A Laboratory Study of Nonlinear Western Boundary Currents, with Application to the Gulf Stream Separation due to Inertial Overshooting*

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ABSTRACT

Various dynamical aspects of nonlinear western boundary currents (WBCs) have been investigated experimentally through physical modeling in a 5-m-diameter rotating basin. The motion of a piston with a velocity $u_p$ that can be as low as $u_p = 0.5 \text{ mm s}^{-1}$ induces a horizontally unsheared current of homogeneous water that, flowing over a topographic beta slope, experiences westward intensification. First, the character of WBCs for various degrees of nonlinearity is investigated. By varying $u_p$, flows ranging from the highly nonlinear inertial Charney regime down to a weakly nonlinear regime can be simulated. In the first case, the dependence of zonal length scales on $u_p$ is found to be in agreement with Charney’s theory; for weaker flows, a markedly different functional dependence emerges describing the initial transition toward the linear, viscous case. This provides an unprecedented coverage of nonlinear WBC dependence on an amplitude parameter in terms of experimental data. WBC separation from a wedge-shaped continent past a cape (simulating Cape Hatteras) due to inertial overshooting is then analyzed. By increasing current speed, a critical behavior is identified according to which a very small change of $u_p$ marks the transition from a WBC that follows the coast past the cape to a WBC (nearly dynamically similar to a full-scale Gulf Stream) that separates from the cape without any substantial deflection, as with the Gulf Stream Extension. The important effect of the deflection angle of the continent is analyzed as well. Finally, the qualitative effect of a sloping sidewall along a straight coast is considered: the deflection of the flow away from the western wall due to the tendency to preserve potential vorticity clearly emerges.

1. Introduction

Western boundary currents (WBCs) are very intense currents that flow along the western boundaries of the oceans and owe their peculiar structure to the sphericity of the earth, which generates the so-called planetary beta effect (Stommel 1948; Munk 1950). The Gulf Stream (GS) and Kuroshio are notable examples of WBCs belonging to the subtropical gyres of the North Atlantic and Pacific Oceans, respectively. The effect of WBCs and of their respective extensions on climate is well known to be due to their huge heat transport, the corresponding air–sea interactions, and the role they play in sustaining the global conveyor belt (e.g., Lynch-Stieglitz et al. 1999; Qiu 2000, 2003; Ganopolski and Rahmstorf 2002; Kelly et al. 2010). International projects such as the U.S. Climate Variability and Predictability Research Program (CLIVAR; http://www.usclivar.org/wbc.php) and the Kuroshio Extension System Study (KESS; http://uskess.org/) have been recently devoted to investigating the main WBC systems both experimentally and theoretically.

Despite the attention devoted to analyzing the main processes ruling WBC structure and variability since the pioneering studies of the early 1950s, some aspects are still the object of scientific debate. In particular, two nonlinear problems have received special attention and have been attacked through numerical simulations, although a general consensus does not seem to have been
reached on their functioning: (i) the dependence of the WBC width on an amplitude parameter in the so-called nonlinear Munk problem (Pedlosky 1996) and (ii) the GS separation from the eastern coast of the United States. In both cases, the model spatial resolution and character of the boundary conditions have turned out to be extremely sensitive parameters, whose choice can considerably affect the numerical results [e.g., see Pedlosky (1996) and Chassignet and Marshall (2008) for (i) and (ii), respectively]. In this context, the physical modeling of those processes performed through laboratory experiments would be free of that conceptual limitation and could therefore provide useful complementary information that could help identify the governing dynamical mechanisms. In fact, rotating tank experiments have proved valuable in the past in providing information on WBC dynamics in the framework of the so-called sliced cylinder/cone models (e.g., Pedlosky and Greenspan 1967; Beardsley 1969, 1975; Beardsley and Robbins 1975; Griffiths and Veronis 1997; Griffiths and Kiss 1998, 1999; Kiss 2000, 2002) and in alternative setups (e.g., Hsueh and Legeckis 1973; Baines and Hughes 1996; Nøst et al. 2008; Pierini et al. 2008, hereafter PEA08).

In this context, laboratory simulations and quantitative analyses of problems (i) and (ii) (and a qualitative analysis of a third problem concerning the effect of sloping sidewalls) are presented in this paper. The experiments have been performed in the 5-m-diameter rotating basin at the Foundation for Industrial and Technological Research at the Norwegian Institute of Technology (SINTEF) in Trondheim, Norway, in the framework of the Integrating European Hydraulic Research Infrastructure (HYDRA-LAB-III) project of the European Community. They represent a substantial extension of previous laboratory experiments performed by PEA08. Basically, the motion of a piston with a velocity that now can be as low as $u_p = 0.5 \text{ mm s}^{-1}$ induces a uniform current of homogeneous water that experiences westward intensification when forced to flow over a topographic beta slope.

