The Structure of Near-Inertial Waves during Ocean Storms

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ABSTRACT

Current meter data from two sites were analyzed for near-inertial motions generated by storms during the ten-month period of the Ocean Storms Experiment in the northeast Pacific Ocean. The most striking feature of the inertial wave response to storms was the almost instantaneous generation of waves in the mixed layer, followed by a gradual propagation into the thermocline that often lasted many days after the initiation of the storm. The propagation of near-inertial waves generated by three storms in October, January, and March was studied by using group propagation theory based on the WKB approximation. It was found that wave frequencies were slightly superinertial, with inertial shifts 1%–3% in October and March and around 1% in January. The phase of near-inertial currents propagated upward below the mixed layer, confirming the downward radiation of energy by these waves. The average downward energy flux during the storm periods was between 0.5 and 2.8 mW m⁻². The vertical wavelengths indicated by the vertical phase differences ranged from 150 to 1500 m. The vertical group velocity was estimated from the arrival times of the groups at successive depths. Using this in the dispersion relation, horizontal wavelengths ranging from 140 to 410 km were obtained. A relation between density and velocity that gives the horizontal directionality of internal waves was derived. During the storm periods examined, the propagation directions of near-inertial waves mainly lay between northeast and south, indicating sources west of moorings. The directions tended to rotate clockwise with increasing depth, consistent with the expected effect of the earth’s curvature. The estimated horizontal wavelength and propagation direction were consistent with the horizontal phase difference between inertial currents at the two sites.

1. Introduction

Storms generate near-inertial oscillations in the upper ocean. Newly generated near-inertial motions can persist for many days in the mixed layer before decaying. This decay is thought to be principally due to propagation of near-inertial internal waves deeper into the ocean (Pollard 1980; Gill 1984). This phenomenon has been observed and studied by many investigators (Webster 1968; Pollard and Millard 1970; Geisler 1970; Kroll 1975; Müller et al. 1978; Price 1983; Gill 1984; Kundu and Thomson 1985; D’Asaro 1985, 1989). The Ocean Storms experiment was conducted in the northeast Pacific Ocean from August 1987 to June 1988 with the aim of improving understanding of upper ocean response to atmospheric forcing. This paper presents the results of analysis of data from two moorings in this experiment. The data are described in section 2.

The Ocean Storms data provide an interesting opportunity to study the generation and propagation of wind-forced near-inertial motions. A most striking feature in the data is the nearly instantaneous generation by storms of near-inertial waves in the mixed layer, followed by the gradual advance of near-inertial wave packets into the thermocline, a process lasting many days after the commencement of the storm. The near-inertial peak is a prominent feature of the internal wave spectrum. Vertical propagation of near-inertial waves below the mixed layer is of particular interest because it transfers the near-inertial energy generated at the surface into the ocean interior (Kroll 1975; Leaman and Sanford 1975; Price 1983; Gill 1984). Presumably, the wind imposes a horizontal length scale on mixed layer inertial currents, though this may be modified by variation of the Coriolis parameter during subsequent propagation (D’Asaro 1989). The rate of vertical transfer of energy through the thermocline varies inversely with this length scale. An accurate estimate of the horizontal scale of near-inertial currents is thus essential for understanding the vertical energy budget.

We concentrated our analysis on three energetic storms that occurred in October, January, and March during the experiment and studied the propagation of...
near-inertial waves from the standpoint of group propagation theory based on the WKB approximation. The characteristics of near-inertial motions during the individual storm events are described in section 3. By examining the relations between the near-inertial currents at various depths, the phase propagation and wavenumber in the vertical direction are estimated. The vertical group velocity is estimated from the arrival times of the groups at successive depths. Applying this to the dispersion relation, the horizontal wavenumber can be obtained. The average downward energy flux during the storm periods is calculated. These tasks are described in section 4. A relation between density and velocity that gives the horizontal directionality of internal waves is derived in section 5. The propagation directions of near-inertial waves during the individual storm events are estimated using this relation. The horizontal wavenumber is independently estimated from the directional information and the horizontal phase differences at two moorings.

2. Observations

The Ocean Storms Experiment was conducted in the northeast Pacific Ocean from August 1987 to June 1988. It consisted of current, temperature, salinity, and surface flux (momentum and heat) measurements. Its main objective was to study the three-dimensional response of the upper ocean to severe storms. Vector measuring current meters (VMCM) were deployed at seven depths on the subsurface mooring at site C1 (47°25.4′N, 139°17.8′W: Fig. 1). They recorded velocity and temperature every 15 minutes at 20-m intervals from 60 m to 160 m and at 195 m (Levine et al. 1990). Five Seacat temperature–conductivity recorders on mooring C1 sampled every 6 minutes until 1200 UTC 25 January 1988 at 70 and 89 m and until 1200 UTC 4 March 1988 at 109, 128, and 150 m, and every 12 min thereafter. The conductivity cell at 150 m failed early in the deployment. The upper two Seacat instruments also contained a pressure sensor, sampling every one hour until 1200 UTC 25 January 1988 and every two hours after that, to monitor the mooring motion. A profiling current meter (PCM) was installed on mooring NP (47°34.7′N, 139°23.3′W). This instrument profiled every 4 hours, averaging current, temperature, and salinity into 5-m depth bins from 195 to 35 m. The data were further filtered with a 20-m wide centered triangular window. The VMCM instrument at 120 m on C1 failed after 11 March 1988. PCM data at 35 m are usually unavailable because currents drew the mooring too deep for the profiling range to reach this depth bin (Eriksen et al. 1982). The useful ranges are from 50 to 195 m in October, from 40 to 190 m from November to May, and from 40 to 195 m in the remaining months. The two moorings (C1 and NP) were 18.5 km apart in water of depth about 4200 m.

