Unstable Frontal Waves along the Kuroshio Extension with Low-Potential Vorticity Intermediate Oyashio Water

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ABSTRACT
A linear stability analysis was conducted for a three-layer primitive equation model including viscosity with a basic state, which modeled the stratification and velocity fields with the vertical and horizontal variations across the Kuroshio Extension. An unstable wave with a wavelength of 220 km and a phase speed of 0.24 m s$^{-1}$ propagating in the downstream direction was found to grow the fastest. Characteristics of this unstable baroclinic wave were similar to those of waves observed along the Kuroshio Extension. The growth rate of the fastest-growing waves became greater with an increase of the cross-stream difference of the potential vorticity (PV) in the intermediate layer. For a cross-frontal stratification structure without the PV gradient in the intermediate layer, which is similar to that in the Gulf Stream, the wavelength of the fastest-growing unstable wave changed to 390 km and the unstable wave had a much different structure. Thus, the unstable frontal waves observed along the Kuroshio Extension occur only for the cases when low-PV Oyashio water exists on the northern side of the main stream in the intermediate layer. The unstable frontal waves revealed in the present study greatly contribute to the formation of a clear salinity minimum in the Kuroshio Extension.

1. Introduction
The Kuroshio Extension is an eastward-flowing jet current that appears after the Kuroshio separates from the east coast of Japan near the Boso Peninsula. This region is one of the main formation sites of the salinity minimum that characterizes North Pacific Intermediate Water (NPIW), which is found at depths of 200–800 m in the subtropical gyre (Yasuda et al. 1996; Talley 1997; Hiroe et al. 2002). Warm saline water is transported into the Kuroshio Extension region by the Kuroshio (Fujimura and Nagata 1992; Yasuda et al. 1996; Hiroe et al. 2002), and cold low-salinity water with low potential vorticity (PV) is transported by the Oyashio, which is the western boundary current of the subarctic gyre in the North Pacific (e.g., Yasuda 1997; Shimizu et al. 2001; Yasuda et al. 2002). These two water masses meet along the Kuroshio Extension, where a strong gradient of water properties is formed, and the two types of waters are rapidly mixed, forming the precursor of the new NPIW at about 150°E (Masujima et al. 2003). Isopycnal mixing by mesoscale eddies has been proposed as an important process in NPIW formation along the Kuroshio Extension, particularly between 140° and 150°E (Yasuda et al. 1996; Okuda et al. 2001). However, the role of frontal eddies along the Kuroshio Extension remains unclear, because there have been few studies of frontal eddy structure and generation in this region. Analyses of infrared satellite images have shown that surface frontal waves with the wavelengths of 10–200 km propagate in the downstream direction with phase speeds of 0.05–0.45 m s$^{-1}$ and that waves with larger wavelengths tend to propagate faster (Hirai 1985; Mizuno 1985). Although Hirai (1985) and Mizuno (1985) speculated that baroclinic instability was a possible cause of the formation of the frontal waves, no
studies have proposed mechanisms that can explain the observed frontal waves along the Kuroshio Extension.

Kouketsu et al. (2005) have shown that frontal waves exist along the Kuroshio Extension not only at the sea surface but also in the intermediate layer, through analysis of data collected by a series of intensive observations (Fig. 1). The frontal waves are associated with a clear salinity minimum at 26.8σt, which appears to be caused by low-salinity water intrusion from the northern side of the main stream into its intermediate layer. Furthermore, the amplitudes of the frontal waves were observed to be greater in the downstream direction. They suggested that these frontal waves may contribute to isopycnal mixing to form new NPIW along the Kuroshio Extension. Frontal waves with wavelengths of about 200 km propagated in the downstream direction, and the phase of the waves in the upper layer precedes that of waves in the intermediate layer by about 1/4 wavelength (Kouketsu et al. 2007). Baroclinic instability may explain observed features of the frontal waves along the Kuroshio Extension.

Investigations of the Gulf Stream include many studies of frontal waves along an oceanic jet. Along the Gulf Stream (75°–45°W), growing meanders with a phase speed of about 0.05–0.15 m s⁻¹ in the downstream direction have been reported. The estimated wavelength of the most energetic meanders is 427 km (Lee and Cornillon 1996). Furthermore, isopycnal floats move horizontally from the southern (northern) to the northern (southern) side of the main stream and vertically upward (downward) from the trough (crest) to crest (trough) of the meanders (Bower and Rossby 1989; Song et al. 1995; Song and Rossby 1997). The upward (downward) motion of the floats corresponds to horizontal divergence (convergence) from the meander troughs (crests) to crests (troughs; Bower 1989).

This phenomenon has been explained by eastward-propagating waves, and it has been suggested that it contributes to cross-stream exchange in the Gulf Stream (Bower 1991). Kouketsu et al. (2007) pointed out that the estimated vertical velocity field along the Kuroshio Extension is consistent with the behavior of the floats in the Gulf Stream. However, the two jet currents have different stratification structures; the Kuroshio has low-PV water at intermediate depths of 200–600 m on the northern side of the main stream, whereas the Gulf Stream does not.