In section 2, the experimental setup and the measuring technique are described. In section 3, problem (i) is analyzed within the general framework of the characterization of WBCs through their zonal length scale. By varying $u_p$, WBCs are analyzed from the strongest nonlinear flows (according to Charney 1955) down to an intermediate weakly nonlinear regime that describes the initial transition to the linear case. In section 4, the qualitative effect of a sloping sidewall on the structure of WBCs is analyzed by adding linear slopes over the beta slope. The deflection of the flow away from the western wall due to the tendency to preserve potential vorticity by following the lines of constant $f/H$ (where $f$ is the Coriolis parameter and $H$ is the local depth) clearly emerges. In section 5, problem (ii) is analyzed by studying WBC separation from a wedge-shaped continent past a cape (simulating Cape Hatteras) due to the inertial overshooting (IO) mechanism. In section 5a, the transition to the separation due to IO for increasing current speed is analyzed: a critical behavior is identified according to which a very small change of $u_p$ marks the transition from a WBC that follows the coast also past the cape to a WBC that separates from the cape without any substantial deflection, as for the GS. Moreover, the dynamic similarity of a specific flow with an idealized GS is emphasized. In section 5b, the important effect of the deflection angle of the continent is analyzed. In section 5c, the results are discussed, with an emphasis on the a priori reasons that make the IO preferable in comparison to other proposed mechanisms. Finally, conclusions are drawn in section 6.

2. Experimental setup and measuring technique

The experimental setup, the measuring technique, and the scaling and formulation in terms of dimensionless parameters used in these experiments closely follow those adopted in the previous experiments of PEA08, so the reader should refer to that study for all the details; in the following only some features of particular relevance are highlighted, whereas the main differences introduced in the setup will be discussed in the subsequent sections. WBCs are simulated along a straight meridional
coast (y axis of Fig. 1) or along wedge-shaped continents (see section 5) in the 5-m-diameter rotating basin at SINTEF, by analyzing the zonal profile of the meridional velocity field as a function of transport intensity and of other dynamical parameters. The return flow that is generated in the real oceans by the surface wind stress curl in the oceanic interior is forced in the rotating basin by the motion of a piston in the absence of any surface stress. The laboratory setup (Fig. 1) consists of two parallel rectangular channels separated by an island and linked by two curved connections. In the first channel, a piston is forced at a constant speed $u_p \approx 0.5 \text{ mm s}^{-1}$ over a distance of 2.4 m, producing a virtually horizontally unsheared current (see Fig. 9U of PEA08) at the entrance of the second channel. In the second channel, a linear reduction of the water depth (shaded gray in Fig. 1) provides the topographic beta effect that is necessary for the development of the westward intensification (the effect of the parabolic free surface resulting from the centrifugal force in this large tank is negligible, as pointed out by PEA08).

After the homogeneous water has come into solid body rotation and before starting the piston, several hundred small buoys are seeded over the entrance of the main channel (the shaded region $A$ sketched in Fig. 1). To avoid effects of laboratory winds, surface films, and surface tension gradients, the buoys are made of 8-mm-diameter, 10-mm-long polyethylene rods with steel tails to keep them submerged to about 90% of their length. These are then advected by the currents to the region of interest. The velocities of the buoys were measured photogrammetrically over two alternative regions of about 1 m$^2$ ($S_1$ and $S_2$), depending on the specific case. Using particle tracking techniques, the particle positions from each image are connected in time traces, allowing the computation of velocity vectors. In addition to the photogrammetry, there was a wide-angle video camera (backup) recorder with time-lapse function to monitor the experiments and confirm that the piston motion and general circulation behaved well. In Fig. 2, some details of the experimental apparatus are shown (see below for details).

3. Western boundary currents with various degrees of nonlinearity

In PEA08 (their section 3b), the scaling of the quasi-geostrophic potential vorticity equation [valid because the Rossby number $\varepsilon_R = U/(f\ell)$ is $\varepsilon_R \approx 0.05 \ll 1$; see below for the definitions of $U$, $f$, and $\ell$] showed the
dynamic similarity between a laboratory simulation and an idealized full-scale GS. Here, we recall the basic aspects of the derivation. The governing equation for the steady state (see section 3a and Fig. 4 of PEA08 for a discussion of the adjustment that leads to virtually steady flows) in dimensionless variables is

\[ e (u u_{xx} - v u_{xy}) + \nu = E u u_{xx} - B u_x, \]  

(1)

where \( u \) and \( v \) are the zonal and meridional velocities, respectively, and the dimensionless parameters \( e \), \( E \), and \( B \) defined as

\[ e = \frac{U}{\beta c^2} = \left( \frac{\delta_I}{c} \right)^2; \quad E = \frac{A_H}{\beta c^3} = \left( \frac{\delta_M}{c} \right)^3; \quad B = \frac{r}{\beta c} = \frac{\delta_S}{c}, \]

(2)

measure the importance of nonlinearities and lateral and bottom friction, respectively. In (2), \( U \) and \( \ell \) are typical zonal velocity and length scales; \( \beta \) is the meridional gradient of the Coriolis parameter \( f \) [see Eq. (4)]; \( A_H \) is either a constant lateral eddy viscosity coefficient in a full-scale schematization of WBCs or the molecular viscosity of water in our experiments; \( r \) is the inverse of the spin-down time due to bottom friction; and \( \delta_I \), \( \delta_M \), and \( \delta_S \) represent the boundary layer length scales for purely inertial and purely viscous Munk and Stommel flows, respectively. In the derivation of (1), a boundary layer approximation has been made: \( x \) and \( y \) are scaled through the width \( \ell \) of the western boundary layer and the meridional width \( L \) of the low-latitude region where the current flows toward the western boundary, respectively, with \( \ell \ll L \).

