Effects of Stratification on the Large-Scale Ocean Response to Barometric Pressure

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ABSTRACT

Single-layer (barotropic) models have been commonly used in studies of the inverted barometer effect and the oceanic response to atmospheric pressure loading. The potential effects of stratification on this response are explored here using a general circulation model in a near-global domain with realistic coasts and bathymetry. Periodic forcing by the diurnal and semidiurnal atmospheric tides and 6-hourly stochastic forcing from weather center analyses are both examined. A global dynamic response (i.e., departures from inverted barometer behavior) is clear in the response to atmospheric tides; for stochastic forcing, the largest dynamic signals occur in shallow and semienclosed regions and at mid- and high latitudes. The influence of stratification in the dynamics is assessed by comparing surface and bottom pressure signals. Baroclinic effects are generally weak, particularly in the response to the large-scale atmospheric tides. Under stochastic forcing, largest differences between surface and bottom pressure signals reach 10%–20% of the surface signals and tend to occur in regions of enhanced topographic gradients. Bottom-intensified, localized interactions with topography seem to be involved. Enhanced baroclinicity is also seen at low latitudes, where stratification effects are also felt in the upper ocean. General implications for modeling the ocean response to high-frequency atmospheric and tidal forcing are discussed.

1. Introduction

With the launching of altimeter missions more than a decade ago, the oceanic response to loading by the gravitational tide potential and by barometric pressure \(P_a\) has been the subject of renewed interest. Driven in part by the need to process and interpret the sea level observations, numerous hydrodynamic tide solutions (Shum et al. 1997) and the first numerical studies of the response under realistic \(P_a\) (Ponte 1993) have been published. Most of these works employ single-layer, so-called barotropic, models under the assumption that stratification is a second-order effect in the dynamics. Their success in explaining the basic characteristics of the observed sea level variability (e.g., Le Provost 2001; Carrère and Lyard 2003) speaks for the validity of that assumption. However, as observations and models continue to improve, the subtle effects of density stratification begin to gain more prominence.

One clear role of stratification was highlighted by the discovery of strong excitation of internal tides in the deep ocean associated with the interaction of the barotropic tide with topography (Ray and Mitchum 1996). Such excitation implied a substantial dissipation of the barotropic tide, which had been missing in single-layer models (Egbert and Ray 2000), and provided added incentive for the recent use of stratified global models of the tides (Arbic et al. 2004; Simmons et al. 2004). In comparison with sea level observations, Arbic et al. (2004) found that their two-layer model solutions were consistently better than the one-layer solutions. The improvement stemmed from differences in the large-scale structure of the two solutions, but how these changes were related to baroclinic internal tide generation was not clear.

Forcing by \(P_a\), being stochastic and broadband in both wavenumber and frequency, samples the full range of possible baroclinic effects in the response to loading. The inclusion of \(P_a\) forcing in realistic numerical models is, however, very rare. Exceptions are those of Webb and de Cuevas (2003), who discuss \(P_a\)-driven signals only in passing, and Tierney et al. (2000), who

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analyze some of the unpublished numerical experiments of Bryan et al. (1999). Based on comparisons of a fully stratified experiment and one with constant temperature and salinity, the latter studies suggest a generally weak influence of stratification except in certain regions (e.g., in the Southern Ocean). Constant temperature and salinity do not necessarily mean constant density (in the sense of one-layer models), however, because of the dependence of density on pressure. Thus, the meaning of the results in Tierney et al. (2000) is not completely clear.

A baroclinic response to loading, even in the absence of topography, is possible on the basis of theoretical arguments. Willebrand et al. (1980) use a vertical structure equation to assess the depth dependence of the midlatitude oceanic response to general forcing. Their analysis suggests that for forcing at sufficiently short (near inertial) periods or small spatial scales (~100 km), the response can be strongly baroclinic. The work of Longuet-Higgins (1968) shows that, even in the case of a vertically constant body force acting throughout the water column as with tidal loading, the response can be barotropic or baroclinic depending on the horizontal scales and frequency of the applied loading. Ponte (1992) obtains similar conclusions for the response of midlatitude and equatorial β-plane oceans to simple $P_a$ forcing. In particular, stratification allows for the possible excitation of baroclinic resonances and a strong dynamic response, which would be absent in a homogeneous ocean. These analytical results are valid for a flat-bottom ocean and thus are not in any way related to topographic effects. The inclusion of topography introduces different spatial scales, allows for new wave modes, and can in principle complicate the problem (Willebrand et al. 1980). It is, thus, not clear how a stratified ocean with realistic topography and coastal geometry may respond under broadband stochastic loading ranging over all spatial and time scales.