Because oceanic jets have strong shear, the fluctuations along with the jets can grow unstably (e.g., Pedlosky 1987). Many studies have used stability analysis to investigate unstable growth of the fluctuations in oceanic fronts and jets. Orlanski (1969) used a two-layer model to investigate a baroclinic jet modeled on the Gulf Stream and demonstrated the effects of bottom topography and stratification on baroclinic instability; he also pointed out that the wavelength of the fastest-growing waves is about 400 km in the deep ocean, for example, where the Gulf Stream separates from the continental shelf. Lozier and Bercovici (1992) performed a linear stability analysis with a simple model with uniform zonal flows and demonstrated that strong cross-stream exchange occurs in the layer deeper than the depth where the cross-stream gradient of PV is at a minimum. Because their analysis was conducted with a one-dimensional model, it cannot explain the behavior of the floats along the Gulf Stream or the structure of frontal waves in the Kuroshio Extension. Furthermore, the meridional shear should be included to investigate the stability of the jets in more realistic situations because the ocean jets like the Gulf Stream and the Kuroshio Extension have strong horizontal shear as well as strong vertical shear. Here, we performed a linear stability analysis for a basic state that modeled the upstream region of the main stream to elucidate the physical mechanism of the observed frontal waves along the Kuroshio Extension. Because the Kuroshio Extension has strong horizontal and vertical shear and has low-PV water at intermediate depths on the northern side of the main stream, we employed a three-layer primitive equation model on a midlatitude f plane with a horizontal eddy viscosity term and sech-type (cosh⁻²) jets for the basic state. In this basic state, we can study the mixed barotropic–baroclinic instability problem and take into account the effects of vertical curvature changes by using a three-layer model, which cannot be included in two-layer models. About the vertical curvature changes, we performed a linear stability analysis and compared the results for several cases in which the intermediate PV gradient was changed. We also per-

![Fig. 1. Salinity distribution at 26.8σt (from Kouketsu et al. 2005). The dashed curve represents the main stream of the Kuroshio Extension. Dots denote the observation stations.](image-url)
formed the stability analysis for cases in which the horizontal eddy viscosity coefficient was changed to check the sensitivity to the growth of frontal waves. Furthermore, we computed all of the energy transfer terms to investigate which effects contribute to the growth of the frontal waves.

2. Methods

a. Observation fields and basic state for linear stability analysis

The Kuroshio Extension is an eastward jet with a peak velocity of more than 1 m s\(^{-1}\). The northern edge of Subtropical Mode Water (STMW) (e.g., Suga et al. 1989), which is low-PV and high-salinity water, is observed on the southern side of the main stream in the density range of 25.2–25.6\(\sigma_t\). On the other hand, in the density range of 26.2–26.9\(\sigma_t\), Oyashio water, which is low-PV and low-salinity water, reaches the northern side of the main stream (e.g., Yasuda et al. 1996; shaded region in Fig. 1). Because these two water types flow concurrently, the sign of the gradients of the layer thickness reverses between 25.5 and 26.6\(\sigma_t\). This type of stratification satisfies a prerequisite for baroclinic instability. Kouketsu et al. (2005, 2007) observed the frontal waves associated with the low-salinity water intrusion from the northern side of the main stream in the intermediate layer (Fig. 1), which they may be caused by baroclinic instability. The frontal waves with a wavelength of about 200 km propagate in the downstream direction at 0.2–0.3 m s\(^{-1}\) and are associated with upwelling (downwelling) of up to 40 m day\(^{-1}\) from the trough (crest) to the crest (trough; Fig. 2). Note that in this paper, the term front does not refer to a feature of the surface mixed layer but to one in the region of strong meridional gradients with jets (Fig. 1).

To model the characteristic velocity and stratification fields (dashed curves in Fig. 3) and to investigate the generation of the intermediate frontal waves, at least three layers are necessary, along with a Rossby number that is not necessarily small. We therefore employed a three-layer primitive equation model on a midlatitude f plane with a horizontal eddy viscosity term.

To represent the observed field as a basic state as simply as possible and to facilitate convergence of the numerical scheme, we chose the following analytic form for the density interfaces:

\[
d_i = \frac{D_F^i}{2} \tanh\left(\frac{(y - Y_i)}{L_i}\right) + D_i. \tag{1}\]

Here, for the \(i\)th interface, \(d_i\) denotes the interface depth (m), \(D_F^i\) is the difference between the depths to interface \(i\) at the northern and southern boundaries, \(Y_i\) is the meridional location of the front to the center of

Fig. 2. Schematic representation of the observed frontal waves (from Kouketsu et al. 2007).

Fig. 3. (a) Basic-state stratification, (b) velocity profiles, and (c) PV gradients for the std case. Model density interfaces [bold curves in (a)] were chosen according to Table 1. The dashed contours superposed on the shaded region in (a) denote the mean stratification structure observed in 2002 (see Kouketsu et al. 2007 for a description of these observations). The velocity in (b) was calculated with the thermal wind relation. The left axis in (c) is for the upper layer, and the right axis is for the intermediate and bottom layers.
domain (km), \( L_i \) is the front width (km), and \( D_i \) is the mean depth of the interface. Note that in our model \( i = 0 \) is at the sea surface on the top of the upper layer, \( i = 1 \) is between the upper and intermediate layers, and \( i = 2 \) is between the intermediate and bottom layers.

The basic flow is assumed to be zonal and geostrophic:

\[
V_i = \begin{bmatrix} U_i(y) \\ 0 \end{bmatrix}, \quad U_i(y) = -\frac{1}{\rho_o f \partial y} \left( \frac{\partial P_i}{\partial y} \right),
\]

\[
\frac{\partial P_i}{\partial y} = \rho_i g \frac{\partial H}{\partial y}, \quad \frac{\partial P_2}{\partial y} = \rho_i g \frac{\partial H}{\partial y} - \rho_o g_1 \frac{\partial H_i}{\partial y},
\]

\[
\frac{\partial P_3}{\partial y} = \rho_i g \frac{\partial H}{\partial y} - \rho_o g_1 \frac{\partial H_i}{\partial y} - \rho_o g_2 \left( \frac{\partial H_1}{\partial y} + \frac{\partial H_2}{\partial y} \right),
\]

\[
g_1 = \frac{\rho_2 - \rho_1}{\rho_o} g, \quad g_2 = \frac{\rho_3 - \rho_2}{\rho_o} g.
\]  

Here, \( g \) is the acceleration of gravity, \( g_1 \) and \( g_2 \) are the reduced gravities at the first and second interfaces, respectively, \( \rho_i \) is an averaged density in each layer, and \( H_i \) and \( U_i \) are \( i \)-th layer basic-state thickness (m) and zonal velocity (m s\(^{-1}\)), respectively. Parameters for the standard case, which is based on actual observations, are listed in Table 1.