PEA08 (see their section 3b) found that the WBC produced by the piston speed \( u_p = 10 \text{ mm s}^{-1} \) (that provides also the correct value of \( U \)) in the channel of width 0.7 m, with a rotation period of \( 4\pi f^{-1} = T = 30 \text{ s} \) and the topographic beta effect \( \beta = 0.14 \text{ rad m}^{-1} \text{ s}^{-1} \), yields \( e_{\text{laboratory}} = 0.28 \) and \( E_{\text{laboratory}} = 5.7 \times 10^{-5} \), whereas an idealized full-scale barotropic GS has \( e_{\text{GS}} \approx 0.25 \) and \( E_{\text{GS}} \approx 5 \times 10^{-3} \) (for \( A_H = 100 \text{ m}^2 \text{ s}^{-1} \)). The Reynolds number is \( Re = e/E = 5000 \) for the laboratory flow and \( Re \approx 50 \) for the GS (the bottom friction term was assumed negligible). Because in both cases \( Re \gg 1 \) and \( e_{\text{laboratory}} \approx e_{\text{GS}} \), the two flows are virtually dynamically similar in the inertial region of width \( \ell \approx \delta_I \), where the predominant nonlinear terms balance the planetary vorticity gradient term (this was also tested by numerically simulating a flow that is fully dynamically similar to the GS: i.e., also with the Reynolds numbers matching in the two cases). However, the dynamic similarity does not hold in the thin viscous boundary layer of width (Pedlosky 1996),

\[ \delta_s = \left( \frac{A_H \delta_I}{U} \right)^{1/2} \approx \frac{\delta_I}{\sqrt{\nu}} \ll \delta_I. \]

This suggests that, for nonlinear WBCs, \( Re \) alone is not an appropriate parameter to characterize the flow in scaling arguments such as those used in laboratory simulations.

Essentially the same flow has also been simulated in this new set of experiments. The topographic beta effect \( \beta \) evaluated by taking an average value of the topography is given by

\[ \beta = \frac{\int dy(y)}{H} \delta(y), \]

(4)

where \( H = 0.275 \text{ m} \) is the average depth and \( \gamma(y) \) is the bottom topography. In these scaling arguments, considering an average \( \beta \) is acceptable thanks to the relatively small variation of \( H \) within \( S_1 \) or \( S_2 \), but it should be considered that in the northernmost latitudes the local \( \beta \approx 1.5 \text{ times larger than that in the southernmost latitudes, which implies a slightly narrower WBC to the north (as evident in Fig. 3). In the present setup, } \beta \text{ is slightly larger than that of PEA08, } \beta = 0.158 \text{ rad m}^{-1} \text{ s}^{-1} \text{, and the piston speed is either the same or slightly smaller, } u_p = 8 \text{ mm s}^{-1} \text{ (in this last case, } e_{\text{laboratory}} = 0.35 \text{ and } Re = 3040 \text{). We will denote this as the dynamically similar GS (DSGS) case. The DSGS flow is shown in Fig. 3c, where the data measured in } S_1 \text{ and binned in a } 2 \text{ cm } \times 2 \text{ cm grid are reported (Fig. 3a shows the flow with } u_p = 4 \text{ mm s}^{-1} \text{). In Fig. 4a, the zonal profile of the corresponding } u \text{ averaged over the whole meridional extension of sector } S_1 \text{ (Fig. 1) and over all the data obtained in the course of the experiment is shown by the solid line } 5. \text{ In the same figure, the lines 6 and 7 show WBCs obtained by PEA08 (their experiments C and D) with } u_p = 20 \text{ and } 30 \text{ mm s}^{-1} \text{, respectively, for which nonlinear effects are even more important than in the DSGS case (in these two cases the time average was performed during the final part of the experiments when an almost steady state was achieved; see section 3a of PEA08). This parameter range lies in the inertial Charney (1955) regime for which the WBC zonal width is expected to be } \ell \approx \delta_I \approx \sqrt{\nu u_p} \text{ (see below).}

Although Charney’s range is the correct one for real WBCs, analyzing the transition to weaker flows, possibly approaching the linear limit, would nonetheless be interesting from a theoretical point of view for at least two reasons. First, that would provide a useful benchmark for experimental validation of classical linear theories of WBCs for which dissipation is dominated by bottom friction (Stommel 1948), lateral friction (Munk 1950), or both (Beardsley 1969), with this last case being likely to
Fig. 3. Examples of velocities measured in $S_1$ and binned in a $2 \text{ cm} \times 2 \text{ cm}$ grid for WBCs flowing along a straight vertical wall for basin rotation period $T = 30 \text{ s}$ with $u_p = (a) 4$ and (c) $8 \text{ mm s}^{-1}$ and for $T = 60 \text{ s}$ with $u_p = (b) 4$ and (d) $8 \text{ mm s}^{-1}$. All the data obtained in the course of the experiment are used to compute the binned values; in blank areas, data may be missing: the flow is not necessarily stagnant.
correspond to our setup (\(\delta_s\) is only slightly smaller than \(\delta_M\) for the DSGS case, PEA08). Second, that would allow us to investigate the transition range, such as the nonlinear Munk problem (Pedlosky 1996), connecting Charney’s range to the linear limit, which, at the best of the authors’ knowledge, has not yet been studied experimentally.

