Interference Pattern and Propagation of the $M_2$ Internal Tide South of the Hawaiian Ridge

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(Manuscript received 18 March 2009, in final form 1 September 2009)

ABSTRACT

Most of the $M_2$ internal tide energy generated at the Hawaiian Ridge radiates away in modes 1 and 2, but direct observation of these propagating waves is complicated by the complexity of the bathymetry at the generation region and by the presence of interference patterns. Observations from satellite altimetry, a tommographic array, and the R/P FLIP taken during the Fairfield Program of the Hawaiian Ocean Mixing Experiment (HOME) are found to be in good agreement with the output of a high-resolution primitive equation model, simulating the generation and propagation of internal tides. The model shows that different modes are generated with different amplitudes along complex topography. Multiple sources produce internal tides that sum constructively and destructively as they propagate. The major generation sites can be identified using a simplified 2D idealized knife-edge ridge model. Four line sources located on the Hawaiian Ridge reproduce the interference pattern of sea surface height and energy flux density fields from the numerical model for modes 1 and 2. Waves from multiple sources and their interference pattern have to be taken into account to correctly interpret in situ observations and satellite altimetry.

1. Introduction

Most of the internal tide energy generated at tall, steep, midocean topography radiates away as low-vertical-mode internal waves (Llewellyn Smith and Young 2003; St. Laurent et al. 2003). These energetic $M_2$ low-mode internal tides have been observed directly (Dushaw et al. 1995; Feng et al. 1998; Lozovatsky et al. 2003; Dushaw 2006; Alford et al. 2007; Rainville and Pinkel 2006a,b) and with satellite altimetry (Ray and Mitchum 1997; Ray and Cartwright 2001; Dushaw 2002; Zhao and Alford 2009), but a clear understanding of their propagation and ultimate dissipation is lacking. The radiated internal tide energy is believed to be one of the main sources of energy into the deep ocean (Wunsch and Ferrari 2004). Specifically at the Hawaiian Ridge, where internal tide generation and associated diapycnal mixing were the focuses of the Hawaii Ocean Mixing Experiment (HOME; Rudnick et al. 2003), most of the energy converted to baroclinic motions was found to escape the generation region (Klymak et al. 2006; Carter et al. 2008). The decay
scale of the mode-1 internal tide propagating away from the Hawaiian Ridge obtained by altimetry is believed to be >1000 km (Ray and Cartwright 2001). A large fraction of the decrease in amplitude of the internal tidal signal detected by altimeters is likely due to wave refraction by mesoscale eddies, which reduce the coherence of the baroclinic internal waves with the barotropic tide (Rainville and Pinkel 2006b). The actual loss of energy from low-mode tides resulting from topographic scattering (St. Laurent and Garrett 2002; Johnston and Merrifield 2003; Johnston et al. 2003) and nonlinear wave–wave interactions (MacKinnon and Winters 2005; Alford et al. 2007) would contribute to the decay of mode 1, but their importance is still debatable. The ultimate fate of these energetic low-mode internal tides is uncertain; they might eventually dissipate all their energy on continental slopes (Nash et al. 2004).

Observation of the low-mode internal tides is further complicated by interference patterns produced by multiple generation sites. For example, two line sources of waves produce nodes and antinodes not only in their sea surface height manifestation but also in energy flux density (Fig. 1). In this example, an arbitrary 90° phase difference between the sources (representing a lag in the maximum barotropic currents between the two source locations) shifts the beams to the right. Despite its spatial variability and presence of strong beams, this interference pattern has zero energy flux divergence. Although wave interference is far from being a new concept in physics and oceanography, there is little documentation of such effects for internal tides, which are crucial for interpreting point measurements at long-term moorings (e.g., Alford and Zhao 2007) or from profiling instruments (e.g., Nash et al. 2006). For example, the interference between internal waves propagating in opposite directions has been considered to explain local variability observed near generation sites (Nash et al. 2006; Martini et al. 2007). The primary goal of this paper is to demonstrate the importance of these interference patterns in interpreting in situ and altimetric observations of internal wave propagation over large distances.

We expanded on previous modeling studies (Merrifield and Holloway 2002; Carter et al. 2008) to examine the propagation of the $M_2$ internal tide away from its sources along the Hawaiian Ridge into the far field, where observations were made with the floating instrument research platform (R/P) FLIP and an ocean acoustic tomography array during HOME. The model setup is described in section 2. Model results are validated by observations from R/P FLIP, the Ocean Topography Experiment (TOPEX)/Poseidon (now replaced by Jason) altimeters, and the tomographic data in section 3. The main generation sites along the Hawaiian Ridge are identified independently using a linear knife-edge model with idealized two-dimensional (2D) topography in section 4. It is shown that four line sources located at “hot spots” on the ridge reproduce most characteristics of the low-mode internal tide wave field south of the Hawaiian Ridge. The conclusions are also presented.