The changing buoyancy frequency profile $N(z; t)$ was calculated from the PCM dataset, averaged over ten-day blocks (Fig. 2). Upper-ocean density structure at the Ocean Storms site was characterized by a double pycnocline consisting of an upper seasonal thermocline.
and a deeper permanent halocline. The mixed layer depths were determined from temperature measured with a thermistor chain with 11 thermistors between 9 and 105 m on mooring W (47°28.0′N, 139°59.2′W) before 1 November (when the mixed layer was shallow, Fig. 3) and from the profiles furnished by the PCM after 1 November. A time series of wind speed and direction was obtained from an anemometer on mooring C0 (47°28.9′N, 139°15.4′W), and was used to calculate a wind stress time series (Fig. 4).

3. Storm-generated near-inertial currents

Some features of near-inertial currents observed during Ocean Storms are described in this section. We first isolate the signals of near-inertial oscillations from the raw time series and then attempt to interpret their spatial and temporal distributions. Near-inertial currents during Ocean Storms manifest all the important features and properties previously documented. The near-inertial responses at moorings C1 and NP are compared.

We used the method of complex demodulation (Perkins 1970; Pollard and Millard 1970) to separate the near-inertial velocities from the total current. From the complex time series \( U(t) = u(t) + iv(t) \), where \( u \) and \( v \) are the eastward and northward velocity components, we calculated the complex amplitude \( D(\tau) \) given by

\[
D(\tau) = \frac{1}{2T} \int_{-\tau}^{\tau} W(t - \tau) U(t) e^{i\omega t} dt, \quad (1)
\]

where \( W(t) \) is a tapered data window of length \( 2T \). The magnitude and argument of \( D \) give the amplitude and phase of the inertial current at time \( \tau \). The complex demodulation was performed at frequency \( \sigma = 0.0625 \) cph rather than the local inertial frequency \( f(0.06153 \) cph at C1 and 0.06169 cph at NP) and on a window length of six demodulation periods (\( 2T = 96 \) h). This makes \( 2T \) an even multiple of the different sampling rates in the two datasets. For \( W(t) \) we used a triangular window, which is very effective in suppressing semi-diurnal and diurnal tides. The window filter has a half-power bandwidth of 0.01329 cph, so that its main lobe encompasses the inertial frequency band.

![Temperature and density profiles during Ocean Storms](image)

**Fig. 3.** The temperature and density anomaly profiles as determined from the thermistor chain and PCM data, which were low-pass filtered (40-h half-power) and decimated to daily values. The thick dashed line indicates the bottom of the mixed layer as determined from the maximum in \( dT/dz \) and \( d\sigma_t/dz \).
The amplitudes of the complex-demodulated inertial currents at corresponding depths at the two moorings for the whole experimental period are shown in Fig. 4. Many storms struck the Ocean Storms instrumental array during the ten-month period. Strong storm-generated inertial responses due to events on 4 October, 16 November, 4 December, 13 January, 4 March, and 2 April are evident, though a storm on 14 September apparently did not generate inertial currents. Storms in October, January, and March generated the most vigorous inertial oscillations. We concentrate on the time periods containing these three storms in the analysis presented in the following sections. The starting times of complex demodulation were set at 1200 UTC 4 October, 13 January, and 4 March respectively, for both moorings. The resulting phase of the complex-demodulated inertial currents was backrotated by \((\sigma - f)\tau\), so that the phase of a purely inertial oscillation would not change with time. The phase of \(D(\tau)\) drifts slowly in time during the main burst of amplitude. From the rate of change of this drift, we can estimate the central frequency \(\omega\) of the near-inertial currents. This usually
differs by a few percent from the local \( f \). We call the proportional difference, \( \nu = (\omega - f)/f \), the inertial shift. If the low-frequency vorticity of mesoscale variability, which would provide background vorticity to bias \( f \) and Doppler shift the frequency, were neglected, a positive inertial shift is indicative of the shallow vertical propagation angle of near-inertial internal waves.

a. October storm

The complex-demodulated inertial currents and the estimated inertial shifts during October are shown in Fig. 5. The prestorm mixed layer base was about 42–45 m (Fig. 3), so the top of the PCM mooring was just at the bottom of the mixed layer, and all the VMCMs were in the thermocline. The storm struck on year day 277 and generated a prominent burst of inertial current amplitude. The VMCM at 60 m responded about one day after the storm. There was even further delay of response at greater depths. At 120 m, for example, the near-inertial response at C1 began on day 285, some 8 days after the passage of the storm. It took about 10 days for the inertial oscillations to propagate from 60 m to 195 m. Inertial currents at 60 m reached their maximum speed (about 0.3 m s\(^{-1}\)) after about a week and persisted at least another two weeks before decaying to the prestorm level. In the early response, the amplitudes decreased with depth. After day 298, a core of maximum inertial energy was formed between 80 and 120 m. The penetration of inertial oscillations became weaker at depths greater than 140 m. The overall pattern of the response was quite similar to the two moorings. The separation of the two moorings (18.5 km) was smaller than the frequently reported horizontal coherence scale of near-inertial currents, which ranges from tens to hundreds of kilometers (Webster 1968; Pollard and Millard 1970).

The inertial currents were dominantly superinertial in the envelope of high amplitudes associated with the storm. The inertial shift during the October storm was 1%–3% and tended to increase with depth. This suggests that the near-inertial oscillations are propagating internal waves and that the waves found at greater depths have a more northerly origin (Kroll 1975). Inertial currents rotated clockwise with increasing depth, indicating upward phase propagation and downward energy flux (Leaman and Sanford 1975).

b. January storm

By January the mixed layer had cooled and deepened. The mixed layer base before the storm on day 378 was between 100 m and 110 m. The response of the mixed layer inertial currents to the storm on day 378 was nearly instantaneous and vertically uniform with only a few hours delay at the mixed layer base (Fig. 6). Another storm on day 389 enhanced the near-inertial currents in the mixed layer. After that, several less intense storms continued to strike the region and the inertial oscillations in the mixed layer persisted over 35 days. The amplitudes and phases were remarkably uniform throughout the mixed layer during this period.