The technical problem encountered in reaching the linear limit was the difficulty in controlling an exceedingly small piston speed. In PEA08, \(u_p\) could not be smaller than \(0.5 \text{ mm s}^{-1}\) because below that threshold the piston had slightly variable speed. In these new experiments, an improved mechanical apparatus (Fig. 2b) has allowed us to gain an order of magnitude, as now speeds as low as \(0.5 \text{ mm s}^{-1}\) could be achieved. This is still not enough to reach the linear limit, but, as we will see, it has at least allowed us to attain the intermediate weakly nonlinear regime. The transition to weaker flows is shown by the solid lines 1–4 of Fig. 4a (from \(u_p = 0.5 \text{ to } 4 \text{ mm s}^{-1}\)).

Figure 4b is analogous to Fig. 4a but shows WBCs with \(b\) halved (\(T = 60 \text{ s}\)), and Figs. 3b,d show the velocity maps for the two examples \(u_p = 4 \text{ and } 8 \text{ mm s}^{-1}\), respectively. To analyze the transition between different dynamical regimes, we introduce two distinct parameters to represent \(\ell\).

First of all, we follow PEA08 in using the zonal location of the stagnation line \(x_s\), which is defined as the point where the velocity changes direction. In Fig. 5, \(x_s\) versus \(u_p\) is shown (by blue dots) for the present experiments reported in Fig. 4a (except the case \(u_p = 4 \text{ mm s}^{-1}\), because the corresponding \(x_s\) cannot be derived) and PEA08’s experiments 6 and 7 with \(u_p = 20 \text{ and } 30 \text{ mm s}^{-1}\).

The error bars drawn for the new experiments are derived by evaluating the stagnation line in profiles obtained by averaging \(v\) over distinct southern and northern \(y\) ranges of sector \(S_1\), so as to take into account the slight variation of \(\beta\) along \(y\). Figure 5 shows that Charney’s power law \(x_s \propto u_p^a\) with \(a = 0.5\) is a good fit for experiments 3 and 5 and PEA08’s experiments 6 and 7, provided that the last two values of \(x_s\) are multiplied by the factor \(\chi = (\beta/\beta_{\text{PEA08}})^{1/2}\) to take into account the dependence \(\delta_f \propto \beta^{-1/2}\) and the small difference in \(\beta\), with \(\chi = 0.942\) and 0.958 for experiments 6 and 7, respectively (see Table 1 of PEA08; the corrected values are shown by squares). This suggests that, for \(u_p \geq 2 \text{ mm s}^{-1}\), we are in the highly nonlinear inertial Charney’s range. A markedly different functional dependence emerges however for the weaker flows with \(u_p = 0.5 \text{ and } 1 \text{ mm s}^{-1}\) (experiments 1 and 2 with Re \(= 70 \text{ and } 160\), respectively), for which \(a = 0.10 \pm 0.08\).

An alternative representation of \(\ell\) is provided by an \(e\)-folding length scale. In a purely inertial WBC along a rectilinear coast, the meridional velocity profile away from the viscous boundary layer decays as \(v \propto \exp(-x/\delta_f)\) (e.g., Pedlosky 1996); thus, the existence of a stagnation line in our experiments belonging to that range is due to the specific setup used, which in some cases generates a weak return flow localized around a restricted \(y\) range centered at \(y \approx 0\) (e.g., see Fig. 3c and the numerical simulation of Fig. 8 of PEA08). To check that the existence of our spurious stagnation line does not affect the theoretical behavior in the inertial range for \(x < x_s\), an exponential for each line of Fig. 4a has been computed (dashed lines) by fitting the first part of the

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**FIG. 4.** Zonal profiles of the meridional velocity \(v\) averaged over the whole \(S_1\) meridional extension for various piston speeds and for \(T = (a) 30\) and (b) 60 s. The profiles 6 and 7 in (a) and 6 in (b) are from PEA08, where a slightly different \(\beta\) was used (see text). The dashed lines in (a) denote the exponential fit of the corresponding solid lines (see text for details).
4. Western boundary currents over shelf topography

The qualitative effect of a shelf topography on the structure of WBCs was analyzed by adding, along the western boundary, linear bottom slopes over the beta slope (Fig. 2c), as done, for instance, by Hsueh and Legeckis (1973) and Sanso`n and Van Heijst (2000) in different setups. Two slopes were considered, both connected to the sidewall along the same line intersecting the water level 64 cm before the end of the beta slope (at y = 56 cm). The topographies “slope1” and “slope2” extend 15 and 8 cm east of the wall and form elevation angles of 56° and 70° with the bottom, respectively.

Figure 6 shows four examples of binned current velocity fields obtained with slope1 (the vertical line delimits the zonal extension of the slope). The correspondence is between Figs. 6a–d (slope1) and Figs. 3a–d (no slope), respectively. The first remarkable effect emerging from this figure is the deflection of the WBC from the line x = 0 for increasing y, which is clearly due to the tendency of the water columns to conserve their potential vorticity by following the lines of constant f/h over the sloping sidewall (where h is the water depth after subtraction of the depth of the beta slope); for example, as explained by Vallis (2006, their section 14.6).

