Noise-Induced Multidecadal Variability in the North Atlantic: Excitation of Normal Modes

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ABSTRACT

In this paper it is proposed that the stochastic excitation of a multidecadal internal ocean mode is at the origin of the multidecadal sea surface temperature variability in the North Atlantic. The excitation processes of the spatial sea surface temperature pattern associated with this multidecadal mode within an idealized three-dimensional model are studied by adding noise to the surface heat flux forcing. In the regime where the internal mode is damped, the amplitude of its sea surface temperature pattern depends on the type of noise forcing applied. While the mode is weakly excited by white noise, only the introduction of spatial and temporal coherence in the forcing, with characteristics of the North Atlantic Oscillation in particular, causes the amplitude of the variability to increase to levels comparable to those observed. Within this idealized model the physical mechanism of the excitation can be determined: the presence of the noise rectifies the background state and consequently changes the growth factor of the internal mode.

1. Introduction

Multidecadal variability in North Atlantic sea surface temperatures is found in proxy data stretching back more than 300 years (Delworth and Mann 2000) and within this data it has a statistically significant peak above a red-noise background. The variability was named the Atlantic Multidecadal Oscillation (AMO) by Kerr (2000), although it may perhaps be more accurate to refer to it as “Atlantic Multidecadal Variability” (AMV). There is now a reasonable consensus among climate scientists that this variability is not a phenomenon simply caused by a slow passive ocean response to noisy atmospheric variability but rather is due to the presence of an internal ocean mode.

The AMO seems to have a definite time-scale range (30–70 yr) and a particular spatial pattern. An AMO index was defined by Enfield et al. (2001) as the 10-yr running mean of detrended Atlantic SST anomalies north of the equator. Warm periods were in the 1950s and from 1995 to the present, whereas during the 1970s the North Atlantic was relatively cold. A first impression of the pattern was obtained from an analysis of SSTs in the North Atlantic over the last 150 years (Kushnir 1994). The difference in SST between the relatively warm years (1950–64) and the relatively cool years (1970–84) shows a pattern with a negative SST anomaly near the coast of Newfoundland and a positive SST anomaly over the rest of the North Atlantic basin.

In modeling and understanding the physics of the AMO, there appear to be two different—and so far quite disjointed—approaches. In the top-down approach, coupled general circulation models (CGCMs) are used to simulate the variability and then deduce the processes from (a mainly statistical) analysis of the different fields. Using the relatively coarse resolution Geophysical Fluid Dynamics Laboratory (GFDL) R15 model, Delworth et al. (1993) were able to simulate the pattern and time scale of the variability. Using regression analysis on the different fields, they concluded that the AMO is associated with variations in the Atlantic meridional overturning circulation (AMOC). After a careful sensitivity analysis with the GFDL R15 model, Delworth and Greatbatch (2000) proposed that multidecadal variability is due to an internal ocean mode that is forced by low-frequency variability of the atmosphere. Numerous more recent analyses have come to similar conclusions, although time scales in models can
differ substantially and statistical relations between different fields have led to different descriptions of the physics of the variability (Eden and Jung 2001; Cheng et al. 2004; Dong and Sutton 2005; Jungelius et al. 2005). The problem with these types of analyses is that statistically many fields are varying in concert, but the chain of processes causing the multidecadal behavior, in particular the pattern and time scale of the variability, can be difficult to extract.

The other approach to studying the AMO is from the bottom up and recognizes that there must be a so-called minimal model that captures the heart of the physics of the AMO. In this context, the term “minimal model” refers to the simplest possible model in which an AMO-like mode can be simulated. Additional physics included in models extending such a minimal model then only quantitatively affect patterns and time scales, without changing the underlying mode. Such a minimal model of the AMO was formulated by Greatbatch and Zhang (1995) and Chen and Ghil (1996) and consists of flow in an idealized three-dimensional northern hemispheric sector model forced only by a prescribed heat flux. It was suggested by Chen and Ghil (1996) that the oscillatory behavior arises from a Hopf bifurcation (an instability of the background steady flow to time-periodic disturbances), but it was not until the work of Huck et al. (1999) and te Raa and Dijkstra (2002) that the existence of this Hopf bifurcation was demonstrated. Analysis near the Hopf bifurcation revealed the precise physical mechanisms of variability as an out-of-phase response of zonal and meridional overturning anomalies to westward propagating temperature (or more generally, density) anomalies (te Raa and Dijkstra 2002).

