The Breaking and Scattering of the Internal Tide on a Continental Slope

JODY M. KLYMAK,* MATTHEW H. ALFORD,+ ROBERT PINKEL,# REN-CHIEH LIEN,+ YUNG JANG YANG,@ AND TSWEN-YUNG TANG&

* School of Earth and Ocean Sciences, University of Victoria, Victoria, British Columbia, Canada
+ Applied Physics Laboratory, University of Washington, Seattle, Washington
# Scripps Institution of Oceanography, University of California, San Diego, La Jolla, California
@ Department of Marine Science, Naval Academy, Kaohsiung, Taiwan
& Institute of Oceanography, National Taiwan University, Taipei, Taiwan

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ABSTRACT

A strong internal tide is generated in the Luzon Strait that radiates westward to impact the continental shelf of the South China Sea. Mooring data in 1500-m depth on the continental slope show a fortnightly averaged incoming tidal flux of 12 kW m\(^{-2}\), and a mooring on a broad plateau on the slope finds a similar flux as an upper bound. Of this, 5.5 kW m\(^{-2}\) is in the diurnal tide and 3.5 kW m\(^{-2}\) is in the semidiurnal tide, with the remainder in higher-frequency motions. Turbulence dissipation may be as high as 3 kW m\(^{-2}\). Local generation is estimated from a linear model to be less than 1 kW m\(^{-2}\). The continental slope is supercritical with respect to the diurnal tide, implying that there may be significant back reflection into the basin. Comparing the low-mode energy of a horizontal standing wave at the mooring to the energy flux indicates that perhaps one-third of the incoming diurnal tidal energy is reflected. Conversely, the slope is subcritical with respect to the semidiurnal tide, and the observed reflection is very weak. A surprising observation is that, despite significant diurnal vertical-mode-2 incident energy flux, this energy did not reflect; most of the reflection was in mode 1.

The observations are consistent with a linear scattering model for supercritical topography. Large fractions of incoming energy can reflect depending on both the geometry of the shelfbreak and the phase between the modal components of the incoming flux. If the incident mode-1 and mode-2 waves are in phase at the shelf break, there is substantial transmission onto the shelf; if they are out of phase, there is almost 100% reflection. The observations of the diurnal tide at the site are consistent with the first case: weak reflection, with most of it in mode 1 and almost no reflection in mode 2. The sensitivity of the reflection on the phase between incident components significantly complicates the prediction of reflections from continental shelves.

Finally, a somewhat incidental observation is that the shape of the continental slope has large regions that are near critical to the dominant diurnal tide. This implicates the internal tide in shaping of the continental slope.

1. Introduction

The tides are believed to be a significant source of energy to the deep ocean, energizing the mixing between water masses that drives the overturning circulation (Huang 1994; Munk and Wunsch 1998). In the deep ocean, an important mechanism of energizing the mixing is via the creation of internal tides. Two end points of internal tide generation have been considered, generation over rough but not very steep topography (St. Laurent and Garrett 2002) and generation over abrupt topographic features (Llewellyn Smith and Young 2003; St. Laurent et al. 2003). The latter process has been studied in fjords (Klymak and Gregg 2004; Inall et al. 2005) and more recently at Hawaii (Merrifield and Holloway 2002; Rudnick et al. 2003). Interestingly, both fjord studies and the observations at Hawaii indicate that most of the energy lost from the barotropic conversion at abrupt topography radiates away as low-mode internal tides rather than dissipating locally (Klymak and Gregg 2004; Klymak et al. 2006a). Recent numerical modeling supports the inferences from observations (Carter et al. 2008) and offers a simple theoretical explanation (Klymak et al. 2010b) in terms of trapped lee waves (Nakamura et al. 2000; Legg and Klymak 2008; Klymak et al. 2010a).