This effect with slope1 is better shown in Fig. 7a (Fig. 7c), where the v profiles for T = 30 s and for different values of u_p integrated in a southern (northern) sector of S1 are reported, respectively, whereas the corresponding profiles in the absence of topographic slope are given by the dashed lines (the same profiles for slope2 are shown in Figs. 7b,d; Fig. 8 is as in Fig. 7, but with T = 60 s: i.e., with halved β). The displacement of the velocity maxima away from the wall in the northern sector is evident, along with a notable increase in the velocity itself, only partly because of the decrease of the water depth to the left of the line that marks the end of the coastal slope (vertical line in the graphs), whereas it is remarkable that the velocity amplification extends well beyond that line. In general, the displacement of the velocity maxima relative to the slope width increases as the bottom slope increases (from slope1 to slope2) and as β increases (from T = 60 s to T = 30 s). Of course, in the real ocean where WBCs are stratified these topographic effects are less relevant, but that case cannot be easily reproduced in rotating tank experiments over the beta plane because the equivalent topographic beta effect is present in (1) for a homogenous fluid only. In conclusion, these results constitute the basis for a future quantitative analysis in a similar but more general context.

5. Western boundary current separation from a wedge-shaped continent due to inertial overshooting

a. Transition to the separation due to inertial overshooting for increasing current speed

In the midlatitude ocean system composed of a subtropical and a (weaker) subpolar gyre, the two corresponding WBCs converge at a given latitude, where they separate...
FIG. 6. Examples of velocity fields measured in $S_1$ and binned in a 2 cm $\times$ 2 cm grid for WBCs flowing along a straight wall with the shell topography slope1 for $T = 30$ s with $u_p = (a)$ 4 and (c) 8 mm s$^{-1}$ and for $T = 60$ s with $u_p = (b)$ 4 and (d) 8 mm s$^{-1}$. The vertical line delimits the zonal extension of the slope. All the data obtained in the course of the experiment are used to compute the binned values; in blank areas, data may be missing; the flow is not necessarily stagnant.
from the coast giving rise to a highly nonlinear free meandering jet (usually termed WBC extension). Understanding the mechanisms that produce the separation is of great importance because WBC extensions have a profound effect on climate. The Gulf Stream Extension, particularly its North Atlantic drift branch, is an integral part of the oceanic conveyor belt; the Kuroshio Extension decadal variability is known to considerably enhance the variability in a vast area of the North Pacific through intense air–sea heat exchanges, strongly affecting North American climate (e.g., Kelly et al. 2010).

Several hypotheses have been put forward to explain the dynamics of the GS separation (Dengg et al. 1996): (i) the dependence on the latitude of vanishing wind stress curl (VWSC), (ii) effects related to the outcropping of isopycnal surfaces, (iii) vorticity crisis, (iv) IO, (v) topographic effects, and (vi) the joint effect of baroclinicity and relief. To study the possibility that, in our idealized framework, IO can be induced for sufficiently strong currents by a coastline like that of America south and north of Cape Hatteras, coastal variations were introduced along the western boundary (in the absence of shelf topography) in the form of wedge-shaped continents with different coastline orientations, whose northern limit corresponds to an idealized Cape Hatteras [see Fig. 2d; for these experiments the photogrammetry system was moved to cover the area S2 (Fig. 1), which required a new calibration of the apparatus].

Figures 9 and 10 show six experiments with the same continent starting at y = 30 cm, forming a deflection
angle of $\phi = 43^\circ$ with the $y$ direction (close to the real GS value) and then becoming again parallel to $y$ (at $x = 41$ cm) north of the idealized Cape Hatteras located at $y = 74$ cm (the thick horizontal line at $y = 119$ cm in Fig. 10 denotes the limit of the topographic beta effect). All the experiments were performed with the same rotation period $T = 30$ s and differed only in the velocity of the piston. In Fig. 9, the path of the WBC is evidenced by a dye stream produced by the apparatus shown in Fig. 2a (middle left) and recorded by the wide-angle video camera (section 2). In all cases, the images are taken at the end of the experiment just before the piston stops: that is, when the spinup is achieved even for the strongest flows. In Fig. 10, the velocities obtained by the photogrammetry system are shown through arrows binned in a 1 cm $\times$ 1 cm grid; the black arrows are averages over the whole experiment (i.e., the average goes from the moment when the piston starts moving until it stops), whereas the red arrows in Figs. 10c–f are computed in the temporal windows $t = 185$–$225$ s (Fig. 10c), 145–178 s (Fig. 10d), 80–150 s (Fig. 10e), and 75–90 s (Fig. 10f) and show the flow in the final part of the experiment when the steady state is fully achieved (for a discussion of the dependence of the spinup time on $u_p$, see PEA08 and their Fig. 4).

The results show a critical behavior. The weaker WBCs (Figs. 9, 10a,b) follow the coast also past the cape, whereas stronger currents separate from the cape with a direction roughly parallel to the inclined barrier (as shown by the paths of Figs. 9c–f and by the red arrows of Figs. 10c–f), similar to the real GS case (e.g., Fig. 9g).