According to this mechanism, the multidecadal time scale of the variability arises from the east–west basin propagation time of the density anomalies. The pattern most favorable to amplification is the one for which flow anomalies caused by density anomalies lead to density anomalies similar to the original ones, and in that sense the mechanism is a generalized baroclinic instability (te Raa and Dijkstra 2002). This description is supported by many model results in the same minimal model and slight extensions of it (Huck et al. 1999). Furthermore, the spectral origin of the multidecadal (normal) mode that eventually obtains a positive growth factor is fully understood by considering the small thermal forcing limit. The mode arises through mergers of stationary modes, called SST modes in Dijkstra (2006). The result of the merging is an oscillatory mode that has a multidecadal time scale under realistic forcing conditions.

Of course, the minimal model does not capture the effects of continental geometry, bottom topography, salinity, wind-driven flow, or dynamical interaction with the atmosphere. The single-hemispheric geometry is also quite constraining as it cannot capture the cross-equatorial or interbasin exchanges. Furthermore, atmospheric damping in the minimal model is assumed to be negligible. This is far from realistic; a reasonable value of the damping time scale of SST anomalies is about 30 days (Barsugli and Battisti 1998). The effect of salinity and wind forcing, continental geometry, bottom topography, and atmospheric damping were considered in several studies (te Raa and Dijkstra 2003; te Raa et al. 2004), which showed that the Hopf bifurcation is robust and that the mechanism as described in te Raa and Dijkstra (2002) still underlies the variability in these extended models (Dijkstra et al. 2006). As expected, the presence of bottom topography and/or atmospheric damping decreases the growth factor of the multidecadal mode, and hence the Hopf bifurcation is shifted to a parameter regime of higher forcing/lower dissipation. The deformation of the pattern of the multidecadal mode in model situations with realistic continental geometry even tends to look like the pattern as in Kushnir (1994). The effect of cross-equatorial flow and interbasin exchange is to localize the variability in the North Atlantic, as was shown in van der Heydt and Dijkstra (2007). The spectral view of the SST mergers leading to the multidecadal modes in these situations was described in Dijkstra and van der Heydt (2007).

One of the missing elements in the minimal model theory as developed so far is the effect of atmospheric damping combined with atmospheric noise. There are important results from the top-down approach using CGCMs, which show that, if noise is absent, there is no multidecadal variability (Delworth and Greatbatch 2000); in the theory from the minimal model, this is interpreted as the multidecadal mode being damped. When only high-pass filtered atmospheric forcing is driving the ocean component of the CGCM of Delworth and Greatbatch (2000), the multidecadal variability is much weaker than in the coupled case. Low-frequency variability in the forcing is needed to obtain multidecadal variability in the ocean-only case. This result motivates the study in this paper, where we consider the physics of the excitation processes of the multidecadal mode in the minimal model for cases in which atmospheric damping is strong enough to damp the multidecadal mode. By introducing noise into the surface boundary condition we can systematically investigate the effects of spatial and temporal correlations in the noise on the amplitude of variability excited, while
the simplicity of the model and boundary conditions allow us to study the excitation of the mode over a whole range of atmospheric damping strengths.

The effect of noise on idealized ocean models was considered in a more limited context by Griffies and Tziperman (1995) and Saravanan and McWilliams (1997, 1998). Saravanan and McWilliams (1998) studied the effect of noise in combination with passive advection in the ocean. Both Griffies and Tziperman (1995) and Saravanan and McWilliams (1997) studied models, a four-box model and a zonally averaged model, respectively, where the multimode mode considered here cannot exist. In these models, the only normal modes consist of two-dimensional loop oscillations for which the presence of both temperature and salt is required (Winton and Sarachik 1993). In more general three-dimensional ocean models these normal modes have a centennial time scale (te Raa and Dijkstra 2003) and appear alongside the three-dimensional normal mode with a multidecadal time scale, which is under investigation in this study. This three-dimensional multimode mode does not exist in a two-dimensional model because the zonal direction is essential for its existence. The three-dimensional mode involves westward propagation of temperature anomalies at the surface, while two-dimensional loop oscillations occur due to the advection of salinity anomalies by the overturning circulation. These differences in propagation mean that the addition of noise in the surface boundary condition can be expected to have significantly different effects on the two modes. The study of the excitation of the multimode mode under different temporal/spatial noise characteristics is hence the main new innovative element of this paper.

It will be shown here that colored noise is needed to excite the pattern of the multimode mode to sufficient amplitude and that the spatial/temporal statistical properties of the North Atlantic Oscillation (NAO) can favorably excite the multimode mode to the amplitudes observed. This excitation is thought to occur through the modification of the growth factor of the multimode mode through rectification of the background state by the noise. In section 2 the minimal model setup is described briefly, with focus on the surface boundary conditions for temperature. In section 3 the results are presented for the deterministic (no noise) case and for the variability appearing when noise, either uncorrelated or correlated in space and time, is included in the forcing. In addition, we take a look at how the background state affects the stability of the mode. This is followed by the discussion and conclusions in section 4.