Because so much of the energy radiates away from abrupt topography, considerable attention has been
placed on understanding where that energy eventually dissipates. One hypothesis is that there are dissipative losses when the tides scatter at isolated seamounts (Johnston and Merrifield 2003) or when they impact continental slopes (Nash et al. 2004, 2007; Martini et al. 2010, manuscript submitted to *J. Phys. Oceanogr.*; Kelly et al. 2010). When internal waves impact a slope, the behavior depends on whether the slope is supercritical ($\beta < \frac{\partial h}{\partial x}$) or subcritical ($\beta > \frac{\partial h}{\partial x}$) with respect to the incoming waves, where $\beta = [(N^2 - \omega^2)/(\omega^2 - f^2)]^{1/2}$ is the aspect ratio of energy propagation of internal waves and $\frac{\partial h}{\partial x}$ is the local topographic slope ($N$ is the local buoyancy frequency, $\omega$ is the tidal frequency and $f$ is the Coriolis frequency). For subcritical slopes, presumably much of the energy scatters upslope onto the continental shelf, where it likely dissipates in shallow water, whereas for supercritical slopes an undetermined amount of energy will reflect back into the basin. In both cases, there is presently no method for predicting a priori how much energy is dissipated locally, though laboratory experiments indicate that near-critical topographies will have significant nonlinearities (McPhee-Shaw and Kunze 2002).

In this paper, we consider observations of the internal tide impacting the continental slope on the west side of the South China Sea (SCS). The site is directly west of the Luzon Strait, a region of very strong internal tide generation (Niwa and Hibiya 2004; Jan et al. 2007). This leads to large-amplitude nonlinear internal waves propagating across the basin (Zhao et al. 2004; Ramp et al. 2004; Klymak et al. 2006b; Alford et al. 2010) and impacting the continental slope (Lien et al. 2005; Chang et al. 2006; St. Laurent 2008). The exact interplay between these nonlinear waves and the internal tide is still being investigated, but it seems the nonlinear waves are largely derived from the steeper semidiurnal tide rather than the diurnal tide (Farmer et al. 2009; Alford et al. 2010; Buijsman et al. 2010).

This paper examines what happens when this strong internal tide impacts the continental slope, beginning with a description of our mooring and ship-based operations (section 2). The observations indicate that there is significant reflection from the continental slope in addition to substantial local dissipation (section 3). These observations compare favorably to a linear reflection model that indicates that for this stratification and continental shelf depth there should be significant reflection of incoming internal tides (section 4), and we conclude with a discussion of the results (section 5).

2. Experiment

We took part in a two-week experiment aboard the *Ocean Researcher I* with observations centered just east of Dong Sha Island (Fig. 1). A line of moorings was deployed along approximately 21°N, with most of the moorings being deployed farther upslope. Here we will concentrate on a mooring deployed near the 1500-m isobath (MP1, Fig. 1b), a ship survey made on the 700-m isobath, and a mooring placed on the Dongsha Plateau in 580 m of water (LR1, Fig. 1b).

The site of mooring MP1 was chosen as part of an array of sensors meant to understand the evolution of solitary waves impacting the slope, the subject of other papers. Here we were interested in the fate of the internal tide and, at sea, decided to occupy a gap in our mooring array. It appears that the east–west profile of the slope at this latitude has a region that is very near critical to the $K_1$ internal tide (Fig. 2). This was only apparent when shipboard bathymetry became available, and it supports the idea of Cacchione et al. (2002) that internal tides smooth
off sharp edges of continental slopes (see section 5). We positioned our ship-based time series directly upslope from this near-critical region with the expectation that the near criticality would drive enhanced mixing at this location.

The slope mooring, MP1, was located at 20.919°N, 117.895°E. Its main instrument was a McLane Moored Profiler (MMP) that moved between 20- and 1450-m depths 14.2 times a day. The profiler measured temperature, salinity, and pressure with a Falmouth Scientific CTD, and velocity with a Sontek acoustic Doppler velocimeter. The mooring also had an RD Instruments 75-kHz acoustic Doppler profiler nominally moored at 80-m depth, though it experienced substantial blowdown. Data were saved every 80 s from 16-m vertical depth bins.

