Dependence of Wind-Driven Current on Wind Stress Direction in a Small Semienclosed, Homogeneous Rotating Basin

HIROFUMI HINATA
Coastal Zone Systems Division, Coastal and Marine Department, National Institute for Land and Infrastructure Management, Yokosuka, Japan

NOBUYOSHI KANATSU
Kokusai Kogyo Co., Ltd., Tokyo, Japan

SATOSHI FUJII
Department of Electrical and Electronics Engineering, University of the Ryukyus, Nishihara, Japan

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ABSTRACT

The dependence of wind-driven current (WDC) on wind stress direction in a small semienclosed, homogeneous rotating basin is investigated using a linear steady-state analytical model based on Ekman solutions. The model is applicable to the middle of the basin (midbasin), and the current is driven by a constant wind stress of an arbitrary direction. The WDC is made up of wind stress–driven current (WSDC) and pressure-driven current (PDC) components. The laterally varying water depth of the basin confines the total volume transport in the longitudinal direction while the wind stress–driven volume transport changes direction according to the wind stress direction. Therefore, the pressure-driven volume transport or, equivalent, the pressure gradient depends on the wind stress direction: the relationship between the pressure gradient and the wind stress is anisotropic. As a result, the midbasin WDC is also dependent on the wind stress direction. The dependence varies according to the lateral position and Ekman number $E$. For large $E$ (small rotation), the longitudinal volume transport is generally proportional to the longitudinal wind stress component. Hence, the ratio of the volume transport driven by the wind stress of direction $\theta$ ($\theta > 0$) to that driven by the longitudinal wind stress ($\theta = 0$) becomes $\cos \theta$. For small $E$ (large rotation), the ratio becomes larger than $\cos \theta$. The extent to which each component of wind stress contributes to the generation of the pressure gradient to satisfy no-net-longitudinal and no-lateral transports is determined by a wind stress–pressure gradient transformation matrix, whose components depend on the lateral position and $E$.

1. Introduction

Currents in small semienclosed basins are greatly affected by wind as well as buoyancy and tides. Numerous studies have examined wind-driven currents under homogeneous conditions in idealized bathymetries with or without the earth’s rotation (e.g., Csanady 1973; Hunter and Hearn 1987; Hearn et al. 1987; Wong 1994; Mathieu et al. 2002; Winant 2004; Sanay and Valle-Levinson 2005). A general feature of the wind-driven currents found in these studies is that the vertically integrated flow (volume transport) is downwind near the shallow sides and upwind, driven by an axial pressure gradient, in the deeper channel. Recent studies by Winant (2004) and Sanay and Valle-Levinson (2005) show that the wind-driven current structure under the effects of the earth’s rotation depends on the ratio of maximum basin depth to Ekman depth ($\delta^{-1} = h_0/h_E$): the current structure becomes axially asymmetric and transverse circulation plays an important role with increasing $\delta^{-1}$, whereas it approaches the nonrotating solution described by Wong (1994) and becomes symmetric when $\delta^{-1}$ approached 1.

The primary interest in the previous studies was the current structure induced by a longitudinal wind, which
is parallel to the basin axis, although in general the actual wind blows from arbitrary directions. Furthermore, the relationship between the wind-driven current and the wind stress is expected to be anisotropic; that is, the current velocities and/or deflection angles from the wind stress directions would depend on the wind stress direction. Let us consider an idealized basin shown in Fig. 1 and a spatially uniform wind stress \( \tau_x \) with an arbitrary direction. The wind stress torque for the vertically averaged field becomes \( \mathbf{V} \times (\tau_x/\rho h) = \partial(\tau_x/\rho h)\partial y \) because of the longitudinally uniform water depth \( h \). Therefore, the lateral wind stress \( \tau_y \) produces no surface wind stress torque, and thus it may generate a much smaller volume transport in comparison with that driven by the longitudinal wind stress \( \tau_x \). Guo and Valle-Levinson (2008) conducted three-dimensional numerical model experiments on the response of Chesapeake Bay water to winds from four directions. Their main interest, however, was the effects on the density-driven (estuarine) circulation. Thus, the objective of this paper is to examine how the wind stress direction contributes to the determination of the wind-driven currents in a small semienclosed, homogeneous rotating basin with triangular cross section by developing a linear, steady-state analytical model. As a first step toward understanding the dependence on wind stress direction, we started the study with a model applicable to the middle part of the basin (midbasin) where effects of the basin end can be neglected.

