A Similarity Analysis of the Structure of Airflow over Surface Waves

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ABSTRACT

Previous field investigations of the wave-induced pressure field have focused on determination of the momentum input from wind to the surface waves. This is useful for the estimation of wave growth rate and, in particular, the wave growth parameter $\beta$. Due to the difficult nature of experimental study of airflow very close to the wave surface, it has been necessary to extrapolate elevated measurements of the wave-induced pressure field to the surface. This practice may be incorrect without adequate knowledge of the complex vertical structure of the pressure field. In addition, the wave-induced pressure and velocity fields are coupled to the near-surface turbulence. Hence, understanding the nature of the wave-induced flow fields is critical for modeling of the near-surface wind and wave fields.

Utilizing a simple similarity hypothesis, detailed vertical structure of the wave-induced pressure and velocity components is examined. Results of this analysis are presented using data obtained in the spring and fall of 1994 during the Risø Air–Sea Experiment program. These results demonstrate that, when compared to theory, simple extrapolation of measurements of the wave-induced pressure field from a fixed height above the surface may contribute to the uncertainty of measured growth rates. In addition, it is demonstrated that an analogous similarity relationship for the wave-induced vertical velocity field yields results that are consistent with previous laboratory studies.

1. Introduction

The air–sea interface is a complex system of interacting waves and atmospheric turbulence over a wide variety of spatial and temporal scales. The wave-induced pressure and velocity fields, as well as their interaction with the marine atmospheric surface-layer turbulence, are integral components of the air–sea coupled system. Despite the fact that the wave-induced pressure at the surface constitutes a major portion of the total energy flux from wind to waves, the nature of the structure of the near-surface airflow over sea waves is largely unknown.

When the sheared wind flows over the sea surface, the downward transport of horizontal momentum produces a wide spectrum of capillary and gravity waves, surface currents, and oceanic boundary-layer turbulence. Thus, the structure of the underlying roughness elements is fundamentally determined by the wind forcing. This dynamic coupling of the airflow and surface is easily seen to be a direct contrast to the flows within land surface layers where the surface roughness and drag coefficient are normally considered constant at a particular location for a given stability. The wind–wave feedback makes a purely wind-based parameterization of the total momentum transport mechanism subject to considerable error.

Very little is known about the structure of the wave-induced flow or how this flow affects the structure of...
the surface-layer turbulence. In fact, a critical uncertainty in wave growth theory is the nature of the modulation of the turbulence structure due to the presence of the waves (Miles 1993). The theoretical treatment of the structure of the wave-induced flow fields was substantially advanced by Miles (1957). Subsequent theoretical and numerical investigations include those of Townsend (1972), Gent and Taylor (1976), Al-Zanaidi and Hui (1984), Jacobs (1987), Chalikov and Belevich (1993), and Belcher and Hunt (1993). These models differ in the manner of turbulence closure, and the results demonstrate sensitivity to the closure scheme (Al-Zanaidi and Hui 1984; Belcher and Hunt 1993). The landmark experimental results of Snyder et al. (1981) are further corroborated by the work of Hasselmann and Boessenberg (1991).

Laboratory investigations, such as Hsu and Hsu (1983) and Papadimitrakis et al. (1986), have yielded information on the structure of the wave-induced velocity and pressure fields, respectively, over mechanically generated waves. These results provide some insight into the nature of flow over waves and the role of this flow in the momentum exchange processes at the interface. Some of the laboratory studies will be used for comparison with the results of the present analysis.

Parameterizations of the neutral stability drag coefficient based on observations vary significantly depending on water depth, fetch, wind and wave direction, and sea state (Geernaert 1990). Because atmospheric and oceanic circulation models and wave development models rely on accurate representations of this momentum flux for predictive confidence, an improved understanding of the structure of the wave-induced flow and the momentum transfer processes over the sea will lead to the reduction in the uncertainty of the numerical model forecasts (Gent and Taylor 1976). Information about these fundamental physical processes at the air-sea interface may also be essential for the interpretation of the backscatter signals obtained with microwave sea-surface radars (Geernaert et al. 1988) and for increased understanding of the gas and heat flux processes at the interface (Papadimitrakis et al. 1984).

The objective of the current study is to provide details about the wavenumber-dependent structure of the wave-induced pressure field. A similar approach is developed to provide a description of the detailed structure of the wave-induced velocity field. The mathematical development of the similarity hypothesis toward that end is presented in section 3.

2. Wave-induced flow

Over the sea, the total wind stress $\tau_w$ is imparted to the ocean surface and manifests itself in the forms of surface currents, which lead to shear instabilities and turbulence in the oceanic boundary layer, and irrotational surface waves. We can describe the individual components of this momentum flux by simple manipulation of the Navier-Stokes equation. Within the atmospheric surface layer, the Navier-Stokes equation can be simplified by scaling out the Coriolis term. Using the usual summation notation,

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = \delta \delta g - \frac{1}{\rho_s} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_i^2},$$

(1)

where $u$ is the velocity component, $t$ is time, $x$ is the spatial distance, $\delta$ is the delta operator, $g$ is the gravitational acceleration, $\rho_s$ is air density, $p$ is pressure, and $\nu$ is kinematic viscosity. Over waves, we can decompose the velocity and pressure into three constituents: the mean, turbulent, and wave-induced components of the flow:

$$u_i = \bar{u}_i + u'_i + \hat{u}_i,$$

$$p = \bar{p} + p' + \hat{p},$$

(2)

where the overline signifies an ensemble average, the prime indicates the departure from the mean, and the tilde denotes the wave-correlated contribution. Using Reynolds averaging rules described in Finnegan et al. (1984), the continuity equation, and (2), we can put (1) into flux form,

$$\frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_i}{\partial x_j} \bar{u}_j + \frac{\partial \bar{u}_i}{\partial x_j} \hat{u}_j = \delta \delta g - \frac{1}{\rho_s} \frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_i^2},$$

(3)

Under the assumptions of stationarity and horizontal homogeneity, it is elementary to obtain the result

$$\frac{\partial}{\partial z} (u' w' + \bar{u}' w' + \nu \frac{\partial \bar{u}}{\partial z}) = 0.$$  

(4)

The three terms on the left-hand side of (4) represent respectively the gradients of turbulent momentum flux ($\tau_v$), wave-induced momentum flux ($\tau_w$), and viscous stress ($\tau_v$). In vector form, we can express the total momentum flux as

$$\tau = \tau_v(z) + \tau_w(z) + \tau_v(z),$$

(5)

where $z$ is height above the mean surface.

Well above the wave surface the wind stress is due primarily to atmospheric turbulence. On the other hand, at the surface the momentum transfer is composed of contributions from viscous stress and the form drag on waves. Belcher and Hunt (1993) present a thorough review of the momentum transfer mechanisms over wave surfaces.

