NOTES AND CORRESPONDENCE

Viscid Eastern Boundary Dynamics and the Spreading of Mediterranean Water along the Portuguese Continental Slope

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ABSTRACT

A linear, continuously stratified model is used to investigate the flows generated by a midlatitude, eastern boundary zonal inflow representing the flux of Mediterranean Water into the North Atlantic. The model allows for time dependence and vertical mixing of density and meridional momentum and assumes a geostrophic balance for zonal momentum. Rossby and Kelvin wave propagation and dissipation away from the inflow region determine the character of the analytical solutions. For both periodic and steady forcing and for a wide range of mixing coefficients, currents have a significant boundary signature. Inflows generate poleward currents at the depth of the forcing and weaker countercurrents above and below. The amplitude of meridional coastal flows can be substantially larger than the amplitudes of the forcing jet, and interior flows are generally weaker. Zonal decay scales depend on the amount of mixing and the relative importance of Kelvin and Rossby wave dynamics in the solutions.

1. Introduction

Excess of evaporation over precipitation in the Mediterranean Sea creates a dense, saline water mass that flows into the North Atlantic through the Strait of Gibraltar. The large-scale influence of Mediterranean Water on the North Atlantic circulation has long been recognized (e.g., Maillard 1986), and a number of studies have addressed its dynamical basis (Arhan 1987; Tziperman 1987), but the nature of the northward spreading along the Portuguese continental slope is less clear. Observations described by Ambar and Howe (1979) and Zenk and Armi (1990) indicate a complex horizontal and vertical structure of the boundary flows with at least two main veins of Mediterranean Water between 600 and 1200 m, approximately. The upper vein seems to follow the coast closely, and the lower vein tends to have a more northwestward path. The integrated geostrophic transport over the depths influenced by the Mediterranean flow is northward, but a number of vertical shear regions is suggested in the hydrographic data of Zenk and Armi (1990). A southward flow beneath the lower vein was also identified by Ambar and Howe (1979).

The existence of complex flows along the eastern boundary is hinted in Tziperman’s (1987) theoretical study of the circulation forced by a mass source representative of the Mediterranean inflow. Tziperman used a layered, geostrophic model, with a nonlinear thermodynamic balance between vertical advection and diffusion of density. Steady solutions show poleward flow at the inflow layer, with equatorward flow in the layers above and below. The circulation is established through long Rossby wave propagation. There is trapping of flows to the boundary, and zonal decay scales correspond to the distance over which Rossby waves traveling from the boundary are dissipated by the presence of diffusion.

The role of Kelvin waves in the alongshore flow adjustment is not apparent in the discussion of Tziperman’s geostrophic solutions. A linear, continuously stratified model is used here to further examine the mechanisms by which the Mediterranean inflow spreads from the source region. The alongshore momentum balance in the model is ageostrophic and allows explicit treatment of Kelvin wave dynamics, bringing to light their role in the solutions. Given the simplicity of the linear model, both steady and oscillatory solutions are explored and their dependence on the various parameters (diffusion coefficients, structure of forcing, etc.) evaluated. We find that boundary currents are part of the solutions under most conditions.

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2. The analytical model

Consider a semi-infinite ocean, unbounded to the west of a north–south wall placed at \( x = 0 \), and forced by a zonal inflow at the boundary with a given vertical and meridional structure. Kundu and McCreary (1986, hereafter referred to as KM) studied the circulation associated with a source flow representing the Indo-Pacific throughflow using the linear, continuously stratified model of McCreary (1981), which assumes a geostrophic zonal momentum balance and includes vertical and horizontal diffusion of density and momentum. The model used here is essentially that of KM with the following differences: For simplicity, horizontal diffusion is not considered and the static stability \( N \) is assumed constant; the \( \beta \)-plane approximation \( (f = f_0 + \beta y) \) is used given the short meridional extent over which solutions are considered; and time dependence is kept to study oscillatory solutions. With the notation as in KM, the basic equations are

\[
\begin{align*}
-fu &= -p_x \quad (1a) \\
v_t + fu &= -p_z + \nu \psi_{zz} \quad (1b) \\
p_z &= -g \rho \quad (1c) \\
\rho_z - N^2 w/g &= \kappa \rho_{zz} \\ u_x + v_y + w_z &= 0. \quad (1e)
\end{align*}
\]