It is worth noticing that for the separated flows a separation bubble is expected to appear past the cape, with corresponding reversed flow near the western boundary. Such a bubble cannot manifest itself in the photogrammetrically derived velocity field, because the particles are seeded upstream (Fig. 1) and are carried along the main stream of the WBC past the separation point.
Moreover, the continent model shaded the near-shore region north of the cape for one of the three cameras, hiding the coastal area from the 3D photogrammetry system. However, in a few separated flow cases, a clear separation bubble was in fact captured by the wide-angle video camera, with particles moving rapidly southward in that region (a video clip is available as supplemental material from the Journals Online Web site at http://dx.doi.org/10.1175/2011JPO4514.s1). In such cases, depending on the specific seeding process, a few particles happened to come from the north, where they were previously advected by the flow during the initial stage of the experiment.

This sensitivity experiment puts in evidence a clear transition from coastally trapped WBCs to separated jets generated by IO for increasing current speed. Moreover, it is important to notice that the transition between these two behaviors occurs for \( u_p = 10 \, \text{mm s}^{-1} \) (Figs. 9, 10c), which, as already noticed, corresponds closely to the DSGS case.

To better understand the character of the separation, in Fig. 11a the zonal profiles of the meridional velocity \( v \) before and after separation are shown in five cases that again differ only by the piston speed. The first two cases (\( u_p = 2.5 \) and \( 5 \, \text{mm s}^{-1} \)) refer to the experiments of Figs. 10a,b, whereas the remaining three cases (\( u_p = 10, 20, \) and \( 30 \, \text{mm s}^{-1} \)) refer to the experiments of section 5b (Figs. 13a–c). The latter correspond to those of Figs. 9, 10c,e,f but are performed with the cape shifted 10 cm northward (which slightly increases the current speed—by \( \sim 5\% \)—because of the reduced depth; see the next section for the motivation of this choice). The thin lines to the left are obtained by averaging \( v \) in the band \( y = 30–35 \, \text{cm} \) for the first two cases and \( y = 40–45 \, \text{cm} \) for the remaining three cases, whereas the thick lines to the right are obtained by averaging \( v \) in the band \( y = 80–100 \, \text{cm} \) for the first two cases and \( y = 90–107 \, \text{cm} \) for the remaining three cases. For the thin lines all the data are used, whereas for the thick lines only the final data within the intervals \( t = 150–225 \, \text{s}, 40–150 \, \text{s}, 30–90 \, \text{s} \) are used for \( u_p = 10, 20, \) and \( 30 \, \text{mm s}^{-1} \), respectively (the green lines are dashed to emphasize that they correspond to the DSGS case; the vertical line marks the maximum zonal extension of the inclined boundary). Although at the beginning of the deflection (thin lines) the zonal extension is virtually the same in all cases, beyond the cape a separation appears between the first two cases (black and blue thick lines) and the remaining three cases. Figure 11a, however, suffers from a nonhomogeneous distribution of the experimental data, with the consequence of hiding somewhat the real nature of the separation and of its critical behavior.

A better way to show the transition is given in Fig. 11b, where the angle formed with the meridional direction by the velocity averaged over zonal bands of 5 cm width as a function of \( y \) are plotted (in the cases \( u_p = 10, 20, \) and...
30 mm s⁻¹, the profiles are shifted 10 cm to the south to ensure correspondence with the first two in connection with the change mentioned above). At \( y \approx 59 \) cm (where side effects connected with the transitions at the beginning and the end of the inclined boundary are minimal), all the angles are virtually coincident, being slightly tilted by \( \approx 10° \) with respect to the 43° inclination of the coast. At the highest latitudes (well beyond the cape, where the coast is meridional), the velocities of the two subcritical cases (black and blue lines) align with the coast (0°), whereas, in the remaining three cases, the flows remain more closely aligned to the inclined boundary.

The critical behavior is strikingly evident: in the cases \( u_p = 5 \) and 10 mm s⁻¹, the current velocities before the cape differ by a mere factor of \( \approx 1.6 \) (thin blue and green lines of Fig. 11a), but this is sufficient to induce a transition from a current flowing along the coast past the cape (blue line of Fig. 11b) to a current flowing virtually along the original direction even past the cape (green dashed line of Fig. 11b, corresponding to the DSGS case). A further notable increase of the current speed does not change the direction of the free jet to the north (red and brown lines).

The most energetic cases considered (\( u_p = 40 \) mm s⁻¹) are finally reported in Fig. 12, with \( T = 20 \) (Fig. 12a), 30 (Fig. 12b), and 60 s (Fig. 12c) (the arrows represent data averaged over the whole experiment). In these experiments (as in those of Fig. 13) the cape is moved 10 cm northward. These are all supercritical cases, because a clear separation is always present, even for small times (there is no need to consider only the data at the end of the simulation to identify separation, as was done for Figs. 9, 10c–f).