It is unclear what proportion each of these terms contributes to the total momentum flux close to the waves (Chalikov and Belevich 1993). Note that the terms on the right side of (5) are generally height-dependent. However, under certain conditions (i.e., when Coriolis effects are negligible and the wind and wave fields are stationary in time and space), it may be assumed that the total wind stress is constant with height.
\[
\frac{\partial \tau_i}{\partial \xi_i} \sim 0. \tag{6}
\]

The form drag is described by the pressure/wave-slope correlation at the surface:
\[
\tau_i = p_i \frac{\partial \eta}{\partial \xi_i}, \quad i = 1, 2, \tag{7}
\]

where the overbar signifies the ensemble average, \(\eta\) is the instantaneous surface elevation perturbation (thus, the spatial derivative defines the wave slope), and \(p_i\) is the atmospheric surface pressure. This wave-induced stress includes contributions from all wave scales, from capillary waves to long gravity waves. From this point, the focus will be on that portion of the form drag induced by the dominant gravity waves. It is also assumed that all the wave motions are linear.

The expression in (7) suggests that the form drag can theoretically be quantified by measurement of the surface wave-induced pressure perturbation, \(p_i\), in phase with the slope of the wave (Snyder et al. 1981). Because of the difficulties in measuring the pressure adjacent to sea waves, essentially no direct field measurements of this wave-induced momentum flux exist. Furthermore, because observations of the vertical structure of the wave-induced pressure field have not been reported up to now, the extrapolation to the surface of elevated pressure measurements in order to estimate the form drag momentum flux is a questionable practice.

In vector wavenumber \(k\) space (7) becomes
\[
\tau_i(k) = k \text{ Im}[C_{p_i}(k)], \tag{8}
\]

where \(C_{p_i}\) is the surface pressure/wave height complex cross-spectral density, and the Im operator indicates the imaginary component of the quantity in brackets. The expressions for the wave-induced momentum flux in (7) and (8) are only valid at the wave surface. The imaginary component of \(C_{p_i}\) in (8) is representative of the momentum transport across the interface.

The wave growth parameter \(\beta\) can be described in terms of the energy input to the waves (Kinsman 1965):
\[
\beta = \frac{E}{E_*}, \tag{9}
\]

where \(E\) is the wave energy per unit surface area, and the overdot indicates the time derivative. Consistent with the expressions in (1)–(8), we will describe the wave growth parameter and the subsequently developed similarity relationship (section 3) in terms of momentum rather than energy flux. As wave momentum is defined as wave energy divided by phase speed, the magnitude of the momentum flux input to gravity waves due to form drag can be parameterized in terms of \(\beta\):
\[
\tau_i(k) = \beta(k) \frac{E(k)}{c(k)}, \tag{10}
\]

where \(c\) is the wave phase speed, and the vector notation for the momentum flux has been dropped. Here we omit the contribution of shear stress to the wave growth defined in Belcher and Hunt (1993). The energy of gravity waves can be described in terms of the wave spectrum \(S_q\) as (Kinsman 1965)
\[
E(k) = \frac{1}{2} \rho_w g S_q(k), \tag{11}
\]

where \(\rho_w\) is the water density. With the definition in (11) and equating (8) with (10), the wavenumber notation is dropped to obtain a one-dimensional form
\[
\tau_i = k \text{ Im}[C_{p_i}] = \frac{\beta \rho_w g S_q}{2c}, \tag{12}
\]

where \(k\) is the wavenumber magnitude.

Using the dispersion relationship for deep water gravity waves,
\[
g = kc^2, \tag{13}
\]

and rearranging, one can obtain an expression for the wave growth rate due to the wind input \(\beta\) in terms of measurable quantities
\[
\beta = \frac{2\text{ Im}[C_{p_i}]}{\rho_w S_q c}. \tag{14}
\]

This expression is valid at the wave surface and theoretically can be used to determine the wave growth rate. However, measurements of atmospheric pressure at the sea surface are technically difficult to acquire.

The alternative approach, which has been utilized by Snyder et al. (1981) and Hasselmann and Bösenberg (1991), is to extrapolate elevated measurements of the quantity in brackets down to the mean sea surface. This strategy requires a priori knowledge of the vertical dependence of the pressure disturbance. Because the vertical structure of the pressure–wave quadrature spectrum is unknown, these investigators have assumed a simple exponential decay with height and empirically determined a decay-rate constant in order to obtain estimates of \(\beta\). Using the notation of this paper, this can be expressed as
\[
C_{p_i}(z) \sim \exp(-akz). \tag{15}
\]

Jacobs (1987) subsequently developed a theoretical basis for this relation with \(a = 1\). However, there is a discrepancy regarding the value of the exponential constant \(a\) (Snyder et al. 1981; Jacobs 1987). Moreover, it should be clear that observational evidence of the vertical structure described in (15) has not been demonstrated to date. This issue will be discussed again after examination of the results of the present analysis in section 6.

Plant (1982) compiled measured values of the wave growth parameter from field measurements and laboratory experimental results. The primary result of Plant’s investigation yields an empirical wind–wave relationship of \(\beta\) with the wave age, \(u_w/c\), as
\[ \beta = (0.04 \pm 0.02) \left( \frac{u^*}{c} \right)^2 \omega \cos \theta, \] (16)

where \( \theta \) is the relative angle between the wind and wave directions, \( \omega \) is the frequency, and \( u^* \) is the friction velocity. This relationship demonstrates the importance of wave age in the characteristics of air–sea interaction. However, it is evident from this expression that a relatively high degree of uncertainty exists in the parameterization of \( \beta \). Plant’s compilation of \( \beta \) values also excludes the possibility of negative wave growth rates, that is, momentum flux from waves to the local atmospheric flow. This is certainly expected to be the case for waves traveling contrary to the mean wind flow (Donelan and Hui 1990).

The wave growth parameter is an important aspect of the wind–wave interaction, and (14) will serve as a guide for the development in section 3 of a unique similarity hypothesis to describe the wave-induced pressure field.

3. Development of the similarity hypothesis

Similarity theory has been utilized to remarkable advantage in research on the structure of atmospheric turbulence (Businger et al. 1971). The primary advantage of similarity theory is that it allows the expression of physical relationships in terms of dimensionless variables, enabling averaging over a variety of stationary environmental conditions. Valid similarity hypotheses show universal self-similar behavior of the dimensionless quantities. It is therefore desirable to utilize similarity theory for the current discussion because the wave-induced flow is superimposed within the ubiquitous surface-layer turbulence.

From (16), the single most important parameter to describe wind–wave interaction is the dimensionless forcing, or wave age \( u^*/c \). Subsequent numerical studies, such as Al-Zanaidi and Hui (1984) and Jacobs (1987) have confirmed that the rate of wave growth is predominantly determined by this forcing parameter. Therefore, it is justified to describe the wave-induced pressure and velocity fields statistically with a relationship that depends on the forcing parameter \( u^*/c \). Furthermore, it is convenient to develop a statistical relation using field-measurable variables to permit the analysis of data obtained within the airflow over sea waves. As in Chalikov and Belevich (1993), the authors define the portion of the surface layer that is significantly affected by the presence of the underlying waves as the wave boundary layer (WBL).

a. Similarity relationship for the wave-induced pressure field

Because the wave-coherent pressure perturbations near the surface are superimposed with the turbulent pressure field, separation of the measured pressure signal into wave and turbulent components is challenging. A similarity relationship is effective in this regard, since it permits averaging over a variety of sea states and wind conditions.