Top and bottom boundary conditions are

\[
\begin{align*}
v_z &= u_z = w = \rho = 0 \quad z = 0, -H, \quad (2)
\end{align*}
\]

where \( H \) is the constant ocean depth. According to (2), there are no surface and bottom Ekman layers. At the meridional wall, a zonal flow with frequency \( \sigma \), amplitude \( U_y \), and meridional and vertical structure given by \( Y \) and \( Z \) is assumed; that is,

\[
u = U_y Y(\gamma) Z(z) e^{-i\sigma t}, \quad x = 0. \quad (3)
\]

The method of solution of (1) by separation of variables is treated in detail by McCreary (1981) and KM and will not be described here. The horizontal currents \( u \) and \( v \), the only fields considered here, are given by a sum over the vertical modes of the system

\[
[u, v] = \text{Re} \sum_{n=0}^{\infty} [u_n, v_n] \psi_n(z) e^{i(k_n z - \sigma t)}, \quad (4)
\]

where \( k_n \) is a function of \( y \). The barotropic and baroclinic modes take the form

\[
\psi_0 = 1, \quad c_0 = \sqrt{gH} \quad (5)
\]

\[
\psi_n(z) = \cos \frac{N}{c_n} z, \quad c_n = \frac{NH}{n\pi}, \quad n = 1, 2, \ldots. \quad (6)
\]

Solution of the horizontal structure problem with boundary condition (3) yields

\[
u_n = \frac{i}{f} \frac{k}{\beta} \left( \frac{\beta}{2\omega_v} \pm \left( \frac{\beta}{2\omega_v} \right)^2 + \frac{\omega_x}{\omega_v} \alpha^2 \right)^{-1/2} \bar{P}, \quad (8)
\]

where subscripts \( n \) on the right-hand side have been dropped for simplicity. In (7) and (8),

\[
k = i \left( \frac{\beta}{2\omega_v} \pm \left( \frac{\beta}{2\omega_v} \right)^2 + \frac{\omega_x}{\omega_v} \alpha^2 \right)^{1/2} \quad \{ (9) \}
\]

\[
U = U_0 \frac{\frac{1}{2} \int_{-H}^Y Z_0 \psi dz}{\frac{1}{2} \int_{-H}^Y \psi^2 dz}, \quad (10)
\]

\[
\bar{P} = -e^{-F} \int_0^\infty f u^t U d\gamma', \quad (11)
\]

with

\[
\omega_j = \frac{N^2}{c^2} - i\sigma \quad \{ (12) \}
\]

\[
F = \frac{1}{f} \int \frac{k \omega_v}{\bar{P}} d\gamma \quad \{ (13) \}
\]

and \( \alpha^{-1} = c/f \), the Rossby radius of deformation. The choice of \( k \) in (9) is determined by requiring bounded solutions as \( x \to -\infty \). The origin of the \( y \) axis is chosen to coincide with the equatorward edge of the source jet region, so that the integration in (11) includes all the forcing effects. Solutions calculated in the following sections assume Prandtl number \( \nu/\kappa = 1 \), in which case \( \omega_y = \omega_z = \omega_v \); the general expressions are given for the purposes of discussing the limits \( \nu \to 0 \) and \( \kappa \to 0 \) later on.

For the solutions considered here, we assume a simple forcing flow of the form

\[
Y(y) = \sin \frac{\pi}{L} y, \quad 0 \leq y \leq L \quad (14a)
\]

\[
Z(z) = \cos \frac{\pi}{L} \left( z + \frac{H}{2} \right), \quad -\frac{H - l}{2} \leq z \leq -\frac{H + l}{2} \quad (14b)
\]

With this spatial structure, integrals in (10) and (11) are solved analytically and numerically, respectively. Evaluation of the expansion coefficients (7) and (8) is then straightforward. The sum over vertical modes in (4) is carried to \( n = 100 \). The presence of diffusion assures, in most cases, the rapid convergence of the sums. Because of the symmetry of the forcing about \( z = -H/2 \), only even mode numbers contribute substantially to the solutions.
3. Oscillatory solutions

To gain insight about the nature of the transient response to variability in the Mediterranean outflow and also to help interpret the steady solutions discussed later, we consider the model response to oscillatory forcing. Solutions are set up through propagation of the wave modes of (1), that is, Rossby and Kelvin waves. The basic nature of the solutions can be inferred from the character of $k$. Assuming for the purpose of this discussion that there is no mixing ($\kappa = \nu = 0$), then (9) simplifies to

$$k = -\frac{\beta}{2\sigma} \pm \left[ \left( \frac{\beta}{2\sigma} \right)^2 - \alpha^2 \right]^{1/2}. \tag{15}$$