The paper is organized as follows: An analytical model of the wind-driven currents for a small homogeneous rotating basin is described in section 2. In contrast to the previous analytical models (e.g., Csanady 1973; Hunter and Hearn 1987; Hearn et al. 1987; Wong 1994; Mathieu et al. 2002; Winant 2004), wind stress of an arbitrary direction is considered. The model results are presented and discussed from section 3 to section 5, with emphasis on the mechanism by which the dependence on wind stress direction is produced and on the dependence of cross-sectional and surface current structures on the wind stress direction. The conclusions are summarized in section 6.

2. Model

a. Ekman solutions

The model basin is shown in Fig. 1 with a right-handed coordinate system \((x, y, z)\). The \( x \) and \( y \) coordinates run along the longitudinal and lateral axes of the basin, respectively, and \( z \) is positive upward. The model basin is longitudinally uniform with a triangular cross section. Thus, the derivatives of variables with respect to \( x \) are equal to 0, except for the sea surface height. The basin width is \( 2B (=19600 \text{ m}) \). The maximum water depth at the center of the basin and the minimum water depth at the lateral boundaries are \( h_0 (=20 \text{ m}) \) and \( 0.02h_0 \), respectively. The minimum water depth of zero results in failure to calculate the integrals in matrix components defined by (11) as given below. The wind direction is \( \theta \) measured counterclockwise with respect to the positive \( x \) direction.

The wind-driven current (WDC) can be divided into the wind stress–driven current (WSDC) and the pressure-driven current (PDC); that is,

\[
V = V_w + V_p,
\]

where \( V, V_w, \) and \( V_p \) are the complex current velocities driven by the wind \((=u+iv)\), wind stress \((=u_w+iv_w)\), and the pressure gradient \((=u_p+iv_p)\), respectively. Following Winant (2004), we consider the linearized steady-state momentum equation with the earth’s rotation. The solution of the equation subject to the boundary conditions of imposed wind stress at the surface and no slip at the bottom is

\[
V = V_w + V_p = \frac{\tau_x}{\rho K a} \sinh[\alpha(z + h)]
\]

\[
+ gN \left[ \frac{\cosh(\alpha z)}{i\alpha } \right] \left[ \frac{\cosh(\alpha h)}{\cosh(\alpha h) - 1} \right],
\]

where \( f \) is the Coriolis parameter \((0.0001 \text{ s}^{-1})\), \( g \) is the gravitational acceleration \((9.8 \text{ m s}^{-2})\), \( K \) is the vertical eddy viscosity \((0.04, 0.004, \text{and} 0.0004 \text{ m}^2 \text{ s}^{-1})\), \( \rho \) is the density \((1024 \text{ kg m}^{-3})\), \( \tau_s \) \((|\tau_s| = 0.1 \text{ Pa})\) is the complex surface wind stress \((=\tau_{sx} + i\tau_{sy})\), \( N \) is the complex pressure gradient \((=\eta_x + i\eta_y = \partial\eta/\partial x + i\partial\eta/\partial y)\), \( \eta \) is the sea surface height, and \( \alpha^2 = i f/K = [(1+i)/h_E^2]^{1/2} \), with the Ekman depth \( h_E = (2Kf)^{1/2} \). The dimensionless parameter that controls the overall velocity distribution in the section is the Ekman number \( E = K/\beta h_0^2 \). The three different values of the vertical eddy viscosity correspond to three Ekman numbers: 1.0, 0.1, and 0.01.
b. Pressure gradient

The vertically integrated momentum and the continuity equations are

$$\frac{|V|}{h} + gN = \frac{\tau_x}{\rho h} - \frac{\tau_y}{\rho h}$$

and

$$\frac{\partial[V_x]}{\partial x} + \frac{\partial[V_y]}{\partial y} = 0,$$

where \([V]\) is the complex volume transport \((=[V_x] + i[V_y])\) and \(\tau_b\) is the complex bottom stress \((=\tau_{bx} + i\tau_{by})\). Because \(\partial[V_x]/\partial x = 0\) and the lateral volume transports at the side boundaries are zero, (4) shows \([V_y] = 0\) for all \(y\). To derive the pressure gradient, we use a continuity condition that assumes no-net-longitudinal and no-lateral transports; that is,