Similar to the approaches of Kitaigorodskii (1973), Benilov et al. (1974), and Phillips (1977), it is initially assumed that the correlation of the wave-induced motion is linearly related to the surface displacements \( \eta(x, t) \). Thus, the wavenumber (and therefore length scale) of the wave-induced atmospheric disturbance is that of the forcing sea wave. As the equations of motion include nonlinear terms, the linearity assumption is only valid for small perturbations of the flow. Therefore, it is inappropriate for application in high sea states, particularly when breaking waves are present. Furthermore, this linearity assumption has been successfully implemented in the work of Hsu et al. (1981), Gent and Taylor (1976), Belcher et al. (1993), and Belcher and Hunt (1993).

Linear similarity functions can be developed between \( \eta \) and the wave-induced pressure, \( \tilde{p} \), and velocity fields, \( \tilde{u} \). Under the linearity assumption, the magnitude of the wave-induced perturbations in the airflow are proportional to the magnitude of the surface wave displacements. Thus, the wave-induced flow is determined by the wave state (which is described with the one-dimensional wave spectrum \( S_\eta \)), the nondimensional atmospheric forcing, \( u^*/c \), the angle between wind and the dominant waves, \( \theta \), and the normalized height, \( k_z \). In the discussion that follows, it is assumed that the wind and waves are aligned, that is, \( \theta = 0 \).

The typical key length scale used to describe the dynamics of wind–wave interaction is the wavelength of the surface waves, \( \lambda \), and the fundamental velocity scale of the surface waves is the wavelength-dependent phase velocity, \( c \). In (14), the wave growth parameter was described in terms of the pressure/wave height quadrature spectrum and wave spectrum. Although this expression is only defined at the surface, one can design a similarity function that is more generally valid at any dimensionless height, \( k_z \).

Thus, in order to examine the structure of the wave-correlated pressure, the pressure/wave height cross-spectrum \( C_{un} \) can be normalized by the wave spectrum, and this ratio can then be nondimensionalized through the use of appropriate scaling variables (\( \lambda \) and \( c \)) to obtain a pressure/wave height correlation similarity function:

\[ N_{un}(u^*/c, k_z) = \frac{C_{un}}{S_{un} \rho \bar{c}^2}. \] (17)

The tilde over the subscript variable has been omitted for notational simplicity. This expression permits the statistical description of the structure of the wave-induced pressure field over sea waves as a function of the wind forcing, \( u^*/c \) and height \( k_z \).
b. Similarity relationship for the wave-induced velocity field

A similarity hypothesis for the wave-induced velocity fields can be analogously developed. For the wave-induced streamwise wind, \( \bar{u} \)

\[
N_{\infty} \left( \frac{u_{\infty}}{c}, k_z \right) = \frac{C_{w \lambda}}{S_n c} \tag{18}
\]

The wave-coherent vertical velocity \( \bar{w} \), similarity relationship is

\[
N_{\infty} \left( \frac{u_{\infty}}{c}, k_z \right) = \frac{C_{w \lambda}}{S_n c} \tag{19}
\]

Thus, the velocity components induced by the presence of the waves can be examined as a function of wave age \( \tau \), and dimensionless height \( k_z \) in a similar manner as the wave-induced pressure field.

As a result of Miles' (1957) theoretical development, previous investigators have used the mean wind defined at a height corresponding to the wavelength \( U_{\infty} \), where \( \lambda = 2\pi k \), as a scaling variable (Snyder et al. 1981; Al-Zanaidi and Hui 1984; Jacobs 1987). However, the mean wind profile close to the wave surface is difficult to quantify. Another reasonable approach to the scaling of the similarity relationships in (17)–(19) is to use \( u_{\infty} \) as the velocity scale. We have investigated this possibility and have concluded that the results are quite similar to those presented in section 6. Our scaling approach also has the advantage of permitting the velocity scale to vary with wavelength. Therefore, for this analysis, the authors choose to use the appropriate velocity scale of the underlying waves.

In section 6, the authors present the results of an application of the similarity expressions in (17)–(19) on an experimental dataset. This analysis demonstrates an easily applicable method to exhibit the detailed vertical structure of the wave-induced flow fields (Hare 1995).

4. The experiment

Ideally, observations of marine atmospheric surface-layer turbulence and wave-induced flow should be made away from the influences of land–sea gradients, shallow water, limited fetch, and other coastal phenomena. Thus, it is preferable to perform these measurements over the open ocean. However, there are few suitable stationary platforms from which to perform research within relatively deep seas. Unfortunately, platform motion correction for ships and buoys, although possible (Hare 1992), is still a relatively expensive endeavor and includes the implementation of a complex compensation algorithm. In addition, flow distortion around large structures, such as oceangoing ships and observational towers, has prohibited the careful measurements of surface waves and turbulence that a comprehensive air–sea surface interaction observational campaign demands.

The adverse effects of flow distortion are difficult to quantify and compensate within high Reynolds number flow (Wyngaard et al. 1985).

Information on the composition of the marine surface layer and WBL is meager. This can be attributed to the scarcity of comprehensive measurements over the sea surface. Because the physical mechanisms responsible for the momentum flux to sea waves differ markedly from those over land, thorough observations of the interaction of the turbulent flow with the air–sea interface on the scale of the surface waves are needed to improve the understanding of these basic processes.

a. RASEX

In order to address this deficiency, the Office of Naval Research (ONR) supported the organization of the Risø Air–Sea Experiments (RASEX) campaign to obtain accurate and comprehensive measurements of the atmospheric surface layer and sea surface variables sufficient to permit an analysis of marine surface-layer turbulence and its interaction with the underlying waves. This series of experiments is part of the ONR Marine Boundary Layers Accelerated Research Initiative (MBL/ARI). An additional experiment aboard the Research Platform (RP) FLIP during the spring of 1995 is also part of the MBL/ARI.

The MBL/ARI experiments are unique because they focus on understanding the physical processes that determine the structure of turbulence within the marine surface layer and its interaction with the wave field below. Scientists from the Risø National Laboratory (Risø) of Denmark, the Woods Hole Oceanographic Institution (WHOI), the National Oceanic and Atmospheric Administration Environmental Technologies Laboratory (NOAA/ETL), and the Pennsylvania State University (PSU) performed the RASEX series of measurements. These air–sea interaction experiments were conducted in the spring (April–May) and fall (October–November) of 1994. The analysis of the resulting dataset contained in this paper is focused on the structure of the wave-induced flow above the interface in terms of statistically averaged variables. The organization of this flow is fundamentally determined by the interaction between the turbulent atmosphere and the surface wave field (Hare 1995).

The RASEX program took advantage of a highly instrumented 48-m mast adjacent to the Vindeby Offshore Windfarm in relatively shallow water (depth 3–4 m) near the village of Vindeby, off the western coast of the island of Lolland, Denmark (see Fig. 1). The experimental site is approximately 2 km from shore within the Great Belt off the Danish coast. The sea mast is equipped with seven levels of cup anemometers and three levels of mean temperature sensors (Barthelmie et al. 1994). In addition, six levels of sonic anemometer/temperature and two levels of infrared hygrometers to measure the turbulent fluxes of momentum (stress), sen-
The instrumentation on the sea mast is mounted to provide coverage for the most likely wind direction, which is primarily westerly. This placement prohibits the use of much of the data from the north and east, as the mast support structure blocks the wind and waves.

