Using an Ocean Model to Examine ENSO Dynamics

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ABSTRACT

An equatorial ocean model with the Kelvin and first six Rossby waves and \( n \) vertical modes is developed to simulate the equatorial sea surface temperature anomalies (SSTAs) and, more importantly, zonal movement of the western equatorial Pacific warm pool associated with an ENSO episode. When forced by observed wind stress, the model reproduces well the SSTAs in both the central and eastern equatorial Pacific. The two gravest baroclinic modes are needed to model the equatorial thermocline depth, zonal current, and SSTAs. The SSTA near the equatorial South American coast is mostly due to upwelling and that in the eastern Pacific from about 150°–100°W is mainly due to both zonal advection and upwelling. The SSTA in the equatorial Pacific near the edge of the warm pool (160°E–160°W) mostly results from the advection of mean SST by the anomalous zonal current and Newtonian damping.

Model calculations show that the growing ENSO disturbance, which propagates eastward from the western equatorial Pacific during an El Niño year, is largely halted by zonal currents advecting the western equatorial Pacific warm pool and the associated atmospheric convection back to their mean position. These zonal currents are mostly associated with equatorial Kelvin and Rossby ocean waves reflected from the eastern and western Pacific boundaries. The model solutions suggest that eastern and western boundary reflections contribute comparably to these negative feedback zonal currents at the crucial region near the edge of the warm pool. The solutions also show that the equatorial central Pacific zonal current due to the wind forcing in the far western Pacific (west of 160°E) only plays a minor role in the negative feedback during El Niño and is negligible during La Niña.

1. Introduction

As has been shown by Gill (1983), Gill and Rasmusson (1983), Fu et al. (1986), Picaut and Delcroix (1995), Picaut et al. (1996, 2001), Clarke and Shu (2000), and Shu (1999), the zonal movement of the western Pacific warm pool is fundamental to ENSO. Deep convection [as represented by outgoing longwave radiation (OLR)] and zonal wind anomalies both follow the zonal displacement of the warm pool on ENSO time-scales (see Fig. 1). The mean position of the eastern edge of the warm pool is near the international date line, and the typical eastward and westward equatorial displacements from this mean position are about 2000 km. Consequently, the interannual coupling of ocean and atmosphere is strongest in the equatorial west-central Pacific from about 160°E to 160°W. It is here that the equatorial interannual heating of the atmosphere by deep atmospheric convection is largest and that the interannual zonal equatorial winds forcing the ocean are strongest. It is thus very important for an ENSO model to be able to simulate the sea surface temperature anomaly (SSTA) in this region.

Since the SSTA changes near the edge of the warm pool are dominated by zonal advection, we first need to be able to calculate equatorial zonal currents accurately. "Intermediate" coupled ocean–atmosphere models have proven to be valuable for understanding ENSO dynamics. They retain the essential physics of the more complicated general circulation model (GCM) and allow for rapid calculation and physical interpretation. But most intermediate ocean models only have a single vertical mode (Zebiak and Cane 1987; Battisti 1988; Kleeman 1993), and the results of McCreary (1981) suggest that more than one vertical mode is needed to describe equatorial currents. Chen et al. (1995) did develop a 2½-layer ocean model to show that both baroclinic modes contribute to equatorial SSTAs. However, they did not examine the calendar year phase-locked warm pool movement and boundary reflections that we will consider here. McPhaden and Yu (1999) and Boulanger and Menkes (2001) have also used models with more than one vertical mode to examine ENSO dynamics, but their analysis is based on just the 1997/98 El Niño

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(McPhaden and Yu) or on a few El Niño events during the TOPEX/Poseidon period (Boulanger and Menkes). We will examine many ENSO events to see what typically happens.

The zonal displacement of the warm pool has been discussed by Picaut and Delcroix (1995), Picaut et al. (1996, 1997), Clarke et al. (2000), and Delcroix et al. (2000). But none of these has examined this dynamics for a typical (composite) El Niño based on a nearly 30-yr-long record. Questions that need to be addressed include: How much of the warm pool movement is due to currents associated with waves that have been reflected at the eastern and western Pacific boundaries? Does the wind variability in the west Pacific (see, e.g., Weisberg and Wang 1997) also play a role in the zonal advection of the warm pool?

The next section describes a multi vertical mode model and its parameters. In section 3, we compare the model results with the observations. The dynamics of the warm pool movement is discussed in section 4. A final section contains the conclusions.

2. The ocean model

The ocean model consists of two parts. One part is a linear wind-forced ocean wave model (see section 2a and the appendix) similar to those of McCreary (1981), Rothstein (1984), and Yu and McPhaden (1999); the other part is the SST model. For given wind stress, the forced wave model estimates the zonal currents and thermocline depth anomalies. These anomalies and the SST climatology will be used in the SST model to estimate the SST anomalies.

a. The forced wave model

Details of the linear forced wave model are given in the appendix. The model is forced by the observed winds and contains the equatorial Kelvin wave and the first six equatorial Rossby waves for vertical modes. Although nonlinearity influences the mean surface flow (see, e.g., Philander and Pacanowski 1980), the time-dependent seasonal surface flow is essentially linear (Yu and McPhaden 1999), and interannual flow and warm pool displacement have been interpreted in terms of linear equatorial Kelvin and Rossby waves (see, e.g., Delcroix et al. 1994; Picaut and Delcroix 1995). Consistent with these results, our comparison of model results and observations in section 3 suggests that, at least to a first approximation, use of our linear model for interannual flow is reasonable. The comparative simplicity of the linear forced wave model enables us to interpret the results easily.

b. SSTA equations

Different physical processes are responsible for SSTAs in different regions of the equatorial Pacific.

In the region near the edge of the western Pacific warm pool (about 160°E–160°W), on ENSO timescales the anomalous SST is mainly changed by zonal advection rather than entrainment of thermocline water (Gill 1983; McPhaden and Picaut 1990; Picaut and Delcroix 1995; Picaut et al. 1996, 1997). The relevant equation for the SST anomaly $T'$ is

$$\frac{\partial T'}{\partial t} + \gamma T' = -u'(\bar{T} + T') - \pi \frac{\partial T'}{\partial x},$$

where $t$ is time, $x$ is eastward distance, $u'$ and $\bar{u}$ are respectively the anomalous and mean zonal equatorial current, $\bar{T}$ is the annual cycle of equatorial SST, and $\gamma^{-1}$ is a thermal damping timescale. The damping term $\gamma T'$ has been added to take into account damping due to anomalous heat flux caused by clouds shading the surface from incoming shortwave solar radiation. For example, when $T' > 0$, cloudiness increases and there is less incoming shortwave radiation, which results in cooling back to equilibrium ($T' \to 0$).