$$\int_{-B/2}^{B/2} V_x dy = \int_{-h}^{0} (u_w + u_p) dz dy = 0$$

and

$$[V_y] = \int_{-h}^{0} (v_w + v_p) dz = 0.$$

Depth integration of Ekman solutions (2) from the bottom to the surface \((z = 0)\) yields

$$[V] = [V_w] + [V_p] = \int_{-h}^{0} (V_w + V_p) dz$$

where \([V_w] = [V_w] + i[V_w] \) and \([V_p] = [V_p] + i[V_p] \) are the dimensionless volume transport by which the Ekman transport \((\tau/r_f)\) and the geostrophic current volume transport \((ghN/f)\) are transformed into the wind stress–driven and pressure-driven volume transports under the effects of bottom stress, respectively. Both dimensionless transports depend only on \(a//h\), which corresponds to the ratio of local depth \(h\) to Ekman depth \(h_E\) or, equivalent, the local Ekman number defined by

$$E_l = \frac{K}{h^2} = \frac{1}{\left(\frac{h}{h_E}\right)^2} = \frac{1}{\left|a//h\right|^2}.$$

The dependence is illustrated in Fig. 2. The angle of \([V_w]\) varies from \(-\pi/2\) to 0 as \(E_l\) increases from 0.01 to 100. The magnitude of \([V_w]\) is a constant value of 1.0 between \(E_l = 0.01\) and \(E_l = 0.3\), whereas it decreases from 1.0 to 0.005 as \(E_l\) increases from 0.3 to 100. The magnitude and direction of \([V_p]\) vary from 1.0 to 0.003 and from \(\pi/2\) to \(\pi\) as \(E_l\) increases from 0.01 to 100, respectively.

For simplicity of explanation, we introduce the concept of hodograph of vectors. For example, the hodograph of the wind stress vectors is a collection of one end of the wind stress vector of all directions. In addition, we shall call the hodographs of the vectors by their shape, such as wind stress circle, surface current circle/ellipse, and volume transport circle/ellipse. It can be seen in (6) that the wind stress and pressure gradient circles are transformed into wind stress–driven and pressure-driven volume transport circles, respectively. The rotation angle and the scaling factor of the transformation are
determined by $E_l$ and are independent of the wind stress/pressure gradient directions.

Substituting the volume transport (6) into (5) yields

$$-\frac{1}{\rho f} \int_{-B}^{B} \{\tau_{xx}[V_{wx}(y)] - \tau_{yy}[V_{wy}(y)]\} \, dy$$

$$= \frac{g}{f} \left\{ \eta_x \int_{-B}^{B} h(y)[V'_{px}(y)] \, dy \right\}$$

$$\text{and}$$

$$-\int_{-B}^{B} h(y)\eta_y(y)[V'_{py}(y)] \, dy \}$$

$$\left( \frac{\eta_x}{\eta_y} \right) = -\frac{1}{\rho g} \left[ \left( \frac{I_1}{I_3} \right) \left( \frac{[V_{wy}(y)]}{[V_{py}(y)]} \right) \right] - \frac{[V'_{py}(y)]}{[V'_{px}(y)]} \right] \, dy,$$

$$I_1 = \int_{-B}^{B} \left( [V_{wx}(y)] + \frac{[V'_{px}(y)]}{[V'_{px}(y)]} \right) \, dy,$$

$$I_2 = \int_{-B}^{B} \left( [V'_{px}(y)] - [V'_{wy}(y)] \, dy,$$

$$I_3 = \int_{-B}^{B} \left( h(y)\frac{[V'_{px}(y)]^2}{[V'_{px}(y)]} \right) \, dy.$$