During the first several days, inertial oscillations were largely confined to the mixed layer. The response at 120 m began gradually on day 385, and even later at greater depths. Correspondingly, the inertial shifts became positive, with values around 1%. The response pattern at the two moorings was quite similar, except that at C1 a local speed minimum was found between 120 and 140 m on days 380–384. This inertial wave hiatus is clearly seen in the raw current time series (Fig. 8). Below the mixed layer, clockwise rotation of inertial currents with depth was seen, suggesting downward energy flux.

c. March storm

In March the mixed layer base deepened to 120 m, which was about its maximum seasonal penetration. The strongest wind speed during the whole experimental period occurred on day 430, though a less intense storm had moved through the array one day earlier. The near-inertial response of the mixed layer resembled that in January, though the amplitudes were significantly smaller (Fig. 7). Inertial response in the mixed layer may have been affected by the combination of wind stress and mixed layer depth (Pollard and Millard 1970), or by the phase of preexisting inertial currents (D’Asaro 1985). Mixed layer depth seemed not to be the factor in this case because the prestorm mixed layer depth was nearly the same in January and March (Fig. 3). Stronger inertial currents existed before the March storm. If the phase of the inertial currents generated by the March storm interfered with that of preexisting inertial currents, the resulting currents in the mixed layer would be decreased. A portion of energy input by the wind stress would be lost to enhanced turbulent mixing. During the first few days after the onset of the storm, there were no near-inertial currents below the mixed layer base at C1. The downward penetration of inertial currents became apparent after day 438. There were large positive inertial shifts associated with this downward penetration, while the inertial shifts were 1%–3% in the envelope of high inertial amplitudes. Negative inertial shifts occurred in the mixed layer between days 438 and 446 at both moorings. Such subinertial frequencies are frequently observed and may be due to the interference of waves of different modes (Kundu and Thomson 1985) or to the location of the moorings on the warm side of a mesoscale eddy that Doppler shifted the inertial wave band (Kunze 1985). The persistence of mixed layer inertial currents was much shorter than in January. As in October and January, the responses at the two moorings were quite similar. Upward phase propagation, suggesting downward energy flux, was seen.
Fig. 5. Complex-demodulated inertial currents (left) and the estimated inertial shifts (right, %) at C1 and NP during the October storm. Inertial speeds greater than 8 cm s\(^{-1}\) and inertial shifts between 0% and 5% are shaded. The inertial current vectors at NP are plotted at 10-m intervals. The cyclone symbol denotes the commencement of the storm on 4 October.
Fig. 6. Complex-demodulated inertial currents (left) and the estimated inertial shifts (right) at C1 and NP during the January storm. Inertial speeds greater than 8 cm s\(^{-1}\) and inertial shifts between 0\% and 5\% are shaded. The inertial current vectors at NP are plotted at 10-m intervals. The cyclonic symbol denotes the commencement of the storm on 13 January.
Fig. 7. Complex-demodulated inertial currents (left) and the estimated inertial shifts (right, %) at C1 and NP during the March storm. Inertial speeds greater than 8 cm s\(^{-1}\) and inertial shifts between 0% and 5% are shaded. The inertial current vectors at NP are plotted at 10-m intervals. The cyclone symbol denotes the commencement of the storm on 4 March.
d. Conceptual picture

A conceptual picture of the generation of inertial waves by a storm is shown in Fig. 8. The inertial wave front that advances into the thermocline crosses each successive depth at a later time. Behind the front is an envelope of near-inertial waves. The response at each depth is made up of wave groups that arrive along almost-horizontal, slanting ray paths (perhaps multiple paths) from distant generation locations at the surface or in the mixed layer.

4. Vertical wave propagation

In this section we use some results from linear internal wave theory to infer certain properties of groups of near-inertial waves. Wave parameters, such as vertical and horizontal wavenumbers and vertical group velocity and energy flux, can be estimated from the Ocean Storms datasets by using these properties.

We take the view that storms generate groups of near-inertial internal waves made up of components of the form

$$\exp\left\{i\left[kx + ly + \int m(z)dz - \omega t\right]\right\},$$  \hspace{1cm} (2)

where \(k, l, m(z),\) and \(\omega\) are wavenumber components and frequency. Vertical wavenumber \(m\) varies in the sense of the WKB approximation as the buoyancy frequency \(N(z)\) varies according to the dispersion relation

$$m^2(z) = K^2 + \frac{N^2(z)}{\omega^2 - f^2},$$  \hspace{1cm} (3)

where \(K^2 = k^2 + l^2\). The WKB approximation is formally valid for vertical wavenumbers large compared to the scale of variation of the medium. This condition will turn out not to be very well satisfied. Even so, we will persist with the estimation. We will return to this point in the discussion. Vertical group velocity is given by

$$C_g = \frac{\partial \omega}{\partial m} = -K \frac{N^2(z)}{\omega m^2(z)}. \hspace{1cm} (4)$$

It is convenient to introduce a stretched vertical coordinate

$$\zeta = \frac{1}{N_r} \int N(z)dz$$ \hspace{1cm} (5)

to eliminate the effect of vertical inhomogeneity (Leaman and Sanford 1975); \(N_r\) is a reference buoyancy frequency (taken to be 3 cph). The advantage of this is that the stretched vertical wavenumber