2. Formulation

All the simulations discussed below were carried out using version 3.1 of the GFDL Modular Ocean Model (MOM). An extensive description of the equations, their discretization, and the solution methods in this model is given in Pacanowski and Griffies (2000). The model domain used here consists of a single hemispheric 64° × 64° basin covering the sector from θ_e = 10° to θ_w = 74°N, φ_w = 74° to φ_e = 10°W. Bottom topography is not included and the basin is 4000 m deep everywhere. The resolution of the model is 4° in both latitude and longitude. A stretched grid with 16 layers is used in the vertical so that the first four layers have a thickness of 50 m, with the thickness then increasing to 583 m in the lowest level. The rigid lid version of the model is used, so the vertical velocity vanishes at the ocean surface. Buoyancy fluxes through the bottom and lateral walls are zero. No-slip conditions are imposed at the lateral boundaries and slip conditions are imposed on the bottom boundary.

Since heat flux forcing has been found to dominate over the freshwater and momentum flux components in causing the SST variability (Delworth and Greatbatch 2000; Eden and Jung 2001), we neglect freshwater and momentum flux forcing in this model. Salinity is set to a uniform value of 35.0 psu, and wind stress forcing has been set to zero in all runs. Horizontal eddy viscosity A_H is set to 1.6 × 10^5 m^2 s^{-1}, vertical eddy viscosity A_V is 1.0 × 10^{-3} m^2 s^{-1}, and horizontal eddy diffusivity K_H is 1.0 × 10^3 m^2 s^{-1}, as in Dijkstra (2006). Vertical eddy diffusivity K_V is 1.0 × 10^{-4} m^2 s^{-1}. This value of K_V is slightly smaller than that used in Dijkstra and gives a more realistic MOC strength. An implicit convective adjustment scheme is used in the model in which vertical eddy diffusivity is increased in areas of unstable stratification.

The model is initially run for 6000 years under restoring conditions for temperature. In these simulations, a restoring heat flux \( Q_{\text{rest}} \) of the form

\[
Q_{\text{rest}} = \frac{\lambda_T}{\rho C_p H_m} [T_S(\theta) - T]
\]

is used, where \( C_p \) is the specific heat capacity of water at constant pressure, \( H_m \) is the depth of the first layer in the model, \( \rho \) is the constant Boussinesq density of water used by the ocean model, and \( \lambda_T \) is an air–sea exchange coefficient. The restoring time scale for temperature \( \tau_T^{\text{rest}} = \lambda_T / (\rho C_p H_m) \) in the model is 30 days. The temperature profile \( T_S(\theta) \) is given by

\[
T_S(\theta) = 15 + 10 \cos \left( \frac{\theta - \theta_s}{\theta_n - \theta_s} \right).
\]
This means that the temperature varies between 25°C at the southern boundary and 5°C at the northern boundary, that is, the meridional temperature difference is 20°C.

The meridional overturning streamfunction of the equilibrium state has a maximum of about 19.2 Sv (Sv = 10⁶ m³ s⁻¹) and is plotted in Fig. 1a. The surface heat flux needed to keep the system in equilibrium is then diagnosed. We call this flux $Q_{\text{pres}}$; it is plotted in Fig. 1b, where a positive value indicates that heat is entering the ocean. Over most of the basin the heat flux into the ocean decreases with increasing latitude. The strongest heat loss from the ocean occurs over the western boundary current.

Prescribing the flux $Q_{\text{pres}}$ as a boundary condition is equivalent to increasing the restoring time scale of the surface temperature to infinity (i.e., reducing the damping to zero). We know that under restoring boundary conditions the multidecadal mode is damped, but under prescribed flux boundary conditions it can have a positive growth rate (te Raa and Dijkstra 2003) so that oscillations spontaneously occur. Restoring and prescribed flux conditions are thus the two limits of damping by the atmosphere of the ocean. To study what happens between these limits, a new general boundary condition for surface heat flux is applied:

$$Q = (1 - \gamma)Q_{\text{rest}} + \gamma Q_{\text{pres}},$$

where $Q_{\text{rest}}$ and $Q_{\text{pres}}$ are the restoring and prescribed heat fluxes, respectively. The relative amounts of these fluxes, and therefore the amount of damping, are changed using the control parameter $\gamma$ in (3).

A value of $\gamma$ representative of the real ocean can be estimated by examining the damping time scale of the upper layer of the ocean. The effective damping time scale $\tau_T$ is defined as

$$\tau_T = \frac{1 - \gamma}{C_p H_m \rho \lambda_T}. \tag{4}$$

Using variables from the model ($\rho = 1 \times 10^3$ kg m⁻³, $C_p = 3990$ J kg⁻¹ K⁻¹, $H_m = 50$ m, $\tau_T = 30$ days) and $\lambda_T = 20$ W m⁻² K⁻¹ as a representative midlatitude value (Barsugli and Battisti 1998) gives a value of $\gamma \approx 0.74$.

