Chaotic Behaviors in the Response of a Quasigeostrophic Oceanic Double Gyre to Seasonal External Forcing

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ABSTRACT
In an oceanic double-gyre system, nonlinear oscillations of the ocean under seasonally changing external forcing are investigated using a 1.5-layer quasigeostrophic model and a simple model related to energy balance of the oceanic double gyre. In the experiments, the variable parameter is the amplitude of external seasonal forcing and the Reynolds number is fixed as 39, at which periodic shedding of inertial subgyres occurs. The authors found that entrainment (at 2 times the period of the forcing) and intermittency (on-off type), phenomena that are often seen in nonlinear systems, emerge with increasing amplitude of the forcing. They seem to be related to the generation mechanism and characteristics of long-term (from interannual to decadal) variations in the strong current region of subtropical gyres such as the Kuroshio and its extension region.

1. Introduction
Wind stress curl at midlatitudes generates anticyclonic subtropical and cyclonic subpolar gyres (i.e., double gyres). Intensive boundary currents in the gyres appear at the western flank and enter into the open ocean as eastward jets. They carry substantial amounts of heat and momentum and strongly affect global climate. A typical example of such currents is the Kuroshio with a 100-km width and 2-m s$^{-1}$ velocity at maximum. In the Kuroshio, we can observe several variations with time scales from a few months to interdecadal. For example, its 80-day variability is related to mesoscale eddies (Matsuura et al. 1996). In addition, the spectrum of variation in the Kuroshio’s large meander is known to have two peaks at 20 yr and 7–8 yr (Kawabe 1987). Moreover, the spectrum of variations in the Kuroshio Extension (KE) was found to have a peak of 2–3 yr (Kakei et al. 2007). Recently, many observational studies have focused on decadal variability of the KE (Qiu 2003; Qiu and Chen 2005) and also nonlinear numerical studies have conducted on low-frequency time scales (Taguchi et al. 2007; Pierini et al. 2009). To clarify the generation mechanism and characteristics of these broad time-scale variations is considered important for the estimation and prediction of eddy heat–momentum transport.

After separating from the Boso Peninsula east of the Japanese islands, the mean path of the KE is characterized by the presence of two northward ridges of quasi-stationary meanders at 144° and 150°E (upstream KE) and by a broadening current downstream of the Shatsky Rise (downstream KE; Mizuno and White 1983; Qiu and Chen 2005). The upstream KE is known to vary with two decadal modes: a stable mode in which the KE jet is accompanied by weak temporal variability and a strong recirculation region and an unstable mode in which the KE path is convoluted and exhibits fluctuations resulting from eddy shedding (Qiu and Chen 2005). However, it is unknown why these two mode phenomena appear in the upstream KE. Intrinsic nonlinear mechanisms are likely to play a major role in determining the meander pattern of the mean flow in a reduced-gravity primitive equation ocean with a schematic coastline at the western side (Pierini 2006).

Although high-resolution oceanic general circulation models can reproduce the separation of western boundary currents, the numerical results are not always easy to...
interpret. Simpler models with middle-range complexity, such as quasigeostrophic (Q–G) models, have been particularly useful in understanding various aspects of the dynamics of midlatitude wind-driven oceanic circulation (Dijkstra and Ghil 2005). In a Q–G model, three dynamic regimes exist: one with basin-scale double-gyre circulation, another with an intense mesoscale recirculation, and the other with a strong western boundary current in each gyre (e.g., Pedlosky 1996). Nadiga and Luce (2001) showed in a Q–G model that shedding of inertial subgyres in an oceanic double gyre can be regarded as the Shilnikov phenomenon seen in nonlinear physics (Shilnikov 1965). When the phase-space trajectories are asymmetric, either anticyclonic or cyclonic inertial subgyres dominate; however, when the attractors are symmetric, anticyclonic and cyclonic inertial subgyres are equally preferred. The mesoscale variability has a time scale of about 110 days, whereas the overall oscillation occurs on a time scale that varies between roughly 1 and 8 yr. Ghil et al. (2002) showed that stable antisymmetric inertial subgyres appeared for some wind stress strengths and can be explained by an analytical modon solution.

