Localization of Multidecadal Variability. Part II: Spectral Origin of Multidecadal Modes

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ABSTRACT

In a companion paper, the authors have shown that in an idealized Atlantic-Pacific Ocean configuration with a conveyor-type overturning circulation, localized multidecadal variability occurs in the Atlantic. Results suggest that the multidecadal variability originates from the instability of the three-dimensional thermohaline circulation and that the physics of the spatial patterns of the SST anomalies can be understood from a study of an Atlantic-only configuration. Specific internal (multidecadal) modes, which obtain a positive growth factor depending on the background thermohaline flow, are associated with the instability. In this paper, the spectral origin of these internal modes is studied using eigensolution continuation techniques. As in the single-hemispheric case, multidecadal modes arise through mergers of so-called SST modes. In the double-hemispheric case studied here, there actually are two types of multidecadal modes that lead to oscillatory behavior. Depending on the background conditions, one of these oscillatory flows is preferred.

1. Introduction

When the linear stability is considered for a background steady ocean flow, usually a set of normal modes appears (Pedlosky 1987). An instability of such a background flow is then associated with at least one of the normal modes having a positive growth factor. This is realized if properties of the background flow, for example, the horizontal or vertical shear, exceed some critical value. If such a normal mode has a nonzero frequency, it is called an oscillatory mode and the instability of the background flow is said to occur through a Hopf bifurcation. If a normal mode has zero frequency, it is called stationary and the instability is associated (generically) with a saddle node, pitchfork, or a transcritical bifurcation (Dijkstra 2005).

Often the normal modes derived from a linear stability problem of the nontrivial background flow are related to the normal modes associated with the linear stability of the motionless (trivial) state. For example, for single- or double-gyre flows in rectangular basins within barotropic quasigeostrophic models, so-called Rossby basin modes obtain a positive growth factor when the horizontal shear (locally) exceeds some critical value (Sheremet et al. 1997; Chang et al. 2001). Sometimes, however, new types of normal modes appear that have no counterpart in the linear stability problem of the motionless flow. An example of such a normal mode is the gyre mode in the double-gyre wind-driven ocean circulation problem. This gyre mode arises because of a merger between two stationary normal modes that occurs through the modification of the double-gyre background flow (Simonnet and Dijkstra 2002).

With this in mind, we now turn to the linear stability problem of the three-dimensional thermally driven ocean circulation in idealized basins (Greatbatch and Zhang 1995; Chen and Ghil 1995; Huck et al. 1999; Te Raa and Dijkstra 2002; Kravtsov and Ghil 2004). Under restoring boundary conditions for temperature the flow is stable, but under prescribed flux conditions oscillatory behavior occurs. In Te Raa and Dijkstra (2002), it was demonstrated that this instability is associated with a Hopf bifurcation and that a particular oscillatory mode, a so-called multidecadal mode (MM), obtains a positive growth factor. For these flows, we may again ask whether the MM is a modification of free normal modes or whether it is a new type of normal mode.

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In this case, the momentum equations are approximately diagnostic and the only normal modes of the motionless flow are eigenfunctions of the linear diffusion equation for temperature. These stationary modes were referred to as SST modes (Dijkstra 2006) for flows in a single-hemispheric configuration. It was demonstrated that, when the background state becomes non-motionless through thermal forcing, some of the SST modes merge and lead to the oscillatory circulation. The growth factor of this mode increases with increasing equator-to-pole temperature difference. The mode mergers appear to be a generic dynamical mechanism leading from normal modes associated with the motionless flow to those associated with the full three-dimensional overturning circulation.

In von der Heydt and Dijkstra (2007, hereinafter Part I) oscillatory behavior on multidecadal time scales was found in both double-hemispheric and double-basin (idealized Atlantic–Pacific Ocean) configurations. It can be expected that this variability also arises through a Hopf bifurcation once a critical forcing strength of the background flow (e.g., controlled by the meridional temperature difference $\Delta T$) is exceeded. This suggests that, again, an instability of the flow is causing the oscillatory behavior. In this paper, we study the linear stability of thermally driven flows in double-hemispheric and double-basin configurations and focus on the problem of the spectral origin of the normal modes that obtain a positive growth factor at sufficiently large $\Delta T$.

In section 2, the new version of our fully implicit ocean model (De Niet et al. 2007) used in this work is briefly presented. The normal modes for the thermally driven three-dimensional circulation in an equatorially symmetric double-hemispheric single basin are presented in section 3. We show that, because of the equatorial symmetry of this configuration, there exist two types of multidecadal modes. The temperature anomaly of one of these modes is equatorially symmetric and the temperature anomaly of the other is anti-symmetric. Both modes arise through SST mode mergers in a slightly more complicated way than for the single-hemispheric case (Dijkstra 2006). In section 3, also the oscillatory transient flows for realistic $\Delta T$ are studied. In section 4, we consider the normal modes in a double-hemispheric single basin with an open southern channel and those in the idealized Atlantic–Pacific configuration used in Part I. A summary and discussion of the results follow in section 5.