### 3. Results

We will first consider the deterministic case (section 3a) and then consider the effect of only spatial coherence of the noise forcing (section 3b). In section 3c, we study the impact of both spatial and temporal coherence in the noise forcing, and then in section 3d we examine the role of rectification of the background state in the excitation of the multidecadal mode. The various runs carried out are listed in Table 1 along with their $\gamma$ values and the temporal and spatial characteristics of the noise included in the heat flux. Variations in SST are analyzed in terms of averages over the surface area (46°–62°N) × (74°–50°W) in the northwestern part of the basin. The meridional overturning circulation (MOC) strength is measured at the location where the MOC is maximal at the end of the spinup run (Fig. 1a). Patterns of variability are analyzed using the Multichannel Spectral Analysis (MSSA) toolkit (Ghil et al. 2002).

#### a. Deterministic case

Under the heat flux (3), we compute equilibrium states using time integration starting from the $\gamma = 0$
equilibrium solution (i.e., the equilibrium solution under restoring conditions). For each solution obtained, the maximum of the meridional overturning streamfunction is determined and its standard deviation is plotted as a function of $g$ in Fig. 2a. For $g \geq 0.85$ there are no oscillations. Near $g \approx 0.85$ the system undergoes a Hopf bifurcation and multidecadal oscillations appear. The amplitude of the oscillations is measured by calculating the standard deviation of the meridional overturning rather than the peak-to-peak amplitude so that comparisons can be made with later simulations where the variability is not regular. The oscillations have periods of about 45 to 50 yr with the period showing a slight dependence on $g$, as seen in Fig. 2b.

The patterns of the MOC anomalies and the temperature anomalies are very similar to those in te Raa and Dijkstra (2002). For comparison with the noise forcing cases later on, the first two EOFs of each of these fields are shown for $g = 0.98$ in Fig. 3. Figure 3a shows the first and second EOFs of the MOC. These two EOFs explain 96.2% of the variance, which is concentrated in the northern half of the basin around the sinking region of the background flow (Fig. 1(a)). These EOFs indicate a strengthening and weakening of the MOC (EOF 1) as well as a shift in the position of the sinking region (EOF 2) during an oscillation. Also shown are the first and second EOFs of the temperature at the surface (Fig. 3b) and in the bottom layer (Fig. 3c), which explain 92.1% and 94.1% of the variance, respectively. We can see that the variance in temperature at the sea surface is concentrated in the north-west of the basin, while at depth the temperature mainly varies in a band along the north of the basin. The anomalies at depth move westward from the eastern boundary and then southward along the western boundary.

The cycle begins with a positive temperature anomaly at the surface in the central western part of the basin. As time progresses the positive anomaly moves northward and a negative anomaly develops to the southeast. The positive anomaly then moves westward and disappears, while the negative anomaly that has developed to the south intensifies. After half a period of the oscillation, the temperature anomalies have the opposite signs to those seen at the beginning of the cycle. There is a lag of $\approx 8$ yr between the maximum temperature [averaged over the area $(46^\circ-62^\circ\text{N}) \times (74^\circ-50^\circ\text{W})$] and the MOC.

**Table 1. Table of runs with noisy forcing.**

<table>
<thead>
<tr>
<th>Run</th>
<th>$g$ value</th>
<th>Type of noise</th>
<th>Spatial pattern of noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{SW}$</td>
<td>0.7–1.0</td>
<td>White</td>
<td>None</td>
</tr>
<tr>
<td>$Q_{SW}$</td>
<td>0.7–1.0</td>
<td>White</td>
<td>Sinusoidal</td>
</tr>
<tr>
<td>$Q_{NAP}$</td>
<td>0.0 and 0.8</td>
<td>NAP index, 30-day time scale</td>
<td>Sinusoidal</td>
</tr>
<tr>
<td>$Q_{NAP}$</td>
<td>0.8</td>
<td>Reversed NAP index, 30-day time scale</td>
<td>Sinusoidal</td>
</tr>
<tr>
<td>$Q_{SW1}$</td>
<td>0.8</td>
<td>White, 1-day time scale</td>
<td>Sinusoidal</td>
</tr>
<tr>
<td>$Q_{SW10}$</td>
<td>0.8</td>
<td>White, 10-day time scale</td>
<td>Sinusoidal</td>
</tr>
<tr>
<td>$Q_{SW30}$</td>
<td>0.8</td>
<td>White, 30-day time scale</td>
<td>Sinusoidal</td>
</tr>
<tr>
<td>$Q_{NAP}$</td>
<td>0.8</td>
<td>Constant NAP+ (no noise)</td>
<td>Sinusoidal</td>
</tr>
<tr>
<td>$Q_{NAP}$</td>
<td>0.8</td>
<td>Constant NAP− (no noise)</td>
<td>Sinusoidal</td>
</tr>
<tr>
<td>$Q_{SW,NAP}$</td>
<td>0.8</td>
<td>NAP+ with added noise</td>
<td>Sinusoidal</td>
</tr>
<tr>
<td>$Q_{SW,NAP}$</td>
<td>0.8</td>
<td>NAP− with added noise</td>
<td>Sinusoidal</td>
</tr>
</tbody>
</table>
with the MOC leading in agreement with the mechanism described in te Raa and Dijkstra (2002).