In contrast to the former circle-to-circle transformation defined by (6), (10) shows a transformation by which the wind stress circle deforms into pressure gradient ellipses. The orientation and axial ratio of the ellipses are determined by the eigenvalues and eigenvectors of the transformation matrix, which depend on $E$ and $E_l$. When the bottom of the bay is flat, the dimensionless volume transports are independent of $y$. Hence, (10) reduces to

$$\left( \frac{\eta_x}{\eta_y} \right) = -\frac{1}{\rho g} \left[ \left( \frac{I_1}{I_3} \right) \left( \frac{[V_{wy}(y)]}{[V_{py}(y)]} \right) \right] - \frac{[V'_{py}(y)]}{[V'_{px}(y)]} \right] \, dy,$$

$$\times \left( \frac{[V'_{wx}(y)]}{[V_{wx}(y)]} \right) \left( \frac{[V'_{wx}(y)]}{[V_{wx}(y)]} \right) \left( \frac{\tau_{xx}}{\tau_{yy}} \right)$$

$$\text{or, equivalent,}$$

$$\left( \frac{[V'_{px}(y)]}{[V_{px}(y)]} \right) \left( \frac{[V'_{py}(y)]}{[V_{py}(y)]} \right) \left( \frac{\eta_x}{\eta_y} \right) = -\frac{1}{\rho g} \left[ \left( \frac{[V_{wx}(y)]}{[V_{wx}(y)]} \right) \left( \frac{[V'_{wx}(y)]}{[V'_{wx}(y)]} \right) \right] \, dy.$$

Equation (12) shows a wind stress circle to pressure gradient circle transformation; that is, the wind-driven current is independent of the wind stress direction. Equation (13) describes a simple balance between the wind stress–driven and the pressure-driven volume transports: at every point of $y$ the two volume transports cancel out each other and thus no longitudinal transport is produced for any wind stress direction.

The goal of this study is to examine how the wind stress direction contributes to the determination of the wind-driven currents for different values of $E$: namely, $E = 1.0, 0.1, 0.01$. In the three cases, the maximum water depth $h_0$ corresponds to $0.71h_E$, $2.2h_E$, and $7.1h_E$, respectively. The water depth is smaller than $h_E$ throughout the section when $E = 1.0$, whereas it is larger in almost the entire area ($|y/B| \leq 0.88$) when $E = 0.01$. When $E = 0.1$, the water depth is larger than $h_E$ in the deeper area of $|y/B| \leq 0.56$ (Fig. 2). The lateral velocity does not satisfy the following boundary conditions because of the sufficiently small but finite water depth at the lateral boundaries: $v = u_x + u_y = 0$ at $|y/B| = 1$. However, this effect is found to be negligibly small in the following section. Hereinafter, we refer to the upwind/downwind direction as the $x$ direction of sign $\tau_x$.

3. Dependence of pressure gradient on wind stress direction

a. Pressure gradient ellipse and momentum balance

Figure 3 shows the pressure gradient ellipses at different lateral positions from $y/B = 0.0$ to $0.8$ for the different values of $E$. The components of $N$ and the deflection angle of $N$ from $\tau_x$ are plotted in the wind stress direction–distance diagram shown in Fig. 4. The pressure gradients are symmetrical about the $z$ axis; in addition, they are almost symmetrical about the $x$ (basin) axis when $E = 1.0$. The axial ratio of the ellipse and the deflection angle depend on $E$ and the lateral position ($E_l$). In general, they
are comparatively large around the basin axis and near the lateral boundaries and increase as $E$ decreases. However, the pressure gradient ellipses are nearly circular in shape and the deflection angles are approximately 0 for any wind stress direction around $|y/B| = 0.2–0.3$ (not shown here).

For all values of $E$, $\eta_x$ is approximately proportional to $\tau_x = (\tau_x \cos \theta)$, whereas the maximum value decreases slightly with decreasing $E$, as found by Winant (2004). When $E = 1.0$, $\eta_y$ is approximately proportional to $\tau_y = (\tau_y \sin \theta)$. The quantity $|\eta_y|$ increases as the water depth decreases, whereas it has a local maximum around the axis of the bay for $E = 0.01$ and 0.1 because of a quasi-geostrophic balance with the longitudinal volume transport, as shown below.

The momentum balances of the vertically averaged momentum equation in (3) in an $E-\theta$ matrix form are shown in Fig. 5. The balances in the $x$ direction for $E = 0.1$ and 0.01 are not shown here, because they are almost the same as those established when $E = 1.0$. In the $x$ direction, the main balance is between the pressure gradient and the stress terms for any wind stress direction because lateral transport is not allowed (Fig. 5a). The values of the terms vary as a function of $\cos \theta$. When the wind stress direction is lateral ($\theta = \pi/2$), $\eta_x$ almost vanishes for all values of $E$ and thus the deflection angles are approximately 0 (Fig. 4).