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**Fig. 8.** Conceptual picture of inertial wave generation by a large-scale storm. The storm strikes simultaneously at the surface over a large area at time \(t_0\). The mixed layer inertial currents respond immediately. An inertial-wave front advances into the thermocline, crossing each current meter at moorings C1, NP at a different time. The envelope behind the front is made up of groups of inertial waves arriving from distant locations along almost-horizontal, slanting ray paths. The \(u\)-component current time series at C1 during the January storm is shown on the right.
\[ \mu = \frac{N_r}{N(z)} m(z) \]  
(6)

is constant, as is the stretched vertical group velocity

\[ C_{g\xi} = \frac{N(z)}{N_r} C_g = -K_s \frac{N_r^2}{\omega \mu} \]  
(7)

Consider the lagged complex inertial-amplitude correlation coefficient

\[ \Gamma_{12}(\tau) = \gamma_{12}(\tau) e^{i \phi_{12}(\tau)} \]

\[ = \frac{D_1^*(t)D_2(t + \tau)}{[D_1^*(t)D_1(t)]^{1/2}[D_2^*(t + \tau)D_2(t + \tau)]^{1/2}} \]  
(8)

based on inertial amplitudes computed from (1) at depths \( z_1, z_2 \). If one assumes that a single wave dominates, the phase of the zero-lagged correlation coefficient \( \phi_{12}(0) \) gives the vertical wavenumber

\[ m = \frac{\phi_{12}}{z_2 - z_1} \]  
(9)

and phase speed \( C_{g\xi} = \omega / m \). By varying the lag \( \tau \) to give maximum \( \gamma_{12}(\tau) \), the time delay of the near-inertial wave group as it propagates vertically can be estimated, and hence its group velocity is

\[ C_{g\xi} = \frac{z_2 - z_1}{\tau_{\max}} \]  
(10)

Given \( C_{g\xi} \), (4) may then be used to estimate the horizontal wavenumber \( K_s \). This formula is better conditioned for \( \omega \approx f \) than (3).

**a. Vertical phase propagation**

We calculated the complex correlation coefficient \( \Gamma_{12}(0) \) for various depth pairs. The time averaging was performed over 20 days, starting at 5 days after the onset of the storm so that the phase difference versus depth was fully developed (Gill 1984). Figure 9 shows the correlations and phases of all depth pairs at mooring C1, as well as their 95% confidence intervals obtained by Monte Carlo simulation based on the bootstrap method (Efron and Gong 1983). For comparison, the corresponding depth pairs at NP are also shown, except that 190 m instead of 195 m was used for January and March because of data gaps. A negative phase means that inertial currents at \( z_2 \) are turned clockwise by that amount relative to \( z_1 \). In almost all cases involving pairs below the mixed layer, \( \phi_{12}(0) \) was negative. Hence, phase almost always propagated upward during the three storm events at both moorings. The only exception occurred at C1 in the 140–160-m pair in October. Pairs within the mixed layer exhibited negligibly small phase differences and correlations close to unity; the mixed layer response was nearly simultaneous. Internal consistency of the vertical turning below the mixed layer, that is, \( \phi_{g\xi} = \phi_{z\xi} + \phi_{g\xi} \), was satisfied only for small separations, indicating that the single wave assumption is locally valid.

In October the clockwise turning from 60 m to 195 m was 107° at C1 and 112° at NP, but the amount of turning from 60 m to 120 m at C1 (83°) was nearly twice that at NP (43°). For pairs above 120 m, the turning at C1 was greater; below 120 m, the relation is reversed. A secondary peak in the buoyancy frequency occurred at 120 m, and this feature of the permanent pycnocline appears to divide the water column and alter the propagation properties of inertial waves. In January most of the vertical turning was greater at C1 than at NP, except for pairs between depths below 140 m. However, the difference between the two moorings was less than 10°. In March inertial currents at C1 consistently showed greater turning. The greatest difference (about 30°) resulted from the largest separation (60–195 m). For the same separation, the vertical turning in January was greater than in March.

In January mixed layer and thermocline currents were highly correlated (0.8), but in March the correlations were less than 0.6. This is perhaps owing to the difference in the amplitude of the vertical group velocity (Pollard 1980). A slowly moving wave group is more likely to be affected by the other variabilities in the upper ocean and exhibit lower correlations.

Least square fits to the local estimates of the stretched vertical wavelength were obtained and are shown in Fig. 10a with their 95% confidence intervals. At NP, only the depths corresponding to those at C1 were used in the estimation. The \( N(z) \)-scaled vertical wavelength was obtained from (6). Corresponding to the maximum and minimum values of \( N(z) \) in each month, the ranges of vertical wavelengths in the thermocline are compared with previous observations (Pollard 1980; Brooks 1983) and with predictions from Price’s (1983) hurricane response model in Table 1. Their results are consistent with the lower and upper ends of our estimates, respectively. Pollard’s estimate suggests higher internal modes, though the Brooks observation and Price model prediction indicate that low-mode structure is significant.