2. Methods
a. Model setup

The Princeton Ocean Model (POM) is a three-dimensional, nonlinear, free-surface, hydrostatic primitive equation model (Blumberg and Mellor 1987) that has been substantially modified to simulate internal tides (Carter and Merrifield 2007; Carter et al. 2008). Forcing is applied at the lateral boundaries with Flather boundary conditions using barotropic tidal currents and elevations of the principal lunar semidiurnal tide ($M_2$; period = 12.42 h) from the TPXO6.2 global tide model (Egbert and Erofeeva 2002). The grid has 0.01° (~1 km) horizontal resolution over a region extending from 19°21′N, 160°W to 22°26′N, 156°W and progressively expands to 0.03° near the boundaries. A 10-cell-wide layer at the boundaries absorbs baroclinic waves by relaxing velocities and displacements to zero (Carter and Merrifield 2007). Model runs last 40 tidal cycles or 20.7 days, with a harmonic analysis and energy terms calculated over the last six tidal cycles. There are 51 or levels (terrain following coordinates) evenly spaced in the vertical. A 20-day run allows modes 1–5 to propagate throughout the domain and contribute to the modeled internal tides in the far field. For example, in 34 tidal
cycles (17.6 days), the time before the harmonic and energy analysis starts, modes 1–4 travel ~9400, 2600, 1700, and 1300 km in deep water, respectively. Topography is obtained from multibeam bathymetry covering the Hawaiian Ridge merged with 1-min Smith and Sandwell (1997) altimetry–sounding-derived data in the southern portion of the domain. Hydrography is uniform throughout the domain and is obtained from a 10-yr average of data from the Hawaiian Ocean time series. This model does not include nontidal mesoscale variability.

b. Observations

During HOME, the R/P FLIP collected data near the generation site in the Kauai Channel (21°41’N, 158°31’W, 1100-m water depth) for 36 days in 2002 and about 430 km to the southwest in the far field (18°24’N, 160°47’W, 5200-m water depth) for 29 days in 2001. Depth profiles of temperature and salinity were obtained to 800 m every 4 min by two CTDs, each profiling 400 m. Currents were calculated as

\[ u(t, x, z) = \frac{-p_n(t, x, y)}{\rho_o g}, \]

where \( \rho_o \) is the density at the surface, \( g \) is the gravitational acceleration, and \( \omega \) is the \( M_2 \) tidal frequency. Here, \( A_2 \) and \( G_2 \) are the amplitude and phase (Greenwich epoch) of \( \zeta \). In the figures where the baroclinic SSH is shown, \( \zeta \) is plotted as a snapshot at \( t = 0 \). For each mode, there is a one-to-one relation between the baroclinic SSH and the maximum interior vertical displacement. For the same vertical displacement magnitude, \( A_2 \) decreases as the mode number increases. The horizontal wavelength for modes 1–3 (given by the spatial structure of \( G_2 \)) is about 150, 80, and 50 km, respectively. Figure 2 shows the total \( M_2 \) baroclinic SSH (no modal decomposition) and that for modes 1–3 from the model. Low-mode propagation from the Hawaiian Ridge is not a simple plane wave but rather a complex pattern of superposition from multiple sources.

We compute baroclinic energy flux density from the model as the product of baroclinic perturbation pressure \( p \) and baroclinic velocity \( \mathbf{u} \), averaged in time over one or several wave periods: \( F = \langle pu \rangle \) (Nash et al. 2005). From the perturbation pressure and velocity for each mode, \( p_n \) and \( \mathbf{u}_n \), the energy flux density of each mode is calculated as

\[ F_n(x, y, z) = \langle p_n(t, x, y, z) \cdot \mathbf{u}_n(t, x, y, z) \rangle. \]

The sum of modal energy flux densities is equal to the total energy flux density \( F \) (see appendix). The depth-integrated \( M_2 \) energy flux densities from the model show relatively complex spatial patterns, with narrow beams.
emanating from generation hot spots at the ridge (Fig. 3). Away from the ridge, fluxes sometimes seem to decrease and pick up again (e.g., fluxes between the S1 and S3 moorings of the tomographic array appear smaller than either north or south of the array) or start in the deep ocean (e.g., south of the islands of Maui and Hawaii, between 18° and 19°N). As in Fig. 1, these patterns are a consequence of the interference of different waves.

Also, what is most noticeable is how different each mode is from the other: specific sites along the ridge seems to be hot spots for mode 1 or 2, but hardly any mode 3 is seen originating from them or vice-versa. Well-defined beams seem to emanate from certain topographic areas (e.g., Kauai Channel, Nihoa), but their direction is not the same for modes 1, 2, and 3.