For air–sea interaction measurements to be representative of the open sea, the chosen site must be open to long fetch and in water deep enough to prohibit the slowing and steepening of waves that result from shoaling. However, the Vindeby sea mast is in relatively shallow water with variable sea fetch directions. The longest fetch directions are roughly from west-northwest (~20 km), north (~70 km), and northeast (~50 km). It is difficult to extrapolate a given segment of turbulence and wave-induced flow measurements to the open ocean under similar stability conditions. Nevertheless, it should be reemphasized that the RASEX program seeks improved statistical descriptions of the physical coupling of the fluctuating wind and wave fields. This can be accomplished with an appropriate similarity hypothesis (see section 3) because the wave-induced flow will be described with respect to the underlying wave field.

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b. Instrumentation

The sea mast siting and configuration are outlined in detail in Barthelmie et al. (1994), Højstrup et al. (1997, unpublished manuscript), and Hare (1995), but some comments about individual instruments are provided in this section. In particular, the analysis in this paper requires the use of accurate turbulence- and wave-measurement instrumentation. Ancillary instruments provide information such as wind stress, wind speed and direction, current, and water depth, which are valuable for the conditional sampling and analysis.

1) Pressure Sensor

The sea mast siting and configuration are outlined in detail in Barthelmie et al. (1994), Højstrup et al. (1997, unpublished manuscript), and Hare (1995), but some comments about individual instruments are provided in this section. In particular, the analysis in this paper requires the use of accurate turbulence- and wave-measurement instrumentation. Ancillary instruments provide information such as wind stress, wind speed and direction, current, and water depth, which are valuable for the conditional sampling and analysis.

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rection for the accelerating frame of reference. Such measurements are also limited to the region very close to the surface. A fixed-height configuration, such as that on the RASEX sea mast, provides a mechanically stable platform for the measurement of the turbulent and wave-coherent pressure fluctuations. Because a fixed-height pressure probe can be mounted only as close to the surface as the crests of the highest waves, only the pressure disturbances due to waves near the spectral peak can be resolved. However, given a proper similarity hypothesis and measurement of the wave-correlated pressure over a wide range of dimensionless wavenumber $k_z$, the detailed vertical structure of the wave-induced pressure perturbation may be resolved (see section 6).

2) Wave wires

Wave height and slope were obtained from a triangular three-element capacitance wave gauge array mounted directly below the lowest sonic anemometer/pressure sensor pair. These wave wires were manufactured at WHOI for use in the RASEX experiments.

Amplitude calibration of the wave wires is accomplished with a subsurface acoustic transducer placed by the Risø science team 20 m from the base of the sea mast. This calibration consists of computation of the half-hour averaged variance spectra of both the acoustic wave meter and the individual wave-wire signals. The sensitivities of the wave wires are computed from the ratio of the spectral estimates of the acoustic wave height sensor and wave wires for points surrounding the wave peak. The offset of the wave wires from the calibrated acoustic transducer is then computed as the difference of the half-hour means of the wave-wire and transducer signals.

The pressure sensors and anemometers are situated as close as possible to the center of the wave-wire array triangle in order to eliminate potential phase lags due to horizontally displaced sensors in the correlation of the signals. In addition, for the one-dimensional analysis presented in this paper, the instantaneous wave height was computed as the average of the three wave-wire measurements. The lowest pair of instruments were located 3 m above the mean sea surface.

5. Processing

The expressions in (17)–(19) were applied to the portion of the RASEX dataset that was not influenced by flow around the mast structure. For the present analysis, only winds from relatively long fetch directions were included. This minimizes the complications of possible internal boundary layers coming from the near shore and provides aligned wind and waves with sufficient wave development to permit measurement of the wave-correlated airflow at the 3-m pressure sensor. The result of the restriction of long fetch and flow away from the mast structure is an analysis on the wind directions between 235° and 305°. An additional benefit of inclusion of these directions is the increase in signal-to-noise ratio at the 3-m measurement height with the more developed, longer fetch sea.

The mean water depth at the RASEX sea mast is 3 m and is relatively uniform around the surrounding area. Tides were estimated by taking the half-hour mean from the underwater acoustic transducer signal. Tidal variations were minimal and of order $\pm 25$ cm (Hare 1995). The mean water depth was used for each half-hour segment in the computation of wave phase speed and sensor height. Current velocity was measured with a current meter located adjacent to the acoustic wave-height sensor. When compared to the wind speed, the currents were relatively insignificant and of order 10 to 20 cm s$^{-1}$ (Hare 1995). This measured mean current was used as a correction for the Doppler frequency shift in the computation of wavenumber.

The data were grouped in half-hour segments. This time period was chosen to be long enough to ensure sufficient statistical reliability in the wind stress and wave spectral estimates, but short enough to minimize the effects of nonstationarity on the dataset. A total of 423 half-hour segments of the total RASEX dataset were utilized for the analysis that is presented in section 6. The distributions of wind speed and direction are found in Table 1.

Table 1. Distributions of (a) wind direction and (b) wind speed for the portion of the total RASEX dataset that was used in the analysis described in this paper. A total of 423 half-hour data segments were acceptable according to the sampling criteria outlined in section 5.

<table>
<thead>
<tr>
<th>(a) Wind direction (deg)</th>
<th>Number of samples</th>
<th>(b) Wind speed (m s$^{-1}$)</th>
<th>Number of samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>235–245</td>
<td>54</td>
<td>2–3</td>
<td>11</td>
</tr>
<tr>
<td>245–255</td>
<td>42</td>
<td>3–4</td>
<td>10</td>
</tr>
<tr>
<td>255–265</td>
<td>45</td>
<td>4–5</td>
<td>32</td>
</tr>
<tr>
<td>265–275</td>
<td>62</td>
<td>5–6</td>
<td>60</td>
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<tr>
<td>275–285</td>
<td>68</td>
<td>6–7</td>
<td>91</td>
</tr>
<tr>
<td>285–295</td>
<td>74</td>
<td>7–8</td>
<td>58</td>
</tr>
<tr>
<td>295–305</td>
<td>78</td>
<td>8–9</td>
<td>42</td>
</tr>
</tbody>
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and minimize this cosine effect, attention is restricted in this analysis to $\theta \leq \pm 30$ deg. Because the dataset is limited, an analysis of the cosine dependence of the wave-induced flow field was not possible.

Auto- and cross-spectra were computed on the 10-Hz and 20-Hz sampled half-hour segments, using a 512-point and 1024-point fast Fourier transform (FFT), respectively, to provide spectra over 51.2 seconds. This period proved sufficient to resolve any significant waves at the sea mast. These spectra were subsequently averaged for each half-hour segment of the dataset and examined for reliability. Figure 2 displays a representative example of the one-dimensional pressure, vertical velocity, and wave spectra that were used in our analysis. A well-defined wave-induced pressure signal is evident centered at the peak of the wave spectral frequency (about 3.3 sec for this example). Time periods with wave spectra having no well-defined peak or multiple peaks were eliminated from the analysis. This procedure ensured that wind-driven waves were present in the field and that unforeseen complications from the influence of wave swell were minimized.