When $E = 1.0$, the main balance in the $y$ direction is also between the pressure gradient and the stress terms (Fig. 5b). The values of the terms generally vary as a function of $\sin \theta$. The considerably smaller lateral pressure gradient for the longitudinal wind stress ($\theta = 0, \pi$) makes the deflection angle almost 0. Although the Coriolis force plays almost no role in the momentum balance...
in both directions, $N$ is deflected by $\tau_s$ for oblique wind stress ($\theta \neq 0, \pm \pi/2, \pm \pi$). The signs for the deflection angles in the shallower and deeper areas are opposite (Fig. 4).

When $E = 0.1$, the Coriolis force plays a major role around the basin axis in the $y$ direction (Fig. 5c). When $\theta = 0, \pi$, a quasigeostrophic balance is established throughout the section. The orientation and axial ratio of the ellipses, and thus the deflection angle, are slightly different from those for $E = 1.0$ in the shallower areas and are significantly different around the basin axis because
of the Coriolis force, which rotates the orientation of the ellipses counterclockwise to establish a quasigeostrophic balance with the longitudinal volume transport (Figs. 3a,b). When \( E = 0.01 \), the Coriolis force plays a major role in the \( y \) direction. The orientation and axial ratio of the ellipses, and thus the deflection angle, are totally different from those for \( E = 1.0 \) throughout the section: upwind (downwind) volume transport in the deeper (shallower) areas rotates the pressure gradient induced by the longitudinal wind stress about \( \pi/2 \) clockwise (counterclockwise) through a quasigeostrophic balance [see the segment from (1) to (3) in Figs. 3a,c]. For the smaller \( E \), the values of the terms do not vary as a function of \( \sin \theta \).

b. Volume transport ellipse

The hodographs of the volume transport components ([\( V \]), [\( V_w \]), and [\( V_p \)]) at different positions for \( E = 0.1 \) are depicted in Fig. 6. In the present model, first we impose the constant wind stress of an arbitrary direction (wind stress circle) on the sea surface. Because no-lateral transport is allowed by the continuity equation in (4), the total volume transport ([\( V \)]) is confined in the longitudinal direction: the hodograph becomes a line segment (Fig. 6a). On the other hand, [\( V_w \)] changes the direction according to (6), and naturally the hodograph of [\( V_p \)] becomes circular in shape (Fig. 6b). These two conditions require the hodograph of [\( V_p \)] = [\( V \)] − [\( V_w \)] to become elliptic (Fig. 6c), and then the hodograph of the pressure gradient must also become elliptic from (6). As a consequence, the wind-driven current in the bay depends on the wind stress direction.

The [\( V_p \)] ellipses are asymmetrical about the \( x \) axis. In comparing the pressure gradient and volume transport ellipses (Figs. 3b and 6c), if we rotate the [\( V_p \)] ellipses in the deeper (shallower) areas \( \pi/2 \) (\( \pi \)) clockwise, their shape is approximately consistent with that of the \( N \) ellipses, because \( E_t = O(10^{-1}) \) around the bay axis and thus the quasigeostrophic balance is generally established, whereas \( E_t = O(10^0-10^1) \) near the lateral boundaries and the balance between the pressure gradient and the surface and bottom stress is dominant when \( E = 0.1 \) (see Figs. 2 and 5).

The dependence of the pressure gradient on the wind stress direction is therefore caused by the laterally varying water depth, which restricts the total volume transport in the longitudinal direction. When the bottom is flat, (10) reduces to (12), and thus the pressure gradient is independent of the wind stress direction.
4. Cross-sectional structures

a. Volume transports and sea surface height anomaly

The cross-sectional structures of the wind-driven currents \((V = V_w + V_p)\) with the longitudinal volume transport components and with the sea surface height anomalies from \(\theta = 0\) to \(\pi\) at every \(\pi/6\) interval are shown in Figs. 7 and 8, respectively. The volume transport \([V]\) distribution has the same general form as the distributions of Winant (2004, see his Fig. 3). It shows downwind flow in the shallower areas near the lateral boundaries and an upwind return flow in the deeper areas, which is known as a “double gyre” pattern flow (Csanady 1973), with the shift of the lateral location of maximum downwind flow into the shallower areas with decreasing \(E\). The ratio of the net downwind/upwind volume transport for \(E = 0.1\) to that for \(E = 1.0\) is 9.4–10.7, which is consistent with that expected from the solution of Winant (2004) for small \(\delta^{-1}\), and it shows almost no dependence on \(\theta\). On the other hand, the ratio of the transport for \(E = 0.01\) to that for \(E = 0.1\) is 3.3–8.4. It depends on \(\theta\), showing the minimum value (3.3) at \(\theta = 2\pi/3\) and the maximum value (8.4) at \(\theta = \pi/3\). The values are between the ratios expected from Winant’s solutions for small and large \(\delta^{-1}\). The ratios for \(\theta = \pi/2\) are excluded from the calculation, because almost no volume transports are produced when \(E = 0.1\) and 1.0.