Along the equator anomalous cloudiness follows the zonal displacement of the edge of the western equatorial Pacific warm pool (see Fig. 1, with anomalous negative OLR as a proxy for anomalous cloudiness and the 28.5°C isotherm as a proxy for the eastern edge of the warm pool). Past work (Gadgil et al. 1984; Graham and Barnett 1987) has shown that usually a necessary condition for deep atmospheric convection is that the total SST must exceed a critical temperature (about 28°C), but Fig. 1 shows that at the equatorial edge of the warm pool a total SST of 28.5°C is both necessary and sufficient. Based on this, the damping term $\gamma$ should be a function of $x$ and $t$ because the shading of the ocean by clouds changes as the warm pool moves zonally. We chose the damping to be large ($\gamma^{-1} = 100$ days) when the water is warm enough (SST > 28.5°C) and small ($\gamma^{-1} = 2500$ days) when the total SST < 28.5°C. The value of 100 days for $\gamma^{-1}$ is similar to the 125-day SST damping used in the Zebiak and Cane (1987) and Battisti (1988) intermediate models. Model results are not very sensitive to $\gamma$ (see section 3c).

While zonal advection and anomalous cloudiness influence SSTAs near the edge of the warm pool, other processes are important to the east. In particular, previous studies have shown that interannual SST changes near the South American coast are mainly due to the anomalous upwelling $w'$ associated with the thermocline displacement rather than the zonal advection and surface heat flux due to the thermal damping. In this region, there is a mean upwelling into the mixed layer so, when $w'$ is negative, there is less upwelling and SST is anomalously high, while, when $w'$ is positive, the anomalous upwelling of water makes $T' < 0$. Thus, we write

$$T' = \lambda h',$$

where $\lambda$ is a positive constant and $h'$ is the thermocline perturbation (positive downward). Observations support (2). Figure 15 of Kessler (1990) shows that eastern equa-
Fig. 1. Zonal displacement of the equatorial 28.5°C SST isotherm (heavy solid line: a proxy of the east edge of the warm pool), zonal displacement of the equatorial −4 m s⁻¹ westerly surface isotach (dash-dot line), and zonal displacement of the 240 W m⁻² equatorial OLR isoline (thin solid line). The “equatorial” SST and the equatorial surface wind are actually an average from 4°N–4°S and the equatorial OLR an average from 5°N–5°S. The vertical dotted line is the mean position of the 28.5°C isotherm (172.2°W). Monthly values of SST, wind, and OLR have been smoothed with a double 5-month running mean filter. The correlation coefficients are 0.94 (the critical correlation coefficient at the 95% level \( r_{\text{crit95%}} = 0.44 \)) for SST and OLR isolines, 0.76 (\( r_{\text{crit95%}} = 0.38 \)) for OLR and wind isolines, and 0.73 (\( r_{\text{crit95%}} = 0.34 \)) for wind and SST isolines. Here and elsewhere we calculate the number of degrees of freedom for the critical correlation coefficient based on Davis (1976).

Fig. 2. (a) Correlation coefficient (solid line) between the equatorial observed thermocline depth anomaly and the observed equatorial SSTA during 1970–99. The correlation was calculated after filtering both time series with a double 5-month running mean. The dash-dot line shows the critical correlation coefficient at the 95% level. (b) The regression coefficient (solid line) between the time series in (a) calculated from the raw data. The dotted line shows the \( \lambda \) values used in Eq. (3).

The equatorial Pacific boundary 20°C isotherm depth (a proxy for \( h \)) is in phase with El Niño/La Niña SST changes. Consistent with this, eastern equatorial boundary sea level height, being in phase with thermocline depth, is also in phase with SST (Hickey 1975).

West of about 160°E the zonal gradient of the mean equatorial SST is small and, as for the region east of the warm pool edge, anomalous upwelling influences SST. Figure 2 shows evidence for this—the correlation of observed thermocline depth anomaly (TDA) and observed SSTA is significant at the 95% level west of about 155°E. Here the observed TDA is the 20°C isotherm depth anomaly calculated from monthly seawater temperature profile data from 1955–99 provided by the Joint Environmental Data Analysis (JEDA) Center at the Scripps Institution of Oceanography (White et al. 1988). The observed SSTA was calculated from the Comprehensive Ocean–Atmosphere Data Set (COADS) (January 1970–October 1981) and Reynolds and Smith (1994) (November 1981–December 1999) SST datasets.

It is necessary to generalize the SST anomaly equation so that a single equation applies right across the equatorial Pacific. We write

\[
\frac{\partial T}{\partial t} + \gamma T' = -u' \frac{\partial (T + T')}{{\partial x}} - \pi \frac{\partial T'}{{\partial x}} + \lambda \frac{\partial h'}{{\partial t}},
\]

where \( \lambda \) is a non-negative function of \( x \). The upwelling parameter \( \lambda \) is 0 near the edge of the warm pool (160°E–160°W) where advection dominates, takes the regression coefficient values in the west and east Pacific, and rises linearly in the central/eastern transition region between 160° and 100°W (see Fig. 2b). This choice of \( \lambda \) implies that (3) reduces to (1) in the 160°E–160°W region. We also have the balance (2) at the South American coast, since there \( u' \to 0, \pi \to 0 \), and the thermal damping term is also negligible, since there the total SST is usu-
ally less than 28.5°C; consequently equatorial ENSO
shading anomalies are insignificant.

c. Wind data and model parameters

To force the ocean model, the Florida State University
(FSU) “pseudo wind stress” (on the Web at http://
www.coaps.fsu.edu/WOCE/SAC/) was converted to the
actual wind stress by using a constant drag coefficient
\( c_D = 1.3 \times 10^{-3} \) and an air density \( \rho_a = 1.2 \, \text{kg m}^{-3} \).
The model domain is limited to the tropical Pacific
(30°N–30°S, 124°E–80°W). The horizontal resolution of
the ocean model is 0.5° latitude and about 3.2° longitude.
The time step is about one month/19 (or 1.58 days) for
both the dynamical model and the SST equation (3).
The eastern boundary is solid with no normal flow, and
the western boundary is such that the integrated mass
transport is zero (Cane and Sarachik 1977). The forced
ocean wave model is solved by the method of charac-
teristics. The details of the solution of (A15)–(A17) and
the boundary conditions are similar to those in Battisti
(1988).