The following expressions were used to compute the relevant phase speed, wavenumber, and wavelength of the points in the spectra. The wavenumber in water of finite depth is

$$k \tanh(kH) = \frac{\omega^2}{g},$$

(20)

where $H$ is the water depth (3 m plus tidal variation for RASEX). Therefore, the dispersion relation is used to find the phase speed,

$$c = \sqrt{\frac{g}{k} \tanh(kH)}.$$  

(21)

We note that for the analysis of the RASEX dataset, the results in section 6 predominantly lie between $1 < k_z < 8$. For the height of the RASEX measurements (3 m), the results in this study are representative of deep water waves.

Finally, the pressure correlation similarity function, $N_{pr}$ as expressed in (17) was computed and averaged over bins of $cl_\ast$, and 95% confidence intervals were estimated using the method outlined in the appendix.

In order to minimize the effect of spurious noise from the auto- and cross-spectra and to ensure sufficient wave-induced pressure signal, only those wavenumbers in which the coherence of the pressure and wave height were above a threshold of 0.1 were included in the final averaging.

6. Results

The fundamental results of the analysis are presented in this section. Recall from section 3 that the similarity hypothesis allows us to plot the wave-induced fields versus dimensionless height ($k_z$) when averaged over bins of $cl_\ast$. The detailed vertical structure of the wave-induced pressure and velocity fields over ocean waves has never been reported. As stated earlier, previous field investigators (Snyder et al. 1981; Hasselmann and Bösenberg 1991) have assumed that the wave-induced pressure perturbations decay exponentially with dimensionless height, similar to the decay rate of the potential flow. In the analysis presented here, no particular a priori profile structure of the wave-induced flow has been assumed.

a. The wave-induced pressure field $N_{pr}$

Within the context of wave-induced flow and wave growth, the authors first focus on the imaginary component of the similarity function, $N_{pr}$, as it represents the deviation of the flow from the leading order inviscid potential flow solution. The expression in (7) indicates that the wave-induced pressure field at the surface that is in phase with the wave slope provides momentum to the waves. In fact, the imaginary component of the wave-induced pressure is purely a result of the interaction between the turbulent flow and the waves.

Previous theoretical and laboratory studies have determined that the pressure profile close to the surface is well behaved (Hsu et al. 1981; Hunt et al. 1988; Belcher and Hunt 1993). No attempt has been made in the present investigation to extrapolate the imaginary component of the wave-induced pressure signal to the surface to obtain estimates of wave growth rate or momentum flux to the surface. Based on the results plotted below, the structure of this flow appears to be more complex than a simple exponential decay with height. This, combined with the width of the confidence intervals in the figures, would indicate a large uncertainty on an extrapolated surface estimate. Furthermore, the
main intent of this research is to provide insight into the structure of the wave-induced pressure field, which extends throughout the WBL, not simply at the surface. However, we note that Belcher et al. (1993) provide theoretical parameterizations of the surface value of the wave-induced pressure that are dependent on the shape of the underlying wave.

1) IMAGINARY COMPONENT OF THE WAVE-INDUCED PRESSURE FIELD

The leading order contribution to the form drag is due to the out-of-phase component of the wave coherent surface pressure (Belcher et al. 1993). Figure 3 shows the results of the analysis of this component of the pressure field using (17). The circles and solid lines in these figures show the vertical profiles of the imaginary component of the wave-induced pressure field. In addition, 95% confidence intervals are displayed as dots in the figures using the estimates outlined in the appendix. For reference, the exponentially decaying profiles, \( \exp(-\kappa z) \), are plotted with dash–dot lines. The surface values of these lines correspond to the extrema of Plant’s (1982) formula in (16) for the range of wave age in each figure and using (14) and (17). Therefore, these dash–dot lines represent the upper and lower limits of the wave-induced pressure field, provided that Plant’s formula and the assumed exponential profile are both valid.

The width of the 95% confidence intervals in this and succeeding figures is a consequence of the low signal level of the wave-coherent pressure. These error intervals will be decreased by increasing the number of measured samples and by placing the measuring probe closer to wave surface. Although the extrema of the Plant’s formula estimates lie within the confidence intervals, the authors feel that the current results are relevant to demonstrate information about the vertical structure of the wave-induced pressure field.

Furthermore, the relative width of the wave age range over which the profiles are averaged demonstrates an additional limitation to the method as applied to the RASEX dataset. It is essential that additional measurements be performed at multiple levels above the surface waves in order to reduce the uncertainty imposed by the width of the wave age intervals. However, we have investigated the possibility that the apparent structure of the profiles may be affected by the width of the wave age intervals. For example, the profiles could be affected at a particular \( \kappa z \) if the distribution of points is not uniform in \( c/u_* \). We have found that the structure of the profiles in Fig. 3 are not appreciably changed after removing those wave ages that are not uniformly distributed (not shown).

These figures demonstrate a dependence of the structure of the profiles on the wave age, \( c/u_* \). Figure 3a shows \( N' \) for wave age below the theoretical threshold of the initiation of wave growth, 20–25 (Volkov 1969; Phillips 1977; Davidson 1974). Over much of the profile, the sign of the dimensionless wave-coherent pressure is negative. Although the lower portion of this profile is not easily extrapolated to the surface, it may be speculated that the net form drag in this case is in the direction of wave to wind, as these waves were decaying. Figure 3a is the first to demonstrate not only the vertical structure of the wave-induced pressure field for conditions below the wave growth threshold, but also to illustrate the distinct negative sign of this field over much of the profile. Thus, the method described in this paper permits the analysis of the airflow over decaying seas in a manner not cited in the literature to date.

Figure 3b provides a profile of the out-of-phase portion of the wave-induced pressure field for slightly greater wind forcing (smaller \( c/u_* \)). Here one can see enhancement of the lower portion of the profile, which is now positive, suggesting momentum transfer from wind to waves.

Likewise, Figs. 3c–d demonstrate the further enhancement of the lower structure of the profiles toward more positive values with a monotonic height dependence. Greater wind forcing provides increased positive values of the lowest levels within the WBL. Above this level, a more complicated secondary structure can be seen. For example, examination of Fig. 3c at \( \kappa z \approx 2 \) shows negative minimum values at this level. Further above, the profile seems to increase again toward positive values. The monotonic nature of the lowest part of these profiles is consistent with the experimental findings of Snyder et al. (1981) and Hasselmann and Bösenberg (1991), as well as the theoretical work of Jacobs (1987). However, it appears that the structure of the overall profile is a superposition of the simpler exponential decay below and the secondary feature above \( \kappa z \approx 1 \).

Regions of constant or increasing wave-induced pressure with height have been observed in laboratory flows (Papadimitrakis et al. 1985) as well as in theoretical treatments (Townsend 1980). These sources have indicated that the nonmonotonic structure is probably a consequence of the interaction of the wave-induced flow with the spatial distribution of the air turbulence.

Consequently, it would be improper to extrapolate measurements of the imaginary component of the wave-induced flow at a dimensionless height of \( \kappa z > 2 \) to obtain the wave growth parameter at the surface using a simple formula such as \( \exp(-\alpha k z) \). Indeed, the presence of these relatively complex features in the profiles demonstrates the necessity for caution before simple exponential extrapolations of elevated measurements to the sea surface are employed for the estimation of transferred momentum or energy (see also Papadimitrakis et al. 1986).