In this work, we take a further look at the effects of stratification on the response to loading. The emphasis is on the case of the $P_a$ forcing, because its broadband characteristics permit an exploration of a wide range of scales and dynamical behaviors. In particular, and following Ponte and Ray (2002), we examine separately the response to the semidiurnal ($S_2$) and diurnal ($S_1$) "air" tides, which share similar spatial and time scales with the gravitational tide potential. Thus, results are also relevant to the ocean tides. A full review of the $P_a$ loading problem is given by Wunsch and Stammer (1997). Here, to study the problem in a realistic setting, we use the Massachusetts Institute of Technology (MIT) general circulation model (MITgcm; Marshall et al. 1997a,b), taking advantage of a recently implemented option to include $P_a$ in its forcing fields.

Besides the stratification issues already discussed, several other factors motivate this work. The importance of $P_a$-driven dynamic signals relative to wind-driven motions at subweekly periods is clear from simple scaling arguments (Philander 1978) or numerical model results (Ponte 1994; Tierney et al. 2000). The simplified numerical experiments here serve as a general test of the behavior of the MITgcm under loading, with the goal of introducing $P_a$ effects in future data-constrained runs (Köhler et al. 2003). As most current modeling efforts omit the effects of $P_a$, our experiments also provide an estimate of those omission errors, which can be useful for data assimilation efforts that examine high-frequency variability. We are also interested in clarifying how stratification affects the relation between surface and bottom pressure signals at high frequencies, an issue of increasing importance for the interpretation and processing of data from the Gravity Recovery and Climate Experiment (GRACE) mission.

In the rest of the paper, after describing the modeling approach and the forcing fields in section 2, we examine separately the response to the air tides (section 3) and realistic stochastic forcing (section 4). A discussion of several relevant aspects of the numerical results is given in section 5. Findings are summarized in section 6.

2. Model, forcing, and method

Our computational tool is the MITgcm. A full model description can be found in Marshall et al. (1997a,b) and further documentation is also available online (http://mitgcm.org/); only a few model details specific to our experiments are mentioned here. The model solves the primitive equations on a sphere under the Boussinesq approximation. The prognostic equations for temperature, salinity, and velocity are integrated on a staggered Arakawa C grid. The internal pressure is computed from the hydrostatic relation with an implicit free surface at each time step.

The basic configuration of our setup is similar to that used by the Estimating the Circulation and Climate of the Ocean (ECCO) Consortium (Köhler et al. 2003). The model grid has 1° horizontal spacing and spans the latitudinal range of ±80° (see Fig. 1); some of the poorly resolved ocean areas near the artificial wall at 80°N (e.g., north of the Bering Strait) are also set to land. There are 23 vertical levels with spacing varying from 10 m at the surface to 500 m at the bottom. The bathymetry is based on the 2′ Gridded Earth Topography
(ETOPO2) dataset provided by the National Geophysical Data Center (information online at http://www.ngdc.noaa.gov/mgg/fliers/01mgg04.html), which has been modified to improve the coarse-grid representation of coastlines, islands, and straits (Köhl et al. 2003). Free-slip bottom and no-slip lateral wall boundary conditions are imposed. Laplacian viscosity is used with horizontal and vertical coefficients of $10^4$ and $10^{-3}$ m$^2$ s$^{-1}$, respectively. For Laplacian diffusivity, horizontal and vertical coefficients are $10^2$ and $10^{-5}$ m$^2$ s$^{-1}$, respectively. Quadratic bottom friction is also used with a drag coefficient of $3 \times 10^{-3}$. Some of the effects parameterized in tide models, such as internal wave drag (Jayne and St. Laurent 2001) and self-atraction and loading (Hendershott 1972), are not considered here.