Finally, it is worth mentioning that, in the companion case of the Kuroshio in the North Pacific Ocean, the inclined coastline south of Japan was found to play a fundamental role in both the path of the Kuroshio Extension and its intrinsic decadal variability (Pierini 2006, 2008). Although consideration of the Kuroshio is beyond the scope of the present study, a detailed analysis of its separation in terms of the IO mechanism (which in such a case is inextricably linked to the observed bimodal behavior) would be of great interest.

b. Effect of different deflection angles of the continent

Here, we analyze the effect of the coastline inclination on flow separation; the results are summarized in Fig. 13. In these experiments (as in those of Fig. 12) the coastal inclination begins at \( y = 40 \) cm: that is, 10 cm to the north of the location used in the experiments of

![Fig. 10. Velocity fields measured in S2 and binned in a 1 cm × 1 cm grid in the six cases shown in Fig. 9. The black arrows show the velocities averaged over the whole duration of the experiments, whereas the red arrows show average over the final part of the experiments, when the spinup is fully achieved (see text). In blank areas, data may be missing: the flow is not necessarily stagnant.](image-url)
Figs. 9 and 10. If such a shift is introduced, the WBC just before the cape is more precisely alongshore and the circulation pattern past the cape is slightly more regular than with the same cape placed farther south (for this reason, we used the experiments of Figs. 13a–c—corresponding to Figs. 10c, 10e, and 10f—to compute the green, red, and brown lines of Fig. 11). In all the experiments shown in Fig. 13, $T = 30 \text{s}$. The piston speed is $u_p = 10, 20,$ and $30 \text{ mm s}^{-1}$ in the first, second, and third columns, respectively; the initial inclination of the continent is $\phi = 43^\circ, 60^\circ,$ and $75^\circ$ in the first, second, and third rows, respectively, whereas the meridional coast is always located at $x = 41 \text{ cm}$. As already mentioned, the three experiments in the first row correspond to the supercritical cases of Figs. 10c, 10e, and 10f, differing from them only in the location of the wedge (the resulting dynamics remaining virtually unchanged). Figure 13a shows the DSGS case.

Comparison of the first (Figs. 13a–c) and third (Figs. 13g–i) rows shows the dramatic effect that a different inclination has on the character of the flow past the cape. For $\phi = 75^\circ$, after an initial separation conserving flow direction, the jet yields a pronounced deflection with consequent realignment to the meridional coast almost immediately north to the cape (this happens even for the strongest supercritical case with $u_p = 30 \text{ mm s}^{-1}$). In the intermediate case with $\phi = 60^\circ$ (second line), the behavior is very similar, but with a tendency of the jet to keep a separated path in the most energetic case (Fig. 13f). Unfortunately, experiments with lower piston speeds were not carried out for the capes with $\phi = 60^\circ$ and $75^\circ$.

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**FIG. 11.** (a) Zonal profiles of the meridional velocity $v$ averaged over the band $y = 30–35 \text{ cm}$ (thin black and blue lines), $y = 40–45 \text{ cm}$ (thin green, red, and brown lines), $y = 80–100 \text{ cm}$ (thick black and blue lines) and $y = 90–107 \text{ cm}$ (thick green, red, and brown lines) for various piston speeds in the presence of the wedge-shaped continent of Figs. 9 and 10. The vertical line denotes the limit of the inclined barrier. (b) Angles formed by the zonally averaged velocity vector with the meridional direction for the cases shown in (a) (see the text for further details).

**FIG. 12.** Velocity fields measured in $S_2$ and binned in a $1 \text{ cm} \times 1 \text{ cm}$ grid for WBCs flowing along a wedge-shaped continent (in gray) with $\phi = 43^\circ$; $u_p = 40 \text{ mm s}^{-1}$; and $T = (a) 20, (b) 30,$ and (c) $60 \text{ s}$. In blank areas, data may be missing: the flow is not necessarily stagnant.
so we are unable to show how the subcritical cases \( u_p = 2.5 \) and \( 5 \) mm s\(^{-1}\) analyzed for \( \phi = 43^\circ \) behave for higher inclinations.

An implicit formula was derived for boundary current separation from a cape by Marshall and Tansley (2001), in which the gradient of the Coriolis parameter in the downstream direction (dependent on the inclination of the continent) and the radius of curvature of the flow could explain the observed dependence of separation on \( \phi \) (see also Munday and Marshall 2005 for a related analysis with a barotropic vorticity model). Direct application of that formula to our experimental results is problematic, because the lack of data very close to the coast prevents a correct evaluation of flow curvature. We can qualitatively hypothesize that the curvature impressed by the coastline inclination combined with the excess of vorticity carried by the jet produces, in the cases discussed above, a complex overshooting after separation that eventually leads to a jet alignment with the meridional coast. The present results may stimulate future theoretical and experimental investigations in which the theories of Marshall and Tansley (2001) and Munday and Marshall (2005) could be tested.