The solution of the model equations (A15)–(A17) re-
quires knowledge of the stratification and other param-
eters. First, we calculated vertical mode eigenfunctions
and eigenvalues \( (c_n, b_n) \) using Eq. (A13) based on the buoy-
cy frequency values provided for the Pacific Ocean
by the National Oceanographic Data Center. The buoy-
cy frequency values were averaged along the equa-
torial waveguide (5°S–5°N) for the Pacific Ocean basin
(120°E–80°W). Then (A18) was used to calculate \( b_n \)
given \( H_{\text{mix}} = 50 \, \text{m} \). The mixed layer depth varies with
\( x \) along the equator, but 50 m is a representative value
(see Ando and McPhaden 1997) and the \( b_n \) are only
very weakly dependent on \( H_{\text{mix}} \). The vertical eigen-
functions for modes 1–4 are shown in Fig. 3 and other
calculated model parameters are listed in Table 1. In
Table 1, we set the first-mode damping scale to be 24
months, the same as that in Minobe and Takeuchi
(1995), by choosing \( A = 1.037 \times 10^{-7} \, \text{m}^2 \, \text{s}^{-3} \) in Eqs.
(A15)–(A17). With this choice, the second-mode damp-
ing timescale (8.26 months; see Table 1) is also close
to a reasonable second-mode damping timescale of
about 7 months or longer (see Table 2 of Reverdin
1987). The damping timescales listed in Table 1 will be
known as the standard model case hereafter.

3. Model results

a. Modeled and observed thermocline depth
anomalies

We now force the ocean with the 1961–99 FSU wind
stress anomalies and compare the modeled TDA with
that observed. The observed TDA is the 20°C isotherm
depth anomaly calculated from monthly seawater tem-
perature profile JEDA data. We define the model ther-
ocline depth anomaly as the depth anomaly at 75 m
since the observed mean 20°C depth is close to 75 m
in the eastern Pacific (east of 160°W) where the TDAs
cause the largest SSTAs in the equatorial Pacific. The
root mean square (rms) of the observed TDAs and the
modeled TDAs for modes 1, 2, 3, and 4 along the equa-
torial Pacific (Fig. 4a) show that, like sea level (Cane
1984), the TDAs are due to the first two baroclinic
modes. The contributions to TDA from modes 3 and 4
are negligible because of the much smaller damping
timescales and \( b_n \) (see Table 1). Henceforth in this study
only the two lowest vertical modes will be used to model
the TDAs.

The first two-mode model captures well the phase of
observed TDA across the Pacific and the amplitude in
the eastern Pacific (see Fig. 4). The observed TDA am-
plitude is underestimated in the central and western Pa-
icbecause model TDA is at 75 m, while the observed
mean 20°C depth in the central and western Pacific is
about 160 m. We could have allowed for this by calcu-
lating the TDA at deeper appropriate depths in the

![Fig. 3. Vertical eigenfunctions as a function of depth for the first four vertical modes.](image-url)
b. Modeled and observed zonal current anomaly

We next check the modeled zonal current anomalies (UAs) with those observed. The observed zonal current is from the Tropical Ocean Global Atmosphere (TOGA)–Tropical Atmosphere–Ocean (TAO) Array along the equator at 147°E, 156°E, 165°E, 170°W, 140°W, and 110°W. The UA monthly time series is the combination of fixed depth zonal current and ADCP profile data, usually at 10- or 15-m depth. In the cases when both data are available, the ADCP data are preferred. At 147°E and 170°W, ADCP data at 30 m are used to check the model since they correspond to the shallowest observation. The model UA is averaged over the top 50 m of the water column. Figure 5 shows the first two-mode model and observed zonal current anomaly time series at the above TAO moorings. Overall, the model well simulates observed UA in both phase and amplitude in the key region from 165°E to 140°W, where the zonal advection dominates the SSTA and thus displacements of the warm pool (Wang and McPhaden 2000). The correlation is poor at 156°E, but there the time series is very short so it is hard to assess model performance (see Fig. 5).

We checked how many vertical modes are needed to
model the observed zonal current using TAO observations from 165°E to 110°W. The results, summarized in Table 2, show that the single vertical mode model is improved by the inclusion of the second mode and that the model with the first two vertical modes gives the best overall correlation skill.

We also checked the first two-mode model UA with UA derived from TOPEX/Poseidon and ERS-1/2 (TPER) sea level data. Figure 6 shows that the model UA is well correlated with the TPER-derived UA including UA at 147°E and 156°E. However, the model amplitudes are bigger at most locations. The inclusion of more modes should improve the model, but Table 2 shows that the model amplitude is even larger then. There are at least two possible explanations for the amplitude discrepancy. One is that the satellite/sea-level-
The monthly SST climatology was obtained from the monthly SST climatology was obtained from the monthly SST climatology was obtained from the monthly SST climatology was obtained from the monthly SST climatology was obtained from