Our intent is to examine the response to loading in the most simplified setting possible. Thus, we neglect both wind and thermodynamic surface forcing. To allow for baroclinic motions, the model is initialized by vertical profiles of salinity and potential temperature typical of midlatitude regions and with no horizontal gradients. Over the course of our experiments, a small drift in thermodynamic variables is observed, mostly in the upper layers. Apart from this drift, the solutions can be considered, to a large extent, adiabatic, with any density fluctuations dominated by advection processes. In any case, our analyses are based on detrended fields and the results presented are not affected by the noted density drifts.

Two types of atmospheric loading were used in this study: periodic forcing by the mean atmospheric tides and 6-hourly residual stochastic forcing. Spatial distributions of the amplitude and phase for $S_1$ and $S_2$ barometric tides are taken from the climatology of Ray and Ponte (2003). These fields were recently used by Ray and Egbert (2004) and Arbic (2005) and are thus appropriate for comparing our results with those works. Both $S_1$ and $S_2$ air tides exhibit westward propagation over the ocean, with maximum amplitudes of $\sim 0.6$ hPa ($S_1$) and $\sim 1.3$ hPa ($S_2$) in the equatorial Pacific. Fields from Ray and Ponte (2003) were bilinearly interpolated to the model grid and provided at every time step.

In the stochastic forcing experiments, we use 6-hourly $P_a$ fields from the National Centers for Environmental Prediction–National Center for Atmospheric Research (NCEP–NCAR) reanalysis, upon removal of
the climatological barometric tides as proposed by Ponte and Ray (2002). For consistency, we use the daily climatology of van den Dool et al. (1997), which is based on NCEP–NCAR reanalysis fields. Bilinear interpolation was used to map 2.5° × 2.5° fields into the model grid. To facilitate running several numerical experiments, only one year (2002) is considered.

As usual in studies of $P_a$ loading, the analysis of model solutions is mostly focused on sea level $\zeta$ and related variables defined in the appendix. In particular, we are interested in dynamic sea level $\zeta_d$ representing departures from the equilibrium or inverted barometer (IB) solution $\zeta^b$. In addition, for checking the influence of stratification, we compare $\zeta_d$ with the equivalent bottom pressure signal $\zeta_b$, also defined in the appendix. If stratification effects are negligible and density is constant in time, from the hydrostatic balance $\zeta_d$ and $\zeta_b$ should be equal. The analysis is also relevant for determining if mass signals inferred from sea level variability actually correspond to true bottom pressure signals.

The experiments with air tides start from a resting state and are integrated forward until an approximately stationary solution is obtained (typically 10–20 cycles). For the runs with realistic NCEP–NCAR forcing, the initial state is the IB solution ($\zeta = \zeta^b$, no flow) computed from the first pressure field in the forcing file. An implicit time-stepping scheme is used for numerical integration. The choice of time step $\Delta t$ is related to numerical stability and damping issues, as well as to computational cost issues. The dependence of the results on $\Delta t$ was tested and is discussed where necessary.

3. Response to air tides

Strong dynamic effects should occur in response to the diurnal and semidiurnal air tides $S_1$ and $S_2$. Their large-scale, high-frequency nature is expected to excite resonant normal modes that play a dominant role in the gravitationally forced ocean tides (e.g., Platzman 1984). Arbic (2005) has recently considered barotropic and baroclinic (two layer) $S_2$ solutions, and Ray and Egbert (2004) discuss barotropic simulations of $S_1$. Comparing tide solutions provides a useful qualitative test of the ability of our numerical model to capture the dynamic response to $P_a$, in particular that mediated by fast gravity waves. Given the likely importance of these waves in the response to the air tides, the dependence on time step was examined in detail.

Stable integrations of the MITgcm in the same ECCO configuration used here, but not including forcing by $P_a$, are commonly done with time step $\Delta t = 1$ h. We tested a range of $\Delta t$ from 1 h to 10 s. At the longest $\Delta t$, the solutions were not well behaved, suggesting that inclusion of $P_a$ makes the stability needs more stringent. Pressure forcing, and in particular its diurnal and semidiurnal variabilities, is more efficient than winds at generating rapid gravity waves (Ponte 1994; Ponte and Hirose 2004), which can cause numerical problems if not properly resolved in time. Solutions with $\Delta t = 20$ min and lower displayed no unstable behavior. Amplitudes generally increased for shorter $\Delta t$; for the implicit time-stepping scheme used, a shorter $\Delta t$ implies a decrease in numerical damping of fast waves. Results with $\Delta t = 5$ min exhibited energy levels comparable to those in Arbic (2005) and Ray and Egbert (2004) and are thus used in this section.