2. Formulation

We consider flows in a spherical sector bounded by longitudes $\phi_w$ and $\phi_e$ and by latitudes $\theta_i$ and $\theta_o$. The ocean basin has a constant depth $D$ and is bounded vertically by $z = -D$ and a nondeformable ocean–atmosphere boundary at $z = 0$. The flows in this domain are forced only by a heat flux $Q_H$ ($\text{W m}^{-2}$), which is proportional to the temperature difference between the sea surface temperature $T$ and a prescribed atmospheric temperature $T_S$; that is,

$$Q_H = -\lambda_T (T - T_S),$$

where $\lambda_T$ ($\text{W m}^{-2}\text{K}^{-1}$) is a constant exchange coefficient. The thermal forcing is distributed as a body forcing over the first (upper) layer of the ocean having a depth $H_m$. As discussed in Haney (1971), the boundary condition (1) is a strong idealization of the atmospheric processes acting to produce the net heat flux at the ocean surface.

Temperature differences in the ocean cause density differences according to

$$\rho_t = \rho_0 [1 - \alpha_T (T - T_0)],$$

where $\alpha_T$ is the volumetric expansion coefficients and $T_0$ and $\rho_0$ are a reference temperature and density.

a. Ocean model

We use the Boussinesq and hydrostatic approximations and neglect inertia in the momentum equations. With $r_0$ and $\Omega$ being the radius and angular velocity of the earth, the governing equations for the zonal, meridional, and vertical velocity $u$, $v$, and $w$ and the dynamic pressure $p$ (the hydrostatic part has been subtracted) become

$$-2\Omega v \sin \theta + \frac{1}{\rho_0 C_p} \frac{\partial p}{\partial \phi} = A_v \frac{\partial^2 u}{\partial z^2} + A_H L_w(u, v),$$

$$2\Omega u \sin \theta + \frac{1}{\rho_0 C_p} \frac{\partial p}{\partial \phi} = A_v \frac{\partial^2 v}{\partial z^2} + A_H L_w(u, v),$$

$$\frac{\partial p}{\partial z} = \rho_0 \alpha_T T,$$

$$\frac{\partial w}{\partial z} + \frac{1}{r_0 \cos \theta} \frac{\partial}{\partial \phi} \left[ \frac{\partial u}{\partial \phi} + \frac{\partial (v \cos \theta)}{\partial \theta} \right] = 0,$$

$$\frac{DT}{dt} - K_v \frac{\partial^2 T}{\partial z^2} - K_H \nabla^2 T = \frac{(T_S - T)}{\tau_T} g(z),$$

where $g(z) = H(z/H_m + 1)$ and $H$ is a continuous approximation of the Heaviside function. Furthermore, $C_p$ is the constant heat capacity and $\tau_T = \rho_0 C_p H_m / \lambda_T$ is the surface adjustment time scale of heat. In addition,
Table 1. Values of fixed parameters used in the numerical calculations in sections 3 and 4.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2\Omega$</td>
<td>$1.4 \times 10^{-4} \text{s}^{-1}$</td>
</tr>
<tr>
<td>$C_v$</td>
<td>$4.2 \times 10^{-1} \text{kg K}^{-1}$</td>
</tr>
<tr>
<td>$T_0$</td>
<td>15.0 K</td>
</tr>
<tr>
<td>$\rho_w$</td>
<td>$1.0 \times 10^3 \text{kg m}^{-3}$</td>
</tr>
<tr>
<td>$K_v$</td>
<td>$1.0 \times 10^4 \text{m}^2 \text{s}^{-1}$</td>
</tr>
<tr>
<td>$A_H$</td>
<td>$2.5 \times 10^5 \text{m}^2 \text{s}^{-1}$</td>
</tr>
<tr>
<td>$r_0$</td>
<td>$6.4 \times 10^6 \text{m}$</td>
</tr>
<tr>
<td>$\tau_F$</td>
<td>75.0 days</td>
</tr>
<tr>
<td>$A_y$</td>
<td>$1.0 \times 10^{-3} \text{m}^2 \text{s}^{-1}$</td>
</tr>
<tr>
<td>$A_z$</td>
<td>$1.0 \times 10^{-3} \text{m}^2 \text{s}^{-1}$</td>
</tr>
<tr>
<td>$g$</td>
<td>9.8 m s$^{-2}$</td>
</tr>
</tbody>
</table>

In (3a)–(3d) $A_H$, $K_H$ and $A_V$, $K_V$ are the horizontal and vertical momentum (eddy) viscosity and diffusivities, respectively, which we will take as constant. Inertial terms have been neglected in (3a)–(3b) because of the small Rossby number for the large scales of interest. Convective adjustment is modeled through an increase in the local vertical mixing coefficient in regions where the stratification is statically unstable.

Slip conditions are assumed at the bottom boundary, while at all lateral boundaries no-slip conditions are applied. At the bottom boundary and all lateral boundaries the heat flux is zero. As the forcing is represented as a body force over the first layer, slip and no-flux conditions apply at the ocean surface. Hence, the boundary conditions are

$$z = -D, \quad 0 \quad \frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = w = \frac{\partial T}{\partial z} = 0,$$

$$\phi = \phi_e, \quad \phi_e \quad u = v = w = \frac{\partial T}{\partial \phi} = 0,$$

$$\theta = \theta_s, \quad \theta_s \quad u = v = w = \frac{\partial T}{\partial \theta} = 0.$$  

The ocean model thus obtained (De Niet et al. 2007) is referred to below as the thermohaline circulation model (THCM).