<table>
<thead>
<tr>
<th>Location</th>
<th>Model vs TAO</th>
<th>One mode</th>
<th>Two modes</th>
<th>Three modes</th>
<th>Four modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>165°E</td>
<td>Correlation ((r_{\text{cor}_1}))</td>
<td>0.58 (0.38)</td>
<td>0.80 (0.40)</td>
<td>0.83 (0.41)</td>
<td>0.84 (0.41)</td>
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<td></td>
<td>Regression  ((\sigma_{\text{model}}/\sigma_{\text{TAO}}))</td>
<td>0.18 ± 0.12</td>
<td>0.83 ± 0.31</td>
<td>1.04 ± 0.36</td>
<td>1.18 ± 0.38</td>
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<tr>
<td></td>
<td>Rms diff (cm s(^{-1}))</td>
<td>30</td>
<td>105</td>
<td>130</td>
<td>147</td>
</tr>
<tr>
<td>170°W</td>
<td>Correlation ((r_{\text{cor}_1}))</td>
<td>0.62 (0.44)</td>
<td>0.62 (0.46)</td>
<td>0.51 (0.55)</td>
<td>0.46 (0.56)</td>
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<td></td>
<td>Regression  ((\sigma_{\text{model}}/\sigma_{\text{TAO}}))</td>
<td>0.25 ± 0.18</td>
<td>0.98 ± 0.71</td>
<td>1.10 ± 1.60</td>
<td>1.16 ± 1.30</td>
</tr>
<tr>
<td></td>
<td>Rms diff (cm s(^{-1}))</td>
<td>18.01</td>
<td>14.65</td>
<td>17.88</td>
<td>20.73</td>
</tr>
<tr>
<td>140°W</td>
<td>Correlation ((r_{\text{cor}_1}))</td>
<td>0.39 (0.31)</td>
<td>0.60 (0.35)</td>
<td>0.55 (0.38)</td>
<td>0.51 (0.38)</td>
</tr>
<tr>
<td></td>
<td>Regression  ((\sigma_{\text{model}}/\sigma_{\text{TAO}}))</td>
<td>0.14 ± 0.12</td>
<td>0.88 ± 0.51</td>
<td>1.05 ± 0.75</td>
<td>1.12 ± 0.89</td>
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<td></td>
<td>Rms diff (cm s(^{-1}))</td>
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<td>14.31</td>
<td>19.44</td>
<td>23.37</td>
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<tr>
<td>110°W</td>
<td>Correlation ((r_{\text{cor}_1}))</td>
<td>0.50 (0.24)</td>
<td>0.36 (0.26)</td>
<td>0.31 (0.24)</td>
<td>0.30 (0.24)</td>
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<tr>
<td></td>
<td>Regression  ((\sigma_{\text{model}}/\sigma_{\text{TAO}}))</td>
<td>0.09 ± 0.05</td>
<td>0.35 ± 0.29</td>
<td>0.41 ± 0.36</td>
<td>0.44 ± 0.40</td>
</tr>
<tr>
<td></td>
<td>Rms diff (cm s(^{-1}))</td>
<td>9.68</td>
<td>11.60</td>
<td>14.40</td>
<td>15.73</td>
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<tr>
<td>Mean</td>
<td>Correlation ((r_{\text{cor}_1}))</td>
<td>0.52 (0.34)</td>
<td>0.60 (0.37)</td>
<td>0.55 (0.40)</td>
<td>0.53 (0.40)</td>
</tr>
<tr>
<td></td>
<td>Regression  ((\sigma_{\text{model}}/\sigma_{\text{TAO}}))</td>
<td>0.17 ± 0.12</td>
<td>0.76 ± 0.45</td>
<td>0.9 ± 0.69</td>
<td>0.98 ± 0.82</td>
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<td></td>
<td>Rms diff (cm s(^{-1}))</td>
<td>12.64</td>
<td>14.63</td>
<td>19.54</td>
<td>23.17</td>
</tr>
</tbody>
</table>

derived currents are more like an average estimate near the equator and have a lower amplitude than currents right at the equator. This is consistent with our results: model and observed amplitudes agree much better for the in situ TOGA–TAO current observations made right at the equator (see Table 2). A second possible explanation is that the model does not include nonlinear terms and therefore does not take into account the mean upwelling induced by the easterly surface trade winds (Philander and Pacanowski 1980). Since the model UA reverses direction near the surface for modes 2 and higher (see Fig. 3), the nonlinear term \(\varpi \partial u \partial z\), if included, would transfer opposite momentum from the subsurface to the surface and thus decrease the model surface zonal current. The UA amplitude discrepancy could also be due to inaccuracy in the wind stress forcing or some other weakness in the model.

We checked the dependence of the model results on damping timescales. Damping increases in the order case 1, standard case, case 2, and case 3 (see Table 3). In case 1 the first-mode damping scale (30 months) is the same as that for the other ENSO ocean models (Zeboik and Cane 1987; Battisti 1988; Kleeman 1993; Chen et al. 1995), while case 3 has first and second mode damping timescales similar to those suggested by Picaut et al. (1993). Overall (see Table 4) regression results are slightly better for case 1 and the standard case than case 2, which is better than case 3, the case of strongest damping. The differences were small in the first three cases, and we wondered if we could distinguish them by testing the models against the long TAO observed time series near the South American coast. We could not—all had correlations \(r = 0.8\).

c. Model and observed SST and zonal displacement of the warm pool

To calculate the SSTAs by using Eq. (3), we must specify some climatological functions and some parameters. The current climatology \(\overline{\varpi}\) is calculated from a forced model run with the climatological winds. We checked this model \(\varpi\) with that observed. The observed \(\varpi\) is constructed from observed buoy and moored current data during January 1987 to April 1992 (Reverdin et al. 1994). Figure 7 indicates that there are some major discrepancies between observed and model currents, showing that there must either be model error, error in estimating the observed flow, or both. Error in the observed flow is possible because the observations were based on only 5½ years of data and interannual currents are often larger and sometimes much larger than \(\varpi\). Since observed \(\varpi\) may be in error because of limited observations and modeled \(\varpi\) may be in error because of model weaknesses, we tried both in our model. Calculations showed that hindcast skill declined using the observed \(\varpi\) instead of the modeled one. Consequently, we will use model \(\varpi\) in our calculations. We note, however, that knowledge of accurate \(\varpi\) is not crucial to warm pool displacement results since it is not part of the main balance for the SSTA near the edge of the warm pool (see section 4a). This is in agreement with observations (Wang and McPhaden 2000) that show the term involving \(\varpi\) in the SSTA equation is not only not important near the edge of the warm pool but also at all equatorial Pacific locations (165°E, 170°W, 140°W, and 110°W) where long TAO current records are available.

The monthly SST climatology \(\overline{T}\) was obtained from...
**Fig. 6.** As in Fig. 5 but for the TPER-derived equatorial zonal current anomalies and the modeled zonal current anomalies at TAO mooring locations. The correlation coefficients are all significant at 95% level.

**Table 3.** Damping timescales $\lambda$ (in months) for cases with weaker and stronger damping than the standard case.

<table>
<thead>
<tr>
<th>Mode $n$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>30</td>
<td>10.23</td>
<td>4.38</td>
<td>1.16</td>
</tr>
<tr>
<td>Standard</td>
<td>24</td>
<td>8.26</td>
<td>3.50</td>
<td>0.93</td>
</tr>
<tr>
<td>Case 2</td>
<td>15</td>
<td>5.16</td>
<td>2.19</td>
<td>0.58</td>
</tr>
<tr>
<td>Case 3</td>
<td>7</td>
<td>2.41</td>
<td>1.02</td>
<td>0.27</td>
</tr>
</tbody>
</table>

the Climate Analysis Center (CAC) at the National Meteorological Center. To determine $\lambda$ in (3), we first calculated the correlation and regression coefficients between the model TDA and observed SSTA (see Fig. 8). The significant positive correlation in the eastern Pacific (east of 160°W) and the western Pacific (west of 160°E) suggests that the upwelling associated with thermocline displacement plays an important role in determining the SSTAs there. The higher regression
coefficient in the eastern Pacific shows that the SSTA in the eastern Pacific is much more sensitive to thermocline displacement than in the western Pacific. This is consistent with the much shallower thermocline in the eastern Pacific.