Basic-state density fields are shown in Fig. 3a, where the model and observed density fields are superposed. The stratification structure was obtained from the observations between 143.5\(^\circ\) and 145\(^\circ\)E in 2002 (see Kouketsu et al. 2007 for details). The basic-state velocity fields were calculated using the geostrophic relation [Eq. (2)]. The peak jet velocity was set to 1.04 m s\(^{-1}\) in the upper layer, 0.27 m s\(^{-1}\) in the intermediate layer, and \(-0.020 \text{ m s}^{-1}\) in the bottom layer (Fig. 3b). The meridional PV gradient, which is important for the requisite condition of baroclinic instability, is not symmetric about the peak of the jet because of the difference of the layer thicknesses between the northern and southern sides of the jet (Fig. 3c). The extrema of the PV gradient are north and south of the center of the jet in the upper and intermediate layers, respectively.

The interface gradients can change temporally and spatially. For example, PV on the northern side of the main stream in the upstream region west of 144\(^\circ\)E, in the intermediate layer, is lower than that in the downstream region (Fig. 4 in Kouketsu et al. 2005), and the meridional gradient of salinity, which corresponds to the PV gradient, becomes smaller in the downstream direction (see Fig. 1). Thus, we employed a basic-state stratification (Fig. 3) as one example and then carried out additional stability analyses for situations in which the stratification structure was changed.

### b. Linear stability analysis

Linear stability analysis has been applied to oceanic jets and mixed layer fronts in many studies (e.g., Moore and Peltier 1987; Shi and Røed 1999). Linearized perturbation equations for the layered primitive equation with a horizontal eddy viscosity are

\[
\frac{\partial u'_i}{\partial t} + U'_i \frac{\partial u'_i}{\partial x} + v'_i \frac{\partial u'_i}{\partial y} = -\frac{1}{\rho_o} \frac{\partial p'_i}{\partial x} + A_h \nabla^2 u',
\]

\[
\frac{\partial v'_i}{\partial t} + U'_i \frac{\partial v'_i}{\partial x} + f v'_i = -\frac{1}{\rho_o} \frac{\partial p'_i}{\partial y} + A_h \nabla^2 v',
\]

\[
\frac{\partial h'_i}{\partial t} + H'_i \left( \frac{\partial u'_i}{\partial x} + \frac{\partial v'_i}{\partial y} \right) + U'_i \frac{\partial h'_i}{\partial x} + v'_i \frac{\partial h'_i}{\partial y} = 0,
\]

\[
\frac{\partial p'_i}{\partial y} = \rho_i g \frac{\partial h'_i}{\partial y} - \rho_o g_1 \frac{\partial h'_i}{\partial y},
\]

\[
\frac{\partial p'_2}{\partial y} = \rho_i g \frac{\partial h'_2}{\partial y} - \rho_o g_1 \frac{\partial h'_2}{\partial y} - \rho_o g_2 \left( \frac{\partial h'_1}{\partial y} + \frac{\partial h'_2}{\partial y} \right),
\]

\[
h' = \sum h'_i.
\]
Here, \( h_i \) and \((u_i', v_i')\) are the \( i \)th-layer thickness and velocity perturbations, respectively. The \( f \) is the Coriolis parameter, \( A_h \) is horizontal eddy viscosity coefficient, and \( \rho_0 \) is the constant reference density.

It is convenient to confine the domain of the basic flow and the disturbance to a zonal channel with walls at \( y = \pm L_d/2 \), which are the northern and southern solid boundaries, respectively, and where \( L_d \) is the meridional domain width. Then the free-slip and no-normal-flow boundary conditions are

\[
\frac{\partial u}{\partial y} = 0, \quad v = 0 \text{ at } y = \pm L_d/2.
\]

For the boundary condition, the variables of \( u_y \) and \( h_i \) can be expanded by finite sums of trigonometric basis functions in \( y \). Thus, we assumed the following forms of solutions for the equations:

\[
\begin{align*}
\bar{u}_i &= \bar{u}_i(y) \exp[i(\alpha t + kx)], \\
\bar{v}_i &= \bar{v}_i(y) \exp[i(\alpha t + kx)], \\
\bar{h}_i &= \bar{h}_i(y) \exp[i(\alpha t + kx)], \\
\bar{u}_i(y) &= \sum_{n=0}^{M} u_{\lambda_n} \phi_{\lambda_n} [\bar{u}_i(y), \bar{h}_i(y)] = \sum_{n=0}^{M} (v_{\lambda_n}, h_{\lambda_n}) \psi_{\lambda_n}, \\
\phi_{\lambda_n} &= \cos \left( \frac{\lambda_\pi(y + L_d/2)}{L_d} \right), \quad \psi_{\lambda_n} = \sin \left( \frac{\lambda_\pi(y + L_d/2)}{L_d} \right).
\end{align*}
\]

\[
\frac{\partial E_e}{\partial t} = -\nabla \cdot \mathbf{F}_e + D + C_1 + C_2 + C_3, \tag{7}
\]

