

\[
\begin{align*}
\alpha_1 & = -0.3^\circ \pm 5.6^\circ, \\
\alpha_2 & = 10.9^\circ \pm 5.6^\circ, \\
\alpha_3 & = 10.4^\circ \pm 5.6^\circ,
\end{align*}
\]

respectively. Hence, between 3 and 1 m, veering in the correct sense (counterclockwise, looking down) was observed.

With $u_w = 0.4$ cm sec$^{-1}$ and $V_w = 10.5$ cm sec$^{-1}$, the mean speed at 14 m during the experiment, Eq. (11) predicts a mean veering throughout the turbulent Ekman layer of

\[
\alpha_{E_k} = -1.2^\circ \pm 1.4^\circ.
\]

In (20) the precision results from estimates of the uncertainty in the above $u_w$ and $V_w$ of $0.05$ and $1.0$ cm sec$^{-1}$, respectively. Eq. (19), with $u_w = 0.4$ cm sec$^{-1}$, gives an Ekman layer depth $k = 25$ m. For a planetary boundary layer of this depth, nearly all of the veering should occur between 14 and 3 m (Dearloff, 1970). Hence, it must be expected that $\alpha_1, \alpha_2, \alpha_3$, but, to within experimental accuracy, this is not the case. All of the observed mean veering occurred in the lowest 3 m, i.e., within the logarithmic layer.

The smallest estimate of $\alpha_1$ is Eq. (20) of $7.8^\circ$ is near the accuracy of the measurement, at which veering can be estimated from the current meter records. It could be argued that it is not surprising that no mean veering was observed above the logarithmic layer. However, the fact that no such mean veering occurred could be a consequence of the current's unsteadiness. There is some indication in the data that this may be the case.

In Fig. 12 are shown sections of the "enough smoothed" (1-min averaged) direction records from 1 and 3 m. Typically, in the logarithmic layer (as illustrated in Fig. 8), veering developed rather quickly and persisted after the current direction had steadied and became northward. The veering at such times was not constant in magnitude but varied proportionally with the current speed.

In Fig. 13a is a time series plot of 50-min averaged values of $\alpha_1$, the veering in the logarithmic layer. For reference, the velocity data similarly averaged from 14 m above the bottom are plotted in Fig. 13b and c. During the periods of steady northward flow, the maximum value of $\alpha_1$ was achieved when the current speed also reached its maximum value, this occurring at time $t = 84$ hr. At this time $\alpha_1$, the veering between 1 and 3 m (see Fig. 13a), was larger than $\alpha_2$, suggesting that an Ekman layer was then extending to at least 14 m above the bottom.

In Figs. 13b and d the veering is generally well behaved and in the correct sense, provided that $V(14$ m) $\geq 10$ cm sec$^{-1}$ and the current direction was steady and northward. At these times, $\alpha_3$ and $\alpha_4$ (see Figs. 13d and e) varied directly with $V(14$ m), in apparent disagreement with Eqs. (10) and (11). A consequence of Eqs. (10) and (11) is that the total Ekman veering, $\alpha_{E_k}$ is

FIG. 13. Time plot of the veering between 1 and 3 m, $\alpha_2$, and the veering between 1 and 14 m, $\alpha_1$, determined from 50-min averaged current directions. (a and b, respectively. For reference, the direction and speed record (50-min averages) are given in c and d, respectively, while $\alpha_3$ and $\alpha_4$ that veering which occurred when the current at 14 m was $\geq 10$ cm sec$^{-1}$ and flowing northward (also includes in Figs. 13b and d, respectively).
a decreasing function of the Rosby number\(^2\) \(\text{Ro} = \frac{V_0}{d/\text{Fr}}\). In Figs. 14a and 15a the veerings displayed in Figs. 13a and b, respectively, are plotted as a function of a Rosby number formed by taking \(V_0 = V_{(14\text{ m})}\) and \(d/\text{Fr} = 0.03\) and then comparing terms. One finds that \(w_0/V_{(14\text{ m})}\) varies inversely with \(\text{Ro}\) in Eq. (11) which implies that \(w_0\) also varies inversely with \(\text{Ro}\).

\(^2\)This can be seen by bringing all terms in \(w_0/V_{(14\text{ m})}\) in Eq. (10) to the left-hand side, substituting a typical value of \(w_0/V_{(14\text{ m})}\) for \(\beta_0\).
Fig. 14. Veering between 1 and 3 m as a function of \( \frac{V(14 \text{ m})}{f_0} \), a Rossby number, for \( \lambda_d = 3000 \text{ cm} \). Those in a are shown in Fig. 13a; those in b are in Fig. 13c.