Solutions for $S_1$ and $S_2$, obtained with forcing by the climatological air tides of Ray and Ponte (2003), are shown in Fig. 2. The amplitude and phase of $\zeta$ are obtained by harmonic analysis of the model solution once it becomes stationary in time. There are clear signs of a nonostatic response, as inferred by the relatively large amplitudes, particularly around some boundaries (e.g., west coast of North America, Arabian Sea), in comparison with forcing amplitudes that barely exceed 1 hPa for both $S_1$ and $S_2$ and are largest in equatorial regions (Ray and Ponte 2003). Our solutions are qualitatively similar to those of Arbic (2005) and Ray and Egbert (2004), reproducing most of the patterns of high and low amplitudes and the propagation and amphidromic point structures in phase. Exceptions occur in regions that are poorly resolved in our model, such as the North Sea and European shelf regions or the Indonesian seas, and regions near artificial boundaries, such as Baffin Bay or the Bering Sea. The noted similarity with previous tide solutions indicates that the dynamics and dissipation in our numerical setup can capture the basics of the response to high-frequency $P_a$ forcing.

The possible influence of stratification should be reflected in differences between $\zeta_d$ and the equivalent bottom pressure signal $\zeta_b$ (see the appendix). The amplitude and phase of $\zeta_d$ and $\zeta_b$ for the case of $S_2$ (Fig. 3) display very similar behavior and indicate that, to a large extent, the response is barotropic (depth independent). In fact, plots of the ratio of the $\zeta_d$ and $\zeta_b$ amplitudes in Fig. 3 and respective phase differences (not shown) indicate that typical amplitude deviations do
not exceed ~1% and phases differ by ~1°, apart from relatively small regions with weak amplitudes where larger differences in phase are possible. Similar findings result from the analysis of $S_1$ solutions (not shown).

Arbic et al. (2004) noticed differences between the large-scale structure of their one- and two-layer model tide solutions, with changes in amplitude and phase being larger than the effects reported here (cf. their Fig. 10). We note that, because of the coarse model resolution, energy transfer between barotropic and baroclinic motions is inhibited in our solutions, as typical internal tide scales (e.g., Simmons et al. 2004) are for the most part poorly resolved. The lack of explicit internal tide generation might explain the weaker effects of barocli-
nicity apparent in our 1° solutions. In addition, our results also suggest that any direct forcing of baroclinic motions by the air tides is very weak.

4. Response to broadband forcing

Experiments forced by stochastic $P_u$ fields for the year 2002 provide a view of the dynamics over the full spectrum from 12 h to 1 yr. As explained in section 2, we remove the climatological air tides of van den Dool et al. (1997) from the forcing. Experiments analyzed in this section were run with $\Delta t = 20$ min, to make more efficient use of computer resources. Test runs with $\Delta t = 5$ min indicate that the primary effect of the longer time step is to decrease the power at periods shorter than approximately 2 days, or at the scales for which gravity waves are most important. With the emphasis being on examining departures from an IB response, the analysis will focus on $\zeta_d$ and $\zeta_b$ fields.

The standard deviation of $\zeta_d$ ($\sigma_{\zeta_d}$) is shown in Fig. 4a. Values range from <1 cm at low latitudes to ~2–3 cm in mid- and high-latitude regions (e.g., Southern Ocean,
northern North Atlantic) over the deep ocean. This pattern mirrors the latitudinal increase in the forcing (e.g., Ponte 1993), with larger $\xi_d$ signals coinciding with storm track regions, but there are visible effects of topography in areas like the Southern Ocean. In addition, there is a clear increase in $\sigma_{\xi_d}$ over shallow depths and semienclosed regions. The largest IB deviations are found over Hudson Bay, with values of $\sigma_{\xi_d}$ reaching more than 10 cm. Variability in $\xi_b$, shown in Fig. 4b, is very similar to $\xi_d$, suggesting generally weak effects of stratification on the response to $P_a$.