\[
\begin{align*}
C_1 &= -\rho_0 \sum_{i=1}^{3} \left[ (U_i \bar{u}_i + H_i \bar{u}_i \cdot \nabla U_i) + (V_i \bar{u}_i + H_i \bar{v}_i \cdot \nabla V_i) \right] \\
C_2 &= -\rho_0 \sum_{i=1}^{3} \left[ U_i \bar{u}_i \nabla \cdot (H_i \bar{u}_i) + V_i \bar{v}_i \nabla \cdot (H_i \bar{u}_i) \right] \\
C_3 &= -\rho_0 g_1 \sum_{i=1}^{3} \left[ H_i \bar{u}_i \cdot \nabla (H_i + H_3) + g_2 \bar{u}_i \cdot \nabla H_3 + \sum_{i=1}^{3} \bar{h}_i \bar{u}_i \cdot \nabla H \right]. \tag{8}
\end{align*}
\]

Here, \( \mathbf{F}_e \) and \( D \) represent energy flux divergence and dumping, respectively. The integral of these terms \((\nabla \cdot \mathbf{F}_e \) and \( D \)) over the whole domain is zero. Because \( C_1 \) is composed of the terms that are proportional to the mean velocity horizontal shear \((\nabla \cdot \mathbf{V})\), \( C_1 \) represents energy conversions corresponding to barotropic instability. The terms in \( C_3 \) are the products of mean horizontal velocity \((\mathbf{V})\) and the correlation between eddy horizontal velocity \((\mathbf{u}'_i)\) and vertical motion \([\nabla \cdot (H_i \mathbf{u}'_i)]\), indicating Kelvin–Helmholtz instability. The terms in \( C_3 \) are proportional to \( \nabla H_i \), and \( C_3 \) represents baroclinic instability.

3. Results

a. Unstable frontal waves in a realistic basic state of velocity and stratification

The numerical solution of the eigenvalue problem yielded growth rates, phase speeds, and modal structures for each eigenmode for a particular zonal wavenumber. The growth rate of the most unstable mode at each zonal wavenumber was determined (Fig. 4); the maximum growth rate of these modes corresponds to a dimensional \( e \)-folding time of 52 days and occurs at the
wavelength of 220 km. The phase speed of this unstable wave at the maximum growth rate is 0.24 m s\(^{-1}\).

The structures of the fastest-growing unstable waves calculated [Eqs. (2) and (5)] are shown in Fig. 5. Because perturbation velocity \(u'\) is obtained as the eigenvector from the eigenvalue problem [Eq. (6)], the magnitude is arbitrary. To show the PV wave fields, we set the magnitude at 5% of the magnitude of the peak jet velocity (≈1.04 m s\(^{-1}\)), because the present linear stability analysis allows only small amplitude perturbations. In each layer, wavelike patterns exist around the PV front where the meridional gradient of the potential vorticity is at a maximum (Fig. 3c). The PV front in the upper layer is located around \(y = 10\) km on the northern side of the current axis at \(y = 0\) km (Fig. 5a). The PV front in the intermediate layer is near the current axis (Fig. 5c). These PV fronts are predominantly determined by the layer-thickness term (1/\(H_i\)). Because the layer-thickness term is not dominant for the PV front in the bottom layer, the magnitude of the PV gradient is large at the same position of the local maximum of |\(U_{yy}'\)| (Fig. 5e).

The wave amplitude is largest in the intermediate layer (Fig. 5c) and smallest in the bottom layer (Fig. 5e). The phase of the upper-layer frontal wave precedes that of the intermediate-layer frontal wave by about 1/4 wavelength, as is evident in Figs. 5a,c where the wave troughs are located at \(x = 30\) km in the upper layer and at \(x = -30\) km in the intermediate layer. This phase difference suggests that this type of wave is caused by baroclinic instability. Furthermore, when energy is transferred from basic currents to perturbations through barotropic instability, the wave crest and trough are horizontally inclined against the zonal basic flow profile so as to decelerate it (Pedlosky 1987). However, our analysis indicates that the wave crest and trough are horizontally inclined with the zonal flow profile (Fig. 5c), suggesting that barotropic instability is not occurring.

The energy conversion [Eq. (8)] also supports the conclusion that baroclinic instability is the main cause of frontal wave development (Table 2). The energy conversion corresponding to baroclinic instability (\(C_3\)) is more than twice as large as the other conversion terms. Thus, baroclinic instability dominates the growth of the unstable waves obtained from the linear stability analysis in the present study. This suggests that the experiments using a basic state without meridional shear (e.g., Lozier and Bercovici 1992) can represent the instability of jets as in the present study to some extent, while the horizontal structure of unstable waves cannot be shown.

The features of the fastest-growing wave resulting from our linear stability analysis are similar to those of the frontal wave observed along the Kuroshio Extension (as shown in Figs. 5a,b for the upper layer and in Figs. 5c,d for the intermediate layer). From the estimates considering the aliasing effect due to wave propagation (Kouketsu et al. 2007), the wavelength of the observed frontal wave was about 200 km, the phase speed was about 0.2–0.3 m s\(^{-1}\), and the upper-layer frontal wave preceded the intermediate frontal wave by about 1/4 wavelength (see Fig. 2). Furthermore, the linear stability analysis showed that downwelling (upwelling) occurs from the crest (trough) to the trough (crest) of the upper-layer frontal wave in the upper layer (Fig. 6), and this pattern is the same as that indicated by the vertical velocity fields estimated by Kouketsu et al. (2007, their Fig. 8). We therefore suggest that the observed frontal wave associated with a clear salinity minimum (see Fig. 2) arose from the baroclinic instability that appeared in our linear stability analysis and that the observed frontal waves can frequently be generated along the Kuroshio Extension.

b. Sensitivity of unstable frontal waves to the basic flow and stratification

In section 3a, we showed that the fastest-growing unstable wave obtained from the linear stability analysis for the Kuroshio Extension was similar to the observed frontal wave. Because the frontal structure along the Kuroshio Extension is variable not only spatially but also temporally, we examined the dependence of wave formation on various basic states. We checked the sen-
sitivity of the baroclinic instability in our linear stability analysis by changing the parameters of the basic state and compared the structures of the resulting unstable waves.