\footnote{This can be seen by bringing all terms in \( \omega_v / V \) in Eq. 10 to the left-hand side, substituting a typical value of \( \omega_v / V \) (e.g., in Figs. 13a and b, respectively, are plotted as a function of a Rossby number formed by taking \( V_0 = V(14 \text{ m}) \) and \( \omega_0 = \omega(14 \text{ m}) \) in \( R_o = \frac{\lambda_d}{2 \pi} \). Hence \( \omega_\nu / V \) varies inversely with \( \omega_0 \); see Eq. (11) and the Rossby number.}

a decreasing function of the Rossby number. \( R_o = \frac{V_0}{\omega_0} \). In Figs. 14a and 15a, the veerings displayed in Figs. 13a and 13b, respectively, are plotted as a function of a Rossby number formed by taking \( V_0 = V(14 \text{ m}) \) and \( \omega_0 = \omega(14 \text{ m}) \) in \( R_o = \frac{\lambda_d}{2 \pi} \). Hence \( \omega_v / V \) varies inversely with \( \omega_0 \); see Eq. (11) and the Rossby number. However, the veerings shown in Figs. 14b and 15b are those which occurred when \( |V(14 \text{ m})| > 10 \text{ cm sec}^{-1} \) and the current direction was northward (i.e., those veerings shown in Figs. 13c and f). Figs. 14b and 15b suggest that the veering was an increasing rather than a decreasing function of the Rossby number.
5. Discussion and conclusions

a. The bottom current

The bottom current was found to be strongly modulated in magnitude and direction by the diurnal tide. Semidiurnal tidal oscillations were relatively weak. These results are in agreement with two recent studies in the Straits of Florida (Smith et al., 1969; Zetler and Hansen, 1970). The amplitude of the diurnal current was large enough to result in direction reversals near the bottom (±3 m) and intervals of 1-3 h at higher levels (105±0.3 m) of near zero current. The long and unexpected periods of steady westerly flow (see Figs. 6 and 7) were associated with the diurnal tide and may be a consequence of the irregular geometry of the Straits of Florida. The mean bottom current direction was very nearly that of a constant depth contour (see Fig. 1).

b. The mean vertical structure of the bottom boundary layer

The mean (over the 64 day experiment) vertical structure was not in complete agreement with stationary theories of planetary boundary layers. A logarithmic layer of depth ~4 m, characterized by a friction velocity, $u^* = 0.4$ cm sec$^{-1}$, was consistently found. However, above the logarithmic layer no Ekman layer, characterized by a mean depth $h = u^*/f = 25$ m and mean total mean veering $\alpha = 10^\circ$ (as predicted by Eq. 11 with $V_{r} = 10$ cm sec$^{-1}$) was found. All of the observed mean veering ($\sim 10^\circ$) occurred within the logarithmic layer, no mean veering was observed above it.

c. Time scales in the logarithmic layer

A time scale for the logarithmic layer is $h^* = u^*/f = 4$ m, (0.4 cm sec$^{-1}$) = 20 min. Hence, one might expect agreement with Eq. (4) for speeds averaged over an interval $\geq 20$ min. This conclusion is substantiated by Fig. 12, which displays $u_{AV}$ values computed from hourly averaged speeds. While the $u_{AV}$ values determined from hourly average speeds were missing, the $u_{AV}$ values were not. The latter varied by two more orders of magnitude than predicted by empirical relations for $u_{AV}$ (see Eqs. 5, 6, 16, 17, and 18). However, for speeds smoothed over intervals $\geq 3$ hr, the inferred $u_{AV}$ values were well behaved and, to within experimental accuracy, were in agreement with the above relations for $u_{AV}$.

The composition of the $u_{AV}$ determined from hourly averaged speeds with (10) is depicted in Fig. 16. Better agreement is obtained in Fig. 16b, where only those values occurring when the bottom current $\left| V_{14}\right| > 2.5$ cm sec$^{-1}$ and flowing northward are depicted. Fig. 16b also indicates that for time scales of $\sim 1$ hr, conditions obtained in the logarithmic layer were in only partial agreement with (10).