Particularly relevant to the far-field measurements discussed here, there is a beam of high energy flux density south of the Kauai Channel. This feature appears to split into two branches just north of the tomographic array, with one branch passing through the far-field R/P FLIP site and to the east half of the array but largely avoiding the west half of the array. Energy density is relatively low throughout the tomographic array but increases again somewhat south of the array (Fig. 3). Many of these patterns will be discussed and interpreted in terms of the interference of several sources (section 4), but first data from R/P FLIP, the tomographic array, and satellite altimetry are used to test the accuracy of the far-field model.

Obtained by integrating the energy flux density in depth and over an along-ridge distance of 400 km (across the width of the dashed box shown in Fig. 3a), the cross-ridge energy flux from the numerical model shows little decrease south of the ridge (Fig. 4). For distances greater than about 150 km from the ridge (one mode-1 wavelength or one bounce in terms of rays), the cross-shelf
flux for each mode is close to being a constant, as it would be for an infinite line source in the absence of dissipation. Note that this holds for integration over a large along-ridge distance, therefore integrating over the minima and maxima of the interference pattern. In this context, to obtain the total $M_2$ energy radiating from the Hawaiian Ridge, one would have to average (or integrate) over all the temporal (between different frequencies) and spatial interferences.

b. Altimetry

The satellite altimeters provide an independent measurement of the SSH resulting from low-mode internal tides. Similar to the classic figure of Ray and Mitchum (1997), Fig. 5 shows the baroclinic SSH and its phase along several ascending tracks nearly perpendicular to the Hawaiian Ridge. The model and satellite altimetry observations agree reasonably well, with both showing a general coherence of baroclinic SSH from one track to the other. There are also regions with little signal (e.g., near 20°N, 162°W), which correspond to the region of destructive interference (section 4).

Looking in more detail along ground track 112 (Fig. 5), the baroclinic SSH in the model appears to have more structure farther away from the ridge, relative to what is observed from the altimetry. The measured baroclinic SSH signal is almost exclusively composed of mode 1, but modes 2 and higher are present (and coherent) for long distances in the model.

c. R/P FLIP

The baroclinic SSH can be calculated from the R/P FLIP observations, which resolve the upper 800 m of the water column very well (Rainville and Pinkel 2006a). Semidiurnal isopycnal displacements are calculated from the R/P FLIP time series and the additional thermistor data from the nearby tomographic mooring S3 (Fig. 6a) by bandpassing the density or temperature time series.
(using a fourth-order Butterworth filter centered at the $M_2$ frequency and a bandwidth of 40 h$^{-1}$) and using the observed mean density or temperature vertical gradients. The amplitude and phases of the first three vertical modes that best explain the variance of the displacement time series are obtained by least squares fit (Fig. 6b). Modal amplitudes and phases are used to calculate the baroclinic perturbation pressure profile and therefore the baroclinic SSH [Eq. (1)].

Most of the variability in the amplitude of the semidiurnal displacements is due to the spring–neap cycle of the forcing at the ridge. Averaged over the spring–neap cycle, the variance in the semidiurnal band should be close to the sum of the $M_2$ and $S_2$ variances. Because the $M_2$ currents dominate the other tidal components by a factor of 2–3, a one-month average is our best approximation to determine the $M_2$ displacement from the data. Holloway and Merrifield (2003) found that the $M_2$ internal tide energy flux is about 6 times the $S_2$ flux.

At the far-field R/P FLIP site, the averaged amplitude of the isopycnal displacement is mainly mode 1, with the first three modes explaining over 80% of the variance (Fig. 6c). The same is true for the displacements at the POM grid point closest to the R/P FLIP site, where the first three modes represent 79% of the total semidiurnal variance, although higher modes are more obvious because they are coherent in the model and do not average out (Fig. 6d). The model overestimates the high mode contribution actually found in the data at the time of the far-field experiment. As demonstrated in previous work (Merrifield et al. 2001; Lee et al. 2006; Carter et al. 2008), this internal tidal model is also good at reproducing observed low-mode velocities.

At both the near-field and far-field sites, the mean values of $\zeta$ over the month-long deployments agree with model predictions (Fig. 5). Also note how the baroclinic SSH differs between track 112 and the R/P FLIP site, despite their proximity (95 km in the along-ridge direction).