Table 2. Energy conversion from mean field energy to perturbation energy (J m⁻³). The form of $C_1$, $C_2$, and $C_3$ is defined in Eq. (8).

<table>
<thead>
<tr>
<th>$C_1$ barotropic instability</th>
<th>$C_2$ Kelvin–Helmholtz instability</th>
<th>$C_3$ baroclinic instability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6</td>
<td>0.5</td>
<td>5.2</td>
</tr>
</tbody>
</table>

1) Sensitivity to Intermediate-Layer Structure

Because the existence of the low-PV Oyashio water north of the main stream in the intermediate layer is a characteristic feature of the stratification in the Kuroshio Extension, we varied the intermediate-layer thickness to check the sensitivity of wave formation to the PV gradient in the intermediate layer (Table 3).

Case 4 in Table 3 is the same as the “standard case” used to model the stratification observed in 2002, the results of which were described in section 3a. We per-
formed the linear stability analysis under conditions (cases) in which the differences between the intermediate-layer thickness at the northern and southern boundaries varied between 0 and 240 m. In each case, we changed only the depth of the second density interface \([d_2 \text{ in Eq. (1)}]\) between intermediate and bottom layers, keeping the mean velocity profiles in the upper and intermediate layers the same as for the standard case and modifying the bottom-layer mean velocity field corresponding to the changes in the stratifications.

The growth rates of the fastest-growing unstable wave for each case increased with the increase of the meridional difference in the intermediate-layer thickness and thus with the increase of the PV gradient in the intermediate layer (Fig. 7a). The dimensional e-folding time at the local maximum growth rate with a wavelength of 200–250 km changed from 4.2 days for case 1 to 7.9 days for case 7. The wavelengths of these unstable waves at the maximum growth rate did not change much, ranging from 210 km for case 1 to 230 km for case 7. The wave structures also did not change much and are quite similar to the standard case of case 4 (Fig. 5). The phase speed for each case was fairly constant at around 0.24 m s\(^{-1}\) at wavelengths of maximum growth rate (Fig. 7b). The short-wavelength unstable waves are caused by the coupling of the upper and intermediate layers. They occur even in the present three-layer model when there is a strong negative PV gradient in the intermediate layer.

In cases 6 and 7, a second local growth rate maximum exists at wavelengths of about 390 km (see Fig. 7a). In these weak PV-gradient cases, the phase speed is 0.03 m s\(^{-1}\). This type of unstable frontal wave has a different structure from the observed wave (see Fig. 2). The phase of these frontal waves of PV distribution in the upper layer is about the same in the intermediate layer and precedes the one in the bottom layer by 1/4 wavelength. These unstable waves may be caused mainly by coupling between the upper and bottom layers. This suggests that the short-wavelength unstable frontal waves similar to the one observed along the Kuroshio Extension require the presence of low-PV Oyashio water on the northern side of the current axis in the intermediate layer.

In each case examined, the energy conversion by the baroclinic instability was larger than that by the barotropic instability in the unstable mode with a wavelength of about 200 km (Fig. 8). Although the cross-stream thickness gradient in the intermediate layer is about 0 in case 7, the baroclinic instability still occurs. This is due to the horizontal velocity shear in the intermediate layer, which makes the PV gradient negative in this case. However, the energy conversion due to baroclinic instability rapidly decreases with the decrease of the thickness gradient in the intermediate layer.

2) SENSITIVITY TO SEA SURFACE HEIGHT AND THERMOCLINE VARIABILITY

The stratification structure of the Kuroshio Extension changes with time, especially in the upper layer. Qiu and Chen (2005) investigated the decadal changes of the Kuroshio Extension jet, its southern recirculation gyre, and their mesoscale eddy fields. The Kuroshio

<table>
<thead>
<tr>
<th>Case</th>
<th>Difference in intermediate-layer thickness</th>
<th>(D_{F2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>240 m</td>
<td>280 m</td>
</tr>
<tr>
<td>2</td>
<td>200 m</td>
<td>320 m</td>
</tr>
<tr>
<td>3</td>
<td>160 m</td>
<td>360 m</td>
</tr>
<tr>
<td>4</td>
<td>120 m</td>
<td>400 m</td>
</tr>
<tr>
<td>5</td>
<td>80 m</td>
<td>440 m</td>
</tr>
<tr>
<td>6</td>
<td>40 m</td>
<td>480 m</td>
</tr>
<tr>
<td>7</td>
<td>0 m</td>
<td>520 m</td>
</tr>
</tbody>
</table>
Extension jet and its recirculation gyre were gradually weakening from 1992 to 1996 and strengthening after 1997. They suggested that the changes were caused by sea surface height (SSH) anomalies, which were generated by curl $\tau$ anomalies corresponding to Pacific decadal oscillation in the central North Pacific and propagating through baroclinic Rossby waves. We examined the sensitivity of the frontal wave generation to variability of the stratification in the upper layer.