d. Time scales for veering

The veering in the logarithmic layer was consistent and in the correct sense when the bottom current was strong $\left| V_{14}\right| > 30$ cm sec$^{-1}$ and flowing northward (see Fig. 13). It developed typically $\sim 15$ min after the current direction had studied and become northward (e.g., see Fig. 8). Veering above the logarithmic layer at 14 m was observed in three periods of strong, steady northward flow only when the bottom current was quite strong $\left| V_{14}\right| > 30$ cm sec$^{-1}$ (see Fig. 13). This suggests that an Ekman layer rarely had sufficient time to form above the logarithmic layer before the current changed. A consequence of (10) and (18) is that the veering in the Ekman layer is a decreasing function of the Rossby number $\left( \frac{V_{r}}{f} \right)$, Because the veering in the logarithmic layer was consistent, one might expect qualitative agreement with this veering and the Rossby number. Fig. 15b indicates that the opposite occurred; the veering in the logarithmic layer was an increasing function of the Rossby number. The results of this experiment imply that quasi-stationary conditions in agreement with Eq. (11) do not occur in a planetary boundary layer in which the velocities are modulated by a function whose frequency is greater than $f$ and whose amplitude is comparable to $V_{r}$.

e. Stratification in the logarithmic layer

In computing the friction velocity $u^*$ from the speed data, it was assumed that Eq. (4) was a suitable representation for the speeds in the logarithmic layer. This is equivalent to assuming that stratification effects are negligible in this region. The Monin-Obukhov length $L$ is the depth at which stratification effects are considered negligible (Monin, 1970) and is defined as

$$L = \frac{u^*}{w^*} \left( \gamma H \right),$$

where $c_p$ is the specific heat capacity, $\gamma$ the buoyancy parameter ($\gamma = -w$ where $u$ is the thermal coefficient of expansion and $g$ is the gravitational acceleration), and $H$ the vertical heat flux taken positive downward. The length $L$ is that height where, in the energy equation for the boundary layer, (Monin, 1970), the ratio of the shearing stress term to the buoyancy term is $1$. Hence, assuming that stratification effects are negligible in the logarithmic layer is equivalent to assuming that $L < h^*$. Below, we estimate $L$ and find it to be comparable to $h^*$. An argument then given as to why this estimate of $L$ is thought to be too small.

To know $L$, the downward heat flux $H$ must be known. It was assumed that

$$H(\rho_0) = -K_h \frac{\partial T}{\partial z},$$

where $K_h$ is the turbulent coefficient of heat exchange and $T$ the mean temperature; it was further assumed that

$$K_h = \frac{u^*}{\nu},$$

estimated from STD profiles; see Fig. 4),

$$H(\rho_0) = 10^4 \text{ cm}^2 \text{ sec}^{-1},$$

$\nu = 0.4$ cm sec$^{-1}$, $\lambda^2 / \nu = 1.5 \times 10^{-8}$ C cm$^{-1}$.
5. Discussion and conclusions

a. The bottom current

The bottom current was found to be strongly modulated in magnitude and direction by the diurnal tide. Semidiurnal tidal oscillations were relatively weak. These results are in agreement with two recent studies in the Straits of Florida (Smith, Zedler, and Hansen, 1970). The amplitude of the diurnal current was large enough to result in direction reversals near the bottom (6 ≤ 3 m) and intervals of 1-3 hr at higher levels (10 ≤ 5 ≤ 30 m) of near zero current. The long and unexpected periods of steady westerly flow (see Figs. 6 and 7) were associated with the diurnal tide and may be a consequence of the irregular geometry of the Straits of Florida. The mean bottom current direction was very nearly that of a constant depth contour (see Fig. 1).

b. The mean vertical structure of the bottom boundary layer

The mean (over the 64-day experiment) vertical structure was not in complete agreement with stationary theories of planetary boundary layers. A logarithmic layer of depth ~4 m, characterized by a friction velocity $u_\text{m} = 4.0$ cm s$^{-1}$, was consistently found. However, above the logarithmic layer no Ekman layer, characterized by a mean depth $h = u_\text{m} / f = 25$ m and total mean veering $u_\text{m} = 10^6$ (as predicted by Eq. (11) with $V_\text{l} = 10^3$ cm s$^{-1}$) was found. All of the observed mean veering ($>10^3$) occurred within the logarithmic layer; no mean veering was observed above it.

c. Time scales in the logarithmic layer

A time scale for the logarithmic layer is $L = u_\text{m} / f = 20$ min. Hence, one might expect agreement with Eq. (4) for speeds averaged over an interval ≥ 20 min. This conclusion is substantiated by Fig. 12, which displays $u_\text{m}$ values computed from hourly averaged speeds. While the $u_\text{m}$ values determined from hourly averaged speeds were reassuring, the $u_\text{m}$ values were not. The latter varied by two more orders of magnitude than predicted by empirical relations for $u_\text{m}$ [see Eqs. (5), (6), (10), (17) and (18)]. However, for speeds smoothed over intervals ≥ 3 hr, the inferred $u_\text{m}$ values were well behaved and, to within experimental accuracy, were in agreement with the above relations for $u_\text{m}$.