Spectral analysis permits the exploration of any dependence of these results on frequency. For a barotropic response one needs similar power levels in $\xi_d$ and $\xi_b$ and a coherent, in-phase relation as well. A good measure of the departures from barotropic behavior is given by examining the spectrum of $\xi_d - \xi_b$, in comparison with that of $\sigma_{\xi_d}$. Globally averaged spectra of $\xi_d$ and $\xi_d - \xi_b$ are shown in Fig. 5. With the possible exception of the semidiurnal period, power in $\xi_d$ is always weaker than that in $\xi_b$ (also shown in Fig. 5), but becomes comparable to it at the shortest periods, signaling the breakdown of the IB assumption. The $\xi_d - \xi_b$ spectrum is in turn much weaker than that of $\xi_d$ at all
periods. Judging from the spectral analyses in Fig. 5, the response is close to barotropic at all time scales resolved, although differences between $\xi_d$ and $\xi_b$ become progressively smaller as frequency decreases.

Some of the latitudinal differences seen in Fig. 4 are further explored in Fig. 6, where we show regionally averaged spectra for equatorial latitudes (10°S–10°N) and for the Southern Ocean (50°–70°S). There are elevated levels in all Southern Ocean spectra relative to equatorial spectra and the failure of the IB assumption (i.e., power in $\xi_d$ similar to that in $\xi_b$) is much more clear near the equator over a wider frequency range (periods <5 days). Comparison of $\xi_d$ and $\xi_d - \xi_b$ confirms that on average the effects of stratification are weak at all frequencies in both equatorial and high-latitude regions.

Spectral results for Hudson Bay and the Japan Sea (semieclosed regions with large IB departures) are also examined in Fig. 6. Both of these regions show a dynamic response over a wider range of frequencies than that found over the deep ocean. Power in $\xi_d$ becomes comparable to that of $\xi_b$ at periods of ~1 month (Hudson Bay) and ~15 days (Japan Sea). Shallow topography, including the shallow constrained connections to the open ocean in the case of the Japan Sea (Fig. 1), play a dominant role in this behavior: enhanced IB deviations in these regions are absent in a numerical experiment with a constant depth of 3700 m (not shown). Differences in the spectra of $\xi_d$ and $\xi_d - \xi_b$ are considerably larger than those in equatorial and Southern Ocean spectra or those in Fig. 5, indicating an even weaker role of stratification in the dynamics of these regions.

5. Discussion

a. Departures from IB behavior

The different treatments of air tides, frictional parameters, topography, and so on prevent a detailed comparison of our results with the few previous modeling studies of the IB effect. Nevertheless, generally speaking, our dynamic response is not fundamentally different from that found in either Ponte (1993) or Bryan et al. (1999, unpublished manuscript). Some of the similar characteristics include the larger $\xi_d$ variability at mid- and high latitudes, the breakdown of the IB assumption at longer periods at low latitudes (Ponte and Gaspar 1999), and the tendency for a stronger dynamic response over shallow depths. The enhanced IB deviations in the Japan Sea and in Hudson Bay are also consistent with previous modeling efforts of these regions (e.g., Nam et al. 2004; Greatbatch et al. 1996).

The $\xi_d$ spectrum in Fig. 9 of Ponte (1993) contains a number of peaks at periods of 1–2 days, which have been shown by Ponte and Hirose (2004) to correspond to fundamental Antarctic Kelvin waves, observable in Southern Ocean bottom pressure records albeit with amplitudes substantially weaker than those in Ponte (1993). The global $\xi_d$ spectrum in Fig. 5 and the Southern Ocean spectrum in Fig. 6 show a broad peak at periods ~1.2–1.4 days, consistent with the time scales and power levels in Ponte and Hirose (2004). The excitation of large-scale gravity wave resonances seen in
previous barotropic studies is thus also present in the stratified experiments examined here.