b. White and spatially correlated noise

We now investigate whether noise forcing from the atmosphere (as represented by noise added to the heat flux) can excite the multidecadal mode in cases when it would normally be damped in the absence of noise, that is, for \( \gamma \leq 0.85 \). Several authors have concluded that multidecadal variability in GCMs is forced by atmospheric phenomena (Delworth and Greatbatch 2000) that involve coherent spatial patterns. The simplicity of the minimal model allows us to systematically explore the effects of spatial and temporal coherence in the noise.

First, we consider the effect of spatial coherence in the noise forcing by comparing the responses of the model under the following two heat fluxes:

\[
Q_W = (1 - \gamma)Q_{\text{rest}} + \gamma Q_{\text{pres}} + \sigma_n Z_{ij}(t),
\]

(5a)

\[
Q_{S,W} = (1 - \gamma)Q_{\text{rest}} + \gamma Q_{\text{pres}} + \sigma_n Z(t) \sin \left( \frac{\pi(i - i_s)}{i_w - i_e} \right) \sin \left( \frac{\pi(j - j_s)}{j_n - j_s} \right).\]

(5b)

In \( Q_W \), \( \sigma_n \) is the amplitude of the noise and \( Z_{ij} \) is a normally distributed random variable that takes on a different value at each grid point \((i, j)\) in space at each time step \(t\). The noise in \( Q_W \) is thus uncorrelated in both space and time. In \( Q_{S,W} \), \( i_s \leq i \leq i_w \) and \( j_s \leq j \leq j_n \) are...
the grid variables in the \( x \) and \( y \) directions, and \( Z(t) \) is a normally distributed random variable. The spatial pattern is chosen as a rough approximation to variations in atmospheric heat fluxes seen over the North Atlantic (Cayan 1992a,b; Grosfeld et al. 2007). However, the pattern has only spatial correlation and the noise is uncorrelated in time. In both \( Q_W \) and \( Q_{S,W} \), the amplitude \( \sigma_n \) of the noise was taken to be 10% of the difference between the maximum and minimum over the basin of the prescribed heat flux \( Q_{\text{pres}} \), which results in \( \sigma_n \approx 20 \text{ W m}^{-2} \). The simulations conducted with different types of noise forcing are listed in Table 1.

Figures 4a and 4b show the effects of noise on the standard deviation of meridional overturning, \( \Psi \) (Sv) and (b) SST [\(^\circ\text{C}\), averaged over the area \((46^\circ-62^\circ\text{N}) \times (74^\circ-50^\circ\text{W})\)] as a function of \( \gamma \), for the no noise, \( Q_W \), and \( Q_{S,W} \) cases. (c) Spectrum of SST over the area \((46^\circ-62^\circ\text{N}) \times (74^\circ-50^\circ\text{W})\) for the \( Q_W \) (thick line) and \( Q_{S,W} \) (thin line) cases with \( \gamma = 0.8 \). The 99% significance levels are also plotted.

FIG. 4. Standard deviation of (a) meridional overturning, \( \Psi \) (Sv) and (b) SST [\(^\circ\text{C}\), averaged over the area \((46^\circ-62^\circ\text{N}) \times (74^\circ-50^\circ\text{W})\)] as a function of \( \gamma \), for the no noise, \( Q_W \), and \( Q_{S,W} \) cases. (c) Spectrum of SST over the area \((46^\circ-62^\circ\text{N}) \times (74^\circ-50^\circ\text{W})\) for the \( Q_W \) (thick line) and \( Q_{S,W} \) (thin line) cases with \( \gamma = 0.8 \). The 99% significance levels are also plotted.