The comparison of the $u_\text{m}$ values determined from hourly averaged speeds with (10) is depicted in Fig. 16. Better agreement is obtained in Fig. 16, where only those values occurring when the bottom current $|V_\text{b} / 14 |$ ≥ 2.5 cm s$^{-1}$ and flowing northward are depicted. Fig. 16 also indicates that for time scales of ~1 hr, conditions obtained in the logarithmic layer were in only partial agreement with (10).

d. Time scales for veering

The veering in the logarithmic layer was consistent and in the correct sense when the bottom current was strong ($|V_\text{b} / 14 | ≥ 10$ cm s$^{-1}$) and flowing northward (see Fig. 13). It developed typically ~15 min after the current direction had steadied and became northward (e.g., see Fig. 8). Veering above the logarithmic layer at 14 m was observed in these periods of strong, steady northward flow only when the bottom current was quite strong ($|V_\text{b} / 14 | ≤ 10$ cm s$^{-1}$). This suggests that an Ekman layer rarely had sufficient time to form above the logarithmic layer before the current changed. A consequence of (10) and (18) is that the veering in the Ekman layer is a decreasing function of the Rossby number $|V_\text{l} / (f a)|$. Because the veering in the logarithmic layer was persistent, one might expect qualitative agreement with this veering and the Rossby number. Fig. 15b indicates that the opposite occurred; the veering in the logarithmic layer was an increasing function of the Rossby number. The results of this experiment imply that quasistationary conditions in agreement with Eq. (11) do not occur in a planetary boundary layer in which the velocities are modulated by a function whose frequency is greater than $1 / f$ and whose amplitude is comparable to $|V_\text{l}|$.

e. Stratification in the logarithmic layer

In computing the friction velocity $u_\text{m}$ from the speed data, it was assumed that Eq. (4) was a suitable representation for the speeds in the logarithmic layer. This is equivalent to assuming that stratification effects are negligible in this region. The Monin-Obukhov length $L$ is that depth at which stratification effects are considered negligible (Monin, 1970) and is defined as $L = \rho_c \alpha / (\gamma H)$, where $\alpha$ is the specific heat capacity, $\gamma$ the buoyancy parameter ($= \rho / \rho_0$ where $\rho$ is the thermal coefficient of expansion and $\rho$ is the gravitational acceleration), and $H$ the vertical heat flux taken positive downward. The length $L$ is that height where, in the energy equation for the bottom boundary layer (Monin, 1970), the ratio of the shearing stress term to the buoyancy term is $1$. Hence, assuming that stratification effects are negligible in the logarithmic layer is equivalent to assuming that $L < \text{Obukhov length}$. Below, we estimate $L$ and find it to be comparable to $\text{Obukhov length}$. An estimate is then given as to why this estimate of $L$ is thought to be too small.

To know $L$, the downward heat flux $H$ must be known. It was assumed that $H = \rho_c \alpha / (\gamma H)$, where $K_\text{n} = (\rho_c \alpha) / (\gamma H)$, and $\alpha$ the mean temperature; it was further assumed that $K_\text{n} = (\rho_c \alpha) / (\gamma H)$.

In Fig. 16 the ratio $u_\text{m} / (\gamma V_\text{b} / (\gamma a))$ as a function of $|V_\text{b} / (\gamma a)| / (\gamma a)$, a Rossby number, for $u_\text{m} / (\gamma V_\text{b} / (\gamma a))$ at 1000 cm s$^{-1}$, speed data from site 5 (see Fig. 15) and current meter 2 (see Fig. 12) were used to calculate $u_\text{m}$ using Eq. (23) with $d = 1$ cm. Part b shows only those values which occurred when $|V_\text{b} / (\gamma a)| / (\gamma a)$ and the current direction was northward. Part a is a straight line of the logarithmic layer. With $u_\text{m} = 0.4$ cm s$^{-1}$, $\partial H / \partial a = 1.5 \times 10^{-5}$ C cm$^{-1}$. $H / \rho_c = 10^{-4}$ C s$^{-1}$. $H / \rho_c = 10^{-4}$ C s$^{-1}$.
Substituting this into the definition of \( L \) gives

\[
L = 4.0 \text{ ms}.
\]