When compared with the $\zeta_d$ spectrum in Fig. 9 of Ponte (1993), power levels in Fig. 5 are generally lower at periods of $<2$ days. Such behavior can be explained by the different levels of damping in the various models. Dissipation in Ponte (1993) might have been on the weak side. Hirose et al. (2001) and also Ponte and Hirose (2004) obtained the best comparisons between barotropic model output and observations for values of bottom friction substantially higher than those tested by Ponte (1993). The present solution seems more consistent with the results of Hirose et al. (2001) and Ponte and Hirose (2004), judging, for example, by the high-frequency power in Fig. 5 versus that in Fig. 8 of Ponte and Hirose (2004), and it also reproduces the amplitudes of the radiational tides well (section 3). Determining what are the most appropriate levels of dissipation is, however, a subject in need of further study. The issue is important because, without better understanding of the dissipation levels, it is difficult to define with certainty a time scale for which the IB assumption becomes compromised, in the sense that $\zeta_d$ power levels are not negligible relative to those of $\zeta_{ib}$.

b. Effects of stratification

Both deterministic, tidelike forcing (section 3), as well as stochastic forcing (section 4), lead to weak departures from a purely barotropic response at all frequencies. To quantify the baroclinic effects, Fig. 7a shows the ratio of the standard deviation of $\zeta_d$ to that of $\zeta_{ib}$ as a function of location for the stochastic forcing case. A pure barotropic response would imply ratios equal to zero. Largest estimated ratios are $\sim$0.1–0.2, with typical values of $\sim$0.01–0.02. Most shallow regions have ratios very near zero. The largest departures from barotropic behavior are confined to relatively small regions and seem to be related to the interaction of flows with topography. Figure 7b shows the amplitude of the gradient of the bottom topography in the

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**Fig. 6.** As in Fig. 5 but averaged for latitudes (top left) $50^\circ$–$70^\circ$S, (top right) $10^\circ$S–$10^\circ$N, (bottom left) the Japan Sea, and (bottom right) Hudson Bay. Error bars are omitted because they are too small to show at this scale.
Fig. 7. (a) Ratio of the std dev of $\xi_H - \xi_d$ to that of $\xi_d$, based on output from the stochastic forcing experiment. (b) Amplitude of the gradient of the model topography, normalized by its maximum amplitude. (c) As in (a) but based on an experiment using the partial step topography.
model. There is a clear correspondence between regions with the largest baroclinic effects and those with enhanced topographic gradients.

The representation of topography in height coordinate models can generate numerical noise (Adcroft et al. 1997) especially in regions of steep gradients, but the main patterns in Fig. 7a seem robust under different numerical treatments of topography. The results were tested by running an experiment using the “partial step” method described in detail by Adcroft et al. (1997). The partial step formulation relaxes the requirement that the topography be fixed to model depth levels, as is the case with the “full step” method used for the results in Fig. 7a, and provides a substantially better handle on the topographic gradients (Adcroft et al. 1997). Examining the results from the partial step experiment, shown in Fig. 7c, one can see that some of the weaker, elongated features in Fig. 7a are eliminated and can indeed be artifacts of the full step topographic representation. However, the areas with the largest baroclinic effects (i.e., red areas in Fig. 7a) are for the most part reproduced in Fig. 7c, indicating a weak dependence of those patterns on the choice of topography scheme. Similar conclusions were drawn from test experiments (not shown) in which we improved the resolution of the topographic gradients by using finer (1/2° and 1/4°) horizontal grids and linearly interpolating the 1° bathymetry to the finer grids.

The patterns in Fig. 7 suggest predominantly local, trapped baroclinic motions, with negligible horizontal propagation away from the regions of sharper topographic gradients. To illustrate the vertical structure of the response in the regions of larger baroclinicity, Fig. 8 shows time series of density anomalies at several depths from one example site in the western Pacific (19.5°N, 147.5°E) and the standard deviation of the density anomalies as a function of depth. The dominant density oscillations, at periods of a few days, are mostly in phase throughout the water column, with largest amplitudes seen toward the bottom, where vertical background density gradients are weaker than closer to the surface. If caused by advection processes, this indicates stronger vertical currents in the lower layers. Such structure is consistent with bottom-intensified baroclinic effects connected to topography.