The insignificant correlation between model TDA and observed SSTA near the edge of the warm pool (160°E–160°W) suggests that the effect of thermocline-induced upwelling on SSTA is negligible there. As noted in section 2a, the mechanism generating equatorial SSTAs gradually changes from thermocline-induced upwelling at the South American coast to zonal advection in the central Pacific. Thus λ is taken to be 0 in the 160°E–160°W region (see Fig. 8b). East of 160°W, λ linearly increases to the regression coefficient between model TDA and observed SSTA at the South American coast [see (2) and Fig. 8b]. West of 160°E, λ is simply set to be the regression coefficient between model TDA and observed SSTA (see Fig. 8b). We tested the model with λ specified in the Fig. 8b and Fig. 2b cases. The model skill is slightly better for the λ from Fig. 8b and therefore, in the following calculations, we will use this λ.

We compared the observed SSTAs with those estimated by forcing the ocean model with the observed FSU wind stress. The following three SST indices were used: SSTAs in ship track 1 (4°–12°S, near 80°W), Niño-3 (5°S–5°N, 150°–90°W), and ship track 6 (4°S–4°N, near 170°W) (see Fig. 9). These indices represent, respectively, the SST along the equatorial South American coast, in the eastern equatorial Pacific, and near the eastern edge of the warm pool. As mentioned earlier in section 2, we took the thermal damping timescale γ⁻¹ to be 100 days over the warm pool where the total SST ≥ 28.5°C and 2500 days at other regions where total SST < 28.5°C. Since the warm pool moves, this implies that γ is a function of both x and t.

Figure 10 shows the time series of the model SSTAs and the observed SSTAs from CAC analysis for ship track 6, Niño-3, and ship track 1 regions. The correlation coefficients between the modeled and observed SSTAs (January 1970–December 1999) for the ship track 6, Niño-3, and ship track 1 regions are 0.83 (τw=99% = 0.85), 0.88 (τw=99% = 0.52), and 0.77 (τw=99% = 0.52), and the rms errors are 0.42°C, 0.50°C, and 0.78°C, respectively. The simulated SSTAs agree well with those observed both in phase and amplitude in all three regions. Note that the results hardly change if we let γ be a constant with γ⁻¹ between 2–4 months or if we let γ⁻¹ be a reasonable function of x alone by, for example, setting it to be 100 days west of 160°W and 2500 days east of 140°W with linearly interpolated values between these longitudes.

The 28.5°C SST isotherm averaged over 4°S–4°N is commonly used to denote the eastern edge of the warm pool. The observed 28.5°C isotherm position time series was calculated by linear interpolation of SST grid values based on COADS (January 1970–October 1981) and the Reynolds and Smith (1994) (November 1981–December 1999) SST dataset. The model 28.5°C isotherm displacement was found by linear interpolation of the total model SST from the sum of SST climatology and model SSTA. Overall, the model 28.5°C isotherm position agrees well with observation (see Fig. 11).

So far we have shown that the model is able to simulate quite well the ENSO equatorial TDA, UA, and SSTAs in both the central and eastern Pacific as well as zonal displacements of the 28.5°C isotherm (a proxy
for the eastern edge of the warm pool). These results suggest that we can use the model to examine the dynamics of the warm pool movement.

4. Dynamics of the warm pool movement

a. Examination of the SSTA equation

Figure 12 shows rms values of the zonal advection $\left(\frac{\partial (T + T')}{\partial x}\right)$, damping $(yT')$, upwelling $\left(\frac{\partial h'}{\partial t}\right)$, and the SSTA tendency $\left(\frac{\partial T'}{\partial t}\right)$ term in Eq. (3) based on model time series during 1970–99. West of about 155°E zonal advection, upwelling, and damping all contribute to the SSTA tendency but between 160°E and 160°W zonal advection and damping dominate. East of the warm pool damping is not important along the equator, so east of about 150°W SSTA change is mainly due to upwelling and zonal advection. Zonal advection dominates near 150°W, but upwelling becomes increasingly important farther east until at 110°W upwelling dominates. These equatorial balances are in approximate accord with those recently found in the observations by Wang and McPhaden (2000) at 160°E, 170°W, 140°W, and 110°W. The main
Fig. 8. As in Fig. 2 but for the equatorial model thermocline depth anomaly and the observed equatorial SSTA.

Fig. 9. The location of the three SST indices: ship track 1 (4°–12°S, near 80°W), Niño-3 (5°S–5°N, 150°–90°W), and ship track 6 (4°S–4°N, near 170°W).

Fig. 10. (a) The observed (solid line) and modeled (dash-dot line) SSTA for ship track 6 during 1970–99. A double 5-month running mean has been applied to both curves. The correlation coefficients and rms errors between the observed and modeled SSTA are shown above each panel. (b) and (c) are as (a) but for the Niño-3 and ship track 1 regions, respectively.

Differences were that Wang and McPhaden found that upwelling was not negligible at 170°W and, while zonal advection dominated over upwelling at 140°W, it was not as great as in the model. Similarly, while at 110°W upwelling was the main cause of SSTA, zonal advection was not negligible.

Now we focus on the SSTA near the edge of the warm pool (160°E–160°W). There λ = 0, and the SSTA equation (3) reduces to

$$\frac{\partial T'}{\partial t} = -\gamma T' - u' \frac{\partial T'}{\partial x} - u \frac{\partial T'}{\partial x} - \frac{\partial T'}{\partial x},$$

where the local change in SSTA (term A) is determined by Newtonian damping (term B), mean temperature advection by anomalous zonal current (term C), anomalous temperature advection by anomalous zonal current (term D), and anomalous temperature advection by mean zonal current (term E). Based on the simulation of the SSTA for 4°S–4°N, 160°E–160°W during 1970–99, the rms values for terms (A), (B), (C), (D), and (E) are 0.11, 0.16, 0.12, 0.06, and 0.09 (°C month^{-1}), respectively. While it seems that every term makes a contribution, the SSTA tendency (term A) is mainly dominated by (B + C) (see Fig. 13). So we write

$$\frac{\partial T'}{\partial t} + \gamma T' = -u' \frac{\partial T}{\partial x}$$

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Fig. 11. The observed and modeled zonal displacements of the equatorial 28.5°C isotherm averaged over 4°S–4°N. Both time series have been filtered by a double 5-month running mean. The correlation coefficient between the time series is 0.84 ($r_{\text{crit95}} = 0.50$).