If \( L = 4.0 \text{ ms} \), as the above estimate indicates, then a better representation than Eq. (4) for the speeds in the logarithmic layer is

\[

\nu_2 = \left( \frac{u_2}{u_1} \right) \left[ \ln \left( \frac{a_2}{a_1} \right) \right],
\]

where \( \nu \) is an experimentally determined constant; the value 5 determined by Webb (1970) is used here. However, such deviations in the lnr profiles typically occurred for \( a_2 > 4 \text{ m} \). Eq. (21) probably deviates appreciably in flat speed profiles from a straight line (i.e., Eq. (4)) for \( a_2 > 2 \text{ m} \). However, such deviations in the lnr profiles typically occurred for \( a_2 > 4 \text{ m} \) (e.g., Fig. 10). An \( a_2 = 20 \text{ m} \) results in the correction term in Eq. (21), \( \nu_2 \), being significant only for \( a_2 > 4 \text{ m} \). Hence, a larger \( L \) is more consistent with the observations. Taking \( K = \nu \nu_2 \) as an overestimate of the eddy thermal diffusivity.

**I. Two implications of the measured values of friction velocity**

It is interesting to note that in Fig. 1 many of the bottom contours in the Straits of Florida are quite straight and evenly spaced, suggesting that the Gulf Stream is a horizontal or transporing sediments along the bottom. For values of \( u_2 = 0.25 \text{ cm sec}^{-1} \), surface creep of bottom materials of diameter \( \approx 0.2 \text{ mm} \) (fine sand) occurs (see Inman, 1940; Fig. 2). Hence, the typical \( \nu_2 \) value of 0.25 cm sec\(^{-1} \) for the mean current at the site of the experiment is too small to account for surface creep. However, when the diurnal tidal current and the mean bottom current reinforced each other, values of \( u_2 = 1.0 \text{ cm sec}^{-1} \) were obtained from the hourly averaged data. During these periods of strong current, it is reasonable to expect the instantaneous \( u_2 \) at times to be equal to the critical value of 0.25 cm sec\(^{-1} \) given by Inman. Bottom photographs made between Miami and Bilini show a fine sandlike material lying on the bottom in most locations. Fig. 5, a bottom photograph made in the immediate vicinity of the experimental site, shows this rather fine material intermixed over a rough bottom. (Note the circular cloud of this material created when the camera ballast weight hit the bottom.) In the upper right portion of this photograph there appears to be some surface creep of the fine material along the bottom. Hence, while the Gulf Stream alone may not be strong enough to transport bottom materials, the combined current resulting from the Gulf Stream and the diurnal tide may be strong enough at times to transport such materials.

Using the experimentally inferred \( \nu_2 \) a rough estimate is made below to determine whether bottom friction under the Florida Current between Key West, Fla., and Cape Fear is a possibly significant dissipative mechanism for that part of the Gulf Stream system which flows through the Straits of Florida. Between Key West and Cape Fear the Florida Current flows along and extends to the bottom of a shelf which is typically \( 1000 \text{ m} \) deep. A two-layer model is considered where the upper layer is \( 1000 \text{ m} \) and the lower layer is at rest.

Bottom friction is assumed to be important only when the upper layer flows over the \( 1000 \text{ m} \) deep shelf between Key West and Cape Fear. The vertically averaged steady momentum and continuity equations for the upper layer are

\[
\begin{align*}
\frac{\partial}{\partial t} \left( \frac{1}{2} \rho u^2 \right) &= \rho f u + \frac{\partial}{\partial x} \left( \sqrt{g h} \right), \\
\frac{\partial}{\partial t} \left( \frac{1}{2} \rho v^2 \right) &= \rho f v + \frac{\partial}{\partial y} \left( \sqrt{g h} \right),
\end{align*}
\]

where \( u \) is the vertically averaged horizontal velocity, \( v \) is the zonal current, \( \rho \) is the density, \( \sqrt{g h} \) is the surface wind stress, and \( \rho f \) is the bottom stress, assumed to be identically zero everywhere except between Key West and Cape Fear, where the upper layer rubs against the bottom. From Eq. (23) a streamfunction \( \psi \) can be defined as

\[
\psi = -\frac{1}{\rho f} \nabla \times \mathbf{v}.
\]

Substituting this into Eq. (22) gives

\[
\frac{\partial}{\partial t} \left( \frac{1}{2} \rho u^2 \right) = \rho f u + \nabla \cdot \left( \sqrt{g h} \right),
\]

where \( \nu \) is the vertically averaged horizontal velocity, \( v \) is the zonal current, \( \rho \) is the density, \( \sqrt{g h} \) is the surface wind stress, and \( \rho f \) is the bottom stress, assumed to be identically zero everywhere except between Key West and Cape Fear, where the upper layer rubs against the bottom. From Eq. (23) a stream function \( \psi \) can be defined as

\[
\psi = -\frac{1}{\rho f} \nabla \times \mathbf{v}.
\]