Parameters that are fixed in the calculations described in section 3 are the same as in typical large-scale low-resolution ocean general circulation models, and their values are listed in Table 1. We choose a relatively large value of $K_v = 10^4 \text{m}^2 \text{s}^{-1}$ to obtain a reasonable meridional overturning strength at realistic $\Delta T$. The boundary condition (1) is only used to generate the background steady flow in the model.

b. Numerical implementation

The equations are discretized in space using a second-order accurate control volume discretization method on a staggered Arakawa B grid in the horizontal with $i = 1, \ldots, N_i$, $j = 1, \ldots, N_j$, and a C grid in the vertical $k = 1, \ldots, N_z$; this combination is called a Lorenz grid. The spatially discretized model equations can be written in the form

$$\mathbf{M} \frac{d\mathbf{u}}{dt} = \mathbf{L}(\mathbf{u}) + \mathbf{N}(\mathbf{u}, \mathbf{u}),$$

where the vector $\mathbf{u}$ contains the unknowns $(u, v, w, \rho, T)$ at each grid point and hence has dimension $d = 5 \times N_i \times N_j \times N_z$. The operators $\mathbf{M}$ and $\mathbf{L}$ are linear and $\mathbf{N}$ represents the nonlinear terms in the equations.

Using THCM (section 2a), we solve steady states and simultaneously determine their linear stability. Steady-state solutions lead to a set of nonlinear algebraic equations of the form

$$\mathbf{F}(\mathbf{u}, \mathbf{p}) = 0.$$  

Here the parameter dependence of the equations is made explicit through the $p$-dimensional vector of parameters $\mathbf{p}$, and hence $\mathbf{F}$ is a nonlinear mapping from $\mathbb{R}^{d+p} \to \mathbb{R}^d$. To determine branches of steady solutions of (6) as one of the parameters, say $\mu$, is varied, the pseudoarclength method Keller (1977) is used. The branches $[\mathbf{u}(s), \mu(s)]$ are parameterized by an “arclength” parameter $s$. An additional equation is obtained by “normalizing” the tangent

$$\dot{\mathbf{u}}_s^\top(\mathbf{u} - \mathbf{u}_s) + \mu(\mu - \mu_0) + \Delta s = 0,$$

where $(\mathbf{u}_0, \mu_0)$ is an analytically known starting solution or a previously computed point on a particular branch and $\Delta s$ is the step length.

When a steady state is determined, the linear stability of the solution is considered and transitions that mark qualitative changes such as transitions to multiple equilibria (saddle-node, pitchfork, or transcritical bifurcations) or periodic behavior (Hopf bifurcations) can be detected. The linear stability analysis amounts to solving a generalized eigenvalue problem of the form

$$\mathbf{A} \mathbf{x} = \sigma \mathbf{B} \mathbf{x},$$

where $\mathbf{A}$ and $\mathbf{B}$ are nonsymmetric matrices and $\sigma = \sigma_r + i \sigma_i$ is the complex growth factor. Solution techniques for these problems are presented in Dijkstra (2005).

At a Hopf bifurcation, a complex conjugate pair of
eigenvalues \( \sigma = \sigma_r \pm i \sigma_i \) crosses the imaginary axis. The corresponding complex eigenfunction \( \mathbf{x} = \mathbf{x}_R + i \mathbf{x}_I \) provides the disturbance structure \( \Phi(t) \) with angular frequency \( \sigma_r \) and growth rate \( \sigma_i \) to which the steady state becomes unstable; that is,

\[
\Phi(t) = e^{\sigma_t}[\mathbf{x}_R \cos(\sigma_i t) - i \mathbf{x}_I \sin(\sigma_i t)].
\]  

(9)

Propagation features of a neutral eigenmode (\( \sigma_r = 0 \)) can be determined by first looking at \( \Phi[-\pi/(2 \sigma_r)] = \mathbf{x}_I \) and then at \( \Phi(0) = \mathbf{x}_R \). The period \( T \) of the oscillation is given by \( T = 2\pi/\sigma_r \).