Thus far, most studies of double gyres using middle-range complexity have been conducted under constant (time independent) wind forcing. In the real atmosphere and oceans, seasonal wind forcing with westerly and trade winds generates subtropical and subpolar gyres arising from western boundary currents and internal currents. Recently, Sakamoto (2006) showed that seasonal wind forcing generates long-term variability through a path toward chaos with a quasi devil’s staircase. However, many problems, especially those with nonlinear aspects, remain unresolved. For example, entrainment as rhythmic phenomena in a nonlinear system (Kuramoto 1984; Pikovsky et al. 2001) has not yet been investigated in the ocean system. Entrainment is an adjustment (e.g., frequency and/or phase locking) of rhythms of two or more self-sustained oscillating systems, which have different periods, because of their nonlinear interaction. We consider that entrainment of a characteristic oscillation of the ocean itself to a seasonal oscillation of an external forcing can occur in the ocean system when both periods are similar. Therefore, it is interesting and important to clarify the nonlinear interaction between a characteristic oscillation of the ocean and a seasonal oscillation of an external forcing, which is the main topic of our study. Moreover, we discuss an intermittency caused by chaotic modulation in the transition from laminar flow to turbulent flow (Berger et al. 1984). In particular, an additional mechanism for generating intermittency is related to the transverse instability of chaotic attractors confined to a manifold whose dimension is smaller than that of the full phase space (Pikovsky 1983; Fujisaka and Yamada

FIG. 1. Classification of inertial subgyre patterns emerging with changing $\alpha$ in our simulation for $Re = 39$. 

Pattern Classification (Re=39)
The intermittency in this case is called on–off intermittency.

This paper is organized as follows: The numerical model and experimental method are presented and discussed in section 2. In section 3, the dependence of numerical solutions on $\alpha$ (the amplitude of seasonal variation) at $Re = 39$ is described. In section 4, entrainment is described and an interpretation of its transition to entrainment is compared with the result of a simple model, Brusselator (Glansdorff and Prigogine 1971). In section 5, the generation of intermittency for $\alpha > 0.4$ is discussed in terms of the on–off intermittency. Finally, in section 6, we summarize our results and state our inferences regarding the Kuroshio.

2. Model and experimental method

The numerical model is the so-called 1.5-layer, reduced-gravity Q–G model (McCalpin 1987, 1995). The modeled ocean has a rectangular basin with a flat bottom. The vertical structure is assumed to be two immiscible, homogeneous layers of slightly different densities. The lower layer is assumed to be infinitely deep and at rest.
Thus, barotropic instability but not baroclinic instability can occur in the modeled ocean.

The governing equation of the model is a standard second-order finite-difference approximation to the Q–G vorticity equation for the upper-layer flow; that is,

\[
\frac{\nabla^2 \gamma^2}{\gamma^2} h_x + \beta h_x = -J(h, \nabla^2 h) - r \nabla^2 h + A_b \nabla^4 h
- A_b \nabla^6 h + \frac{f_0}{\rho_0g^2 H} \text{curl} \tau, \tag{1}
\]

where \( h \) is the interface anomaly (positive upward), \( \gamma \) is the coefficient of internal friction, \( A_b \) is the coefficient of eddy viscosity, \( A_b \) is the coefficient of high-order (biharmonic) viscosity, \( \tau \) is the wind stress on the sea surface, \( \rho_0 \) is the mean density of seawater, and \( H \) is the reference thickness of the upper layer. The subscripts \( x \) and \( t \) indicate partial derivatives, and \( \nabla \) and \( J \) are the nabla and Jacobian operators, respectively. Other definitions include \( \gamma^2 = f_0^2/g^2 H, g' = g\Delta \rho/\rho_0 \), and \( \beta = \partial f/\partial y \).

Here, \( f \) is the Coriolis parameter (assumed to be a function of \( y \)), \( f_0 \) is the value of the Coriolis parameter at the middle of the domain, \( g \) is the gravitational acceleration, and \( \Delta \rho \) is the difference between the layer densities. As fixed parameters, \( A_b = 0.0 \text{ m}^4 \text{ s}^{-2}, r = 1.0 \times 10^{-7} \text{ s}^{-1}, \)
\( f_0 = 7.3 \times 10^{-5} \text{ s}^{-1}, \beta = 2.0 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1}, g' = 2.0 \times 10^{-2} \text{ m s}^{-2}, \) and \( H = 1000 \text{ m} \) are used.