Substituting this into Eq. (22) gives

\[
\frac{\partial}{\partial t} \left( \frac{1}{2} \rho u^2 \right) = \rho f u + \nabla \cdot \left( \sqrt{g h} \right),
\]

where \( \nu \) is the vertically averaged horizontal velocity, \( v \) is the zonal current, \( \rho \) is the density, \( \sqrt{g h} \) is the surface wind stress, and \( \rho f \) is the bottom stress, assumed to be identically zero everywhere except between Key West and Cape Fear, where the upper layer rubs against the bottom. From Eq. (23) a stream function \( \psi \) can be defined as

\[
\psi = -\frac{1}{\rho f} \nabla \times \mathbf{v}.
\]

Substituting this into Eq. (22) gives

\[
\frac{\partial}{\partial t} \left( \frac{1}{2} \rho u^2 \right) = \rho f u + \nabla \cdot \left( \sqrt{g h} \right),
\]

where \( \nu \) is the vertically averaged horizontal velocity, \( v \) is the zonal current, \( \rho \) is the density, \( \sqrt{g h} \) is the surface wind stress, and \( \rho f \) is the bottom stress, assumed to be identically zero everywhere except between Key West and Cape Fear, where the upper layer rubs against the bottom. From Eq. (23) a stream function \( \psi \) can be defined as

\[
\psi = -\frac{1}{\rho f} \nabla \times \mathbf{v}.
\]

Substituting this into Eq. (22) gives

\[
\frac{\partial}{\partial t} \left( \frac{1}{2} \rho u^2 \right) = \rho f u + \nabla \cdot \left( \sqrt{g h} \right),
\]

where \( \nu \) is the vertically averaged horizontal velocity, \( v \) is the zonal current, \( \rho \) is the density, \( \sqrt{g h} \) is the surface wind stress, and \( \rho f \) is the bottom stress, assumed to be identically zero everywhere except between Key West and Cape Fear, where the upper layer rubs against the bottom. From Eq. (23) a stream function \( \psi \) can be defined as

\[
\psi = -\frac{1}{\rho f} \nabla \times \mathbf{v}.
\]
Substituting this into the definition of $L$ gives

$$L = 4.0 \text{ m}$$

If $L = k_2$, as the above estimate indicates, then a better representation than Eq. (4) for the speeds in the logmie layer is

$$v(x) = (u(x)/L) \ln [(u(x)/v) + c_v/L]$$

where $u$ is an experimentally determined constant, the value of which is determined by Webb (1970) and used here. With $L = 4.0 \text{ m}$, Eq. (21) predicts appreciable deviation in the speed profiles from a straight-line [Eq. (4)] for $x > 2 \text{ m}$. However, such deviations in the bed profiles typically occurred for $x > 2 \text{ m}$ (e.g., Fig. 10). An $L = 20 \text{ m}$ results in the correction term in Eq. (21), $u(x)$, being significant only for $x > 2 \text{ m}$. Hence, a larger $x$ is more consistent with the observations. Taking $K = u(x)/v$ may be an overestimation of the eddy thermal diffusivity $K$.

### Two Implications of the Measured Values of Friction Velocity

It is interesting to note that in Fig. 1 many of the bottom contours in the Straits of Florida are quite straight and evenly spaced, suggesting that the Gulf Stream is scouring the bottoms or transporting sediments along the bottom. For values of $u > 1.5 \text{ cm} \cdot \text{s}^{-1}$, surface creep of bottom materials of diameter $0.2 \text{ mm}$ (fine sand) occur (see Inman, 1949, Fig. 2). Hence, the typical $u$ value of $0.4 \text{ cm} \cdot \text{s}^{-1}$ for the mean current at the site of the experiment is too small to account for surface creep. However, when the diurnal tidal current and the mean bottom current reinforced each other, values of $u = 1.0$ were obtained from the hourly averaged data. During these periods of strong current, it is reasonable to expect the instantaneous $u$ at times to be equal to the critical value of $1.5 \text{ cm} \cdot \text{s}^{-1}$ given by Inman. Bottom sediments, photographs made between Miami and Bimini show a fine sandlike material lying on the bottom in most locations. Fig. 5, a bottom photograph made in the immediate vicinity of the experimental site, shows this rather fine material interspersed over a rough bottom. (Note the circular cloud of this material created when the camera ballast weight hits the bottom.) Hence, while the Gulf Stream alone may not be strong enough to transport bottom materials, the combined current resulting from the Gulf Stream and the diurnal tide may be strong enough at times to transport such materials.