3. The equatorially symmetric case

In this section, we will study the variability of flows in a double-hemispheric configuration, set by \( \theta_s = 60^\circ \)S, \( \theta_n = 60^\circ \)N, \( \phi_w = 286^\circ \)E, and \( \phi_e = 350^\circ \)E, with \( D = 4000 \) m. The background atmospheric temperature is chosen as

\[
T_s(\theta) = T_o + \frac{\Delta T}{2} \cos \left( \frac{\pi}{\theta_n} \frac{\theta}{\theta_n} \right),
\]

and \( \Delta T \) will be our control parameter. In section 3a we first determine the SST modes for this configuration. Next, in section 3b steady solutions of (3)–(4) are determined (under restoring boundary conditions) versus \( \Delta T \). The linear stability of each steady solution is calculated subsequently under prescribed flux conditions. In section 3c, the resulting transient flow above the stability boundary (the first Hopf bifurcation) will be considered.

a. SST modes

For \( \Delta T = 0 \) the steady solution is given by \( u = v = w = 0 \) and \( T = T_o \). When the linear stability of this solution is considered under prescribed flux conditions, the governing equation for the temperature perturbations reduces to the problem

\[
\frac{\partial T}{\partial t} = \frac{K_H}{r_0^2 \cos \theta} \left[ \frac{\partial}{\partial \phi} \left( \frac{1}{\cos \theta} \frac{\partial T}{\partial \phi} \right) + \frac{\partial}{\partial \theta} \left( \cos \theta \frac{\partial T}{\partial \theta} \right) \right]
\]

\[+ K_V \frac{\partial^2 T}{\partial z^2}, \quad z = -D, 0 : \frac{\partial T}{\partial z} = 0, \]

\[\phi = \phi_w, \phi_e : \frac{\partial T}{\partial \phi} = 0, \quad \text{and} \]

\[\theta = \theta_s, \theta_n : \frac{\partial T}{\partial \theta} = 0. \]

(11a)

(11b)

(11c)

(11d)

The general solution of this problem, for a given initial condition, is found by separation of variables as

\[
T(\phi, \theta, z, t) = \Gamma(t) \Phi(\phi) \Theta(\theta) Z(z). \]

(12)

This leads to four problems:

\[
\frac{\Phi^*}{\Phi} = -\mu^2, \quad \text{and} \quad \frac{\Theta^* \cos \theta \sin \theta - \Theta (\mu^2 - \sin^2 \theta)}{(\nu \cos \theta - \mu^2)},
\]

\[
Z' = -(v + \lambda) \frac{K_H}{K_V r_0^2} \quad \text{and} \quad \Gamma' = \lambda \frac{K_H}{r_0^2},
\]

where \( \nu, \lambda, \) and \( \mu \) are separation constants. The boundary conditions are given by \( \Phi(\phi_w) = \Phi(\phi_e) = 0, \Theta(\theta_n) = \Theta(\theta_s) = 0, \) and \( Z'(-D) = Z'(0) = 0. \)

The eigenvalues \( \mu \) are easily determined from (13a) to give

\[
\nu = \frac{n \pi}{\phi_e - \phi_w}, \quad n = 0, 1, \ldots.
\]

(14)

Under special cases, the problem (13b) can be solved in terms of associated Legendre functions but, in general, it has to be solved numerically. We discretize the equations and boundary conditions centrally in space on a grid \( \{ \theta_0, \theta_1, \ldots, \theta_{M-1}, \theta_M = \theta_n \} \) and solve the corresponding \( (M + 1) \times (M + 1) \) matrix eigenvalue problem. For each value of \( \nu_n \), we hence find \( M + 1 \) eigenvalues \( \nu_{n,m}, m = 0, \ldots, M. \) Since the problem is self-adjoint, all eigenvalues \( \nu_{n,m} \) are real. If we define \( \alpha = \nu + \lambda, \) then the eigenvalues of (13c) are given by

\[
\alpha_l = (ln\pi^2) \frac{K_V r_0^2}{K_H D^2}, \quad l = 0, 1, \ldots
\]

(15)

and hence we finally find for the dimensional damping rates \( \lambda_{n,m,l}^\phi \) (s\(^{-1}\))

\[
\lambda_{n,m,l}^\phi = \frac{K_H}{r_0^2} \alpha_l \lambda_{n,m,l} = -\left[ \frac{K_V}{D^2} (ln\pi^2) + \nu_{n,m} \frac{K_H}{r_0^2} \right].
\]

(16)

This shows that part of the damping of the eigenmodes is determined by the vertical diffusion time scale and part by the horizontal diffusion time scale. The corresponding eigenfunctions \( T_{n,m,l}(\phi, \theta, z) \) are easily determined from (13) once the eigenfunctions \( \Theta_{n,m} \) are calculated from (13b).

The discrete version of the eigenvalue problem (13b) was solved numerically by a standard library [Numerical Algorithms Group (NAG)] routine. In the fourth column of Table 2, the damping factors \( \lambda_{n,m,l}^\phi \) (yr\(^{-1}\))
for the least damped eigenmodes are given for $M = 32$. The values in column 4 are those determined from (16); the values in column 5 are those determined with THCM for $\Delta T = 10^{-2}$ K.