The domain in which (1) is solved is 3600 km \( \times \) 2800 km in extent (180 \( \times \) 140 grids). A nonslip boundary condition is used. The time interval of integration is \( 7.2 \times 10^5 \text{ s} (2 \text{ h}) \), and the horizontal grid spacing is \( 2.0 \times 10^4 \text{ m} (20 \text{ km}) \). Therefore, the model can resolve oceanic eddies because the internal deformation radius \([g' H]^{1/2}/f \) is about \( 6.1 \times 10^4 \text{ m} (61 \text{ km}) \), which is larger than the horizontal grid spacing.
The forcing has seasonal variation in time and north-south variation in space, as follows:

\[ \tau(t, y) = -\tau_0(1.0 + \alpha \cos \omega t) \cos(2\pi y/L). \]  

(2)

Here, \( \tau_0 \) is the amplitude of wind stress (\( =0.1 \text{ N m}^{-2} \)), \( \alpha \) is the amplitude of seasonal variation, \( \omega \) is the period of seasonal variation, \( y \) is the location in the north-south direction, and \( L \) is the length of the region in the north-south direction. This represents a simplified time-space distribution of wind stress in northern midlatitudes.

Control parameters in the experiments are \( \alpha \) [in (2)] and \( \text{Re} \), which is defined as

\[ \text{Re} = 2\pi \tau_0 / \rho_0 A_h B H. \]  

(3)

The only variable parameter in the right-hand side of (3) is \( A_h \). With increasing \( A_h \), \( \text{Re} \) decreases. As a result, the control parameters are \( \alpha \) and \( A_h \). We performed experiments for \( \alpha = 0.0 \) and \( A_h = 1.2 \times 10^5 \) to \( 1.0 \times 10^7 \text{ m}^2 \text{ s}^{-2} \) (\( \text{Re} = 26-314 \)) and for \( \alpha = 0.0-1.0 \) and \( A_h = 8.0 \times 10^2 \text{ m}^2 \text{ s}^{-2} \) (\( \text{Re} = 39 \)). The integration period is 250 yr for all experiments.

In the case with no seasonal variation (\( \alpha = 0.0 \)), we can see stable nonsymmetric patterns for \( \text{Re} = 26 \), periodic shedding of inertial subgyres for \( \text{Re} = 31-41 \), and stable modon-like (antisymmetric) patterns for \( \text{Re} = 42-70 \). For \( \text{Re} > 100 \), a jet on which subtropical and subpolar gyres meet becomes unstable and oscillates irregularly. Then, for \( \text{Re} = 314 \), eddies separate from the jet.

In this study, we focus not on the dependence on \( \text{Re} \), but on the dependence on the amplitude of seasonal variations of wind stress \( \alpha \) for a fixed \( \text{Re} = 39 \), at which shedding of inertial subgyres appears. We analyze the obtained results using flow patterns, time series of total energy, and trajectories in two-dimensional phase space of kinetic energy \( K \) and available potential energy \( P \) in the following three sections.

3. Flow pattern of double gyres depending on \( \alpha \)

The classification by \( \alpha \) of patterns emerging in our simulation for \( \text{Re} = 39 \) is summarized in Fig. 1. As shown in Fig. 1, entrainment occurred at \( \alpha = 0.18 \); also, though development to chaos by periodic-doubling bifurcation (\( \alpha = 0.25 \)) and chaotic eddy shedding (\( \alpha > 0.3 \)), intermittency emerges for \( \alpha > 0.4 \). In this section, we discuss the transition of the flow patterns in an oceanic double gyre with dependence on \( \alpha \) for Q–G experiments.
a. Case with no seasonal variation ($\alpha = 0$)

In the case with no seasonal variation ($\alpha = 0$) for $Re = 39$, when the amplitude of variation of anticyclonic inertial subgyres become large, anticyclonic inertial subgyres start to be shed regularly with a period of 2.45 yr (29.4 months; see Figs. 2, 5a). The time series of total energy shows a periodic pattern (Fig. 3a), and the trajectory in two-dimensional phase space of kinetic and available potential energy shows a limit cycle (Fig. 4a).