Using the semidiurnal baroclinic perturbation pressure and the measured baroclinic velocities, Rainville and Pinkel (2006a) calculate that the total depth-integrated $M_2$ energy flux density observed at this site is $1.7 \pm 0.3$ kW m$^{-1}$, with practically all the flux in mode 1. The one-month-averaged energy flux density associated with mode 2, calculated from the FLIP data, is about 5 times less than that of mode 1, and it is not aligned away from the ridge, as one would expect and is seen in the model. During the R/P FLIP far-field deployment, modes 2 and higher were most likely refracted west of the far-field site by mesoscale currents (Rainville and Pinkel 2006b), which are not present in the model. The modeled total flux density at the R/P FLIP far-field location is $2.2$ kW m$^{-1}$, with $1.8$ kW m$^{-1}$ in mode 1, which is within the range of the values calculated from the direct observations.

d. Tomographic array

The tomographic array forms a directional antenna for the low-mode internal tides in the far field of the Hawaiian Ridge (Dushaw et al. 1995; Dushaw 2003). The sum of the reciprocal travel times for each eigenray between acoustic transceivers gives the average sound-speed (temperature) along the acoustic ray path (Munk et al. 1995). The vertical averaging of the ray paths suppresses the higher internal tide modes that oscillate with depth, providing a natural filter for low-mode sound-speed variability. The horizontal averaging along each acoustic path forms a directional antenna that is sensitive to internal tides with wavenumbers perpendicular to the path (i.e., wave crests parallel to the path) and that discriminates against internal tides propagating in other directions, making the tomographic data less sensitive than point data to interference effects. The tomographic array consisted of multiple acoustic paths that were aligned with respect to the axis of the Hawaiian Ridge to provide good angular resolution of wavenumbers emanating from the ridge (Fig. 7).

The tomographic sources transmitted 8 times per day, every other day. After obtaining the high-frequency variations of sum travel times by subtracting daily means on each transmission day, a linear inverse was applied to
the travel-time data from each path to determine the time series of the low-mode amplitudes. The model for this inverse was the set of internal tide sound-speed modes, given by the product of the internal tide displacement modes and the vertical profile of the potential sound-speed gradient. For the oceanographic conditions south of Hawaii, the ray-path structure is such that only mode 1 is well resolved. The rms of mode-1 SSH on each path was 0.6–0.7 cm, with the largest rms on the acoustic path approximately parallel to the ridge (S2–S4).

A weighted least squares fit for the amplitudes and phases of eight tidal constituents (M\textsubscript{2}, S\textsubscript{2}, N\textsubscript{2}, K\textsubscript{2}, O\textsubscript{1}, K\textsubscript{1}, P\textsubscript{1}, and Q\textsubscript{1}) was then made to the time series of mode-1 amplitude (Dushaw et al. 1995; Dushaw 2006). The tidal analysis accounted for 70% of the variance on S2–S4 but only 11%–30% of the variance on the other paths (Table 1). The implication is that most of the mode-1 variability on the other acoustic paths was not a result of internal tides or that it was a result of internal tides that were not coherent over the approximately 195-day record length.

The path-averaged harmonic constants for the mode-1 M\textsubscript{2} internal tide, expressed in terms of SSH, are listed in Table 1. We note that, although the model mode-1 SSH along the acoustic paths has amplitudes of up to 3–4 cm (Fig. 2), their spatial averages over the paths (\langle \xi \rangle) are on the order of 0.5 cm in both the model and from the tomographic data. The observations indicate that the internal tide propagating perpendicular to the main diagonal of the array (S2–S4) has the largest amplitude. This path is roughly parallel to the Hawaiian Ridge, and the dominant internal tides are therefore propagating directly away from the ridge. The averaged amplitude and phase from POM for this path are in good agreement...
with the observations, with the amplitudes differing by only about 10%. The averaged amplitudes and phases from POM do not agree as well with the observations for the other paths, although even in these cases the agreement is generally within a factor of 2. The model amplitudes tend to be larger than the observed values for the paths that are not parallel to the ridge. The tomodographic data indicate that the coherent $M_2$ internal tides are more anisotropic at the location of the array than is the case in the model. Analysis combining all these data in a consistent way to obtain a best estimate for the internal tide and its energy flux through the array is beyond the scope of this paper.

4. Generation and interference patterns

a. Line source model

Internal tide generation along the ridge is not uniform (Merrifield and Holloway 2002; Rudnick et al. 2003; Lee et al. 2006). In particular, submarine ridges in the Kauai and Kaulakahi Channels and near Nihoa Island are strong sources (Figs. 2 and 3). Here, we derive a simple model of superposition of waves that reproduces most features seen in the baroclinic SSH and energy flux density pattern in the domain of this study. The parameters for the line source model are chosen independently from the POM simulations, using primarily the knife-edge model of St. Laurent et al. (2003).