### Table 2. Damping factors $\lambda_{n,m,l}^*$ of the seven least damped SST modes for $M = 32$. The values in column 4 are those determined from (16); the values in column 5 are those determined with THCM for $\Delta T = 10^{-2}$ K.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$m$</th>
<th>$l$</th>
<th>$\lambda_{n,m,l}^*$ (yr$^{-1}$)</th>
<th>$\lambda_{n,m,l}^*$ (yr$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>$-1.925 \times 10^{-3}$</td>
<td>$-1.927 \times 10^{-3}$</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>$-2.227 \times 10^{-3}$</td>
<td>$-2.281 \times 10^{-3}$</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>$-4.152 \times 10^{-3}$</td>
<td>$-4.183 \times 10^{-3}$</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>0</td>
<td>$-7.566 \times 10^{-3}$</td>
<td>$-7.546 \times 10^{-3}$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>2</td>
<td>$-7.703 \times 10^{-3}$</td>
<td>$-7.688 \times 10^{-3}$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>$-8.085 \times 10^{-3}$</td>
<td>$-8.394 \times 10^{-3}$</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>1</td>
<td>$-9.986 \times 10^{-3}$</td>
<td>$-9.472 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

The modes with other indices $(n, m, l)$ are more damped than those shown in Table 2. There are two classes of modes: those that have equatorially symmetric temperature anomaly patterns (for which $m$ is even) and those with antisymmetric patterns (for which $m$ is odd). The temperature field of an SST mode is simply given by

$$T(\phi, \theta, z, t) = e^{n\lambda_{n,m,l}(\text{yr}^{-1})} \cos \left( \frac{n\pi(\phi - \phi_w)}{\phi_e - \phi_w} \right) \times \cos \left( m\pi \frac{\theta - \theta_n}{\theta_m - \theta_n} \right) \cos(lz), \quad (17)$$

and so the spatial patterns of these modes are clear. The ordering of the modes (according to their growth rates) is different than those for the single-hemispheric basin case (Dijkstra 2006). In particular, in the case here, the $(1, 0, 0)$ mode is more damped than the $(0, 1, 1)$ mode.

#### b. SST mode merging

The damping factors (16) of the SST modes form a nice check of the numerical eigenvalue solver of the full THCM model. The resolution of THCM is taken $4^\circ$ horizontally and 16 equidistant layers ($N_x = N_y = N_z = 16$). Using pseudoarclength continuation, we determine

![Fig. 1](image-url)
a branch of stably stratified equilibrium solutions with $\Delta T$ as a control parameter starting from the trivial solution at $\Delta T = 0$ K. For each steady solution, we solve the linear stability problem using the Jacobi–Davidson QZ method (JDQZ) (Sleijpen and Van der Vorst 1996). For $\Delta T = 10^{-3}$ K, the growth factors of the first seven SST modes are given in the fifth column of Table 2, showing an excellent agreement with the ones calculated from (16).

The strength of the meridional overturning circulation (MOC)—measured here by the maximum value of the meridional overturning streamfunction $\psi_M$—of the background flow is plotted versus $\Delta T$ in Fig. 1a. Plots of the meridional overturning streamfunction, the sea surface temperature field, and a meridional section of the temperature are shown for $\Delta T = 24$ K in the Figs. 1b–d. The flow has the characteristic equatorially symmetric MOC (Fig. 1b) pattern with a near zonally independent SST pattern (Fig. 1c) and a stably stratified temperature distribution (Fig. 1d). The strength of the MOC increases approximately with a 1/3 power-law dependence up to a little more over 16 Sv ($Sv = 10^6$ m$^3$ s$^{-1}$) for $\Delta T = 24$ K (Fig. 1a). Note that Fig. 1a is plotted using about 80 equilibrium (steady) solutions.

In the range $\Delta T \in [2, 24]$ K, the most unstable modes (those with the largest growth factors) are computed with the JDQZ method, and the result for the growth rates and angular frequencies is plotted in Figs. 2a and 2b, respectively. There are two modes that obtain a positive growth factor at sufficiently large $\Delta T$. The first mode has an equatorially antisymmetric SST pattern (Fig. 3b) and a symmetric MOC anomaly (Fig. 3a); we will refer to this as the antisymmetric multidecadal mode (AMM). Its oscillation period (Fig. 2b) is slightly smaller than that of the second mode, referred to as the symmetric multidecadal mode (SMM), which has a symmetric SST pattern (Fig. 3d) and an antisymmetric MOC anomaly (Fig. 3c). The growth rate of the AMM becomes positive near $\Delta T = 12$ K, while that of the SMM becomes positive near $\Delta T = 16$ K (Fig. 2a). Hence, the AMM destabilizes the thermohaline flow more easily than the SMM.

We now trace both eigenmodes back in $\Delta T$ to see how they connect to the stationary SST modes. The result for the growth factor is plotted in Fig. 4. The AMM enters the picture at the labeled location in Fig. 4; it splits into two real modes near $\Delta T = 0.95$ K (at the black diamond labeled 1) and these modes eventually connect to the (0, 1, 1) and the (0, 1, 0) modes, which indeed both have antisymmetric SST patterns. The SMM enters the bottom of Fig. 4 and it remains oscillatory down to $\Delta T = 0.43$ K (at the black diamond labeled 2) where it splits into two stationary modes. The first mode is the (0, 2, 1) mode and the other connects eventually to the (1, 0, 0) and (0, 0, 2) modes. All of these modes have symmetric SST anomaly patterns. There are small intervals where an oscillatory mode appears (between the black diamonds labeled 3 and 4), but these are not relevant for the origin of the AMM and SMM.