The dependence of the volume transport on \(\theta\) and \(E\) is well described by the ratio of the volume transport driven by the wind stress of direction \(\theta \geq 0\) to that driven by the longitudinal wind stress \((\theta = 0)\) defined by \(\beta = \overline{[V]}_{\theta \geq 0}/\overline{[V]}_{\theta = 0}\), where the overbar denotes cross-sectional average.

FIG. 6. Volume transport ellipses normalized by \(\mid r_i \mid / \rho f\) for \(E = 0.1\): (a) total volume transport, (b) wind stress–driven volume transport, and (c) pressure-driven volume transport. Numerals in parentheses correspond to the wind direction: 1 is \(\theta = 0\), 2 is \(\theta = \pm \pi/2\), 3 is \(\theta = \pm \pi\), and 4 is \(\theta = -\pi/2\). Crosses are depicted to allow easy recognition of the deformation. Note that the crosses generally do not correspond to the axes of the ellipses.
When $E = 1.0$ and $0.1$, the downwind and upwind flows are generally driven by the wind stress and the pressure gradient, respectively. The Coriolis force does not play a major role in the momentum balances (Figs. 5a,b). The term $\beta$ is nearly equal to $\cos \theta$, which indicates that $[V]$ is generally proportional to $t_x$ and is independent of $t_y$, which balances with the pressure gradient (wind setup/setdown) and the bottom stress in the $y$ direction. The dependence of $[V]$ on the value of the longitudinal wind stress $t_x$ is consistent with the solution of Winant (2004) for small $\delta^{-1}$ and with the previous nonrotating solutions of Csanady (1973), Wong (1994), and Mathieu et al. (2002).

When $E = 0.01$, it is interesting to note that both the upwind and downwind flows are generally driven by the pressure gradient in almost the entire cross section for $\theta = 0, \pi/6, 5\pi/6$, and $\pi$. Also, $[V_u]$ contributes to the downwind flow just beside the lateral boundaries $|y/B| \approx 0.85$, where the water depth is nearly equal to $h_E$ or less (see Fig. 2), and $\beta$ is larger than $\cos \theta$, namely, 0.97 ($\theta = \pi/6$), 0.68 ($\theta = \pi/3$), 0.21 ($\theta = \pi/2$), $-0.32$ ($\theta = 2\pi/3$), $-0.76$ ($\theta = 5\pi/6$), and $-1.0$ ($\theta = \pi$). The difference between $\beta$ and $\cos \theta$ is proportional to $\sin \theta$: the difference becomes the largest at $\theta = \pi/2$. This suggests that $\tau_y$ plays an important role in the generation of $[V]$. Even when $\theta = \pi/2$, double-gyre pattern flow appears to cancel out the longitudinal Ekman transport $[V_{wx}]$, although it is considerably weaker when $E = 0.1$: $[V_{wx}]$ is almost canceled out by $[V_{p}]$ at each point of $y$. The extent to which each component of the wind stress ($t_x, t_y$) contributes to the generation of the pressure gradient to satisfy the continuity condition (5) is determined by the $\tau-N$ transformation matrix (10), whose components depend on $E$ and $E_l$.

When $E = 1.0$ and 0.1, the sea surface setup and setdown driven by the lateral wind stress component are dominant, and $\eta$ rises/falls rapidly near the side boundaries, because the water depth approaches $0.02h_0$. Exceptions
are around the basin axis when $E = 0.1$ and $\theta = 0, \pi$. The double-gyre pattern flows vary the distribution slightly through a quasigeostrophic balance in the $y$ direction (Fig. 5c). This becomes apparent when $E = 0.01$: the sea surface height anomalies are made up of the wind setup/setdown near the lateral boundaries plus the component induced by the longitudinal volume transport through a quasigeostrophic balance in the $y$ direction.