In cylindrical coordinates, the baroclinic SSH associated with a cylindrically spreading wave is

$$\zeta(t, r, \theta) = \xi_o \left(\frac{r}{r_o}\right)^{1/2} \exp(i k \cdot r - i \omega t + \phi_o),$$

for $|\theta - \theta_0| < a_o/2r_o$,

$$\text{(3)}$$

where $r$ is the distance from the spreading center, $\xi_o$ is the amplitude of baroclinic SSH at distance $r_o$ (the ridge), and $\phi_o$ is an arbitrary phase. This equation is only evaluated for $r > r_o$. Furthermore, the wave is prevented from radiating in all directions by assigning an “arc length” $a_o$ and a propagation direction $\theta_0$, such that Eq. (3) is only valid for propagation angles $\theta$ in the range $\theta_0 \pm (a_o/2r_o)$ and $\zeta = 0$ elsewhere. Baroclinic pressure, velocities, etc., can be derived from the baroclinic SSH with knowledge of the modal vertical structure.

Using the same bathymetry as used in the model, the Hawaiian Ridge is first approximated by an infinitely narrow ridge with its height determined by the shallowest depth across the ridge (Fig. 8a). Following St. Laurent et al. (2003), we use a uniform buoyancy frequency ($N_0 = 0.0021 \, \text{rad s}^{-1}$), a deep ocean depth of 4500 m, and scale the depth of the ridge appropriately (for a scaling example, see Gill 1982). The amplitude and phase of the barotropic tidal currents across the ridge are determined from the TPXO tidal model (Egbert and Erofeeva 2002). We use a knife-edge model to estimate the baroclinic wave field generated at the ridge (Fig. 8b). Integrating the knife-edge energy flux along the ridge (Fig. 8c), four dominant generation sites are apparent.

Keeping in mind the many simplifications associated with such a calculation [e.g., two-dimensional, flow strictly
across a zero-width ridge, nondissipative, see St. Laurent et al. (2003), the results compare well with the model. Similar general agreement has also been found between direct observations and a knife-edge model (St. Laurent and Nash 2004). Note that, because of some flexibility in the choice of the knife-edge parameters (total ocean depth relevant to a knife-edge ridge, buoyancy frequency, and barotropic flow), the magnitude of the energy flux can be tuned by a factor of 2 or so. This does not affect significantly the magnitude of the sites relative to each other.

The wave propagation model described here can have various levels of complexity. For example, one can imagine only a few sources at the major hot spots, or a much larger number of two-dimensional sources in each channel, superposed to create a finite line source. Here, we retained only the four dominant sources: located near the islands of Nihoa and Hawaii and in the Kaulakahi and Kauai Channels. Finite line sources are chosen as a compromise between unrealistic point sources (there is not a single point source at each hot spot) and an equally unrealistic representation of the ridge as an infinite line source. The length \( a_o = 80 \text{ km} \) and radius of curvature \( r_o = 150 \text{ km} \) of the sources are chosen to be uniform to retain simplicity. These are reasonable estimates, given the geometry of the major channels of the Hawaiian Ridge.

Propagation angles \( \theta_o \) are established by the direction of the strongest barotropic tidal currents in each channel. The barotropic tide also determines the phase \( \phi_o \), such that the baroclinic currents at \( r_o \) have the same phase as the maximum cross-ridge barotropic tide (St. Laurent et al. 2003). The wavelength for each mode is calculated using the stratification used in the model. The final parameter of the line source model \( \zeta_o \) is chosen so that the wave generated by the line source carries the same amount of energy as predicted across the entire channel by the knife-edge model. The depth-integrated energy fluxes across the ridge from the knife-edge model are integrated over the entire width of the enhanced generation area (Fig. 8a), and the total is redistributed along a uniform line source of length \( a_o \). The corresponding baroclinic SSH is readily obtained from linear dynamics. Specific values are given in Table 2. For simplicity, values are rounded to 3, 2, 3, and 1 cm for the four sources.

### b. Sea surface height

The baroclinic SSH fields for each of the four major generation sites are shown in Figs. 9c–f, where thick black (thin gray) lines are crests (troughs) of these idealized waves. The four sources are summed in Fig. 9b to be compared to the SSH for mode 1 for the full numerical model (Fig. 9a). To compare the results from the numerical and knife-edge models, the same crests and troughs of the idealized waves are plotted in Fig. 2b, and corresponding ones for mode 2 in Fig. 2c. Crest–crest or trough–trough intersections in the wave propagation model correspond to constructive interference (thick–thick or thin–thin black line intersections in Figs. 2b,c), which can be seen in the numerical model as regions of higher wave amplitude. Crest–trough intersections are locations of destructive interference (thick–thin line

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**Table 1.** Path-averaged amplitudes and phases for the mode-1 \( M_2 \) internal tide from the tomographic data and from POM. The phase is Greenwich epoch. The percentages of the mode-1 variance accounted for by the tidal analysis of the tomographic data are also given.