The main result from Fig. 4 is that it demonstrates that—as in the single-hemispheric case (Dijkstra 2006)—the oscillatory modes that destabilize the steady thermohaline flow at large $\Delta T$ originate from the mergers of SST modes.
c. Transient supercritical flows

According to the results in Fig. 2a, the first Hopf bifurcation occurs near $\Delta T = 12$ K, while a second one appears near $\Delta T = 16$ K. To investigate the periodic orbits that arise from these bifurcations, we use the implicit transient solver in THCM (De Niet et al. 2007). From the steady state computed under restoring conditions at some value of $\Delta T$ above the Hopf bifurcation, we diagnose the heat flux. As initial state of the transient integration, we choose the computed steady state and perturb it into the direction of the first eigenmode (which is the AMM for each $\Delta T$). Next, we integrate the governing equations in time, using the diagnosed heat flux as forcing. A time step of one month is used (note that the fully implicit code has no intrinsic stability numerical limitations on the time step), which provides a sufficient accuracy in the results.

The type of flow is diagnosed by plotting the positive maximum value of the MOC in the Northern Hemisphere ($\psi_N$), the negative minimum of the MOC in the Southern Hemisphere ($\psi_S$), and the difference between both quantities. If an oscillation is due to the AMM, then $\psi_N$ and $\psi_S$ are in phase as the AMM is associated with a symmetric MOC anomaly (Fig. 3a). If an oscillation is due to the SMM, then $\psi_N$ and $\psi_S$ are out of phase as the SMM is associated with an antisymmetric MOC anomaly (Fig. 3c).

For $\Delta T = 15.7$ K, which is just before the location where the SMM obtains a positive growth factor (Fig. 2a), the values of $\psi_N$ and $\psi_S$ are in phase (Fig. 5a), indicating that the periodic orbit that arises is due to the growth of the AMM. The amplitude of the oscillation is still relatively small (the peak-to-peak amplitude is about 4 Sv) and the period is about 100 yr. This is slightly shorter than the value based on the value of $\sigma_i$ at the Hopf bifurcation (at $\Delta T = 12$ K) for the AMM, which is $\sigma_i = 0.05$ yr$^{-1}$ (Fig. 2b) giving a period of about 125 yr. For the single-hemispheric case, it was shown in Te Raa et al. (2004) that the period of the finite-amplitude oscillation decreases with increasing distance from the Hopf bifurcation, and this behavior is expected here too.

For $\Delta T = 19$ K, which is beyond the value of the second Hopf bifurcation where the SMM mode starts to grow, the periodic orbit is again dominated by the AMM as $\psi_N$ and $\psi_S$ are still approximately in phase (Fig. 5b). The period has decreased to about 80 yr and the peak-to-peak amplitude is about 4 Sv in each hemi-
sphere. The decrease in period with increasing distance to the Hopf bifurcation can again be seen in the time series shown in Fig. 5c, which is the case for $\Delta T = 24$ K. Here the presence of the SMM is seen as the more or less quasiperiodic signal and the small shift in phase between $\psi_N$ and $\psi_S$. From dynamical systems theory, we know that periodic orbits can become unstable through a Neimark–Sacker bifurcation leading to quasiperiodic behavior. This is likely the case here but we are currently lacking the tools to provide a more convincing case for this scenario. The mean period has decreased to about 60 yr and the peak-to-peak amplitude has increased slightly to about 5 Sv.

4. Equatorially asymmetric configurations

In the previous section, we have seen that in the equatorially symmetric case there are two modes that eventually destabilize the thermally driven steady flow. In Part I, equatorially asymmetric configurations were also considered with an open southern channel (to allow for a circumpolar flow) and with a second basin added to represent the Pacific. It was found that, in the double-basin case with a conveyor-type circulation, the multidecadal variability was localized in the North Atlantic.

In this section, we investigate how the patterns of AMM and SMM are modified by asymmetries of an open southern channel and the presence of a second basin. From the results above, we can conclude that we do not need to compute the path of these modes up to realistic $\Delta T$. Since the SST mode mergers already occur at small $\Delta T$, we can only consider the eigenmodes associated with the linear stability of an asymmetric flow at relatively small values of $\Delta T$.

For the open-channel case, we again choose the domain as in the equatorially symmetric case, but now there is a $20^\circ$ latitudinal channel at the southern boundary of the domain. Periodic boundary conditions are prescribed at the zonal boundaries of the channel. Again, steady solutions are computed versus $\Delta T$ under restoring conditions and the linear stability of each steady flow is determined under prescribed flux conditions. For $\Delta T = 3.5$ K, the MOC and SST pattern of the steady state are plotted in Figs. 6a,b, respectively.

Because of the open channel and the Drake Passage
effect (Toggweiler and Samuels 1995), it is more difficult to achieve downwelling of water in the south, and hence the MOC is slightly weaker in the Southern Hemisphere (Figs. 6a,b). The asymmetry clearly has its signature on the SST pattern of the least stable mode (Figs. 6c,d). In one phase, the pattern is still nearly antisymmetric and looks like that of the AMM mode (Fig. 6d) but one quarter of a period later (Fig. 6c), the pattern is localized in the northern part of the basin. The resulting mode is the deformation of the AMM in this slightly asymmetric case. Also the SMM (not shown) undergoes the same type of deformation and becomes localized in the northern part of the basin during one phase of the oscillation.