At a given forcing frequency and latitude, for vertical modes with $\alpha < \beta / 2\sigma$, $k$ is real and equal to the zonal wavenumber of Rossby waves described by (1); the factor $e^{(i\kappa - \kappa)\nu}$ in the solutions is purely oscillatory and represents propagating Rossby waves. In this regime, Kelvin wave energy can disperse away from the boundary by Rossby wave radiation. For vertical modes with $\alpha > \beta / 2\sigma$, $k$ is complex, free Rossby waves are not possible, and solutions have a mixed oscillatory–evanescent character in $x$. Modes with $\alpha \gg \beta / 2\sigma$ behave essentially like Kelvin waves with decay scale given by deformation radius $\alpha^{-1}$. According to (15), for seasonal and shorter period fluctuations at midlatitude, all but the barotropic and possibly the first baroclinic modes are trapped to the boundary. Thus, one expects the baroclinic response to have strong boundary signals in general.

To illustrate the character of the seasonal response, we discuss solutions calculated assuming a forcing inflow oscillating at the annual period ($\sigma = 2 \times 10^{-7} \text{ s}^{-1}$) with an amplitude $U_0 = 1 \text{ cm s}^{-1}$, and with meridional extent $L = 10^6 \text{ m}$ and vertical extent $l = 600 \text{ m}$. With these values, the fluctuation in transport is approximately 0.2 Sv ($Sv = 10^8 \text{ m}^3 \text{ s}^{-1}$) or an order of magnitude smaller than typical strength of the (steady) Mediterranean inflow. For the other model parameters, we take $H = 2500 \text{ m}$, $f_0 = 8.36 \times 10^{-5} \text{ s}^{-1}$, $\beta = 1.87 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1}$ (\beta-plane centered at 35$^\circ$), $N = 2 \times 10^{-3} \text{ s}^{-1}$ (typical of subthermocline levels), and $\nu = \kappa = 10^{-2} \text{ m}^2 \text{ s}^{-1}$ (similar to values used in models of coastal currents; e.g., McCreary 1981).

Figure 1 shows the $y-z$ structure of the meridional flow field at $x = 0$, and the $x-z$ structure at $y = 200$ km, at the time of maximum inflow. A poleward current is present at the depth of the forcing jet (maximum speed of $\sim 10 \text{ cm s}^{-1}$) with weaker opposite flows above and below. Flows reach a maximum at the northern limit of the forcing region and decay poleward of this latitude. The amplitudes are significantly larger than that of the forcing jet. In the $x-z$ plane, currents are strongly trapped to the coast, with decay scales of a few kilometers. Zonal currents (not shown) are much weaker overall.

A number of features in the solutions of Fig. 1 are worth interpreting. The trapped nature of the flows indicates the importance of Kelvin waves in the solutions, as anticipated in the discussion of (15). These waves propagate energy along rays with a slope relative to the horizontal given by $\pm \sigma / N$. For the solutions in Fig. 1, $\sigma / N = 10^{-4}$ and energy propagates only 50 m in the vertical over horizontal distances of 500 km. Thus, significant flows are confined to depths close to the forcing depth. The product $k\overline{P}$ determines the amplitude of coefficients $v_0$ in (8) and hence the vertical structure of the meridional flows. The amplitude of $k$ goes approximately as $\alpha$ or $c_n^{-1}$ and increases with baroclinic mode number. However, $\overline{P}$ decays rapidly with mode number because of its dependence on $k$ and $U_n$. (The forcing jet projects more strongly onto low baroclinic modes.) Thus, solutions exhibit more energy at low and intermediate vertical mode numbers. Finally, the vertical scale increases away from the boundary because higher modes have shorter zonal decay scales ($k^{-1} \sim c_n$).

Solutions with different amounts of dissipation were explored. Summarizing results, the vertical scale of flows increases slightly with stronger mixing because of the weaker contribution of higher baroclinic modes to the solutions ($\omega \sim c_n^{-2}$). The strength of the flows varies inversely with the strength of the mixing but the relation is less than linear (i.e., maximum speeds behave as $\nu^{-\gamma}$ with $\gamma < 0.5$, approximately). One point to emphasize is that, unless very large values of $\nu (\sim 1 \text{ m}^2 \text{ s}^{-1})$ are used, solutions exhibit boundary currents with amplitudes substantially stronger than those of the source jet.