It is also of interest to compare our observational results with recent theoretical predictions. In particular, Belcher and Hunt (1993) have developed an asymptotic theory of turbulent shear flow over waves in the limit of
Fig. 3. Profile (in $kz$) of the imaginary component (circles with solid line) of the wave-induced pressure field $N_{pn}$ as defined in (17). The dots indicate 95% confidence intervals (see appendix). Dashed-dot lines are extrema plotted from Plant's formula (16), with constant $(0.04 \pm 0.02)$ and assuming an exponential decay with height, $\exp(-kz)$. (a) $25 < clu_s < 294$, (b) $20 < clu_s < 25$, (c) $16.7 < clu_s < 20$, (d) $10 < clu_s < 16.7$, and (e) $2.4 < clu_s < 10$. 

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Since Belcher and Hunt provide analytical solutions for the leading order wave-perturbed flow, it is feasible to compare their results with our observations.

According to the theory in Belcher and Hunt (1993), the leading order solution for the imaginary part of the pressure is a simple exponential profile with height. The surface value of the imaginary part of the profile is always positive, and its magnitude varies significantly depending on the surface roughness parameterization. However, as the condition \((u^* + c)/U \approx O(1)\) is approached, the shear stress layer (i.e., the layer where the turbulent shear stress is significant) becomes thicker and the asymptotic solution breaks down.

The criterion given in (22) is marginally satisfied for the conditions of Figs. 3d–e of this study, where our results for the imaginary component also show positive values over much of the profile. However, the profiles in these figures show the structure to be not simply exponential, although the large uncertainty intervals prevent us from making a definitive statement. For Figs. 3a–c, where the condition in (22) is not satisfied, the vertical profile of the imaginary part of the wave-induced pressure seems more complex. This may be a consequence of the growing shear stress layer predicted by the Belcher and Hunt model.

2) REAL COMPONENT OF THE WAVE-INDUCED PRESSURE FIELD

The in-phase wave-correlated pressure signal profiles are shown as circles with solid lines in Figs. 4a–e. As before, the dots indicate the 95% confidence intervals. The structure displays an exponential dependence on height, closely related to the theoretical potential flow. The dash–dot lines are plotted from the potential flow solution of Hasselmann and Bösenberg (1991):

\[
\text{Re}(N_p) = -\left(\frac{U}{c} - 1\right)^2 \exp(-kz),
\]

with the approximation, \(U \sim 25u^*\) (only valid for developing seas, \(c/u^* < 25\)). For stronger winds, the agreement is reasonable. However, at lower wind speeds, where the potential solution becomes small, the agreement is poor, as expected.

The dependence of the real signal on the dimensionless wave age, \(c/u^*\), appears to be consistent with the field data of Snyder et al. (1981) and Hasselmann and Bösenberg (1991). The magnitude of the real part of the pressure signal is significantly greater than that of the imaginary component.

We also note the recent combined theoretical and laboratory work by Mastenbroek et al. (1996), which reports on the vertical structure of the wave-correlated pressure field for \(kz < 1.2\) and \(c/u^* = 4.3\). These results have not been included on our Figs. 3e and 4e due to the relatively wide uncertainty in our results below \(kz = 1\). However, the Mastenbroek et al. (1996) results are consistent with those presented here.

3) MAGNITUDE AND PHASE OF THE WAVE-INDUCED PRESSURE FIELD

Figures 5 and 6 show the magnitude and phase, respectively, of some of the wave age ranges of the wave-induced pressure field from the analysis of the RASEX dataset. For comparison, selected points from the results of the laboratory analysis of Papadimitrakis et al. (1986) are plotted in the figures within the specified wave age range. Although the profiles shown in Figs. 5 and 6 are redundant given the imaginary and real components shown in Figs. 3 and 4, we present the results so that direct comparison can be made with the laboratory work. The lack of reported field measurements of the wave-induced flow components compels us to make the comparison of our products with the laboratory results.

It is necessary to rescale the laboratory results to obtain a product in the same form as (17). Using the notation of Papadimitrakis et al. (1986), the magnitude of the pressure coefficient \(C_p\) becomes

\[
|N_p| \approx |C_p| \frac{\pi U_{\infty}^2}{c^2},
\]

where \(U_{\infty}\) is the freestream velocity. Furthermore, as the friction velocity was not reported for the Papadimitrakis et al. (1986) experiments, the authors have assumed a relationship with the freestream wind speed \(U_{\infty} \sim 30u^*\). This approximation is not critical for interpretation of the plots given the width of the averaging intervals of wave age, \(c/u^*\), in the figures.

The laboratory results are described within a wave-following coordinate system. However, given that the range of fixed-height variation of the wave-following coordinates is at most twice the wave amplitude (\(\alpha = 2.5\) cm) and that the variability of the laboratory profiles reduces with increased dimensionless height, we have elected to include the laboratory results in Figs. 5 and 6 above the height \(kz = 1\).

In Fig. 5a, it is shown that the laboratory results for decaying wave fields are very similar to those of this field study. The general trend of increasing wave-induced pressure as the surface is approached and the overall shape of the profiles are consistent between the datasets. The RASEX results generally exhibit smaller magnitudes for a given dimensionless height than those of the laboratory study in Figs. 5a–c.

Comparison between the laboratory and the current results in the phase plots of Fig. 6 are largely inconclusive given the relatively broad uncertainty of the phase. However, the field and laboratory profiles are quite similar for decaying waves, as seen in Fig. 6a.
Fig. 4. Profile (in $k_z$) of the real component (circles with solid line) of the wave-induced pressure field $N_{pu}$ as defined in (17). The dots indicate the 95% confidence intervals (see appendix). Dash–dot lines are extrema plotted from formula (23) of Hasselman and Bosenberg (1991). (a) $25 < clu_s < 294$, (b) $20 < clu_s < 25$, (c) $16.7 < clu_s < 20$, (d) $10 < clu_s < 16.7$, and (e) $2.4 < clu_s < 10$. 
4) **Effect of Wave Scattering**

Wave scattering from nearby support structures has been indicated as a potential error source in previous investigations (Snyder 1974; Snyder et al. 1981; Papadimitrakis et al. 1985, 1986; Hasselmann and Bösenberg 1991). The RASEX sea mast is supported by three 1-m diameter cylindrical posts. Waves of the scale of the post and smaller will be efficiently reflected contrary to the wind into the measurement domain. From potential theory, the error magnitude can be estimated from the ratio of the pressure signals of the reflected (subscript \( R \)) and incident (\( I \)) waves (Papadimitrakis et al. 1985) as

\[
\frac{P_R}{P_I} = \frac{a_R}{a_I} \left( \frac{U + c}{U - c} \right)^2, \tag{25}
\]

where \( a \) is the wave amplitude and \( U \) is the mean wind speed. However, this expression ignores the effects of directional spreading, angle of incident propagation, length scale of scattering, and distance from the scattering object. Most importantly, this expression is a statement from potential theory that strictly describes only the in-phase, or real component of the wave-induced pressure field.