Fig. 12. The rms of SSTA tendency $\partial T'/\partial t$ (solid line), zonal advection $u' [\partial T'/\partial x + T'/\partial x] = \pi \partial T'/\partial x$ (dash–dot line), upwelling $\lambda \partial h'/\partial t$ (dashed line) and damping $\gamma T'$ (dotted line) terms in the equatorial Pacific (4°S–4°N) based on the model time series during 1970–99.

Fig. 13. The model SSTA tendency $\partial T'/\partial t$ (solid line) and zonal advection of mean SST by anomalous zonal currents plus a Newtonian damping $-u' \partial T'/\partial x - \gamma T'$ (dash–dot line) near the edge of the warm pool (4°S–4°N, 160°E–160°W). The correlation coefficient is 0.61 ($r_{\text{crit95}} = 0.44$) after both time series are filtered by a double 5-month running mean.

If there were no Newtonian damping ($\gamma = 0$), $T'$ would lag $u'$ by a quarter period or several months for ENSO periodicity. Physically, it takes that long for particles to be advected by the anomalous current and cause anomalous SST. But with Newtonian damping included, the lag is shorter. By Taylor series expansion we have, approximately,
The error in omitting the higher-order terms is order $\omega^2 / (2\gamma^2)$ where $\omega$ is a typical ENSO frequency. This error is less than 15% for $\gamma^{-1} \sim 100$ days and $\omega^{-1} \sim 42$ months/2$\pi$. Thus, (5) may be approximately written

$$T'(t + \frac{1}{\gamma}) = T'(t) + \frac{1}{\gamma} \frac{\partial T'(t)}{\partial t}. \quad (6)$$

The error in omitting the higher-order terms is order $\omega^2 / (2\gamma^2)$ where $\omega$ is a typical ENSO frequency. This error is less than 15% for $\gamma^{-1} \sim 100$ days and $\omega^{-1} \sim 42$ months/2$\pi$. Thus, (5) may be approximately written

$$T'(t + \frac{1}{\gamma}) = \frac{\partial T'(t)}{\partial x} \gamma. \quad (7)$$

and $T'$ lags $u'$ by $\gamma^{-1} \sim 3$ months near the edge of the warm pool. This is consistent with the observation that the peak lag correlation ($r = 0.71$) between observed SSTA and TPER-derived zonal current anomalies in the region 160$^\circ$E–160$^\circ$W occurs at $u'$ leading $T'$ by 3 months (see Fig. 14).

b. Examination of the dynamics of the zonal current anomaly

So far we have shown that the SSTA near the warm pool edge and the associated displacement of the warm pool is mainly due to the zonal advection of mean background SST by the anomalous zonal current. But what causes the zonal current anomaly? We will examine the cause of the zonal current anomaly using the composite technique of Rasmusson and Carpenter (1982).

Since the zonal current anomaly is forced by the zonal equatorial wind stress anomaly, we first composite the anomalous zonal wind stress over the last eight El Niños as in Clarke and Shu (2000) (see Fig. 15a). The time within the composite is labeled as “Month(Yr)” relative to the El Niño year Yr(0). Yr(−1) and Yr(+1) are the year before and the year after the El Niño year, respectively. For the last eight El Niños, Yr(0)s are taken to be: 1965, 1969, 1972, 1976, 1982, 1987, 1991, and 1997. Thus, the Jul(0) composite is computed by averaging the anomalous July values from 1965, 1969, . . . , 1997. Although the 1982 El Niño propagated eastward and was the biggest of the twentieth century, it is incorrect to conclude that the eastward propagation of westerly wind stress seen in Fig. 15a is due to bias by this huge event. Figures 15b–e show that the calendar year phase locking and eastward propagation are similar for all the major El Niños (1972, 1982, 1997) during the record and also for a composite of the other weaker ones (1965, 1969, 1976, 1987, 1991).

This eastward propagation of westerly wind stress anomalies is accompanied by the calendar year phase-locked eastward propagation of negative OLR anomalies representing anomalous deep atmospheric convection [see Fig. 16 and Wang and Weisberg (2000)] as well as an eastward movement of the warm pool (see Figs. 1 and 17). The surface wind anomaly is eastward and slightly to the west of the maximum anomalous heating in agreement with numerical computations and physical arguments (Clarke 1994).

Gill and Rasmusson (1983) suggested that the eastward movement was due to a coupled ocean–atmosphere instability. They proposed that if there is anomalous deep atmospheric convection, then this will cause westerly winds west of the convection zone. These winds will drive the warm pool eastward and will cause the convection region above it to also migrate eastward, resulting in farther eastward displacement, etc. Gill and Rasmusson could not say what initiated the feedback process and did not identify where and when it begins. Recent analysis (Clarke and Shu 2000) suggests that it typically begins in the far-western equatorial Pacific around November. There the winds are phase locked to the calendar year with quasi-biennial (2-yr) periodicity.

Various negative feedbacks limit the growth of the above coupled instability. The delayed oscillator theory, a leading theory for ENSO, suggests that the nonlinear flow reflected from the western boundary is a critical negative feedback. Specifically, the westerly winds generate long equatorial Rossby waves that propagate to the western Pacific boundary, reflect as equatorial Kelvin waves, and, after the appropriate wave propagation delay, reach the edge of the warm pool. The reflected equatorial Kelvin waves are associated with a westward flow tending to return the warm pool to its original position; that is, there is delayed negative feedback. The delayed oscillator theory was originally proposed by Schopf and Suarez (1988) and has been widely discussed by Battisti (1988), Battisti and Hirst (1989), Clarke (1991), du Penhoat and Cane (1991), Li and Clarke (1994), Mantua and Battisti (1994), Kessler and McPhaden (1995), Boulanger and Fu (1996), and Clarke et al. (2000). However, Delcroix et al. (1994) and Boulanger and Menkes (1995) document an example of the failure of Rossby wave reflection at the western Pacific boundary as a negative feedback mechanism. They not-

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**Fig. 14.** The lag correlation coefficients between the observed SSTA and TPER-derived zonal current anomalies near the edge of the warm pool (160$^\circ$E–160$^\circ$W). Negative time means that $u'$ leads $T'$. The correlation peak ($r = 0.71$) occurs with $u'$ leading $T'$ by 3 months.
ed that most of the returning negative feedback by the equatorial Kelvin wave seems to be forced by an easterly wind anomaly located in the western Pacific, rather than by reflection of an upwelling first Rossby wave at the western boundary. Consistent with this, observation also shows that the negative (easterly) phase of the quasi-biennial wind stress begins in the western equatorial Pacific near the end of the El Niño year [see Fig. 15 and Clarke and Shu (2000)]. This will generate Kelvin waves with a westward current that will tend to advect the warm pool back toward its equilibrium position. In addition, Picaut and Delcroix (1995), Boulanger and Fu (1996), and Picaut et al. (1997) suggest that delayed negative feedback can also occur because of ocean wave reflection at the eastern ocean boundary. Equatorial Kelvin waves directly generated by the wind forcing propagate eastward to the eastern ocean boundary and reflect as equatorial Rossby waves with westward equatorial currents. These currents act to advect the warm pool westward toward its undisturbed position.