FIG. 5. Power spectra of total energy for (a) $\alpha = 0.0$, (b) $\alpha = 0.1$, (c) $\alpha = 0.18$, (d) $\alpha = 0.25$, (e) $\alpha = 0.3$, (f) $\alpha = 0.35$, (g) $\alpha = 0.4$, (h) $\alpha = 0.45$, (i) $\alpha = 0.5$, (j) $\alpha = 0.6$, (k) $\alpha = 0.7$, and (l) $\alpha = 1.0$. 
Nadiga and Luce (2001) showed that three patterns exist in shedding of inertial subgyres in a 1.5-layer Q–G model with biharmonic viscosity ($A_h = 0, \gamma > 0, A_p > 0$): shedding of cyclonic inertial subgyres only, anticyclonic inertial subgyres only, and both types of gyres. This case is considered to correspond to the second case. They explained this shedding by the homoclinic attractor suggested by Shilnikov (1965).

b. Case with seasonal variation ($0.1 < \alpha < 1.0$)

For $\alpha = 0.1$, the time series of total energy has two periods, 1 yr with a seasonal oscillation of the external forcing and 2.3 yr (27.7 months) with a characteristic oscillation of the ocean itself, and it is slightly reduced from that at $\alpha = 0$ (Figs. 3b,h, 5b); the trajectory shows a torus (Fig. 4b). For $\alpha = 0.12$, cyclonic inertial subgyres are shed instead of anticyclonic inertial subgyres; however, for $\alpha = 0.16$, anticyclonic inertial subgyres are shed again. For $\alpha = 0.14$, the time series of energy starts to show entrainment. Then, for $\alpha = 0.18–0.2$, entrainment occurs between a characteristic oscillation of the ocean itself and a seasonal oscillation of the external forcing, and the shedding period becomes 2.0 yr (23.8 months; Figs. 3c, 4c, 5c). Details of the entrainment appear in section 4. The periodic doubling appears at $\alpha = 0.25$ (see Fig. 5d) and switches to chaos, as shown in Figs. 3d, 4d, and 5e for $\alpha = 0.3$. At $\alpha = 0.4$, anticyclonic eddies are shed irregularly (cf. Fig. 10a).

For $\alpha > 0.45$, stable modon-like (antisymmetric) patterns (cf. Ghil et al. 2002) with cyclonic and anticyclonic inertial subgyres appear in addition to shedding of cyclonic and anticyclonic inertial subgyres. Interannual to decadal variations appear in the power spectrum (Figs. 5h–l). For $\alpha > 0.7$, the terms of the stable modon-like patterns become extended, and the time series of total energy show intermittent variations (Fig. 3f). Details of the intermittency appear in section 5. In these parameter ranges, stable modon-like patterns and cyclonic and anticyclonic eddy shedding patterns are mixed. For $\alpha = 1.0$, annual oscillation with external forcing becomes remarkable (Figs. 3f, 4f, and 5l).

4. Entrainment

It is known in nonlinear dynamics that entrainment (or synchronization) is crucial to the understanding of
self-organization phenomena occurring in the fields of coupled oscillators of the dissipative type (Kuramoto 1984; Pikovsky et al. 2001). In a non-self-oscillating (forced) system such as the ocean, entrainment can occur between an intrinsic oscillation of the ocean and a seasonal oscillation of an external forcing. In our Q–G experiments, after \( \alpha = 0.16 \) (cf. Fig. 1) entrainment begins. In this section, we discuss entrainment in our results of the Q–G model related to a simple model. With increasing \( \alpha \), the trajectory changes from a limit cycle (Fig. 4a) to a torus (Fig. 4b) and then returns to a limit cycle (Fig. 4c). For \( \alpha = 0.18 \), the spectrum has a maximum peak at 2 yr and a secondary peak at 1 yr (Fig. 5c). That is, with an external forcing oscillation with a period of 1 yr, the internal oscillation, which originally has a period of 2.45 yr, is modulated to a 2-yr period (i.e., twice that of the external oscillation).

Next, we investigate entrainment with a simple model, Brusselator, a hypothetical chemical reaction model proposed by the Brussels school (Glansdorff and Prigogine 1971) and first applied to oceanic general circulation by Seidov (1986). Figure 6 shows a conceptual figure of the model. The governing equations of the model follow Seidov (1986, 1989) but are adjusted for seasonally changing wind stress:

\[
\begin{align*}
x_t &= x^2y - (B + 1)x + A(1.0 + \alpha \cos \omega t) \quad \text{and} \quad (4) \\
y_t &= -x^2y + Bx. \quad \text{(5)}
\end{align*}
\]

Here, \( x \) and \( y \) indicate the kinetic energy and available potential energy, respectively, integrated within the whole region. The subscript \( t \) indicates partial derivative. The last term on the right-hand side of (4) is the work (per unit time) of the seasonally changing wind stress. The second term in both (4) and (5), \( Bx \), is a “slow” process (global scale) of exchange between the total kinetic energy and the available potential energy. The first term in both (4) and (5) is a “fast” process (mesoscale eddy genesis) of exchange between the total kinetic energy and the available potential energy. The third term in (4), \( -x \), is the dissipation (DIS) of the kinetic energy; in this phenomenological approach, it is assumed to be proportional to the kinetic energy itself. The terms \( A \) and \( B \) are parameters related to the amplitude of external periodic forcing and the exchange.