Because the RASEX instrumentation configuration was similar to that of Hasselmann and Bösenberg (1991), an error analysis was performed in a similar fashion to that outlined in their paper (for details, see Hare 1995; Hasselmann and Bösenberg 1991). The result of this analysis of the reflected wave error using the RASEX configuration parameters and peak wave phase speeds is shown in Fig. 7. This plot shows the profile of the near-field error as expressed by the ratio of the scattered and incident pressure perturbations. Inspection of Fig. 7 reveals that the majority of the wave reflection errors are less than 20%. Furthermore, examination of Figs. 4a–e does not reveal any effect of errors due to scattering from the sea mast supports, which would appear preferentially around \( k_z = 2–3 \). It is imperative to emphasize that these errors have been stated in terms of potential theory. As this will only
describe the real part of the complex wave-induced pressure field, no explicit statement can be made about the validity of the error argument to the pressure field in quadrature to the underlying waves. In addition, the authors have recently developed an efficient algorithm to estimate the directional wave spectrum from an array of wave wires based on the Data-Adaptive Spectral Estimation (DASE) technique of Davis and Regier (1977). The method can resolve secondary waves in propagation directions that differ from that of the dominant waves, even if the wave energy of the secondary waves is smaller by an order of magnitude (Hanson 1996). This algorithm has been applied to several typical wave conditions during RASEX, and no evidence of reflected waves has been found. It is therefore reasonable to assume that the scattered waves did not introduce significant errors in our analysis.

5) Effect of Breaking Waves

Although quantification of errors due to nonlinear effects on the presented results is difficult, some comments are in order about the potential uncertainties due to random wave breaking. Over the open ocean, wave breaking modifies the exchange of turbulent fluxes across the air–sea interface. Furthermore, the presence of breaking waves is associated with local airflow separation and a dramatic increase in the momentum transfer from both form drag and shear stress mechanisms (Banner 1990). Therefore, one would expect that the structure of the pressure and velocity fields is modified from the linear case.

Banner (1990) examined the influence of wave breaking on the surface pressure and momentum transfer distribution in a laboratory flow. Relative to the nonbreaking wave, he found a distinct downwind phase shift between the surface pressure and the surface elevation due to airflow separation. In addition, a large increase (~100%) in the total wind stress was observed over the breaking waves relative to the nonbreaking case. Banner found that the pressure disturbance was more persistent in the vertical than for flow over nonbreaking waves.

Maat and Makin (1992) found a 100% increase in...
total wind stress for modeled breaking waves similar to the Banner laboratory waves. They also found that the surface pressure maximum shifted forward of the wave crest, consistent with the Banner results.

Despite a few carefully executed experiments in the laboratory and with models, very few observations exist for atmospheric flow over breaking sea waves. The intermittency of breaking events and the difficulty of quantifying the percent coverage of whitecaps on the local surface have limited the ability to assess the flow field deviations resulting from local breaking.

There were no sustained visual or video observations of the sea surface during the RASEX deployments. However, it was observed that white-capped breaking waves were present in the field during periods of higher mean wind speeds (above 10 m s\(^{-1}\)). This is roughly consistent with the observations of Monahan et al. (1983), where a 1% white cap areal coverage is estimated for 10 m s\(^{-1}\) winds and about 4% for winds of 15 m s\(^{-1}\). Examination of Table 1 shows that these higher wind speeds (above 10 m s\(^{-1}\)) represent about 15%–20% of the total RASEX dataset. Accordingly, our results at lower \(c/u_\infty\) values can be affected by breaking waves. However, for each half-hour period of the RASEX dataset, each wave spectrum, pressure spectrum, and velocity spectrum were visually examined for shape and noise contamination. Those periods with poor spectra were eliminated from the analysis presented here (Hare 1995).

### b. Wave-induced vertical velocity \(N_w\)

Figures 8 and 9 show the results of the parallel analysis of the wave-induced vertical velocity field. A similar analysis was performed for the streamwise component of the wave-induced velocity, but due to the relatively high uncertainty of the results is not included in this presentation. The plots in Figs. 8 and 9 demonstrate the effectiveness of the technique outlined in this paper despite the reduced signal levels of wave-induced velocities relative to the pressure signal. The confidence intervals for the velocity profiles are significantly wider than those of the wave-induced pressure field. This is partly due to the sensitivity of the sonic anemometer and to the significant level of turbulence near the sea surface that acts as noise to mask the wave-induced signal. It is anticipated that further research using these methods will reveal significant detail about the wave-induced vertical velocity field in a manner similar to that demonstrated for the wave-induced pressure field.

Figures 8a–e show the magnitude of the wave-induced vertical velocity field as described by the similarity relationship in (19). Wave-induced vertical velocities were previously measured by Hsu et al. (1981), Hsu and Hsu (1983), and Mastenbroek et al. (1996) in laboratory wind flumes. Although the scale of waves were much smaller in their experiments, the range of wave age was comparable to our RASEX field observations. In Fig. 8, the laboratory results of Hsu and Hsu (1983) and Mastenbroek et al. (1996) are also plotted. In Figs. 8a–d, our field results show a monotonic decrease with of the wave-induced vertical velocity with height. The overall magnitude of the wave-induced velocity decreases as the wind forcing increases. Although the confidence intervals for \(N_w\) are quite wide, the general trend is very consistent with the laboratory results.

The phase plots of the vertical component of the wave-induced velocities are also included here as Figs. 9a–e. The arctangent function is quite sensitive to the noise inherent in the ratio of two relatively small, noisy numbers. Only the lower dimensionless heights are shown in these figures due to the large scatter of points above. In addition, the width of the confidence intervals and the scatter of this data makes interpretation difficult. Yet, there is a clear trend of increasing phase with decreasing wave age, from approximately \(-90^\circ\) in Fig. 9a to approximately \(+50^\circ\) in Fig. 9e. Again, the agreement with laboratory results is encouraging.

The reasonable agreement between the laboratory results of Hsu and Hsu (1983), Mastenbroek et al. (1996), and our field measurements, despite the large differences, suggests that our similarity hypothesis is applicable to a rather large range of scales and that extrapolation of laboratory results to open ocean conditions may be feasible. It is apparent that the technique outlined in this paper, originally developed to expose the structure of the wave-induced pressure field, can be extended to a similar analysis of the wave-induced velocities. However, further accumulation of field data is necessary to confirm the validity of our similarity hypothesis.

### 7. Conclusions and future work

This investigation of the wave-induced flow field above sea waves is the first implementation of a sim-
Fig. 8. Profile (in $kz$) of the magnitude (solid line) of the wave-induced vertical velocity field $N_{w}$ as defined in (19). The dots indicate the 95% confidence intervals (see appendix) and “X” are points from the laboratory investigations of Hsu and Hsu (1983) and Mastenbroek et al. (1996). (a) $25 < cl/u_{*} < 294$ (X are at $cl/u_{*} = 36.3$ and 27.9), (b) $20 < cl/u_{*} < 25$ (X are at $cl/u_{*} = 21.4$), (c) $16.7 < cl/u_{*} < 20$ (X are at $cl/u_{*} = 18.2$), (d) $10 < cl/u_{*} < 16.7$ (X are at $cl/u_{*} = 14.2$), and (e) $2.4 < cl/u_{*} < 10$. 
Fig. 9. Profile (in $k_z$) of the phase (solid line) of the wave-induced vertical velocity field $N_w$ as defined in (19). The dots indicate the 95% confidence intervals (see appendix) and “X” are points from the laboratory investigations of Hsu and Hsu (1983) and Mastenbroek et al. (1996). (a) $25 < c/\nu^{*} < 294$ (X are at $c/\nu^{*} = 36.3$ and 27.9), (b) $20 < c/\nu^{*} < 25$ (X are at $c/\nu^{*} = 21.4$), (c) $16.7 < c/\nu^{*} < 20$ (X are at $c/\nu^{*} = 18.2$), (d) $10 < c/\nu^{*} < 16.7$ (X are at $c/\nu^{*} = 14.2$), and (e) $2.4 < c/\nu^{*} < 10$.