**b. Vertical group velocity**

We assume that the near-inertial wave groups originate from the mixed layer base, so only the depths below the mixed layer were used for January and March. At NP, only the depths corresponding to those at C1 were used. The time averaging at \( z_1 \) in (8) is always performed over the period of a prominent burst of inertial amplitudes in order to trace the maximum inertial energy. Figure 10b shows the least square fits to the local estimates of the stretched vertical group velocity \( C_{g\xi} \). The magnitude of \( C_{g\xi} \) was successively smaller in October, January, and March. The un-
stretched vertical group velocity in the permanent pycnocline showed the same property because the $N(z)$ values remained nearly unchanged with seasons. The $N(z)$-scaled vertical group velocities are listed in Table 1. The group velocity $C_g$ was greater at C1 than at NP in October, but smaller in January and March.

c. Horizontal wavelength

Horizontal wavelength, $\lambda_h = 2\pi/K_h$, is recorded in Table 1. In October near-inertial waves had comparable horizontal wavelength at the two moorings. The shortest horizontal wavelengths were found in January at both moorings. The horizontal wavelengths of near-inertial waves in March differed markedly. Since their 95% confidence intervals correspond to the vertical wavelength estimations, the uncertainty in the estimates was also quite large.

d. Vertical energy flux

The energy input from strong atmospheric forcing to the surface mixed layer is an energy source of the deep-ocean internal wave field. The energy transfer from mixed layer to thermocline is accomplished by the downward propagation of near-inertial internal waves. The estimation of the vertical energy flux is thus important for understanding the energy budget of the
where $S(\omega)$ is the clockwise rotary current spectrum; we use $\delta \omega = 0.08f$ for the bandwidth. The rotary spectrum was computed on a 20-day time series, starting at the group arrival time determined by (8), so that the maximum near-inertial energy was followed. A boxcar window was applied to reduce the smearing effect. Total energy differs from horizontal kinetic energy $E_h$ by only $\nu E_h$ (Fofonoff 1969), where $\nu$ is the inertial shift, a few percent. The vertical energy flux in the thermocline was then estimated as $F = E_h C_{ge}$.

The spline-fitted profiles of energy density are shown in Fig. 11. In the thermocline the inertial currents were most energetic in January because the energy transfer is more efficient with larger horizontal wavenumber. The subsurface energy peak in October appeared at C1, but was smeared at NP due to the 20-day averaging. The energy level was higher at C1 than at NP in October and January, and mostly higher at C1 in March except at the anomalous depths between 140 m and 160 m.

The profiles of the vertical energy flux in the thermocline are also shown in Fig. 11. They vary with depth by a factor of 3–5 depending on month and location, indicating that the WKB scaling assumption was not entirely fulfilled. The vertical energy fluxes were averaged over depths 60–190 m at C1 and 50–190 m at NP in October, 100–190 m in January, and 115–190 m in March, respectively. They are recorded in Table 1 and show temporal and spatial variability. The estimates of vertical energy flux for the October, January, and March storms at C1 are 2.66, 2.79, and 0.51 mW m$^{-2}$, respectively; the values at NP are 1.30, 1.68, and 0.47 mW m$^{-2}$. The typical rate for sources and sinks of internal waves is about 1 mW m$^{-2}$ (Bris-

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**Table 1. Comparison of near-inertial wave parameters estimated from the Ocean Storms experiment with previous observations and predictions from Price's hurricane response model.**

<table>
<thead>
<tr>
<th></th>
<th>Oct C1</th>
<th>Oct NP</th>
<th>Jan C1</th>
<th>Jan NP</th>
<th>Mar C1</th>
<th>Mar NP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelengths (Vertical (m))</td>
<td>230–1300</td>
<td>240–1400</td>
<td>150–780</td>
<td>150–810</td>
<td>190–1100</td>
<td>260–1500</td>
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<td></td>
<td>250</td>
<td>290</td>
<td>140</td>
<td>140</td>
<td>260</td>
<td>410</td>
</tr>
<tr>
<td>Vertical group velocity (m day$^{-1}$)</td>
<td>7–39</td>
<td>6–34</td>
<td>5–25</td>
<td>5–27</td>
<td>4–20</td>
<td>4–22</td>
</tr>
<tr>
<td>Vertical energy flux (mW m$^{-2}$)</td>
<td>2.66</td>
<td>1.30</td>
<td>2.79</td>
<td>1.68</td>
<td>0.51</td>
<td>0.47</td>
</tr>
<tr>
<td>Decay timescale (days)</td>
<td>16</td>
<td>19</td>
<td>23</td>
<td>29</td>
<td>54</td>
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<td>$\zeta$ (m)</td>
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<td>60</td>
<td>105</td>
<td>105</td>
<td>115</td>
<td>115</td>
</tr>
</tbody>
</table>

* Moorings, N, C, and S in the Gulf of Mexico.
The annual average energy flux into the mixed layer by midlatitude storms has been calculated as 1.44 mW m⁻² (D’Asaro 1985). If one assumes that dissipation takes place within the depth of the pycnocline (about 200 m), these vertical energy fluxes imply dissipation rates of $2 \times 10^{-6}$ to $1 \times 10^{-5}$ W m⁻³. Such values compare reasonably with other observations during Ocean Storms (Crawford and Gargett 1988) and to previous upper-ocean observations (Osborn 1980).

If the loss of inertial energy from the mixed layer is balanced by energy gain in the thermocline, a decay timescale $\tau_d$ can be estimated from the ratio of the total energy above a certain depth $z_d$, chosen as at 60 m in October, 105 m in January, and 115 m in March, to the energy flux at $z_d$:

$$\tau_d = \int_{z_d}^{zd} \frac{E_n(z)dz}{F(z_d)}.$$  \hspace{1cm} (12)

We assumed uniform energy level in the mixed layer and used whatever current meters were available in the mixed layer. In the October storm, when the shallowest current meter was below the mixed layer, the energy density was extrapolated up to the mixed layer base. Horizontal energy flux divergence is neglected in this account of near-inertial energetics. The decay time-scales are listed in Table 1. They were comparable to the observed durations of inertial oscillations in the October and January storms. However, they were much longer than the inertial-oscillation duration in the March storm. This suggests that wave dispersion alone
was insufficient to deplete the upper-layer near-inertial energy and that some small-scale dissipative mechanism such as mixing played an important role in reducing the energy.