<table>
<thead>
<tr>
<th>Path</th>
<th>Tomography ( \langle \zeta \rangle )</th>
<th>POM ( \langle \zeta \rangle )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Variance (%)</td>
<td>(cm)</td>
</tr>
<tr>
<td>S1–S2</td>
<td>27</td>
<td>0.36 ± 0.03</td>
</tr>
<tr>
<td>S1–S3</td>
<td>11</td>
<td>0.08 ± 0.03</td>
</tr>
<tr>
<td>S1–S2</td>
<td>14</td>
<td>0.16 ± 0.03</td>
</tr>
<tr>
<td>S2–S3</td>
<td>24</td>
<td>0.38 ± 0.03</td>
</tr>
<tr>
<td>S2–S4*</td>
<td>70</td>
<td>0.73 ± 0.02</td>
</tr>
<tr>
<td>S3–S4</td>
<td>30</td>
<td>0.26 ± 0.03</td>
</tr>
</tbody>
</table>

* The main diagonal of the array.
intersections in Figs. 2b,c) and reduced amplitude. The agreement for modes 1 and 2 is remarkable for such a simple wave propagation model. The comparison becomes poorer for mode 3, because its sources on the ridge are more widely distributed and greater in number (Fig. 3). Mode 3 idealized waves are not plotted, because their wavelengths are too small and their generation is likely to be occurring at different sites.

The interference pattern caused by these very few sources explains some of the complicated structure seen in the baroclinic SSH observed by the satellite altimeters. For example, there is constructive interference around 20°N along satellite ground track 112 because of the superposition of the mode-1 internal tide from the Kauai and Kaulakahi Channels sources; however, there is destructive interference of waves from the same sources around 17°N along satellite ground track 23. The mode-1 waves from Nihoa probably contribute to the node observed near 20°N along satellite ground track 74 (Figs. 2b and 5a).

c. Baroclinic energy flux density

Not only is generation localized along the ridge, but each section of the ridge also generates internal tides with different vertical or modal structures. Thus, the pattern of internal wave modes emanating from the ridge is far from uniform (Fig. 3); again, however, at least for modes 1 and 2, it is fairly well represented by only a few sources (Fig. 10).

The complex pattern found in the energy flux density distribution from the model highlights the importance of considering how several sources distributed along the ridge interfere with each other. In our idealized model, four sources are necessary: for mode 1, the Kaulakahi and Kauai Channels sources are necessary to reproduce the narrow beam coming from the Kauai Channel. The Nihoa source contributes to the emergence of a beam in the far field and creates a null in the energy flux density just west of the location of R/P FLIP. Finally, the source near the island of Hawaii makes the checkered pattern south of the eastern Hawaiian Islands, and its superposition with the Kaulakahi source creates weak beams starting in the middle of the ocean, as in the first example we presented (Fig. 1).

The depth-integrated energy flux density map from the idealized line sources, shown in Fig. 10b, reproduces many of the characteristics found in the high-resolution model (Fig. 10a). The spatial pattern and variations in the beams of energy are well represented. Fortunately, the far-field R/P FLIP site was located in a region where the energy flux density and displacement are generally consistent with the simple geometry of a plane mode-1 wave propagating from the Hawaiian Ridge. This suggests that, if R/P FLIP had been moored 50 km

<table>
<thead>
<tr>
<th>Source</th>
<th>Total integrated energy flux (GW)</th>
<th>Line depth-integrated energy flux (kW m⁻¹)</th>
<th>Baroclinic SSH (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kauai Channel</td>
<td>0.37</td>
<td>4.1</td>
<td>2.9</td>
</tr>
<tr>
<td>Kaulakahi Channel</td>
<td>0.17</td>
<td>2.2</td>
<td>2.1</td>
</tr>
<tr>
<td>Nihoa Island</td>
<td>0.36</td>
<td>4.5</td>
<td>3.1</td>
</tr>
<tr>
<td>Island of Hawaii</td>
<td>0.04</td>
<td>0.5</td>
<td>1.0</td>
</tr>
</tbody>
</table>
farther west, hardly any energy flux density would have been measured. Note that the location of the R/P FLIP far-field site was in part chosen based on a previous version of the POM simulation used here. That previous simulation, however, was run with smaller domains and missed a lot of the interferences. In particular, the mode-1 wave generated near Nihoa is essential to explain the variations of SSH and energy flux density observed across the area sampled by the tomographic array, but it was outside the domain used to model this section of the Hawaiian Ridge.

The mode-2 energy density splits into numerous branches much closer to the ridge (Figs. 10c,d). Also, there are several other notable regions of elevated mode-2 energy density near the ridge—for example, at the north point of Oahu. This pattern becomes more dispersed again for mode 3.