Previous investigations into the wave-induced pressure field (Snyder et al. 1981; Hasselmann and Bösch...
enberg 1991) have focused on determination of the surface wave growth parameter \( \beta \). This paper has highlighted the more fundamental structure of the profile of the wave-correlated flow field. The scaling arguments that were used to derive the similarity hypothesis (section 3) have roots in the Miles (1957) theory of wave development. However, there is a clear distinction between the wave-induced flow [hypothesized as \( N_{\eta} \) in (17)–(19)] and that aspect of the flow (parameterized with \( \beta \)) responsible for wave growth. Based on the profiles in Fig. 3, it is concluded that the structure of the wave-induced pressure field in quadrature with the waves is probably more complex than previously assumed. Therefore, extrapolation of elevated measurements to the sea surface to determine wave growth rates or momentum transfer should only be attempted with caution.

In addition to the wave-induced pressure field, the authors have developed and successfully tested a similarity hypothesis to describe the wave-induced vertical velocity. Although the uncertainty in the velocity components is relatively large, the reasonable agreement with previous laboratory results implies that our similarity hypothesis is valid for a wide range of scales. It is anticipated that future experimental investigations over deep-water waves will yield more details about this integral part of the coupled air–water system. In addition, it is essential that the measurements of the wave-induced flow field be performed at multiple levels above the sea surface. Assuming that the similarity hypotheses in (17)–(19) are valid, multiple-height measurements will provide information necessary to fully describe the profiles of the wave-induced pressure and velocity fields. An analysis to investigate the wave-correlated flow is planned for data obtained during two field programs on the RP FLIP off the U.S. West Coast during the spring and fall of 1995.

As indicated in (7), the wave-induced momentum flux is most appropriately defined in terms of two-dimensional wavenumber and, therefore, two-dimensional wave slope and cross-spectral density. Thus, the next step in this investigation is to perform a carefully designed experiment and analysis of the type indicated in this paper using the cross-spectrum of wave slope in the similarity hypothesis of (17). Also of interest is the effect of wave-directional spectral spreading on the wave-induced flow. These investigations will also lead to insight into the directional attributes of the wave-induced momentum flux and drag coefficient (Geernaert et al. 1993).

Further development of this work will include investigation into potential nonlinear interaction between wind and waves. A particular focus will be on those conditions when breaking waves are in the field and directly within the wave-wire measurement array. It is important to reiterate that a primary assumption for the research within this paper is that a linear relationship exists between the waves and the wave-induced flow. Of course, this hypothesis must be adequately tested to provide complete confidence in the results of this paper.

The information provided by analyses such as presented in this paper is crucial to the understanding of the physical processes occurring at the air–water interface. The wave-induced flow field is intimately coupled to the marine surface-layer turbulence. Experimentalists must begin to examine the effects of the interaction between the wave-induced flow field and the turbulence. Results such as presented in section 7, combined with a full similarity analysis of the marine surface-layer turbulence, will be instrumental in improving our understanding of the structure of this interaction.

As mentioned in section 1, the output of current numerical models for flow over waves is sensitive to the applied closure method. The results of the study reported in this paper may enable modelers to develop closure techniques that can more precisely resolve the structure of the wave-induced flow.

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APPENDIX

Error Analysis

The 95% confidence intervals for the dimensionless similarity products are estimated with the relationships found in Bendat and Piersol (1986):

\[
\hat{a}(1 - 2\epsilon) \leq a \leq \hat{a}(1 + 2\epsilon),
\]

(A1)

where \( \hat{a} \) is the estimated value, \( a \) is the true value, and \( \epsilon \) is the normalized random error estimated for the magnitude of the similarity relationship in (17) as

\[
\epsilon|N_{\eta\eta}| = \left( \frac{1 - \gamma_{\eta\eta}^2}{2\gamma_{\eta\eta}^2} \right)^{1/2},
\]

(A2)

where \( \gamma_{\eta\eta} \) is the averaged coherence and \( n_d \) is the number of records within the averaging bin. To estimate the standard error of the real and imaginary components of the similarity function, one can take advantage of the expansion

\[
N_{\eta\eta} = |N_{\eta\eta}^1|\sin\phi + |N_{\eta\eta}^2|\cos\phi,
\]

(A3)
where $\phi$ is the phase angle. Using $\delta$ as the non-normalized error,

$$
\delta[N_p] = i[\delta[N_p] \sin\phi + |N_p| \cos\phi \delta\phi] + [\delta|N_p| \cos\phi - |N_p| \sin\phi \delta\phi].
$$

(A4)

For notational simplicity, the real and imaginary components of the similarity function are denoted as $C$ and $Q$, respectively. Then, using only the imaginary part of this expression along with (A3), after some obvious manipulation an expression for the standard error of the imaginary component of $N_p$ is obtained:

$$
\frac{\delta Q}{Q} = \frac{\delta|N_p|}{|N_p|} + \frac{C}{Q} \delta\phi.
$$

(A5)

The first term is simply the error expressed in (A2). The second term includes the ratio of the real and imaginary components, as well as the error of the angle between the two. From Bendat and Pierson, the normalized random error for the phase angle is

$$
\frac{\delta\phi}{\phi} \approx \epsilon(|N_p|),
$$

(A6)

where $\phi$ can be estimated from the arctangent of the individual components. A similar expression for the standard error of the real component is written

$$
\frac{\delta C}{C} = \frac{\delta|N_p|}{|N_p|} - \frac{Q}{C} \delta\phi.
$$

(A7)

Finally, in the same form as (A2), the standard errors of the real and imaginary components of $N_p$ can be written as

$$
\epsilon(\text{Im}(N_p)) = \left(1 - \frac{\gamma_{2}^2}{2 \gamma_{2}^2 a}ight)^{1/2} \left[1 + \frac{\text{Re}(N_p)}{\text{Im}(N_p)} \frac{\phi}{\epsilon}\right],
$$

(A8)

$$
\epsilon(\text{Re}(N_p)) = \left(1 - \frac{\gamma_{2}^2}{2 \gamma_{2}^2 a}ight)^{1/2} \left[1 - \frac{\text{Im}(N_p)}{\text{Re}(N_p)} \frac{\phi}{\epsilon}\right].
$$

(A9)

These estimates of the normalized random error are then used in the analysis of the wave-induced pressure and velocity fields. The expressions in (A8) and (A9) were used to compute the 95% confidence intervals that are displayed in the figures shown in section 7.

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