5. Horizontal wave propagation

In this section we show how the horizontal directionality of internal waves can be obtained from the phase relation between density and rotary current at a single location. Conventionally, if a horizontal array of current meters is available, directionality of internal waves can be determined from horizontal phase relations between currents in the array (Schott and Willebrand 1973; Müller et al. 1978). By combining horizontal direction with the horizontal phase relation between near-inertial waves at two sites, an independent estimate of horizontal wavelength can be calculated and compared to the estimates obtained from the method in section 4.

Neglecting the effects of mean shear, friction, and horizontal inhomogeneity, propagating internal waves are governed by the following linearized equations of motion:

\[
\left( \frac{\partial}{\partial t} + if \right)(u + iv) = - \left( \frac{\partial}{\partial x} + i \frac{\partial}{\partial y} \right) p \quad (13)
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (14)
\]

\[
0 = - \frac{\partial p}{\partial z} - \frac{\rho}{\bar{\rho}} g \quad (15)
\]

\[
\frac{\partial \bar{\rho}}{\partial t} - \bar{\rho} N^2 w = 0. \quad (16)
\]

In these equations \( p \) is pressure perturbation divided by the average density \( \bar{\rho} \), \( \rho \) the density perturbation, \( g \) the acceleration due to gravity; \( u, v, \) and \( w \) are velocity components corresponding to Cartesian coordinates \( x, y, \) and \( z \) measuring distance eastward, northward and upward respectively and \( t \) is time. The hydrostatic and Boussinesq approximations have been made. By assuming plane-wave solutions of the form

\[
\begin{pmatrix} u + iv \\ \rho \\ p \end{pmatrix} = \begin{pmatrix} U_0 \\ \rho_0 \\ p_0 \end{pmatrix} e^{i(kx + ly + mz - \omega t)}, \quad (17)
\]

the relation between density and rotary current amplitudes is

\[
U_0 = \frac{g \rho_0}{(\omega - f) m \bar{\rho}} K_h e^{i(\theta + \pi/2)}, \quad (18)
\]

where \( K_h \) is the magnitude of horizontal wavenumber vector, and \( \theta = \arctan(1/k) \) is the direction of horizontal phase propagation. Hence, the phase difference between rotary current and density gives \( \theta \), by means of

\[
\phi_U - \phi_p = \theta + \frac{\pi}{2}. \quad (19)
\]

The complex correlation between potential density anomaly and rotary current at the inertial frequency is

\[
\Gamma = \frac{\sigma_\theta(t) D(t)}{[\sigma_\theta(t) \sigma_D(t)]^{1/2} [D^*(t) D(t)]^{1/2}}, \quad (20)
\]

where \( \sigma_\theta(t) \) is the complex-demodulated potential density anomaly, similar to \( D(t) \) [Eq. (1)]. The magnitude of \( \Gamma (\leq 1) \) measures the overall correlation. If the near-inertial band were a sum of waves of the form (17) over all horizontal directions, the expected value of \( \Gamma \) would be zero. If the near-inertial band were dominated by a wave from one direction, the magnitude of \( \Gamma \) should be large, \( O(1) \), and its phase would give an estimate of the left side of (19), and hence the wave direction.

a. Mooring motion correction

The above reasoning presumes that observations of current and density were made at fixed depths. Because of mooring motion, current−density time series from instruments fixed on the mooring line were made at a variable depth. By considering the correlation between pressure and density at fixed instruments on the mooring it is possible to correct the preceding analysis for the effect of mooring motion. The PCM time series, because they are constructed by binning and averaging the data as the instrument passes through a pressure range, do not need to be so corrected.

Assume that the near-inertial internal wave field currents and potential density anomaly consist of a clockwise rotating component at frequency \( \omega \) and vertical wavenumber \( m \):

\[
(u + iv)(z, t) = U_0 e^{i(mz - \omega t)} + n_U \quad (21)
\]

\[
\sigma_\theta(z, t) = \sigma_{\theta_0} e^{i(mz - \omega t)} + \sigma_\theta(t) + n_{\sigma_\theta}, \quad (22)
\]

where \( U_0 = |U_0| e^{i\phi_U}, \sigma_{\theta_0} = |\sigma_{\theta_0}| e^{i\phi_{\theta_0}}; \sigma_\theta(t) \) is the mean potential density anomaly profile. The dependence on horizontal coordinates has been suppressed. The contribution by the mean current shear is negligibly small. On an oscillating mooring, current and density are observed at oscillating depths

\[
z(t) = \bar{z} + z_0 e^{i(mz - \omega t + \phi_z)} + n_z \quad (23)
\]

(the term \( m\bar{z} \) has been added to the phase \( \phi_z \) for convenience). The terms \( n_U, n_{\sigma_\theta}, n_z \) are uncorrelated noises in the observations. Hence, the time series observed at an instrument on the mooring is approximately
\[
(\hat{u} + i\hat{v})(t) = |U_0| e^{i(m\epsilon - \omega t + \phi_U)} \left[ 1 + O(mz_0) \right] + n_U,
\]
\[
\sigma_\theta(t) = |\sigma_{\theta_0}| e^{i(m\epsilon - \omega t + \phi_{\theta_0})} \left[ 1 + O(mz_0) \right] 
+ \frac{\partial \sigma_\theta}{\partial z} z_0 e^{i(m\epsilon - \omega t + \phi_{\theta_0})} + n_{\sigma_\theta},
\]
where \( n'_{\sigma_\theta} = n_{\sigma_\theta} + n_{\sigma_\theta} \partial \sigma_\theta / \partial z \). The pressure data at 70 and 89 m on mooring C1 indicate that the rms and maximum values of mooring oscillation are about 1 and 10 m, respectively. Since the characteristic vertical scale of near-inertial waves is at least \( O(100 \text{ m}) \), the \( O(mz_0) \) terms are \( \sim 0.01 \) to 0.1 and can be neglected.