It should be noted that the spatial distribution of modes emanating from the ridge may be affected by topographic scattering. Flux directed along the ridge may scatter off other topography oriented across its path into a higher-mode, cross-ridge flux that is a short
distance from the main mode-1 generation sites. This scattering process may explain why energy densities for modes 2 and 3 have a more scattered pattern than mode 1: one example is found south of Oahu. The large mode-1 energy flux density resulting from the superposition of the Kauai and Kaulakahi sources corresponds to a region where mode-2 and mode-3 flux seem to diverge (Figs. 3c,d), suggesting that topographic scattering might contribute to the overall generation of higher modes.

5. Summary

This paper focuses on the structure and propagation of the $M_2$ internal tide southward from the Hawaiian Ridge. The generation of internal tides is known to be nonuniform along the ridge; however, the model reveals that each mode is generated in slightly different locations on the ridge and propagates away with a different spatial pattern. In the model domain, internal tides are primarily generated at four submarine ridges near the islands of Hawaii and Nihoa and in the Kauai and Kaulakahi Channels, creating interference patterns in sea surface height and energy flux density that are different for each mode. Energy flux density, in particular, appears as a composite of narrow beams, which might be misleading when trying to get a location of the sources of the waves. As demonstrated in Fig. 1, a narrow energy flux beam or filament does not imply the presence of a directed source of internal waves at its origin.

Because the baroclinic wave field in the far field is a superposition of several waves generated at discrete spots along the complex ridge topography, a local plane wave description is not adequate. Ray and Cartwright (2001) estimated energy flux in this region from SSH. They allowed for quasi-standing wave components (i.e., waves propagating in opposite directions), but not for waves propagating at an arbitrary angle. An approach that considers a multidirectional wave field is required (Dushaw 2002; Zhao and Alford 2009).

In situ observations from R/P FLIP and the tomographic array agree reasonably well with the numerical simulation but highlight the strong spatial variability of the $M_2$ internal tide, even in the “far field.” It is likely that some mode-1 waves are missing from the POM simulated field because of its limited domain. The importance of considering multiple sources and their interference is emphasized for point measurements. For example, if R/P FLIP had been moored 50 km to the west during the HOME Farfield Program, hardly any $M_2$
internal tides would have been measured. Our results also demonstrate that distant sources need to be considered when comparing direct observations with models.

In the numerical model, the energy flux density in modes 2–3 displays a larger along-ridge component. This may be a result of scattering from topography near the generation site of the much stronger along-ridge component of the mode-1 energy flux density. When compared with direct measurements and satellite altimetry data, the POM simulation consistently overestimates the high-mode contribution. In the real ocean, refraction by the mesoscale more than likely rapidly makes mode 2 and higher incoherent (Rainville and Pinkel 2006b), and/or their dissipation might not be adequately captured by a numerical model. In fact, there appears to be very little decay of the cross-ridge energy flux when averaged across the model domain. Long-term averages of direct observations of mode-1 internal tide are consistent with a negligible energy decay within 1000 km of the generation area, although the large spatial and temporal variability of the modal structure complicates the interpretation of in situ data. Dissipative processes—such as subharmonic instability (Hibiya and Nagasawa 2004; MacKinnon and Winters 2005), critical layer absorption (Munk 1981), or topographic scattering (Johnston and Merrifield 2003)—would lead to an actual decay of the internal tide signal and are therefore not dominant for the lowest modes south of Hawaii within 1000 km of the ridge. The prevalence of the interference pattern in the far field suggests that trying to establish whether there is a real decay of mode-1 energy in the open ocean from one or a few point measurements is a daunting task, whereas integrated measurements (such as tomography) might be more adequate.

Finally, and perhaps most importantly, the apparently complex patterns in observations and a high-resolution numerical model around the Hawaiian Ridge can be reproduced by a few line sources of internal tides. The location and strength of the sources are derived independently using simplified bathymetry and a linear knife-edge model forced with barotropic currents. Here, we demonstrate that idealized models can help understanding and interpreting numerical and observational results. Long-time series observations, such as tomographic data or long mooring time series, generally exhibit more variability in the amplitude of the semidiurnal internal tide than can be explained by the barotropic forcing (Rainville and Pinkel 2006b). The ability of a simple wave propagation model to reproduce the complexity of the mean mode-1 internal wave energy flux density is promising, suggesting that idealized models or numerical simulations with low horizontal and vertical resolution can capture most of the energetics of propagating internal waves and can be used to study their variability. Ultimately, an approach that combines analytical models, numerical simulations, and direct observations will be necessary to understand where and how the energy associated with low-mode internal tide dissipates, which remain as open questions.