Figure 7 also shows enhanced baroclinicity along the equator. Ponte (1992) discusses the possibility of stronger baroclinic effects at low latitudes because of a rich baroclinic equatorial wave spectrum. An example of the density anomalies from a location in the western equatorial Pacific (0.5°N, 143.5°E) is also shown in Fig. 8. The difference in phase (and time scale, at places) between the variability in the upper and lower layers near the equator contrasts with the behavior noted at 19.5°N, 147.5°E and suggests shorter-scale vertical structures in the equatorial response consistent with baroclinic mode excitation. The standard deviation profile, showing upper-ocean density variability as large as that seen at depth (cf. off-equatorial example in Fig. 8), also supports this interpretation.

On theoretical grounds, quasi-resonant excitation is expected to be an important cause of IB deviations, and
the presence of stratification is thought to increase such possibilities (Ponte 1992; Wunsch and Stammer 1997). There is little evidence, however, for strong excitation of baroclinic resonances in our solutions, either in the values and spatial structure of $\sigma_n$ in Fig. 4, which are not that different from the results with single-layer models (e.g., Ponte 1993), or in the spectra of Figs. 5 and 6. Some regional effects may be present but the large enhancements of $\xi_d$ variability over shallow areas are clearly not of a baroclinic nature. We should note, however, that results can be affected by horizontal resolution and density structure. For example, a full exploration of the low-latitude response likely involving equatorially trapped waves should be possible only with finer grids and more realistic stratification than what we have used here. More realistic numerical experiments at higher resolution will be needed to assess the full potential for near-resonant, baroclinic regimes.

c. Relation to gravitational tides

The parallel between tidal and $P_a$ forcing has been drawn numerous times for the case of a homogeneous ocean (e.g., Gill 1982). In this case, it is clear that both types of forcings enter the horizontal momentum equations exactly in the same manner, as an extra pressure gradient. In the more general case of a stratified ocean, tide loading still enters the horizontal momentum equations as a vertically constant body force (any weak depth dependence of the tide potential can be ignored for our purposes). For the case of $P_a$ forcing enters the problem as part of a surface boundary condition that assures continuity of pressure. The commonly used linearized form is (e.g., Ponte 1992; Wunsch and Stammer 1997)

\[ p - g \rho_s \xi = P_a \quad \text{at} \quad z = 0, \]

where $p$ is pressure (other symbols are defined in the appendix). Through the hydrostatic relation, the effect of this boundary condition is to add a vertically constant pressure term corresponding to $P_a$ at every depth in the water column. Thus, forcing by $P_a$ is equivalent to having a depth-independent body force on the stratified system and is dynamically equivalent to the case of gravitational tidal forcing.

In this regard, solutions under barometric tide forcing in section 3 are relevant for the gravitational forcing problem as well. For the case of $S_2$, the relation between gravitational and barometric components is very close, as the spatial structure of the two forcings is very similar. The close similarities in $\xi_d$ and $\xi_b$ highlighted in Fig. 3 are consistent with the expected mainly barotropic character of the gravitationally forced tides, although the effects related to internal tide generation are not well captured here, as discussed in section 3.

d. Response to residual air tides

As can be seen in Fig. 5, the removal of the mean air tides from the NCEP–NCAR $P_a$ fields reduces the forcing power at the diurnal and semidiurnal periods by one order of magnitude. Nevertheless, clear peaks still remain in the forcing, particularly for $S_2$. The residual peaks reflect the strong day-to-day variability of the daily cycle in $P_a$.

Corresponding peaks are seen in $\xi_d$ (Figs. 5 and 6). The semidiurnal period still corresponds to the largest $\xi_d$ signal, but calculation of the respective amplitude and phase (not shown) reveals a dynamic response much attenuated and much less structured in phase, with no signs of organized large-scale phase propagation, relative to that in Fig. 3. Similar considerations apply for the diurnal period. Thus, the residual dynamic variability at the tidal periods examined in section 4 is characterized by spatial scales shorter than those of the periodic solutions in section 3—a consequence of the stochastic nature of the residual forcing, which lacks the spatial coherence of the air tides in Ray and Ponte (2003).