**b. Current structure**

The Ekman numbers 1.0, 0.1, and 0.01 correspond to $\delta^{-1} = 0.71, 2.2,$ and $7.1$, respectively, for the study of Winant (2004). The horizontal velocity distribution for $\theta = 0$ is found to be almost the same as the midbasin flow pattern shown in Fig. 10 of Winant (2004). The effects of the finite water depth at the lateral boundaries cannot be recognized. Thus, the general features of the currents driven by the longitudinal wind stress described in his study can be applied to the present results for $\theta = 0$, except for the vertical velocity.

As a whole, the distribution patterns of both the longitudinal and lateral current components for $E = 1.0$ and $E = 0.1$ are very similar for all of the wind stress directions. For the longitudinal currents, upwind and downwind currents prevail in the deeper and shallower areas, respectively. The velocity gradually becomes small as $\theta$ approaches $\pi/2$, whereas the distribution pattern generally holds (Figs. 7a,b). When $\theta = \pi/2$, the distribution pattern becomes very different from that for the other wind stress directions: the pressure-driven upwind current around the basin axis disappears and the two-layered weaker longitudinal circulation appears instead in the deeper area because of the longitudinal weaker surface Ekman transport. For the lateral current field, the overall circulation, consisting of two counterrotating gyres, which is described in Winant (2004), disappears when $\theta$ is between $\pi/6$ and $5\pi/6$ (Figs. 8a,b). Instead, the clockwise circulation, which is toward the positive $y$ direction in the upper layer with a compensating return flow caused by the pressure gradient in the lower layer,
prevails in the section, and it gradually develops as $\theta$ approaches $\pi/2$.

When $E = 0.01$, the locations of the maximum velocities in the downwind and upwind directions shift to shallower areas and to the subsurface layer of the basin axis, respectively (Fig. 7c). The subsurface core of the upwind velocities, which is apparent when $\theta = 2\pi/3$, is a result of weakening of the upwind PDC at the surface layer and strengthening at the subsurface layer by the WSDC in the Ekman spiral form. The longitudinal velocity gradually weakens as $\theta$ approaches $\pi/2$, whereas the distribution pattern generally holds. When $\theta = \pi/2$, the distribution pattern becomes very different from that for the other wind stress directions: the pressure-driven upwind current around the axis disappears and the two-layered weaker longitudinal circulation appears instead throughout the section because of the longitudinal surface Ekman transport, because the water depth is larger than $h_E$ in almost the entire region ($|y/B| \leq 0.88$). For the lateral circulation, the surface layer becomes distinct (Fig. 8c). The overall lateral circulation, which has the same flow pattern as that found in Figs. 8a and 8b when $\theta = 0$ and $\pi$, weakens as $\theta$ approaches $\pi/2$. The directions of the two counterrotating gyres for $\theta = 0$–$\pi/6$ are opposite to those for $\theta = 2\pi/3$–$\pi$. When $\theta = \pi/3$, the four-layered flow structure, consisting of three counterrotating gyres, appears. When $\theta = \pi/2$, the simple two-layered structure, which is toward the positive $y$ direction in the thin surface Ekman layer with a compensating return flow resulting from the pressure gradient in the lower layer, prevails in the cross section, except for a small portion of the middle layer around the basin axis.

5. Surface current ellipse

Figure 9 shows the hodographs of the WSDC, PDC, and WDC at the surface for all values of $E$. The shape of
the PDC hodographs is basically elliptic because of the dependence of the pressure gradient on the wind stress direction, whereas that of the WSDC is, naturally, circular. As a consequence, the WDC hodographs become elliptic. The WDC ellipses are basically located inside the WSDC circles, because the WSDC and PDC tend to oppose each other. However, when $E = 0.01$ and the wind stress direction is approximately longitudinal, the magnitude of the WDC is larger than that of the WSDC in the shallower areas, because the longitudinal quasigeostrophic current (PDC) enlarges the downwind WSDC.

Figure 10 shows the dependence of the surface WDC on the wind stress direction. When $\theta = 0$ or $\pm \pi$, the velocity maximum appears around $|y/B| = 0.5$ for $E = 1.0$ and 0.1 and around $|y/B| = 0.7$ for $E = 0.01$, whereas much smaller currents are derived around the basin axis. In contrast, when $\theta = \pm \pi/2$, larger velocity and comparatively smaller velocity appear around the basin axis and in the shallower areas, respectively.