**Acknowledgments.** We thank Richard Ray for providing altimeter data and Sam Kelly, Jonathan Nash, and Jody Klymak for their numerous helpful comments. This work was supported in part by grants from the National Science Foundation (OCE-9819519, OCE-9819525) and the National Oceanic and Atmospheric Administration (NA67RJ0154). R/P FLIP data were collected with the help of the Ocean Physics Group at UCSD, and we thank Captain Golfinos and the crew. We acknowledge the invaluable support of Matthew Dzieciuch and of the captain and crew of the R/V Roger Revelle during the tomographic component of the HOME Farfield Program.

**APPENDIX**

**Energy Flux of a Superposition of Waves**

This paper describes the total depth-integrated energy flux density as the sum of the depth-integrated energy flux density from each mode. Here, we demonstrate that the superposition of several waves with the same and different mode numbers, propagating in different directions, can be separated in a single pressure and velocity field for each mode and that the sum of the energy flux density associated with each mode is equal to the total energy flux density. First considering for waves that all have the same frequency \( \omega \), if there are \( M \) different waves for each mode \( n \), the total baroclinic pressure is

\[
p(t, x, y, z) = \sum_{m=1}^{M} \sum_{n=1}^{\infty} p_n(t, x, y, z) = \sum_{m=1}^{M} \sum_{n=1}^{\infty} F_n(z) A_{mn}(x, y) \exp(i k_{mn} \cdot x - i \omega t),
\]

where \( A_{mn}(x, y) \) is a complex amplitude, which includes the phase that is written out explicitly in the main text, and \( k_{mn} \) is the horizontal wavenumber vector of the \( m \)th wave of mode number \( n \), whose vertical structure is described by \( F_n(z) \). Waves with the same mode number \( n \) have equal \( |k_{mn}| \) but different directions and amplitudes. Similar equations for baroclinic velocities are found from the polarization relations (Gill 1982).
Even if several mode-1 waves are added, all with the same \( |k_m| \), the spatial pattern for the amplitude and phase of the superposition does not have the same horizontal wavelength. Indeed, the addition of two plane waves cannot be represented by a single plane wave. However, for a given position \((x_0, y_0)\), the specific wave-number of each individual wave contributes an additional phase offset, which can be incorporated into \( A_{mn}(x_0, y_0) \):

\[
p(t, z) = \sum_{m=1}^{M} \sum_{n=1}^{\infty} F_n(z)P_{mn} \exp(-iot), \tag{A2}
\]

where \( P_{mn} = A_n(x_0, y_0) \exp(i k_{mn} \cdot x_0) \) is a complex number. Baroclinic velocities have similar amplitude terms \( U_{mn} \) and \( V_{mn} \). The sum of those complex numbers is linear; therefore, we can sum all the waves with the same mode number and write \( p \) as

\[
p(t, z) = [P_1 F_1(z) + P_2 F_2(z) + \cdots] \exp(-iot) = p_1(t, z) + p_2(t, z) + \cdots, \tag{A3}
\]

where \( P_1 \) is the phase and amplitude of mode 1 (with contribution from all \( M \) mode-1 waves), whose vertical structure is given by \( F_1(z) \).

When modes are fitted to the baroclinic field of the POM simulation, the complex amplitudes \( P_m, U_m, \) and \( V_m \) are found. From these, one can compute the energy flux density. For example, the east component of energy flux density is

\[
\int_0^H \langle pu \rangle \, dz = \frac{1}{2} \int_0^H P_1 U_1 \, dz + \frac{1}{2} \int_0^H P_2 U_2 \, dz + \cdots
\]

\[
= \int_0^H \langle p_1 u_1 \rangle \, dz + \int_0^H \langle p_2 u_2 \rangle \, dz + \cdots, \tag{A4}
\]

where \( \langle \cdot \rangle \) is long-time average. The cross terms in the last equation, \( \langle p_m u_n \rangle \) where \( m \neq n \), cancel out because

\[
\int_0^H F_m(z) F_n(z) \, dz = 0 \quad \text{for} \quad m \neq n \tag{A5}
\]

by virtue of the modes being orthogonal to each other.

For the analysis of the model where a single frequency was used to run the model, the sum of the energy flux density associated with each mode is therefore the total energy flux density. When several frequencies are present, all the cross terms between different frequencies and mode number are also vanish, although the time average has to be taken over a longer period of time than when only one frequency is present. This is a concern for the R/P FLIP time series (less than one month).

The variability of the semidiurnal energy flux density observed at R/P FLIP was described by Rainville and Pinkel (2006a,b). It is assumed here that the monthly averages of the semidiurnal baroclinic SSH and of the energy flux density are dominated by the \( M_2 \) signals (Holloway and Merrifield 2003).

REFERENCES