c. Discussion

Following the results shown in the preceding two subsections, we discuss some a priori reasons that make the IO mechanism for GS separation preferable in comparison to other proposed mechanisms. With the only notable exception of the IO hypothesis, in the theories reviewed by Dengg et al. (1996) (see section 5a) the separation is always associated (i) with a marked change...
in the direction of the current at the separation point and (ii) with the dependence of the latter on low-frequency wind variability. In the Gulf Stream case, however, it is quite clear that neither of these two properties occur. As far as property (i) is concerned, the GS flows almost parallel to the coast and flows past Cape Hatteras without any substantial deflection (e.g., as shown in the satellite thermal image of Fig. 9g). Moreover, as for property (ii), it is well known that the separation always occurs within a few tens of kilometers from Cape Hatteras, being therefore independent of variations of the wind forcing. These are two evident and notable features of the GS separation that do not seem to have been taken into proper account in the application of the other theories (section 5a). The IO mechanism appears therefore to be the only one to be able to explain such a peculiar phenomenology (Dengg 1993 provided interesting evidence of this possibility in a numerical process study). Indeed, if the water columns moving northward need to dissipate, along the boundary, the gain of negative relative vorticity induced by the beta effect, nonlinear effects may become so important that such a process may be locally negligible, so that a sudden disappearance of the coast (as happens at Cape Hatteras) would almost not be felt by the current. It would then continue flowing undisturbed along its path into the open ocean, at least sufficiently close to the cape (e.g., see Batchelor 1967, section 5.10, for a discussion on nonrotating fluids). In this sense, one might say that it is not the GS that separates from the coast but rather the coast that leaves the GS.

As a matter of fact, the separation point of the GS is virtually coincident with the mean position of the line of VWSC (around 35°N), whose direction, moreover, follows substantially the path of the GS Extension. Thus, the VWSC theory (i.e., the linear Sverdrup theory (e.g., Pedlosky 1996) that implies a vanishing meridional velocity along the VWSC line in the oceanic interior) could at first sight provide a plausible explanation for the separation and subsequent direction of the free jet, but Rhines and Schopp (1991) demonstrated that this would not be possible for a nonzonal VWSC line, so that theory cannot be correctly applied to this case. In summary, in view of the lack of any substantial change in direction of the GS past Cape Hatteras and of the independence of the separation point on GS changes, the IO mechanism appears to us to be a priori the best candidate for the main cause of GS separation. Our results in sections 5a and 5b are in line with this hypothesis.

The paleoceanographic study of Matsumoto and Lynch-Stieglitz (2003) also supports this view. Their reconstruction of the GS separation during the Last Glacial Maximum (LGM) and the Holocene on the basis of stable oxygen isotope ratio (δ18O) measurements on deep-dwelling planktonic foraminifera has shown that the separation latitudes were similarly located, with the glacial latitude of the separation within just one degree of that of the Holocene. This contradicts the VWSC theory because a southerly shift of several degrees of the line of maximum westeries during the LGM can be argued. Matsumoto and Lynch-Stieglitz (2003) concluded that, among all the proposed separation mechanisms, the IO is probably the one to be preferred also in view of the fact that the geometry of the ocean boundary around Cape Hatteras (the main factor conditioning the IO) remained unchanged over glacial–interglacial time scales.

In conclusion, there are several reasons to consider the IO mechanism as the primary cause of the GS separation. Our results in sections 5a and 5b support this view in a very idealized setting and may stimulate analogous but more realistic studies devoted to better understanding the real GS separation process.

6. Conclusions

In this paper, rotating-tank experiments have allowed us to analyze aspects of nonlinear western boundary currents that are still the object of scientific debate and have revealed new dynamical features that can help understand the real oceanographic phenomena and that might even stimulate new theoretical and numerical studies on the subject. Two main results have been achieved.

The first result, concerning WBCs flowing along a straight meridional coast, is the identification of a weakly nonlinear regime marking the transition from the highly nonlinear inertial Charney regime to the linear, viscous case. Although the appropriate range for real western boundary currents (e.g., the Gulf Stream and Kuroshio, in particular) is Charney’s, the characterization of a barotropic western boundary current zonal length scale in terms of its zonal velocity scale over the range of scales derived here is appealing from a theoretical point of view and may even have direct application to coastal waters where the intensification due to the topographic beta effect could locally fall into the weakly nonlinear range.

The other main result concerns the analysis of western boundary current separation from a wedge-shaped continent past a cape (simulating Cape Hatteras) due to the inertial overshooting mechanism. By analyzing flow behavior for increasing current speed, a critical behavior is identified according to which a small change in current velocity (by less than a factor 2) marks the transition from a western boundary current that follows the coast
also past the cape to a current (nearly dynamically similar to a full-scale Gulf Stream) that separates from the cape without any substantial deflection (Fig. 11b), as with the Gulf Stream Extension. Although the real Gulf Stream separation is a much more complex phenomenon than that simulated here, the present result suggests a major role played by the inertial overshooting mechanism that after all is the only one (among those proposed) for which there is no change in the direction of the current at the separation point and no dependence of the latter on low-frequency wind variability, as is the case for the Gulf Stream.

Another, more qualitative result concerns the effect of a sloping sidewall on the structure of western boundary currents studied by adding, along the western boundary, linear bottom slopes over the beta slope. A deflection of the flow away from the western wall due to the tendency to preserve potential vorticity by following the lines of constant f/β is observed.

We believe that the present laboratory study is an example of how these scaled studies of basic oceanic mechanisms can produce benchmarks and be a valuable tool even today, when the high-resolution numerical modeling of oceanographic phenomena is usually considered to be the only means available for attacking nonlinear problems.

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