When $E = 1.0$ and 0.1, the surface WDC is deflected by the wind stress between $-\pi/2$ and $+\pi/3$. In general, the sign of the deflection angle in the deeper area ($|y/B| \leq 0.2$) and that in the shallower areas ($|y/B| \geq 0.4$) are opposite. In the deeper area, when $0 \leq \theta \leq \pi/2$ ($\pi/2 \leq \theta \leq \pi$) and $-\pi \leq \theta \leq -\pi/2$ ($-\pi/2 \leq \theta \leq 0$), the deflection angle is basically positive (negative). When $E = 0.01$, the deflection angle is frequently between $-\pi/6$ and $-\pi/4$. Nevertheless, it is physically less meaningful to compare the deflection angle and/or the ratio of surface current velocity to wind speed with those from Ekman theory. Yoshikawa et al. (2007) successfully extracted the Ekman spiral (WSDC) from the current velocity data acquired in the Tsushima Strait by subtracting the geostrophic current velocity (PDC), which was inferred from the interior velocity profile measured by a lower-frequency ADCP, from the velocity profile measured by a higher-frequency ADCP and high-frequency radar, and by using the EOF analysis of the wind stress and residual velocity. The WDC is made up of the WSDC and anisotropic PDC, and the theory should be compared with the WSDC.

Linear correlations between the surface current velocity measured using high-frequency radar and the wind velocity have been used to examine the WDC for open coastal oceans (e.g., Prandle 1987; Prandle and Matthews 1990; Kohut et al. 2004; Kaplan et al. 2005). However, from Fig. 9, it can easily be recognized that the method approximates the WDC ellipses to circles having a smaller radius than those of the WSDC, and thus the resultant circles would not represent any physical process for the small basin. These results suggest that close attention is required when trying to extract the WDC from current data obtained in actual small basins by using a linear correlation analysis.

**FIG. 10.** Dependence of the wind-driven surface current: (left) velocity and (right) deflection angle from wind stress. Velocity is normalized by $|r/\rho f h_0|$. Deflection angle of $-\pi/4$ is indicated by thick line. Other negative angles are dotted.
6. Conclusions

The dependence of the WDC on the wind stress direction in a small homogeneous rotating basin was investigated using a linear steady-state analytical model based on the Ekman solutions of the wind stress–driven and pressure-driven currents. The model is applicable to the midbasin. In contrast to the previous model studies, the current is driven by a constant wind stress of an arbitrary direction.

The laterally varying water depth of the basin confined the total volume transport in the longitudinal direction, whereas the wind stress–driven volume transport changed direction according to the wind stress direction. Therefore, the pressure-driven volume transport, or equivalently the pressure gradient, depended on the wind stress direction. As a result, the midbasin WDC was also dependent on the wind stress direction. The dependence varied with the Ekman number and the lateral position, namely, the local Ekman number.

For large Ekman number (small rotation), the lateral wind stress component did not play a crucial role in the generation of the longitudinal volume transport [\( V \)]. Hence, the ratio of the volume transport driven by the wind stress of direction \( \theta (\theta > 0) \) to that driven by the longitudinal wind stress \( (\theta = 0) \) became \( \cos \theta \). For small Ekman number (large rotation), the ratio became larger than \( \cos \theta \) and the difference between the ratio and \( \cos \theta \) was proportional to \( \sin \theta \). This indicates that \( \tau_y \) plays an important role in the generation of \( [V] \). The extent to which each component of the wind stress \( (\tau_x, \tau_y) \) contributes to the generation of the pressure gradient to satisfy the no-net-longitudinal and no-lateral transports is determined by the \( \tau-N \) transformation matrix defined by (10), whose components depend on \( E \) and \( E_t \).

Linear correlations between surface currents and winds have been used to examine the WDC in open coastal oceans. However, the analysis of the model-derived surface current suggests that the comparison would be physically less meaningful for the real small basins. The surface currents in the basins include anisotropic PDC component, which also depends on coastal and bottom topography. The winds should be compared with the WSDC component.

A comparison of the model results with field observations and accurate measurements of the anisotropic pressure fields are required to understand better the WDC in actual small basins. These are left for future studies.

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