We find that, in cases with noisy forcing, for values of \( \gamma > \gamma_c \), the spatial patterns of variability closely resemble those in the no-noise case. When \( \gamma \) is decreased below \( \gamma_c \), noise dominates the patterns seen in the MSSA analysis. However, if the data is low-pass filtered to allow periods from 30 to 100 yr, then the patterns of multidecadal variability are recovered. Figures 5a and
show the EOFs for the $Q_W$ and $Q_{S,W}$ cases with $\gamma = 0.8$. In the $Q_W$ case the southern half of the basin is dominated by small-scale variations due to the white noise forcing. In the northern half of the basin there are similar patterns to those seen in the EOFs for the no-noise case (Fig. 3b). Figure 5b shows the first four EOFs of SST for the $Q_{S,W}$ case, with the EOFs accounting for 15.6%, 15.4%, 9.9%, and 9.5% of the variance, respectively. As in the $Q_W$ case, the northern part of the basin is dominated by a pattern similar to that in Fig. 3b. In the first EOF (and to a smaller extent in the third), the southern part of the basin shows a signal from the sinusoidal spatial pattern of the noise forcing. These two cases show that the noise has excited the same multidecadal mode as in the cases with $\gamma > \gamma_c$, a mode which is damped under these boundary conditions in the absence of noise.

In other words, there is a multidecadal mode present in the system with a growth rate that increases with increasing $\gamma$. For $\gamma < 0.85$ the growth rate is negative; that is, the mode is damped and the damping of the mode increases as $\gamma$ decreases. This means that for $\gamma$ below 0.85 the steady equilibrium state of the system is stable. As we have seen, however, the multidecadal mode can still be excited in the presence of noise. At $\gamma = 0.85$ the growth rate of the multidecadal mode is zero and the system undergoes a Hopf bifurcation. For $\gamma > 0.85$ the growth rate is positive, so the steady equilibrium state becomes unstable to the mode and multidecadal oscillations are observed even in the absence of noise.
c. The impact of temporal coherence in the noise

While both white noise \((Q_W)\) and noise with spatial coherence \((Q_{S,W})\) have been shown to excite the multidecadal mode for \(\gamma\) less than the critical value, neither can excite it to an amplitude comparable to that seen in observations [cf. the observed variation in SST of \(-0.5^\circ C\) (Delworth et al. 1993) and the values in Fig. 4b]. Since atmospheric phenomena can persist on time scales much longer than the time step used in the model, we next investigate the effect that temporal coherence in the noise has on the multidecadal variability.

To this end, the sinusoidal spatial pattern in Fig. 5b is multiplied by a monthly white noise index \(Z(t)\) so that the noise has a temporal coherence of 30 days; the resulting flux is indicated as \(Q_{S,W,30}\). This case of noise that has a sinusoidal spatial pattern but is white in time mimics the spatial variations of the NAO but not the temporal variations. Delworth and Greatbatch (2000) found that the low frequency part of the heat flux forcing was most effective at exciting multidecadal variability, so it is interesting to investigate the effect that an NAO with a more realistic temporal signature would have on the model. The monthly NAO index of Luterbacher et al. (2002) is thus also used to force the sinusoidal spatial pattern (with resulting flux \(Q_{S,NAO}\)). As the Luterbacher et al. (2002) NAO index has nonzero mean, we also force the model using an index of the opposite sign (with resulting flux \(Q_{S,r,NAO}\)). In each case the index is scaled so that it has the same variance as the time series used in the white noise index run.

Figure 6a compares a time series of model SST variability for the \(Q_{S,W,30}\) case to \(Q_{S,NAO}\) and \(Q_{S,r,NAO}\), which are the two cases involving the NAO index of Luterbacher et al. (2002). The temperatures for the \(Q_{S,NAO}\) case have been reversed so that they can be easily compared to the \(Q_{S,NAO}\) case. It is clear from the two NAO index runs that the small offset in the mean of the Luterbacher et al. (2002) NAO index does not greatly affect the resulting temperature variability. The variability in the \(Q_{S,W,30}\) case has a smaller amplitude than the two NAO index cases, which indicates that temporal coherence in the noise is having an effect on the amplitude of the variability.

Next, we investigate the effect of the time scale of temporal coherence more systematically. Figure 6b shows the spectra for cases in which white noise with sinusoidal spatial patterns and time scales of 1, 10, and 30 days (runs \(Q_{S,W,1}, Q_{S,W,10}\) and \(Q_{S,W,30}\)) are compared to the NAO index case (\(Q_{S,NAO}\), which also has a coherence time scale of 30 days). The multidecadal peak increases as the coherence time scale in the noise forcing increases. Forcing using the NAO index also increases the power in the multidecadal peak over that when white noise forcing with the same time scale is used. Since the spectrum of the NAO index shows slightly increased power at low frequencies compared to white noise, this result agrees with Delworth and Greatbatch (2000), who found that multidecadal variability in their coupled climate model was driven by the low frequency variations in heat flux. Note, however, that the time scale of coherence here is at most 30 days, whereas Delworth and Greatbatch (2000) used a 20-yr filter to separate high and low frequency atmospheric forcing. When spatial coherence is removed (so that the noise added to each grid point is independent) the temporal coherence still causes large variations in temperature, but the power at multidecadal frequencies is greatly
reduced (not shown), which indicates that the spatial coherence is necessary to effectively excite the mode.