4. Steady solutions

The general character of steady solutions to (1) is treated extensively by KM. Here, before examining specific results, we only highlight the role of Kelvin and Rossby wave dynamics in the solutions by addressing the nature of wavenumber $k$. With $\sigma = 0$ in (12), $k$ is always imaginary and solutions, evanescent in $x < 0$, are obtained by choosing the minus sign in (9). Consider first the effect of having a Prandtl number different than one, in particular, the limiting cases of zero ($\nu \to 0$) and infinite ($k \to 0$) Prandtl number. The former limit is essentially that of Tziperman’s (1987) model. In this case $k \sim -i\alpha^2 \omega_0 / \beta$, which is basically the long Rossby wave dissipation scale discussed by Tziperman. In the absence of mixing of meridional momentum, all baroclinic modes adjust to a geostrophic balance in the steady limit, and the only wave signature left in the solutions is that of geostrophic Rossby wave decay. In the case $k \to 0$, as discussed by KM and Tziperman (1987), solutions have vanishing boundary flows and consist simply of a zonal flow mimicking that of the source. Basically, in the absence of density mixing, (1d) implies zero vertical velocities in the steady limit and because of vorticity conservation no
meridional flows are allowed. Zonal flows at the source must continue along constant latitude circles.

The Kelvin wave zonal decay scale is not apparent in the solutions with zero and infinite Prandtl number. It does become apparent, however, when both density and momentum mixing are present. Then, the $\epsilon$-folding trapping scale depends essentially on the ratio $\epsilon = \alpha / (\beta / 2\omega)$ (for simplicity we take Prandtl number $\nu/\kappa = 1$ here). For $\epsilon^2 \ll 1$, then $k \sim -i \alpha^2 \omega / \beta$, which is again the long Rossby wave dissipation scale. However, for $\epsilon^2 \gg 1$, then $k \sim -i \alpha$ and the decay scale is that of Kelvin waves and independent of $\nu$. In general, for $\nu$ in the range $10^{-2} - 10^{-4}$ m$^2$ s$^{-1}$, only the lowest vertical baroclinic modes adjust to the Rossby wave scale with higher modes adjusting to the Kelvin wave scale. (The barotropic mode has $k \sim 0$ and very long decay scales because $c_0$ is large and the barotropic adjustment is very fast and not affected by mixing, as explained by KM.)

As a standard case, we calculate solutions with parameters equal to those used in the solutions of Fig. 1 but assume a steady source inflow ($\sigma = 0$) of amplitude $U_0 = 5$ cm s$^{-1}$, equivalent to a transport of 1 Sv similar to the observed order of magnitude for the Mediterranean inflow. Figure 2 shows the meridional current in the vertical planes along $x = 0$ and along $y = 200$ km. Maximum poleward flows (over 40 cm s$^{-1}$) occur at the depth of the forcing with weaker equatorward flows above and below; amplitudes decay slowly poleward of the forcing region due to dissipative effects. The vertical integrated flow associated with any baroclinic mode is zero, and the transport of the inflow jet is carried by the barotropic mode, which adjusts to a state of no meridional flow [i.e., $k_0 \sim 0$ and so is $v_0$ by (8)]. Thus, in our solutions, the mass flux at the source is carried westward by a weak barotropic flow.

Flows in Fig. 2 are strongly trapped to the boundary over scales on the order of the deformation radius
Meridional velocity

$\alpha_n^{-1}$ of the lowest baroclinic modes, similar to the periodic solutions in Fig. 1. The adjustment to the Kelvin wave scale is expected given that, for the values of $\nu$ used, $\epsilon$ is order one or greater for all (even) baroclinic modes. Thus, the zonal trapping is due primarily to the Kelvin wave dynamics underlying the setup of the steady flows along the boundary, rather than the dissipative decay of Rossby waves propagating away from the boundary as in the solutions by Tziperman (1987).

The dependence of the vertical scale of steady solutions on mixing is similar to that discussed for periodic solutions, but the same is not true for their horizontal structure and strength. In particular, in the limit $\nu = \kappa = 0$ periodic solutions exhibit the strongest meridional flows along the boundary, but steady solutions have vanishing boundary flows as discussed previously. To examine when this limit is approached, Fig. 3 shows the horizontal velocity field at mid-depth for the standard case and for $\nu, \kappa = 10^{-4}$ m$^2$ s$^{-1}$, a canonical value for interior oceanic mixing. The latter solution exhibits weaker flows along the boundary and a stronger zonal flow at the latitude of the forcing. While most of the inflow bends poleward in the standard solution, the canonical mixing case shows much less bending. For $\nu, \kappa = 10^{-5}$ m$^2$ s$^{-1}$, values of $\nu$ become negligible and the inflow jet just continues as a geostrophic zonal flow into the interior. Note also that the solution with canonical mixing has longer $\epsilon$-folding zonal scales. In this case, baroclinic modes with $n < 10$ adjust to the Rossby wave scale. Thus, Rossby wave dynamics are more important in defining the zonal structure of flows in Fig. 3b than for the standard solution in Fig. 3a.