**TABLE 1. Relation among amplitude of external forcing \( \alpha \), number of peaks lower than the maximum peak \( n \), angular frequency of external forcing \( \omega_\text{e} \), and angular frequency of the maximum peak \( \omega_{m} \). See text for details.**

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( n(\alpha) )</th>
<th>( \omega_{m}/\omega_\text{e} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>9</td>
<td>0.474</td>
</tr>
<tr>
<td>0.41</td>
<td>10</td>
<td>0.476</td>
</tr>
<tr>
<td>0.42</td>
<td>12</td>
<td>0.481</td>
</tr>
<tr>
<td>0.43</td>
<td>16</td>
<td>0.485</td>
</tr>
<tr>
<td>0.44</td>
<td>24</td>
<td>0.495</td>
</tr>
<tr>
<td>0.45</td>
<td>40</td>
<td>0.498</td>
</tr>
<tr>
<td>0.5</td>
<td>Entrainment</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Fig. 8. Power spectrum of \( K \) obtained from energy balance model for (a) \( \alpha = 0.4 \), (b) \( \alpha = 0.41 \), (c) \( \alpha = 0.42 \), (d) \( \alpha = 0.43 \), (e) \( \alpha = 0.44 \), and (f) \( \alpha = 0.5 \), which are used to explain Table 1. The x axis is the frequency normalized by \( \omega_\text{e}/2\pi \), where \( \omega_\text{e} = 2.2 \) is the angular frequency of the external forcing. Therefore, 0.35 (\( = 2.2/2\pi \)) corresponds to 1/12 month in the Q–G model (see Fig. 5).
rate from kinetic energy to available potential energy, respectively, and $a$ and $v$ indicate the relative amplitude and frequency of external periodic forcing, respectively. We choose $A = 1.0$, $B = 2.5$, $\omega = 2.2$, and $0 < \alpha < 1.0$. Seasonal forcing in the ocean is realized for $\alpha = 1.0$ because the external wind stress is positive.

The trajectory in two-dimensional phase space of kinetic and available potential energy changes from a limit cycle for $\alpha = 0$ (Fig. 7a) to a torus for $\alpha = 0.3$ (Fig. 7b) and returns to a limit cycle for $\alpha = 0.5$ (Fig. 7c) while merging multiperiodic bands (see Figs. 7c,d). We can consider that the return to a limit cycle is due to entrainment. The power spectrum (Fig. 8) clearly explains the transition to entrainment because the frequency of the maximum peak $\omega_a$, for $0.4 < \alpha < 0.5$, can be expressed as $\omega_a = \omega_e/[2 + 1/2n(\alpha)]$, where $\omega_e$ is the angular frequency of the external forcing and $n = n(\alpha) = 8 + 2^{(\alpha-0.4) \times 100}$ is the number of the peaks at a higher frequency than (i.e., to the left of) the maximum peak. These relations can be inferred from Table 1 and Fig. 8.

For $\alpha = 0.5$, $\omega_a \sim \omega_e/2$, which corresponds to entrainment. That is, for $\alpha = 0.5$, many small peaks around $\omega_a$ and $\omega_e$ for $\alpha < 0.5$ are vanished and entrained to $\omega_a$ and $\omega_e$. This result coincides with the entrainment appeared in the Q–G experiment at $\alpha = 0.18$ (cf. Figs. 4c, 5c). Therefore, it can be expected that a favorable period (2 yr) appears in the KE current region.