To demonstrate that the variability is indeed caused by the excitation of the multidecadal mode and not by direct forcing of the low frequency component of the NAO index, we compare the cases of NAO index forcing ($Q_{\text{NAO}}$) for two values of $g$. Figure 7a shows the variations in temperature seen for the cases with $g = 0.8$ and $g = 0.0$ (pure restoring conditions). The spectrum of the $g = 0.0$ case (not shown) does have a small peak at multidecadal frequencies but it is dwarfed by the variability seen in the higher $g$ case. This illustrates that the further that the system is from the Hopf bifurcation point (in the stable regime), the smaller the variability excited by forcing of a particular strength; that is, the external forcing is unable to generate multidecadal variability in the absence of the multidecadal mode, or even when it is present but strongly damped.

Next we perform two additional tests to study the decay of the multidecadal mode through the relaxation of the system back to a steady state after forcing (which excites the multidecadal mode) has been removed. The surface flux pattern associated with the NAO+ state ($Q_{\text{NAO}+}$: flux positive in the southern half of the basin and negative in the north) was applied to the ocean model in equilibrium (with $g = 0.8$) for a duration of 10 years and then removed so that the model could relax back to equilibrium. A similar test was carried out with the flux pattern of the NAO− state ($Q_{\text{NAO}−}$: flux negative in the southern half of the basin and positive in the north). The overturning strength and surface temperature are plotted in Fig. 7b. After the additional forcing is removed, the ocean undergoes a series of oscillations with a multidecadal period, with the oscillations decaying as the system relaxes back to its equilibrium state.

![Figure 7](image_url)
d. Mechanism of excitation

The results in the previous sections show that a multidecadal mode that destabilizes the background state for values of $\gamma > \gamma_c$ can also be excited under conditions where $\gamma < \gamma_c$ by noisy forcing. To identify the mechanism of this excitation we take a look at background states. Noise can rectify the background state and the background state has an effect on the stability of the multidecadal mode. Thus, the effect of the background state on the stability of the multidecadal mode is investigated. Figure 8a shows the MOC of the background state with steady forcing (i.e., no noise) with $\gamma = 0.8$. In one simulation, the model is then forced with the (steady) surface heat flux pattern associated with a permanent NAO+ state until a steady state is reached. Figure 8b shows the difference between the MOC of this new time-mean state and the steady state under standard no-noise forcing. In another simulation, this is repeated for a steady permanent NAO− forcing, and the difference between this time-mean state and the standard no noise case is shown in Fig. 8c.

The pattern of permanent NAO+ forcing has positive heat flux over the southern half of the basin and negative heat flux over the northern half and, thus, acts to increase the north–south surface temperature gradient. This leads to an increase in the MOC as well as a slight northward shift of the sinking region, as seen in Fig. 8b. The opposite occurs with permanent NAO− forcing, with a decrease in the north–south temperature gradient leading to a decrease in MOC (Fig. 8c). According to Dijkstra (2006), the growth rate of the multidecadal mode increases with increasing strength of the MOC, which in turn increases with increasing $\Delta T$, where $\Delta T$ is the north–south temperature difference. This means that the permanent NAO+ and NAO− forcing has alternately increased and decreased the growth factor of the multidecadal mode; that is, under permanent NAO+ forcing the mean meridional overturning circulation is more unstable to the multidecadal mode than the mean meridional overturning circulation under permanent NAO− forcing. This difference in stability of the background states can also be seen in Fig. 7b, where the amplitude of the oscillations is larger when relaxing back to the equilibrium state from permanent NAO+ forcing than from permanent NAO− forcing.

This difference in stability under NAO± forcing can be most clearly illustrated by superimposing spatially coherent, temporally white noise on the two equilibrium states reached under steady NAO+ or NAO− forcing (runs $Q_{\text{S,W,NAO+}}$ and $Q_{\text{S,W,NAO−}}$). Figure 9a shows SST anomalies (relative to their respective equilibrium states) for the two cases. The same type of forcing is used in each case, yet the amplitude of variability excited in the NAO+ case is larger. This is also seen in the spectra of the two cases in Fig. 9b, where the larger peak at multidecadal frequencies in the NAO+ case shows that the mean state under permanent NAO+ conditions is more unstable to the multidecadal mode than the mean state under permanent NAO− conditions.