In summary, the main proposed negative feedbacks tending to restore the warm pool back to its original position seem to be the nonlocal flow associated with the remote far western Pacific wind forcing and the reflection of equatorial Rossby and Kelvin waves from the western and eastern equatorial Pacific boundaries.

Before examining those negative feedbacks associated with the zonal current anomaly, we will first compare the model UA composite with the warm pool displacement. The anomalous eastward currents advect the warm pool eastward from December in Yr(−1) to February in Yr(+1) and then westward for the rest of Yr(+1) (see Fig. 17). Consistent with Eq. (7), the phase of UA leads that of the warm pool displacement (i.e., SSTA in the central Pacific) by about three months. In addition, the balance in (7) enables us to understand why the eastward propagating coupled ocean–atmosphere disturbance begins at about 160°E (see Figs. 15 and 16). The mechanism requires the eastward advection of warm water, which, by (7), can only occur when $\frac{\partial T}{\partial x}$ is sufficiently negative. Figure 18 shows that $T$ decreases east of about 160°E, suggesting that the eastward propagation should start about there.

The model UA at any point is a sum of the UA fields of equatorial Kelvin and Rossby waves (see, e.g., Gill and Clarke 1974; Clarke and Liu 1993). For example, in the crucial region at the edge of the warm pool (160°E–160°W), for each vertical mode the equatorial Kelvin wave UA field is due to the equatorial Kelvin wave forced by the zonal wind stress between the western boundary and the warm pool edge plus the freely propagating damped equatorial Kelvin wave generated by Rossby wave reflection at the western boundary. Similarly, the Rossby UA field consists of the Rossby waves forced by the zonal wind stress.
between the eastern boundary and the edge of the warm pool plus the freely propagating damped equatorial Rossby waves generated by equatorial Kelvin wave reflection at the eastern boundary. Since the zonal wind stress anomalies east of the warm pool edge tend to be negligible to those over it, the Rossby wave contribution is mainly due to the damped waves generated at the eastern boundary. Figure 19a shows UA due to the waves generated at the western and eastern boundaries plus UA only forced by wind in the far west Pacific (west of 160°E). Comparing Fig. 19a with Fig. 17a, one sees that the strong eastward zonal currents during the second half of Yr(0) in Fig. 17a are mostly due to the local wind forcing in the central Pacific. The nonlocal westward currents oppose the local eastward current near the end of Yr(0) and reverse it during the first half of Yr(+1). By (7), this westward current will result in a negative SSTA in about three months, that is, near the middle of the Yr(+1). Associated with this change in sign of the SSTA near the edge of the warm pool, the wind shifts from westerly to easterly (see Fig. 15) and El Niño conditions change to those for La Niña. The nonlocal currents are responsible for this phase transition.

Figures 19b–d show the contributions to nonlocal UA. The equatorial Kelvin wave generated at the western boundary (Fig. 19b) can be seen to propagate across the basin, especially toward the end of Yr(0) and the first half of Yr(+1). Similarly, Rossby waves generated at the eastern boundary (Fig. 19c) propagate westward across the basin, especially during the last half of Yr(0) and most of Yr(+1). The Rossby waves decay as they propagate, so that by the time they reach the edge of the warm pool, boundary-generated equatorial Kelvin and Rossby waves make a similar contribution to the UA field (see the standard 4°S–4°N El Niño case in Table 5). Comparison of Figs. 19b–d shows that nonlocal UA is mostly due to boundary wave reflection rather than wind forcing in the western equatorial Pacific. The sizable contribution from Rossby waves generated at the eastern boundary is consistent with the results of Picaut et al. (1997) and the TOPEX/Poseidon data analysis and linear model research by Delcroix et al. (2000), who have suggested that the Rossby waves reflected from eastern boundary play a role in terminating warm events.

The La Niña case (see Figs. 20 and 21) is similar to the El Niño case one but of opposite sign. One difference in the total UA fields (cf. Fig. 20 with Fig. 17) is that in the La Niña case the currents advecting the warm pool back to its original position in Yr(+1) are much weaker. This means that the nonlocal currents due to the reflected waves and the far western Pacific forcing (see Fig. 21a and Fig. 20a) are only slightly stronger than the locally generated currents and occur during most of Yr(+1). As in the El Niño case, the nonlocal currents are mostly due to the reflected waves from
Fig. 19. (a) As for the El Niño composite in Fig. 17a, but for wave reflection from the western and eastern ocean boundaries and UA due to far western Pacific (west of 160°E) wind forcing. (b), (c), and (d) as in (a) but for UA due to wave reflection from western ocean boundary, wave reflection from the eastern ocean boundary, and the far western Pacific wind forcing alone, respectively. The small arrows in (b) and (d) are eastward near the end of Yr(-1)/beginning of Yr(0) and westward at the beginning of Yr(+1).

Table 5 shows that, near the edge of the warm pool, negative feedback current anomalies due to western boundary reflection are greater than those associated with eastern boundary reflection if we average over 8°S–8°N instead of 4°S–4°N. The wider latitudinal average is closer to the total transport estimated by linear theory. Table 5 also shows that the negative feedback due to western boundary reflection is increasingly important the stronger the dissipation. Physically, this is due to the warm pool being closer to the western boundary, so the western-boundary-reflected waves do not have to travel as far as those reflected from the eastern boundary.

In summary, our model results suggest that the reflections from both the eastern and western boundaries are usually responsible for the reduction of the instability growth and the eventual termination of the warm and cold ENSO events. The wind forcing in the far western Pacific (west of 160°E) typically plays a small role in the negative feedback.