Using the experimentally inferred $u$, a rough estimate is made below to determine whether bottom friction under the Florida Current between Key West, Fla., and Cape Fear is a possible significant dissipative mechanism for that part of the Gulf Stream system which flows through the Straits of Florida. Between Key West and Cape Fear the Florida Current flows along and extends to the bottom of a shelf which is typically $1000 \text{ m}$ deep. A two-layer model is considered where the upper layer is $1000 \text{ m}$ and the lower layer is at rest. Bottom friction is assumed to be important only when the upper layer rubs over the $1000 \text{ m}$ deep shelf between Key West and Cape Fear. The vertically averaged steady momentum and continuity equations for the upper layer are

$$v \cdot \nabla v = -\frac{1}{ho_1} \nabla p_1 + F$$

$$v \cdot \nabla v = v_1$$

where $v$ is the vertically averaged horizontal velocity; $v_1$ is the component of the wind stress; $u$ is the x, y unit vector; $p_1$ the vertically averaged pressure; $c_v$ the constant density of the upper layer; and $v_1$ is the surface wind stress. For $v_1$, $u_1$ is usually assumed to be identically zero everywhere except between Key West and Cape Fear, where the upper layer rubs over the bottom.

From Eq. (23) a stream function $\psi$ can be defined as

$$\psi = -v \times \nabla \psi$$

Substituting this into Eq. (24) gives

$$\nabla \psi = -\nabla \psi + \nabla \psi + \nabla \nabla \psi + F$$

Eq. (24) is now integrated along the streamline $\psi$ which passes over the site of the experiment. At $x = 0 \text{ m}$ and along any closed contour $C$ for any scalar $\psi$ and $\phi$, $\psi \cdot d\phi = 0$ along a streamline, Eq. (24) when integrated along $\psi$ becomes

$$\int_{\phi(x_1)}^{\phi(x_2)} \Delta \psi(\phi) \, d\phi = 0$$

It is convenient to consider the contour of integration in Eq. (25) as consisting of two sections. The superscript $A$ will denote that section where the upper layer rubs over the bottom; and $B$ the remainder, the midocean and deep-water section. On $\phi = \theta_0$, $\theta_1$ is assumed to be zero. Hence Eq. (25) becomes

$$\int_{\phi(x_1)}^{\phi(x_2)} \Delta \psi(\phi) \, d\phi = 0$$

Hence, if bottom friction is the most significant dissipative mechanism, as is assumed in this model, Eq. (26) should nearly apply.

Using the yearly averaged wind stress data given in Hellerman, (1967) the term on the left-hand side of Eq. (26) is estimated to be

$$\int_{\phi(x_1)}^{\phi(x_2)} \Delta \psi(\phi) \, d\phi = 6 \times 10^{19} \text{ dyn cm}^{-1}$$

Using $u_1 = 0.4 \text{ cm} \cdot \text{s}^{-1}$, a typical value for this experiment, as a representative friction velocity along $\phi_0$, the bottom stress term in Eq. (26) is estimated to be

$$\int_{\phi(x_1)}^{\phi(x_2)} \Delta \psi(\phi) \, d\phi = (0.4)^3 \text{ dyn cm}^{-1} \left[\left(\frac{\text{cm}}{\text{m}}\right)^{10} \text{ dyn cm}^{-1}\right]$$

$$= 0.2 \text{ dyn cm}^{-1} \left[\frac{\text{m}}{10 \times 10^4 \text{ cm}}\right]$$

$$= 0.3 \times 10^{15} \text{ dyn cm}^{-1}$$

$\psi$ is the length of $\phi$. Therefore, Eq. (26) is not satisfied, as the bottom stress term is about 1/20 that of the wind stress term. This analysis suggests that the role of bottom friction in the dynamics of the upper 1000 m portion of the Gulf Stream system is rather small.

**Appendix**

### Estimating the Friction Velocity from Spectra

For a speed record obtained in the logarithmic layer the friction velocity $u_t$ is related to the frequency energy spectrum of the speed fluctuations $f(\omega)$ by (Wimbush and Munk, 1970)

$$u_t = \left\{ \frac{2}{3} (\bar{\omega})^2 \left| f(\bar{\omega}) \right| \right\}^{1/3}$$

where $\bar{\omega}$ is the frequency in the inertial subrange, i.e.,

$$f(\bar{\omega}) \propto \bar{\omega}^{-1/3}$$

In (A1) $K$ is a universal constant; the value of $0.56$ determined by Nasmyth (1970) is used here.