4. Discussion and conclusions

A systematic study was performed on the noise-induced variability in the North Atlantic using an idealized—so-called minimal—model. The effect of atmospheric damping was considered by introducing a parameter $\gamma$ in the heat flux, such that at $\gamma = 1$ there are sustained oscillations while at $\gamma = 0$, there is no oscil-
latory behavior. The use of the minimal model with simple boundary conditions allowed us to perform calculations for a range of $g$ values rather than being restricted to the value inherent to one particular model. This means that we can follow the mode and study the effect of the noise as the growth rate changes. In the deterministic case, the transition value between no oscillations and sustained oscillations is found near a critical value $g_c \approx 0.85$. From a mathematical point of view, we know that the system undergoes a Hopf bifurcation at this critical value of $g$. In our case we have used $g$ as the control parameter, but the critical value also depends on other model variables. In te Raa and Dijkstra (2002), for example, both horizontal diffusivity $K_H$ and vertical diffusivity $K_V$ were used as control parameters.

With the addition of noise a so-called Stochastic Hopf bifurcation occurs. In this case, oscillatory variability with an amplitude depending on the noise arises for values of $g \leq g_c$; while for $g > g_c$ the variability does not differ much from the deterministic case. For $g \leq g_c$, oscillations are damped in the deterministic case and, in the absence of further excitation, decay back to the stable steady state. The presence of noisy forcing continuously excites the variability, and a spectrum shows that the variability has the same multidecadal period as in the cases where $g$ is greater than the critical value, while EOFs confirm that the same spatial pattern of the variability also reappears. This clearly shows that for $g < g_c$, the noise excites the normal multidecadal mode, which in the deterministic case obtains a positive growth factor only for $g > g_c$.

It was demonstrated that both spatial and temporal coherence in the random part of the heat flux forcing are important to excite the multidecadal variability to reasonable amplitude. The spatial coherence is important as it modifies the surface heat flux so as to cool the northern part of the basin and warm the southern part, or vice versa. For temporally uncorrelated white noise, these two situations have the same probability and the background state is only slightly rectified, which can only slightly alter the effective growth factor of the multidecadal mode. When temporal coherence is introduced, by using the NAO index for example, the noisy heat flux directly influences the background state and hence a relatively large modification of the growth factor can occur.

Together with results in previous papers where the effects of salinity, wind, continental geometry, bottom topography (te Raa and Dijkstra 2003; te Raa et al. 2004), cross-equatorial flow, and interbasin exchange (von der Heydt and Dijkstra 2007; Dijkstra and von der Heydt 2007) were considered, we have now, with the effect of noise presented here, considered all relevant extensions of the minimal model. With this framework, we can try to interpret results of CGCM studies on this topic, in particular those by Delworth and Greatbatch (2000). Their CLIM simulation (the ocean model forced with only climatological atmospheric fluxes) nicely demonstrates that the flow regime is not supercritical; that is, there is no sustained variability and the multidecadal mode is damped. The TOTAL (ocean forced by total fluxes of the coupled model run) and RANDOM (ocean forced by only the annual-mean atmospheric fluxes chosen at random) simulations demonstrate that coupled feedbacks and atmospheric noise on time scales $<1$ yr are not central to the generation of multidecadal variability. As the multidecadal variability has a much smaller amplitude under HEAT_HP (only the low frequency part $<20$ yr) than under HEAT_LP (only the low frequency part $>20$ yr) forcing, it appears that the low frequency component of the atmospheric variability is driving the multidecadal variability as is, indeed, the interpretation in Delworth and Greatbatch (2000). However, as we have shown here, it is both the...
spatial and temporal correlations in the noise that are important for the excitation of the multidecadal mode, with the amplitude of the mode increasing with increasing temporal correlation.

Although both Delworth and Greatbatch (2000) and our study find that low frequency variability excites the mode, there is a striking difference in the time scales involved. Delworth and Greatbatch found that atmospheric variability on time scales less than a year were not necessary to excite the mode, whereas here we find that the variability can be excited to an appreciable amplitude using noise forcing with a temporal coherence on the time scale of days. This is related to the relative stabilities of the modes in the two models. The mode in the model of Delworth and Greatbatch may have a lower growth rate than the mode in our simple model [topography, e.g., is observed to stabilize the mode (te Raa et al. 2004)], which is equivalent to the Delworth and Greatbatch model having a value of \( \gamma \) farther away from the bifurcation point. Increasing levels of noise (and coherence in the noise) are then necessary to excite the variability.

In conclusion, according to our results, multidecadal variability in the North Atlantic is not directly driven by the low-frequency atmospheric variability (in particular the NAO). Instead the variability results from the excitation of a multidecadal internal ocean mode through atmospheric noise. This noisy forcing rectifies the background state that affects the growth rate of the underlying multidecadal mode and, hence, determines the amplitude of the resulting multidecadal variability.

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