From the comparison of the cases involving SSH ($d_0$) changes (Table 4), a large (small) difference in SSH between the northern and southern sides tended to cause less (greater) growth of unstable frontal waves with a wavelength of about 200 km (Fig. 9). On the other hand, a large (small) difference in the depth to the first interface ($d_1$; Table 5) between the northern and southern boundaries tended to cause strong (weak) evolution of unstable frontal waves with wavelengths around 200 km (Fig. 10). This suggests that a stronger Kuroshio Extension jet may not always generate more unstable frontal waves, although a strong vertical shear tends to generate faster-growing unstable waves.

3) SENSITIVITY TO HORIZONTAL VISCOSITY

We also checked the sensitivity of wave generation to the horizontal eddy viscosity. In the linear stability analyses presented so far, we set the horizontal viscosity ($A_h$) at $10^4$ cm$^2$ s$^{-1}$. The growth rate of the unstable waves depends on the horizontal viscosity (Fig. 11).

**TABLE 4.** Case settings for SSH change. Std case is in case 2.

<table>
<thead>
<tr>
<th>Case</th>
<th>$D_{F0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>$-0.86$ m</td>
</tr>
<tr>
<td>Case 2</td>
<td>$-0.68$ m</td>
</tr>
<tr>
<td>Case 3</td>
<td>$-0.50$ m</td>
</tr>
</tbody>
</table>

**FIG. 7.** (a) Growth rate and (b) phase speed for the cases listed in Table 3.
When the horizontal viscosity was set at $10^5 \text{ cm}^2 \text{s}^{-1}$, the growth rates of the unstable waves became smaller for the waves with shorter wavelengths (long-dashed curve in Fig. 11). The growth rates of the unstable waves when the horizontal viscosity was set at $10^3 \text{ cm}^2 \text{s}^{-1}$ (short-dashed curve in Fig. 11) are similar to those of the standard case. The unstable wave with a wavelength of about 220 km grows fastest when $A_h \leq 10^5 \text{ cm}^2 \text{s}^{-1}$ (Fig. 11), indicating that the unstable waves consistent with observations result from the linear stability analyses where $A_h \leq 10^5 \text{ cm}^2 \text{s}^{-1}$.

### 4. Discussion and conclusions

Linear stability analysis was performed for a basic state that modeled the velocity and stratification fields of the Kuroshio Extension. We found that the short-wavelength instability was caused mainly by baroclinic instability, although we simplified the stratification by using a three-layer model. Although the basic state used in this study was much simpler than the state of the real Kuroshio Extension because of assumptions such as a zonally uniform basic state and three-layer stratification, the fastest-growing unstable waves that resulted were similar to the observed frontal waves (Kouketsu et al. 2005, 2007). The wavelength of 220 km predicted by the model is similar to the observed wavelength of about 200 km. The predicted phase speed of $0.24 \text{ m s}^{-1}$ is also similar to that observed at about $0.2-0.3 \text{ m s}^{-1}$. The phase difference between the upper and intermediate layers is also common to both in the linear analysis and the observations. This kind of unstable

<table>
<thead>
<tr>
<th>Case</th>
<th>$D_{f1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>480 m</td>
</tr>
<tr>
<td>Case 2</td>
<td>520 m</td>
</tr>
<tr>
<td>Case 3</td>
<td>560 m</td>
</tr>
</tbody>
</table>

**Fig. 9.** Growth rate for the cases listed in Table 4.

**Fig. 10.** Growth rate for the cases listed in Table 5.

**Fig. 11.** Wave growth rates for the std-, high-, and low-viscosity cases.
wave results from baroclinic instability, because the intermediate-layer PV anomalies precede the upper-layer ones by 1/4 wavelength, and the energy conversions corresponding to baroclinic instability are dominant in comparison with the other conversion terms (Table 2). The wavelength, phase speed, and wave structure are not very sensitive to the degree of the intermediate-layer PV gradient as long as the low-PV Oyashio water exists on the northern side of the Kuroshio Extension. We therefore conclude that the observed frontal wave is an unstable wave in the presence of the low-PV Oyashio water in the intermediate layer. Kouketsu et al. (2007) showed that the vertical phase lag created situations where the low-salinity Oyashio water in the intermediate layer was positioned below the upper-layer, high-salinity Kuroshio water, thus forming a vertical salinity minimum in the Kuroshio Extension main stream. The results of the present study suggest that the observed structures of the frontal waves are also caused by baroclinic instability. The development of the wave can encourage the isopycnal mixing that forms new NPIW in the Kuroshio Extension region through nonlinear effects such as eddy shedding and eddy breaking. Furthermore, eddy shedding may play an important role in cross-frontal transport across the Kuroshio Extension.

The growth rate of the unstable waves became greater with the increase of the PV gradient in the intermediate layer. Along the Kuroshio Extension, low-PV Kuroshio water is distributed on the southern side of the current axis in the upper layer, and low-PV Oyashio water is distributed on the northern side in the intermediate layer. We showed that unstable frontal waves similar to the ones observed could form only for the cases in which low-PV water existed on the northern side in the intermediate layer. This indicates that the low-PV Oyashio water originating from the Okhotsk Sea is essential both as the low-salinity source of NPIW and for the mixing with the high-salinity Kuroshio water. On the other hand, in cases with weak intermediate-layer PV gradients, frontal waves with longer wavelengths and slower phase speeds grow quickly. Wavelengths of about 390 km and phase speeds of 0.03 m s$^{-1}$ are near the value reported for the Gulf Stream meanders (Lee and Cornillon 1996). The meanders in the Gulf Stream may be interpreted in light of the results from the weak intermediate-layer PV-gradient cases.