We next consider the double-basin configuration similar to the type of geometric configuration for which multidecadal oscillations were found in Part I. When the basins are thought to be unconnected, one would expect two multidecadal modes in each basin, say labeled AMM\(_A\), SMM\(_A\), AMM\(_P\), and SMM\(_P\). A southern connection introduces a coupling between these modes, so different connections between these four modes are expected. The asymmetry of the background steady flow subsequently determines which of these mixed modes obtains the largest growth factor.

The steady-state MOC pattern at \(\Delta T = 6.1\) K is approximately the same in each of the basins, and the global MOC is plotted in Fig. 7a. As in the open southern channel configuration, the northern MOC is slightly stronger than the southern one; the SST pattern of this steady flow is nearly equatorially symmetric in each basin (Fig. 7b). For \(\Delta T = 6.1\) K, there is already a mode with \(\sigma_1 > 0\) in this configuration. The SST pattern at phase \(t = 0\) is near equatorially antisymmetric in both basins and it has an opposite sign in the Atlantic and Pacific (Fig. 7d). One-quarter of a period later, localization of the SST patterns of these modes occurs in both basins while keeping opposite signs (Fig. 7c). Hence, the most unstable mode here is a pairing of two AMM modes, as in Fig. 6, with opposite signs.

5. Summary and discussion

In this paper, we studied the linear stability of thermally driven three-dimensional flows in an equatorially symmetric double-hemispheric basin; our main control parameter was the equator-to-pole temperature difference \(\Delta T\). Two oscillatory multidecadal modes were found to destabilize the equatorially antisymmetric MOC state at large enough \(\Delta T\); these were the antisymmetric multidecadal mode and the symmetric multidecadal mode.

We showed that the origin of these normal modes
was a merger of modes that can be connected to the free modes of this configuration, the SST modes. As was seen in Fig. 4, there are two classes of SST modes: those with a symmetric SST pattern \([n, 2m, l], m = 0, 1, \ldots]\) and those with an antisymmetric SST pattern \([n, 2m + 1, l], m = 0, 1, \ldots]\). Mergers only occur between modes of similar symmetry class and already at small \(\Delta T\), and lead to the AMM and SMM. As the antisymmetric \((0, 1, 0)\) SST mode is always less damped than the symmetric \((0, 2, 0)\) SST mode (because it has a more complicated meridional structure), the AMM always has a larger growth rate than the SMM.

This would imply that one always expects a periodic oscillatory flow for which the SST anomaly is antisymmetric, as is indeed seen in the calculations in section 3c. In Part I, however, a periodic orbit with a symmetric SST anomaly pattern was found. The differences between the model setup here and in Part I is the choice of the vertical grid and the presence of wind forcing and freshwater forcing. In the appendix, we show that the different effective forcing in both models is a plausible explanation of this difference.

The equatorially symmetric double-hemispheric configuration serves as a reference case to understand the patterns of multidecadal variability under more realistic conditions. An open southern channel introduces an equatorial asymmetry in the mean MOC, which induces a modification of the SST pattern of the AMM. The net result is a localized pattern with the largest SST anomalies in the strongest sinking region of the mean flow. In Part I, this asymmetry and localization is explained with the help of the mechanism of the multidecadal mode propagation and growth (Te Raa and Dijkstra 2002). The result here shows that this asymmetry is already present in the patterns of the internal multidecadal modes. When a second basin is added, combinations of the AMM and the SMM in each basin can occur. The first Hopf bifurcation is associated with a mixed AMM in both basins, which is partly localized in the Northern Hemispheric areas where the sinking of the mean flow is largest.

Having this theoretical framework for the multidecadal modes and the results in Part I, the question arises how this connects to observed patterns of (multi) decadal variability. Obviously, the two main candidates of such variability patterns are the Atlantic “multidecadal oscillation” (AMO) and the Pacific “decadal oscillation” (PDO).
tion” (PDO). First of all, the underlying assumption here is that internal modes of variability play an important role in the observed variability. Although there is support from GCM results (Delworth et al. 1993; Delworth and Mann 2000), this need not be true. Other preferred patterns such as nonnormal modes may play a role; it may simply be that the observed features are an oceanic response to atmospheric variability, or mesoscale eddy variability in the ocean may be essential for the multidecadal variability.

Under the assumption, however, that patterns of variability arise through the normal modes (possibly through excitation by atmospheric noise), the only relevant normal modes are, indeed, the ones arising from SST mode mergers. As the present thermohaline circulation is “basin asymmetric” with the sinking in the North Atlantic and not in the North Pacific, a localization of the MM with a dominant pattern in the North Atlantic is expected (as the results in Part I clearly indicate). While this can serve as a prototype for the AMO, it cannot explain the presence of the PDO in the North Pacific.