The complex correlation between the potential density anomaly and rotary current is
\[
\Gamma = \frac{\sigma_\theta^* (\hat{u} + i\hat{v})}{|\sigma_\theta^* (\hat{u} + i\hat{v})|^{1/2} |\hat{u} + i\hat{v}|^{1/2}}
\]
\[
e^{i(\phi_U - \phi_{\sigma_\theta})} \left[ 1 + \frac{\partial \sigma_\theta}{\partial z} \gamma e^{i(\phi_{\sigma_\theta} - \phi_U)} \right]
\]
\[
= \left[ 1 + 2 \frac{\partial \sigma_\theta}{\partial z} \gamma \cos(\phi_{\sigma_\theta} - \phi_U) + \left( \frac{\partial \sigma_\theta}{\partial z} \right)^2 \gamma^2 \right]^{1/2}
\]
\[
\times \left[ 1 + O\left( \frac{n_{\sigma_\theta}^2}{|U_0|^2} \right) + O\left( \frac{n_{\sigma_\theta}^2}{|\sigma_{\sigma_\theta}|^2} \right) \right]^{-1}.
\]
\[
= |\Gamma| e^{i(\phi_U - \phi_{\sigma_\theta})_{\text{obs}}},
\]
where \( \gamma \) is the coherence between potential density anomaly and mooring oscillation, and \( S_{\sigma_\theta} \) and \( S_{\sigma_\theta U} \) are the autospectra of mooring oscillation and potential density anomaly, respectively. If the mooring oscillation and density are in phase, that is, \( \phi_U = \phi_{\sigma_\theta} \), no modification to the observed phase difference between rotary current and density is needed. Likewise, if there is no coherence between density and mooring oscillation, \( \gamma = 0 \), no modification is required. However, if the mooring oscillation and density are coherent and not in phase, the true phase difference between rotary current and potential density anomaly is obtained from the apparent phase difference \( (\phi_U - \phi_{\sigma_\theta})_{\text{obs}} \), which is estimated from (20), by means of
\[
(\phi_U - \phi_{\sigma_\theta}) = \arctan \left[ \frac{\tan(\phi_U - \phi_{\sigma_\theta})_{\text{obs}} - \xi}{\xi \tan(\phi_U - \phi_{\sigma_\theta})_{\text{obs}} + 1} \right],
\]
where \( \xi \) is given by
\[
\xi = \frac{\partial \sigma_\theta}{\partial z} \gamma \sin(\phi_{\sigma_\theta} - \phi_U) + \frac{1 + \partial \sigma_\theta}{\partial z} \gamma \cos(\phi_{\sigma_\theta} - \phi_U).
\]

b. Inference of density time series at VMCM depths at C1

Since there were no salinity measurements at the seven VMCM depths at mooring C1, we infer the density time series at these depths from the Seacat data and from the \( \sigma_\theta - T \) relation. The time series of the potential density anomaly calculated from the Seacat data were linearly interpolated every 15 minutes to make the sampling rate consistent with the current measurements. The permanent halocline lay in the depths 90–150 m. Within the halocline, the time series of \( \sigma_\theta \) at 100, 120, and 140 m were obtained by using a second-degree polynomial to vertically interpolate and extrapolate \( \sigma_\theta \) from depths 70, 89, 109, and 128 m. The use of a second-degree polynomial is justified by experimenting with the PCM data for the smallest phase difference between the measured and the inferred \( \sigma_\theta \). Outside the halocline, the time series of \( \sigma_\theta \) at 60, 80, 160, and 195 m were derived from the temperature data measured by VMCMs by using the \( \sigma_\theta - T \) relation fitted by fifth-degree polynomials during the 24-day period at the same depths at NP.

c. Horizontal direction of propagation

We computed the inertial correlation and phase over record lengths of 20 days encompassing the response from each storm. At mooring C1, the directions of propagation were estimated with mooring motion correction. The differences of the direction of propagation estimated with and without mooring motion correction are in no case larger than 26° and mostly less than 10°. This is because the magnitude of dimensionless parameter \( \gamma \partial \sigma_\theta / \partial z \), which figures in the correction, is never larger than 0.46, and usually nearer 0.1 or smaller. Figure 12 shows the correlations between potential density anomaly and velocity, and the estimated directions of propagation, during the periods of storm, as well as their 95% confidence intervals obtained by Monte Carlo simulation using the bootstrap method.

At NP, the estimates are shown at 10-m depth intervals starting 10 m below the mixed layer. Certain qualitative information on the sources of near-inertial waves may be obtained from the direction of propagation estimated at one location. During all three events near-inertial waves mainly propagated in directions between northeast and south, indicating sources west of the moorings. Waves propagating toward the northeast will eventually be reflected at their turning latitudes where the north–south component wavenumber \( l \) becomes zero. Waves propagating toward the southeast, however, may come directly from sources in the northwest, or they may have been reflected at their turning latitudes.

The horizontal direction of propagation tends to rotate clockwise with increasing depth. This phenomenon is believed to be due to the \( \beta \) effect proposed by
D’Asaro (1989). The latitudinal variation of $f$ produces a temporal drift of north–south wavenumber, $d\|/dt = -\beta$. North–south wavenumber therefore decreases with time, while east–west wavenumber $k$ remains constant, causing a southerly rotation with the age of the wave.