In the limit of very strong mixing, one expects weak flows everywhere as all modes generated in the source region do not propagate far before being strongly damped. Thus, maximum poleward currents are obtained for intermediate values of the mixing coefficients. For the parameters chosen here, solutions with $\nu = \kappa = 10^{-2}$ m$^2$ s$^{-1}$ (Fig. 2) show maximum poleward flows ($\sim 40$ cm s$^{-1}$). For mixing coefficients of $10^{-1}, 10^{-3}$ m$^2$ s$^{-1}$, the strongest
poleward flows decrease to 30 cm s\(^{-1}\). As for the periodic solutions, the dependence on the strength of mixing is less than linear over these range of values.

Dependence of results on the meridional and vertical extent of the forcing jet was explored by examining solutions with different values of \(L\) (50 and 200 km), and \(l\) (300 m) in (14), but keeping the transport of the inflow always equal to 1 Sv by adjusting the amplitude \(U_s\). The choice of \(L\) has little impact on the structure of the flows. Effects of smaller \(l\) are more important. As expected, solutions exhibit shorter vertical scales and stronger flows along the boundary. Note, however, that the strengthening of the flows is not simply due to the larger \(U_s\): it is also a result of the stronger projection of the thinner forcing jet onto the baroclinic modes. The ratio of high to low mode amplitudes is larger in the thin jet forcing case. Thus, dependence of results on mixing coefficients is different. Solutions with maximum poleward flows along the boundary and with flow structure characteristic of the limit \(\kappa \to 0\) are obtained for values of \(\nu\), \(\kappa\) smaller than those for solutions with \(l = 600\) m.

5. Discussion

Before discussing the results in the context of the observations, we compare them with those of Tziperman (1987) and KM. The vertical structure of Tziperman’s solutions is fairly similar to what we find here, but the zonal decay scales can be different. The difference occurs because of Tziperman’s geostrophic assumption in both zonal and meridional momentum balance; that is, \(\nu = 0\). In this case, the zonal decay scale for all vertical modes is the geostrophic Rossby wave dissipation scale. One possible interpretation is that when \(\nu = 0\) and \(\sigma \to 0\), there is no cutoff vertical mode number for Rossby waves.
and propagation of energy from the boundary can occur at all latitudes and vertical scales. In contrast, for \( \nu \neq 0 \) and \( \sigma \rightarrow 0 \), not all vertical modes are able to disperse into the interior as Rossby waves. The modes above the cutoff for Rossby wave propagation are trapped to the coast over a zonal scale associated with Kelvin wave dynamics.

Concerning the study of the Indo–Pacific throughflow by KM, one important difference with our results is that, for similar mixing strengths, the present solutions show significantly more trapping of energy to the boundary. Although KM use rather different forms of stratification and forcing, the main reason for the weaker coastal trapping in their results seems to be the low latitude of the inflow (5°–10°) compared to ours (35°). The propagation speeds of Rossby waves increase toward the equator and the number of Rossby wave modes available also increases [see discussion of (15)]. Thus, less energy is carried by Kelvin-like modes, and for similar dissipation rates one expects the response to low-latitude inflows to have less of a boundary character.

In general, the periodic and steady solutions studied here are characterized by weak interior flows and coastally trapped meridional flows with amplitude larger or comparable to that of the source jet. Observations by Ambar and Howe (1979) and Zenk and Armi (1990) indicate that, after reaching Cape São Vicente, the Mediterranean source flow turns generally northward and continues along the boundary, in qualitative agreement with the present findings. The countercurrents typical of the model solutions may also explain the return flows below the main core of the Mediterranean flow reported by Ambar and Howe.

The zonal structure of the observed alongshore flows is not well determined, but density fields described by Zenk and Armi suggest a complicated mixture of mean flow and eddies with a number of different zonal scales. One interesting question is whether the observations indicate the presence of boundary currents with decay scales of a few kilometers as found in the solutions with strong mixing. Zenk and Armi’s data show some enhanced sloping of isopycnals near the boundary that may represent such currents, but they do not emphasize those features in their discussion. Processes such as interaction with topographic features, instabilities, and eddy shedding most certainly affect the spreading along the boundary. Ultimately, a numerical model must be used to explore the full complexity of these flows. In this case, if the narrow boundary currents suggested in the solutions are indeed part of the flow structure, resolution of these currents may require rather small grid spacing.

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