With respect to the Pacific situation it is interesting, however, that the theory on the El Niño–Southern Oscillation (ENSO) has clearly shown that a coupled ocean–atmosphere internal mode is involved in the interannual tropical variability. Jin and Neelin (1993) demonstrated that this mode arises through a merger of a symmetric SST mode and an ocean dynamics mode. Here, the only relation between the equatorial SST modes and the midlatitude ones is that they mathematically originate from the time derivative of the SST equation. This symmetric equatorial SST mode has the same pattern in the equatorial zone as the SMM that was found in this study. Hence, ENSO variability may project on the SMM and induce off-equatorial SST anomalies, providing a subtle relation between the PDO and ENSO. This may explain why the spatial structure of the PDO is more equatorially symmetric than that of the AMO. While these ideas are pure speculations at the moment, it would provide, if true, an elegant framework on the organization of low-frequency climate variability. Further research must show whether this framework is useful for understanding this variability.

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APPENDIX

Equidistant versus Stretched Vertical Grids

In Part I a periodic orbit with a symmetric SST anomaly pattern was always found, whereas here we find an antisymmetric SST anomaly. The differences between the model setup here and in Part I is the choice of the vertical grid and the presence of wind forcing and freshwater forcing.

This result motivated us to perform simulations with both a nonequidistant and an equidistant grid for \( \Delta T = 24 \) K and with temperature forcing only. These results are computed with the Modular Ocean Model (MOM), and details of the computational procedure can be found in Part I. For the equidistant grid, indeed a periodic orbit with an antisymmetric SST pattern is found (Fig. A1) and the period is about 50 yr. For the nonequidistant grid, a periodic orbit with a symmetric SST pattern is found (Fig. A2) with a slightly larger period. As expected, the results for the equidistant grid are fully compatible with those in section 3c.

Hence, it appears that under a stretched vertical grid, the symmetric periodic orbit (from the SMM) is preferred and the antisymmetric periodic orbit (from the AMM) is preferred under an equidistant vertical grid (at the same \( \Delta T \)). As the AMM always has a larger growth factor, independent of the structure of the ver-

![Fig. A1. Results of simulations with the MOM (version 3.1) under the same forcing and configuration as the THCM results in section 3 for an equidistant vertical grid: (a) Mean MOC field under prescribed flux conditions, (b) time series of the maximum MOC strength, (c) SST difference pattern between oscillation extremes, and (d) Hovmöller diagram of the multidecadal variability in MOM for \( \Delta T = 24 \) K.](image-url)
Fig. A2. As in Fig. A1 but for a nonequidistant vertical grid.

Fig. A3. Diagnosed heat flux from the MOM model for $\Delta T = 24$ K (a) on an equidistant grid scaled by the upper-ocean layer thickness $H_m = 250$ m and (b) on a nonequidistant grid scaled by the upper-ocean layer thickness $H_m = 50$ m.
tical grid, it is also expected that the first Hopf bifurcation is associated with the antisymmetric periodic orbit. The only possibility is that this periodic orbit becomes unstable at smaller $\Delta T - \Delta T_H$ than for the equidistant grid; here $\Delta T_H$ is the value of $\Delta T$ at the Hopf bifurcation.

A reason for this instability is proposed with the help of Fig. A3 in which the heat flux (diagnosed for the same $\Delta T$) is plotted for both equidistant and nonequidistant grids. For the same value of $\Delta T$, the strength of the MOC under the equidistant grid (Fig. A1a) is slightly weaker than that of the nonequidistant grid (Fig. A2a), the difference being about 2 Sv. In the diagnosed heat flux (Figs. A3a,b), this difference is reflected in the fact that the maximum amplitude is shifted poleward in the case of a nonequidistant grid. For easier comparison of the heat fluxes in the two shifted poleward in the case of a nonequidistant grid. For the same value of $\Delta T$, this difference is re-(Fig. A2a), the difference being about 2 Sv. In the di-

We subsequently note that there is a marked asymmetry between a finite-amplitude periodic signal associated with the AMM and that of the SMM. For the SMM, the oscillation of the MOC anomaly is antisymmetric and, hence, at each phase of oscillation when the southern MOC is amplified (more negative $\psi_S$), the northern MOC is also amplified (more positive $\psi_N$). So, the total MOC remains equatorially antisymmetric and it remains compatible with the symmetric heat flux. On the contrary, a periodic orbit associated with the AMM has a symmetric MOC variation. At each phase of the oscillation, when the southern MOC is amplified (more negative), the northern MOC is weakened (less positive). As a consequence, the total MOC becomes asymmetric and this flow is not compatible with the symmetric heat flux that forces the flow.

Consider now a finite-amplitude antisymmetric AMM-type periodic orbit that leads to an oscillatory flow with an asymmetric MOC, as described above. As the asymmetry increases, the MOC is more and more incompatible with the heat flux forcing. For example, if the Southern Hemispheric MOC becomes weaker, this flow is counteracted by the heat flux forcing in the south, which provides the tendency for sinking. As this tendency is larger for the nonequidistant grid, the asymmetric MOC flow is destabilized earlier and the preference for the symmetric periodic orbit appears at a smaller value of $\Delta T$. This provides a plausible explanation for the subtle difference between the MOM results for the equidistant and nonequidistant grids.

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