Inconsistency with the beta dispersion occurred at about 80–100 m in October and below 150 m at NP in January and March. In October, modeling efforts (D’Asaro 1995c) showed that eddy activity played an important role in drawing mixed layer energy rapidly downward to form the energy maximum at about 100 m. The phase structure of near-inertial waves might be modulated by the eddy activity over the 20-day period. In January and March at NP, the correlations at depths below 150 m were low and the uncertainty in the estimates was relatively large.

d. Horizontal wavelength

Combining the directional information about the near-inertial waves with the horizontal phase differences observed between the two moorings furnishes an independent estimate of horizontal wavelength. The horizontal phase difference between the two moorings is given by the direct application of (8) to the complex-demodulated inertial currents at the same depth at C1 and NP (represented by subscripts 1 and 2, respectively). Backrotation of the phase of the demodulated inertial currents is not necessary in order to compare
the phase relation at the same frequency. The phases and correlations at corresponding depths are shown in Fig. 13. The inertial currents were highly correlated because the mooring separation is well within the estimated horizontal wavelengths. The inertial currents at NP led in phase at all depths during all three storm events. At least three moorings are needed to estimate horizontal direction and wavelength from current–current phase differences. Failing this, a horizontal wavelength may be estimated from the horizontal phase difference

$$\phi_2 - \phi_1 = K_0(\Delta x \cos \theta + \Delta y \sin \theta),$$

(30)

where $(\Delta x, \Delta y) = (-6.9$ km, 17.3 km) is the separation vector between the two moorings, and $\bar{\theta} = (\theta_1 + \theta_2)/2$ is the mean direction of propagation. The estimation was carried out only at 60, 80, 100, and 195 m in October because the near-inertial waves at these depths propagated nearly in the same direction at the two moorings. The estimated horizontal wavelengths are shown in Table 2. They are comparable in order of magnitude to the estimates in Table 1.

6. Summary and discussion

Current meter data at two sites in the Ocean Storms Experiment in the northeast Pacific were analyzed for near-inertial motions generated by storms. The data were taken during a ten-month period from August 1987 to June 1988. We concentrated on three major energetic storms, which occurred in October, January, and March, for which we studied the propagation of near-inertial internal waves into the thermocline.

The most striking feature of the inertial-wave response to the storms was the almost instantaneous generation of waves in the mixed layer, followed by a gradual propagation into the thermocline that lasted many days after the initiation of the storm. Our conceptual picture of what happened is summarized in Fig. 8. A storm strikes simultaneously over a wide area sending off groups of near-inertial waves along rays in many directions. These rays propagate at a very slight angle below the horizontal. In Fig. 8 we show just those rays that reach a given depth on a mooring, such as C1, at a given time. Examination of the relations between the near-inertial currents at various depths revealed the following properties of the near-inertial wave groups that traveled along these rays.

1) Wave frequencies were slightly superinertial, with inertial shifts 1%–3% in October and March and around 1% in January.

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>$\theta_1$ (deg)</th>
<th>$\theta_2$ (deg)</th>
<th>$\phi_1 - \phi_2$ (deg)</th>
<th>$\lambda$ (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>6</td>
<td>8</td>
<td>-22</td>
<td>80</td>
</tr>
<tr>
<td>80</td>
<td>-9</td>
<td>-21</td>
<td>-26</td>
<td>150</td>
</tr>
<tr>
<td>100</td>
<td>14</td>
<td>1</td>
<td>-16</td>
<td>100</td>
</tr>
<tr>
<td>195</td>
<td>-60</td>
<td>-45</td>
<td>-35</td>
<td>180</td>
</tr>
</tbody>
</table>

Table 2. Horizontal wavelengths estimated from the horizontal phase differences between near-inertial currents at the same depth at two moorings during the October storm.

Fig. 13. Horizontal correlations and phases between inertial currents at the same depth at moorings C1 and NP. A negative phase means the inertial currents at NP lead in phase. The 95% confidence limit is obtained from the bootstrap method.
2) Inertial responses in the mixed layer were nearly in phase. The phase of near-inertial currents propagated upward below the mixed layer, confirming the downward radiation of energy by these waves. The average downward energy flux during the storm periods was between 0.5 and 2.8 mW m$^{-2}$.

3) The phase propagation speed and wavelength in the vertical direction were estimated. For the latter we obtained values in the range 150–1500 m.

4) The vertical group velocity was estimated from the arrival times of the groups at successive depths. From the dispersion relation, horizontal wavenumber can then be obtained. We observed inertial wavelengths in the range 140–410 km.

5) A near-inertial response in density, due to isopycnal displacement, was observed. The phase difference between near-inertial current and density indicates the direction of propagation: We found the dominant directions of propagation were between northeast and south, indicating sources west of the moorings. The dominant directions tended to rotate clockwise with increasing depth.

6) The horizontal wavelength from 4) and the propagation direction from 5) were consistent with the phase difference between inertial currents at the two moorings C1, NP.

The estimation of these wave parameters is based on linear internal wave propagation theory and the use of the WKB approximation and overlooks dissipative processes. However, mixing may affect the generation and propagation of near-inertial waves, especially in the upper ocean. D'Asaro's calculations (1995b,c) indicate that linear wave dynamics are insufficient to account for the evolution of near-inertial motions observed during Ocean Storms. The WKB scaling assumptions may not be fully satisfied in the upper ocean. Generally, when the vertical variation of the buoyancy frequency is rapid, wave reflection, which the WKB approximation does not countenance, may occur. The validity of the WKB approximation where wave scales are comparable to or larger than the scales of inhomogeneity is questionable. Kunze (1985) encountered a similar situation. Still, he showed that the errors due to the application of the WKB approximation beyond its formal limits of validity were small.

The spectral densities of the internal wave band are remarkably uniform in space and time, although the likely sources are very nonuniform. This suggests that low-frequency internal waves propagate horizontally to homogenize the internal-wave energy distribution. D'Asaro (1991) pointed out that only low-mode, low-frequency internal waves can propagate substantial distances (>1000 km) in the ocean. Near-inertial internal waves are notably anisotropic in the upper ocean and are a strong candidate for accomplishing this redistribution.

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