5. Intermittency

It is known that, when entrainment is broken in a nonlinear system, intermittency often emerges (Berge et al. 1984). In our Q–G experiments, intermittency emerges for $\alpha > 0.4$. For $\alpha > 0.45$, two typical flow patterns appear, one with unstable cyclonic and anticyclonic eddy shedding (Figs. 9b,c, respectively; Nadiga and Luce 2001) and another with stable modon-like pattern (Fig. 9a; Ghil et al. 2002). Eddy shedding patterns are anticyclonic for $\alpha = 0.45$, cyclonic for $\alpha = 0.5$, and anticyclonic again for $\alpha = 0.6$ (see Fig. 1). With
increasing $\alpha$, the stable modon-like pattern is maintained longer (see Fig. 10). For $\alpha = 0.7$, both cyclonic and anticyclonic eddy shedding patterns appear, and the stable modon-like pattern is inserted between them. The stable modon-like pattern appears within the regular oscillation of energy with a 1-yr period, and unstable cyclonic and anticyclonic eddy shedding patterns appear within an irregular sudden decrease in energy (see Fig. 12; the regular 1-yr oscillation is filtered out).

Figure 10 shows that intermittency becomes noticeable with increasing $\alpha$; that is, the coupling factor is smaller between the intrinsic oscillation of the ocean and the seasonal oscillation of an external forcing (Fujisaka and Yamada 1985). With increasing $\alpha$, small (stable) oscillations are maintained longer; between them, appearances of large (unstable) oscillations become random. The small (stable) and large (unstable) oscillations are corresponded to stable modon-like patterns and unstable cyclonic and anticyclonic eddy shedding patterns, respectively. This intermittency can be considered on–off intermittency, which is an aperiodic switching between static or laminar behavior (off state) and chaotic bursts of oscillation (on state; Pikovsky 1983; Fujisaka and Yamada 1985; Platt et al. 1993). It can be generated by systems with an unstable invariant (or quasi-invariant) manifold, within which a suitable attractor is found. In our case, the stable modon-like and unstable eddy shedding patterns are considered to correspond to the off state and on state, respectively. Typical on–off intermittency involves behaviors around a specific state, but our case shows a jump between two states, cyclonic and anticyclonic eddy shedding.

We would like to discuss these results in terms of homoclinic bifurcation. Simonnet et al. (2005) have explained transition among three steady-state branches—subtropical (cyclonic), subpolar (anticyclonic), and antisymmetric (modon-like) patterns—by homoclinic bifurcation. Our case—transition among cyclonic eddy shedding, anticyclonic eddy shedding, and modon-like patterns—can also be understood as homoclinic bifurcation. States with $\alpha > 0.4$ appear on three curves, two oscillatory curves and a middle stationary curve, as Hopf bifurcation (see Fig. 4 of Nadiga and Luce 2001). For $\alpha = 0.5$, transitions occur homoclinically between an

![Fig. 10. Time series of height of the first layer for (a) $\alpha = 0.4$, (b) $\alpha = 0.5$, (c) $\alpha = 0.6$, and (d) $\alpha = 0.7$.](image)
oscillatory curve (cyclonic eddy shedding) and the middle stationary curve (Fig. 11b). For $\alpha = 0.4$ and 0.6, homoclinic transitions occur between the other oscillatory curve (anticyclonic eddy shedding) and the middle stationary curve (Figs. 11a,c). For $\alpha = 0.7$, transitions among the three curves occur (Fig. 11d). With increasing $\alpha$, the time spent on the middle curve, which shows a stable modon-like pattern, is longer, and the intermittency becomes remarkable.

Figure 12 shows time series of available potential energy, kinetic energy, and total energy for $\alpha = 0.7$, in which periods under 18 months are filtered out. From around year 85, a quasi-stable modon-like vortex pair becomes unstable and vortex shedding starts to occur. This instability of the vortex pair is not caused by seasonal oscillation of the external forcing, because it occurs for $\alpha = 0$. From year 85 to year 250, a total of 31 vortexes are shed. The shedding occurs once every 5 yr on average, but the period for frequent shedding (once in about 3 yr) and the period for rare shedding (once in about 10 yr) are mixed. Flow patterns for points a–c in Fig. 12b correspond to Figs. 9a–c. During periods for frequent shedding, vortex pairs of inertial subgyres become unstable and vortex shedding occurs successively. Before vortex shedding, large energy transports from available potential energy to kinetic energy occur; just before the shedding, kinetic energy takes a local maximum (Figs. 13a,b). The earlier half period (during January–December of year 223) in Figs. 13a,b includes a peak without shedding, and the later half period (during January–December of year 224) in Figs. 13a,b includes a peak with shedding of anticyclonic vortexes. In the later half period, the local maximum of available potential energy (March of year 224; point 1 in Fig. 13a), the local maximum kinetic energy (April of year 224; point 2 in Fig. 13b), and vortex shedding (May of year 224; point 3 in Fig. 13c) occur in order. The figure also shows that, when vortex shedding occurs, the total energy is reduced compared to the case with no vortex shedding. Moreover, the timing of vortex shedding...
corresponds to the season when the total energy reaches a maximum. For cyclonic vortexes, the same can be seen, for example, during years 233–235 (not shown).