5. Conclusions

An equatorial ocean model with the Kelvin and first six Rossby waves and n vertical modes was developed to simulate the SSTA associated with ENSO episodes. As in McCreary (1981), the Rayleigh damping coeffi-
The linearized model equations for a stratified ocean of mean density \( \bar{\rho}(z) \) on the equatorial \( \beta \) plane are

\[
\begin{align*}
\left( u_x - \beta v_y + \frac{p_x}{\rho_0} \right) &= \left( \nabla \bar{\rho} \right)_x + F_x \\
\left( v_x + \beta u_y + \frac{p_y}{\rho_0} \right) &= \left( \nabla \bar{\rho} \right)_y + F_y \\
\rho_i + \omega p_z &= (\kappa \rho_i)_z
\end{align*}
\]

where \( \beta = \frac{d\bar{\beta}}{dz} \) is the meridional gradient of the Coriolis parameter, \( \kappa \) is the meridional gradient of the stratification, \( \rho_i \) is the density anomaly, and \( \omega \) is the vertical velocity.

The boundary conditions are

\[
\begin{align*}
\nu u_x &= \nu v_x = 0, \\
\nu w &= \eta = \frac{p_x}{\rho_0 g}, \\
\rho_i + \omega p_z &= (\kappa \rho_i)_z = 0 \\
\end{align*}
\]

at \( z = 0 \) (A6)

\[\nu u_x = \nu v_x = w = (\kappa \rho_i)_z = 0 \quad \text{at} \quad z = -H, \]

where \( u, v, \) and \( w \) are the zonal, meridional, and vertical velocity anomalies, respectively, \( \rho_i \) and \( \rho \) are the pressure and density anomalies, \( g \) is the acceleration due to gravity, \( \rho_0 \) is the reference density, and \( \eta \) is sea level perturbation. The body force vector \((F_x, F_y)\) is explicitly given by

\[
(F_x, F_y) = \begin{cases} 
\left( \tau^x, \tau^y \right) / \rho_0 H_{\text{mix}} & \text{for } -H_{\text{mix}} \leq z \leq 0 \\
0 & \text{for } z < -H_{\text{mix}}.
\end{cases}
\]

Here \( \tau^x \) and \( \tau^y \) are the eastward and northward surface wind stress and \( H_{\text{mix}} \) is the surface mixed layer thickness.

Following McCreary (1981), eddy coefficients are assumed to be

\[
\nu = \kappa = A / N^2, \quad \text{and} \quad \lambda = \frac{g \bar{\rho}}{\rho_0},
\]

where \( A \) is a positive constant and \( N \) is the Brunt–Väisälä frequency.
FIG. 20. (a) Average model equatorial surface $U_A$ from 4°S to 4°N for the La Niña year [Yr(0)] and the years before [Yr(−1)] and after [Yr(+1)] the La Niña year. The average is based on the five most recent La Niña events (1973, 1975, 1984, 1988, and 1995) and $U_A$ time series filtered by a double 5-month running mean. (b) is as for (a), but for the La Niña equatorial 28.5°C isotherm composite (solid line). The vertical dash–dot line is the mean position of the 28.5°C isotherm.

these present changes should not greatly alter the basic dynamical balances. Eliminating $w$ and $\rho$ from (A3), (A4), and (A5) and using (A9) and (A10), we get

$$u_z + v_y + \frac{1}{\rho_0} \left( \frac{(A_p I_N^2)}{N^2} - \frac{p_s}{N^2} \right) = 0. \quad (A11)$$

Using (A4), (A5), (A9), and (A10) with the boundary conditions (A6) and (A7) enables the boundary conditions for $p$ to be rewritten:

$$\frac{p_s}{N^2} + \frac{p_s}{g} = 0 \quad \text{at} \quad z = 0 \quad \text{and} \quad (A12a)$$

$$p_s = 0 \quad \text{at} \quad z = -H. \quad (A12b)$$

Now we express the $z$-dependent part of the solution of equations (A1), (A2), and (A11) in terms of the eigenfunctions $F_n(z)$, each $F_n$ satisfying the Sturm–Liouville problem:

$$\left( \frac{F_n}{N^2} \right) + \frac{F_n}{c_n^2} = 0 \quad (A13a)$$

$$\frac{F_n}{N^2} + \frac{F_n}{g} = 0 \quad \text{at} \quad z = 0 \quad (A13b)$$

$$F_n = 0 \quad \text{at} \quad z = -H, \quad (A13c)$$

where $c_n$ is the phase speed of vertical mode $n$. The vertical modes form a complete set of eigenfunctions, so we may write

$$u = \sum_{n=1}^{\infty} u_n F_n, \quad v = \sum_{n=1}^{\infty} v_n F_n, \quad \frac{p}{\rho_0} = \sum_{n=1}^{\infty} p_n F_n,$$

$$X_n = \sum_{n=1}^{\infty} X_n F_n, \quad Y_n = \sum_{n=1}^{\infty} Y_n F_n \quad (A14)$$

where the expansion coefficients $u_n$, $v_n$, $p_n$, $X_n$, and $Y_n$ are functions only of $x$, $y$, and $t$. The barotropic mode ($n = 0$) flow and pressure fluctuations are negligible, so the sums in (A14) begin at $n = 1$. It follows from (A13b,c) that the boundary conditions (A12) are satisfied. Substituting (A14) into (A1), (A2), and (A11) and using (A9), (A13), and the orthogonality condition for eigenfunctions, we get

$$\left( \frac{\partial}{\partial t} + \frac{A}{c_n^2} \right) u_n - \beta y v_n + p_{nx} = X_n \quad (A15)$$

$$\frac{A}{c_n^2} v_n + \beta y u_n + p_{ny} = Y_n \quad (A16)$$

$$\left( \frac{\partial}{\partial t} + \frac{A}{c_n^2} \right) p_n + u_{nx} + v_{ny} = 0, \quad (A17)$$

where

$$X_n = \frac{b_n}{\rho_0}, \quad Y_n = \frac{b_n}{\rho_0} \quad \text{and} \quad (A18)$$

$$b_n = \frac{1}{H_{\text{max}}} \int_{-H_{\text{max}}}^{0} F_n \, dz.$$

Note that the long-wave approximation has been made in (A16) to filter out the short or slow Rossby waves. As in McCreary (1981), Eqs. (A15)–(A17) are mathematically equivalent to the ENSO ocean model equations of Zebiak and Cane (1987), Battisti (1988), and Kleeman (1993) except that they only considered a single vertical mode and have an additional surface frictional layer. Also, in our case the Rayleigh damping coefficient $(A/c_n^2)$ depends on the vertical mode number,
whereas in Cane and Patton (1984) and Chen et al. (1995) the Rayleigh damping coefficient is constant.

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