Horizontal eddy viscosity affected the growth rates of frontal waves in our model, although the local maximum of the growth rates existed at the shorter wavelengths even when $A_h$ was large. In field observations (see Fig. 1), the amplitude of the frontal waves rapidly became large in the downstream direction. When the frontal waves propagate with a phase speed of 0.2–0.3 m s$^{-1}$, an $A_h$ of $10^4$–$10^5$ cm$^2$ s$^{-1}$ may be consistent with the growth rate estimated from the field observations, although long-term continuous observations are needed to determine the actual values.

Our sensitivity analysis suggested that the activity of the frontal waves may change with the decadal variability of the Kuroshio Extension, which was investigated in previous studies (e.g., Qiu and Chen 2005), and may influence cross-frontal exchange. However, studies using more realistic states considering nonlinear effects are needed to clarify the relation between the long-term variability and eddy fields in the Kuroshio Extension region.

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APPENDIX A

Components of Coefficient Matrix (E)

After substituting solution forms [Eqs. (5)] into Eq. (3), we obtained nondimensional perturbation equations with the following scales (subscripted asterisk denotes dimensional values):

\[
(x_0, y_0) = L(x, y), \ t_0 = \frac{1}{f} t, (u_0, v_0) = U_0(u, v), h_0 = \frac{D_s U_0}{fL}, h, \\
\nabla p_0 = fU_0 \nabla p, U_0 = V_0 U, V_0 = \max(|U_i(y)|), \\
\alpha_1 = \frac{g_1}{g}, \ \alpha_2 = \frac{g_2}{g}, \ D_d = \sum_i H_i, \ \epsilon = \frac{V_0}{fL}, \ S = \frac{gD_d}{f^2L^2}, E_h = \frac{A_h}{fL^2}. \tag{A1}
\]

Here, $U_0$ is an arbitrary value of the characteristic scale of the velocity perturbation, $\epsilon$ is a Rossby number, $S$ is a Burger number, and $E_h$ is a horizontal Ekman number.
Using these scales, we obtained the following eigenvalue problem equations:

\[
\sigma u_{\lambda i} = \left[ iE \left( k^2 + \frac{\lambda^2 \pi^2}{L_d^2} \right) \delta_{\lambda \lambda} - \frac{1}{1 + \delta_{\alpha 0}} e k(\phi_{\alpha} U_{\phi_{\alpha}}) \right] u_{\lambda i} + \frac{1}{1 + \delta_{\alpha 0}} \left[ e \left( \phi_{\alpha} \frac{\partial U_i}{\partial y} \psi_{\lambda} \right) + 4 \lambda \frac{\text{mod}_2(\kappa - \lambda)}{(\kappa^2 - \lambda^2)\pi} \right] v_{\lambda i} - kS g_{\lambda i},
\]

(A2)

\[
\sigma v_{\lambda i} = -4k \frac{\text{mod}_2(\kappa - \lambda)}{(\kappa^2 - \lambda^2)\pi} u_{\lambda i} + \left[ iE \left( k^2 + \frac{\lambda^2 \pi^2}{L_d^2} \right) \delta_{\lambda \lambda} - \frac{1}{1 + \delta_{\alpha 0}} e k(\psi_{\lambda} U_{\phi_{\alpha}}) \right] v_{\lambda i} + S \frac{\kappa \pi}{L_d} g_{\lambda i}.
\]

(A3)

\[
\sigma h_{\lambda i} = -\frac{1}{1 + \delta_{\alpha 0}} e k(\phi_{\alpha} H_{\phi_{\alpha}}) h_{\lambda i} + \frac{1}{1 + \delta_{\alpha 0}} \left[ \frac{\lambda \pi}{L_d} (\phi_{\alpha} H_{\phi_{\alpha}}) \right] v_{\lambda i} + \frac{1}{1 + \delta_{\alpha 0}} e k(\psi_{\lambda} h_{\lambda i}) u_{\lambda i}.
\]

(A4)

where \( \text{mod}_2 \) is a function that gives the remainder when the argument is divided by 2 and \( \kappa \) denotes the number of each mode equation. Note that the summation rule [Eq. (A5)] is applied about the subscripts \( \lambda \).

**APPENDIX B**

**Derivation of Eddy Energy Equation**

The three-layer \( f \)-plane primitive equations with Boussinesq approximation are

\[
\frac{\partial \text{KE}}{\partial t} \left[ \times \frac{\partial}{\partial x} \sum_{i=1}^{3} \frac{1}{2} \rho_o h_i (u_i + v_i) \right] = \sum_{i=1}^{3} \left[ -\nabla \cdot \left[ \frac{1}{2} \rho_o h_i (u_i + v_i)^2 \right] - \rho_o \nabla \cdot \left( h_i u_i \right) - \rho_o A_i h_i \nabla \cdot \left( u_i \right) \right].
\]

(B4)

Here, \( \bar{a} \) is an ensemble mean of \( a \). The total potential energy equation is obtained from the summation of \( h_3 \times \Sigma_{i=1}^{3} \left( B_3 \right), h_2 + h_3 \times \Sigma_{i=1}^{3} \left( B_3 \right), \) and \( h \times \Sigma_{i=1}^{3} \left( B_3 \right) \). The total potential energy (PE) equation is then

\[
\frac{\partial \text{PE}}{\partial t} \left[ \times \frac{\partial}{\partial x} \sum_{i=1}^{3} \frac{1}{2} \rho_o g h_i (u_i + v_i)^2 \right] = \frac{1}{2} \frac{\partial}{\partial t} \left[ \rho_o g h_i (h_2 + h_3)^2 + \rho_o g \frac{h_i}{h_3} \frac{h_2}{h_3} \right]
\]

\[
= -\rho_o g \nabla \cdot h \sum_{i=1}^{3} (h_i u_i) - \rho_o g \frac{h_i}{h_3} \nabla \cdot (h_2 + h_3) \sum_{i=1}^{3} h_i u_i - \rho_o g \frac{h_i}{h_3} \nabla \cdot \left( h_i u_i \right) - \rho_o g \frac{h_i}{h_3} \nabla \cdot \left( h_i u_i \right).
\]