After $\alpha = 0.4$, the power spectra of total energy have peaks with periods longer than 10 yr (see Fig. 5). In particular, peaks of 13.8 yr for $\alpha = 0.45$; 0.5, 10.4, and 42 yr for $\alpha = 0.7$; and 12 and 42 yr for $\alpha = 1.0$ are notable. We consider that these peaks show decadal and interdecadal variability in the oceanic general circulation and that they are determined by the intermittency inherent to the ocean system. Therefore, the decadal and interdecadal variability observed in the ocean may be considered to be generated with the intermittency. This will be discussed in the next section.

6. Summary and discussion

In this paper, we investigated oceanic double gyres with periodic eddy shedding of inertial subgyres by a 1.5-layer Q–G model and an energy balance model forced by an external seasonal wind stress. The results show that the ocean system causes entrainment between an intrinsic frequency of the system ($\omega_0$; 2.45 yr) and half the frequency of the external forcing ($\omega_e/2$; 2 yr), and they show that, with increasing amplitude of the external forcing, the system causes irregular variations (intermittency) between stable modon-like patterns and unstable cyclonic and anticyclonic eddy shedding. We suggest that decadal and interdecadal variability in the double-gyre ocean arises through the intermittent modulation of recirculation regions resulting from nonlinear interaction between the intrinsic oscillation of eddy shedding and seasonal oscillation of external forcing.

The coupling factor of two chaotic oscillators is the strongest when entrainment occurs. With weakening the coupling factor, intermittency appears, which is an aperiodic switching between static or laminar behavior (off state, in which two chaotic oscillators move as unite system) and chaotic bursts of oscillation (on state, in which two chaotic oscillators move as independent systems). This is one of typical routes to chaos (e.g., Pikovsky et al. 2001). In our cases, with small $\alpha$ (i.e., large coupling factor), entrainment occurs; with increasing $\alpha$ (i.e., decreasing coupling factor), intermittency appears. If the external forcing is constant, entrainment cannot occur and the way to reach intermittency will be different (Pierini et al. 2009). Though further studies are needed on this point, we consider that the seasonally changing external forcing is important for the real situation.

As stated in the introduction, recently, many observational studies have focused on decadal variability of the KE (Qiu 2003). The cause of this variability is not yet

FIG. 12. Time series of (a) $P$, (b) $K$, and (c) total energy for $\alpha = 0.7$. Frequencies lower than $\frac{1}{18}$ month are filtered out. Flow patterns for points a, b, and c in (b) correspond to Figs. 9a–c.
FIG. 13. Time series of (a) $P$ and (b) $K$ and (c) flow (height of the first layer) patterns for $\alpha = 0.7$ (contour interval is 30 m). See text for an explanation of points 1–3.
fully understood, but various suggestions have been made: decadal variability of the atmosphere (Deser et al. 1999; Qiu 2002, 2003), changes in recirculation with propagation of Rossby waves (Qiu and Chen 2005, 2006), and a self-sustained oscillation (Pierini 2006). Our results suggest that the decadal variability is caused by intermittency between the stable modon-like and unstable eddy shedding modes resulting from nonlinear interaction between an intrinsic oscillation of the ocean and seasonal oscillation of an external forcing. Our results are consistent with a suggestion by Primeau and Newman (2008) that time-dependent forcing may be important for rapid switching between two stable and unstable modes, which appeared in their primitive equation ocean model with time-independent forcing.

Long-term variations in the strong current region of subtropical gyres such as the Kuroshio and its extension region are interesting also in terms of whether the oceanic circulation itself can excite long-term oscillations and how it is related to variations in a western boundary current. It has often been considered that variations in the western boundary current are caused by local instability. An increasing number of observations, however, indicate that the variations are also considered to be part of the instability of the entire gyre (Qiu 2000; Pierini 2006; Matsuura and Fujita 2006). The entrainment and intermittency observed in this study are also considered to be related to the entire system. Therefore, these phenomena may be crucial to understanding the generation mechanism and characteristics of long-term variations in the strong current region of subtropical gyres such as the Kuroshio and its extension region.

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