Spectra of the Sauter motor array speed records (e.g., see Fig. 17) are in quantitative agreement with (A1) for $x > 2 \text{ m}$ and $x < 4 \text{ m}$ in that the slopes are nearly equal to $-0.5$ and the spectral estimates vary inversely with $x$. However, the $u_t$ values computed using (A1) are typically one-half as large as the value of $0.4 \text{ cm} \cdot \text{s}^{-1}$ determined earlier in Section 4 using Eq. (4). This lack of quantitative agreement in the $u_t$ values is not surprising as condition (A2) is not satisfied for any of the spectra of speed records obtained in the logarithmic layer. For example, for the speed record at $x = 65 \text{ cm}$ ($x = 7.76 \text{ cm}$) condition (A2) is satisfied.

70 cycles hr$^{-1}$

The highest resolvable frequency for any of the rotor array data is $21.5 \text{ cycles hr}^{-1}$. Hence, no spectral estimate from the inertial subrange is found in this spectrum. Including the effects of stratification (as estimated in Section 5) into (A1) given $u_t$ values that are slightly smaller than those found ignoring stratification.

### REFERENCES


The Structure of the Residual Flow in an Offshore Tidal Stream

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Abstract

A theoretical account is given of the turbulent boundary layer at the ocean floor beneath a continuously oscillating tidal flow. A parameterization, involving the techniques described by Johns and Dyke, is used to obtain an assessment of the contribution of the nonlinear advective terms in the system. In particular, the theory is applied to assess the representation of the Lagrangian residual flow immediately above the ocean floor. This is evaluated numerically with parameters representative of the conditions in Sydney Harbor. It is found that the combined effect of the M2 and S2 tidal constituents leads to a predominantly north-westerly residual flow which can attain a magnitude of almost 3 cm s\(^{-1}\) within 1 m of the ocean floor.

1. Introduction

Tidally induced residual currents in shallow seas have been considered by Hunt and Johns (1965) who discussed several interesting examples that result from an imposed basic tidal oscillation. Although of undoubted value as a first study, the underlying assumptions in their model are open to serious criticism and may well be untenable. The most important of these is related to their specification of a completely laminar oscillatory boundary layer flow at the ocean floor which, in their applications, is implicitly identified with a turbulent layer in which the kinematic viscosity has an enhanced value. The implications of this procedure are serious. First, it is known to reproduce an unsatisfactory structure for the tidal flow in the boundary layer. Since this structure is used in calculating the residual flow, the resulting current system may not be an adequate representation of the process that exists in practice. Second, Hunt and Johns obtained the residual current system at the upper limit of the boundary layer and then used this quantity to assess the flow just above the ocean floor. However, the thickness of the boundary layer at the bar is at least of order \(\sqrt{\nu/\nu_{0}}\), where \(\nu\) is the radius frequency of the basic tidal oscillation. Thus, by considering the \(\nu_{0}\), tidal constituent and supposing that \(\nu_{0} = 14\) cm s\(^{-1}\), say, we find that \(\nu/\nu_{0} \approx 3\). Consequently, at such a position, the magnitude of the residual flow may not be a reliable indication of the current closer to the ocean floor where sedimentation processes become important. The present work has been undertaken with a view to obtaining a more satisfactory representation of tidally induced residual current systems in offshore regions. The description of the basic tidal flow in the turbulent boundary at the ocean floor follows that given by Johns and Dyke (1971). This is then used as a first approximation in developing a solution of the nonlinear equations that yield the residual flow. As an illustration, the solutions are evaluated numerically by using parameters representative of the Liverpool Bay region. Our computations show that the combined effect of the M2 and S2 tidal constituents leads to a predominantly northerly residual current system which may attain a magnitude of almost 3 cm s\(^{-1}\) within 1 m of the ocean floor. We propose, therefore, that such a process should be taken into account when assessing the long-term net movements of loose bottom debris. Moreover, an interpretation of observational work relating to the movement of sea-bed drifters would include a reference to the mechanism which, in entering circumstances, may act either to reinforce or to oppose that associated with other causes. It must be emphasized that the work in the present paper disregards local variations in the mean water depth; neither does it include a representation of the complicated system of embayments and estuaries along the eastern land boundary of Liverpool Bay. The inclusion of such topographical complexities, which would necessarily involve the development of a large-scale numerical model, must be expected to have an effect upon the local structure of the residual current system. Consequently, the present work must be regarded as applying to the gross features to be found in the process, the evaluation of the details being reserved for further study. Finally, the fluid is treated as homogeneous.

2. Formulation and method of solution

Conditions in the system are referred to a set of rectangular Cartesian coordinates \(x, y, z\) whose origin