6. Summary and final remarks

Experiments with the MITgcm in a simplified configuration have been used to explore the response of a stratified ocean to $P_a$ loading. Similarities with previous barotropic studies indicate that the fundamental character of the response to loading does not change in the presence of stratification, in terms of the dependence on the location or time scale. Baroclinic effects are generally weak at all periods. This is a nontrivial result given that, with realistic forcing and topography, many vertically dependent regimes are in principle possible, as for example in the response to storms (e.g., Gill 1982; van Dam and Wahr 1993) and subinertial forcing (Willebrand et al. 1980) and in the presence of baroclinic wave mode excitation (Ponte 1992; Wunsch and Stammer 1997). The numerical results indicate that realistic pressure loading fields do not project strongly onto possible large-scale baroclinic waves or resonances, or else that dissipation, as parameterized in the model, can render such baroclinic processes unimportant in general.

In our solutions, the strongest departures from barotropic behavior tend to be connected to flow interactions with bottom topography in regions of enhanced topographic gradients and in low-latitude regions.
where equatorial waves are possible. In these regions of stronger induced baroclinicity, differences between the dynamic surface and bottom pressure variability are on the order of 10%–20% of that in surface signals, under stochastic forcing. Errors of this amplitude are thus possible in loading simulations that assume a constant-density, homogeneous ocean. Consideration of baroclinic effects could lead to improvements in modeling the ocean dynamic response to stochastic pressure loading at weekly and shorter time scales. Given the importance of bottom pressure for gravity missions like GRACE, consideration of stratified models could also improve the procedures to dealias such data. A complete investigation of these issues is left for future study.

Figure 4 highlights the dynamic character of the response to \( P_a \) in shallow and semienclosed regions, where \( \xi \) amplitudes can easily reach several centimeters. Departures from IB behavior are also not negligible at many mid- and high-latitude deep-ocean regions. Modeling efforts that attempt to simulate weekly and shorter periods would benefit from the inclusion of \( P_a \) forcing. Conversely, data assimilation efforts that assume an IB response need to consider \( P_a \)-driven dynamic signals as an additional “representation” error. Values in Fig. 4 provide a crude estimate of those errors.

As a final point, we return to the exploratory nature of this work and some of its shortcomings. In particular, baroclinic influences are likely to depend on horizontal variations in stratification and especially on the horizontal resolution, of both model and forcing fields. Further investigation of the ocean response to \( P_a \) and tidal loading would benefit from improved vertical and horizontal resolution, to be able to span shorter scales and the full influence of topography in inducing baroclinic effects, and from longer periods of integration, to better assess the behavior at the low-frequency end of the spectrum. Such efforts are currently being pursued.

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APPENDIX

Sea Level and Related Variables

Sea level and bottom pressure are the two main variables analyzed in this work. When sea level gradients balance the applied loading, one has the so-called equilibrium or inverted barometer (IB) solution. Sea level is then given by

\[
\xi^{ib} = \frac{P_a - P_a}{g \rho_a},
\]

(A1)

where \( \rho_a \) is the ocean surface density, \( g \) is the acceleration of gravity, and \( \overline{P_a} \) is the spatial average of \( P_a \) over the global ocean. Deviations from the IB solution can be studied in terms of adjusted or dynamic sea level defined as

\[
\xi_d = \xi - \xi^{ib}.
\]

(A2)

These surface variables can be easily related to bottom pressure \( p_b \). Vertical integration of the hydrostatic pressure balance from the bottom depth \( H \) to the surface yields

\[
p_b = g \rho_s \xi + g \int_0^H p \, dz + P_a.
\]

(A3)

Rewriting \( \xi \) as \( \xi^{ib} + \xi_d \) from (A2), and using (A1) gives

\[
p_b = g \rho_s \xi_d + g \int_0^H p \, dz + \overline{P_a}.
\]

(A4)

If \( \rho \) is constant and the response is isostatic (\( \xi_d = 0 \)), then \( p_b \) is spatially constant and equal to \( \overline{P_a} \). To explore the effects of stratification, it is useful to work with the “dynamic” bottom pressure \( p_b - \overline{P_a} \) or in terms of the equivalent sea level:

\[
\xi_b = \frac{p_b - \overline{P_a}}{g \rho_s}.
\]

(A5)

Whereas in a barotropic case, \( \xi_b \) is simply equal to \( \xi_d \), a variable \( \rho \) allows for other behaviors. Differences between \( \xi_b \) and \( \xi_d \) can be taken as a measure of the effects of stratification.