(B5)

The equations for the mean fields are

\[
\frac{\partial U_i}{\partial t} + V_i \cdot \nabla U_i + \bar{u}_i \cdot \nabla U_i - fV_i = -\frac{1}{\rho_o} \frac{\partial P_t}{\partial x} + \bar{A}_h \nabla^2 U_i,
\]

(B6)

\[
\frac{\partial V_i}{\partial t} + V_i \cdot \nabla V_i + \bar{u}_i \cdot \nabla V_i + fU_i = -\frac{1}{\rho_o} \frac{\partial P_t}{\partial y} + \bar{A}_h \nabla^2 V_i.
\]

(B7)

The mean kinetic energy (KE\(_m\)) equations are

\[
\frac{\partial H_i}{\partial t} + \nabla \cdot (V_i H_i) + \nabla \cdot (u_i \bar{h}_i) = 0.
\]

(B8)

Here, \( a' = a - \bar{a} \). Note that \( \bar{a}' = 0 \) and the triple correlation term \( a''b''c'' \) are assumed to be zero.

The mean kinetic and potential energy equations are derived from Eqs. (B6) to (B8) in a way similar to that used to derive the total energy equations.
\[
\frac{dKE_m}{dt} = \sum_{i=1}^{3} \left\{ -\nabla \cdot \left[ \frac{1}{2} \rho_H \nabla_i (U_i^2 + V_i^2) \right] - \nabla \cdot \left[ \frac{1}{2} \rho_o \nabla_i (U_i^2 + V_i^2) \right] - \rho_o \nabla \cdot (U_i H \mu_i \nabla i) - \rho_o \nabla \cdot (V_i H \psi_i) \right. \\
+ \rho_o (U_i H \mu_i + H_i \mu_i) \cdot \nabla U_i + \rho_o (V_i H \psi_i) + H_i \psi_i) \cdot \nabla V_i + \rho_o U_i \nabla_i \cdot (H_i \mu_i) + \rho_o V_i \nabla_i \cdot (H_i \psi_i) \\
- \left. H_i \nabla_i \cdot \nabla P_i + \rho_o A_h H_i \cdot \nabla^2 V_i \right\}. 
\]

(B9)

The mean potential energy (PE\textsubscript{m}) equation is

\[
\frac{dPE_m}{dt} = -\rho_o g \nabla \cdot \left[ H \sum_{i=1}^{3} H_i \nabla i \right] - \rho_o g_1 \nabla \cdot \left[ (H_2 + H_3) \sum_{i=2}^{3} H_i \nabla i \right] - \rho_o g_2 \nabla \cdot \left[ H_3 \sum_{i=3}^{3} H_i \nabla i \right] - \rho_o g \nabla \cdot \left[ H \sum_{i=1}^{3} \nabla i \right] \\
- \rho_o g_1 \nabla \cdot \left[ (H_2 + H_3) \sum_{i=2}^{3} \nabla i \right] - \rho_o g_2 \nabla \cdot \left[ H_3 \sum_{i=3}^{3} \nabla i \right] + \rho_o g_3 \nabla \cdot \left[ H_3 \nabla^2 H_i + g_i (H_2 + H_3) \right] + H_i \nabla i \cdot \nabla P_e. 
\]

(B10)

The eddy energy (E\textsubscript{e}) equation [Eq. (8)] is obtained by subtracting the mean energy equation from the total energy equation \([(B4) + (B5)] - [(B9) + (B10)]]:

\[
\frac{dE_e}{dt} = -\rho_o g \nabla \cdot \left[ h_i (H_i \mu_i) \right] - \rho_o g_1 \nabla \cdot \left[ (H_2 + H_3) (h_i u_2 + h_i u_3) \right] - \rho_o g_2 \nabla \cdot \left[ h_3 (h_i u_3) \right] - \rho_o \nabla \cdot (U_i H \mu_i \nabla i) + \rho_o \nabla \cdot (V_i H \psi_i) \\
+ \rho_o g \nabla \cdot (HH_i \nabla i) + \rho_o g_1 \nabla \cdot \left[ (H_2 + H_3) (H_2 V_2 + H_3 V_3) \right] + \rho_o g_2 \nabla \cdot (H_3 H_3 V_3) + \rho_o g \nabla \cdot (HH_i \nabla i) \\
+ \rho_o g \nabla \cdot \left[ (H_2 + H_3) h_i \mu_i + h_i \psi_i \right] - \rho_o g_2 \nabla \cdot \left[ H_3 h_3 \mu_3 \right] + \rho_o A_h h_i \mu_i \cdot \nabla i \psi_i - \rho_o A_h H_i \nabla i \cdot \nabla^2 V_i \\
- \rho_o (U_i H \mu_i + H_i \mu_i) \cdot \nabla U_i - \rho_o (V_i H \psi_i + H_i \psi_i) \cdot \nabla V_i - \rho_o U_i \nabla_i \cdot (H_i \mu_i) - \rho_o V_i \nabla_i \cdot (H_i \psi_i) \\
- \rho_o g h_i \nabla U_i \cdot \nabla H - \rho_o g h_i \nabla U_i \cdot \nabla (H_2 + H_3) - \rho o g \nabla \nabla g_i (H_2 + H_3). 
\]

(B11)

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