The plateau mooring, LR1, was located at 21.087°N, 117.347°E in 580 m of water and consisted of a single upward-looking 75-kHz broadband ADCP that was placed on the seafloor, saving data every 90 s from 16-m vertical bins. This instrument had been deployed 4 months previously and was recovered at the end of the MP1 deployment cruise, so there is only an 11-day overlap with the data from MP1.

The shipboard measurements were made with a fast-profileing CTD. This system drops at approximately 3 m s⁻¹, tethered to a Kevlar cable, and transited the 700-m depth every 8.4 min. These observations were supplemented by acoustic Doppler velocities from a 75-kHz ADCP mounted on the ship. This Doppler collected 16-m velocity measurements in narrowband mode to approximately 600 m deep in the water column. The ship transit was made at 2 kt north-northeast along the 700-m isobath.

Dissipations are calculated from Thorpe-sorted profiles (Thorpe 1977) on MP1 and from the shipboard measurements. The T–S relationship in the SCS is such that temperature dominates the stratification and there are no salinity-compensated intrusions (Fig. 3). Therefore, the overturning analysis was performed using just temperature sensors. Potential temperature profiles were sorted vertically, and the vertical displacements Δz were used to calculate a dissipation rate,

\[ \varepsilon = (\Delta z)^2 N_s^3, \]

where \( N_s \) is the buoyancy frequency calculated from the density profile under the same sorting (Alford and Pinkel 2000; Nash et al. 2007; Klymak et al. 2008). Overturns that did not meet a noise criteria of \( \Delta T > n^{-1/2}0.01°C \) were discarded, where \( \Delta T \) is the temperature difference across an overturn and \( n \) is the number of temperature samples within the overturn; 0.01°C is much greater than the noise level of the thermistors. This screening step made very little difference in the observed dissipation rate because temperature sensors are very precise (as compared to calculating density overturns, which can suffer from salinity spiking) and because the dominant overturns were very large.

3. Experimental results

a. Overview

First, we briefly overview the data collected before proceeding to more quantitative analyses. The offshore mooring, MP1, shows a strong baroclinic tidal signal (Fig. 4a), with a pronounced near-bottom intensification.
Isopycnals near the seafloor demonstrate very large vertical excursions, over 200 m during spring tides, and velocities exceed 0.7 m s\(^{-1}\). The strong near-bottom forcing drives large overturning events, some exceeding 200 m in height, allowing us to infer large turbulence dissipations (Fig. 4b). The overturning events demonstrate a clear spring–neap cycle in phase with the barotropic forcing for the region, though perhaps lagged by two days, consistent with propagation from Luzon Strait (as indicated in Fig. 4d).

The structure of the internal tide at this site is very reminiscent of the observations at Hawaii and the Oregon shelf (Fig. 5). On-slope flow causes the isopycnals to steepen and eventually break, and the appearance of large overturns tends to be phase locked to the tide. There is also a phase change to the tide between 600- and 1000-m depths. Although this is close to the zero crossing of the mode-1 tide in the open basin (though not the local mode 1; Fig. 6), it is also near the depth of the shelfbreak and is perhaps indicative of a beam like structure at this depth (see below). The upward phase propagation near the seafloor indicates downward energy propagation rather than a simple mode-1 response.

The character of the tide farther up the slope, near the 700-m isobath, is demonstrated in the ship-based time series (Fig. 7). The diurnal tide dominates at this location, with downward propagating phase. Solitary wave events

![Figure 4. Observations at mooring MP1. (a) East–west velocity and isopycnals spaced 200 m in the background density profile. Blue is westward (on slope). (b) Dissipation rate estimated via overturns. (c) Depth-integrated dissipation rate, raw and smoothed by 1 day. (d) Depth-integrated baroclinic westward energy flux smoothed by 24 h: raw (black), sum of flux based on bandpassed \( K_1 \) and \( M_2 \) components (blue), and flux based on \( K_1 \) components (cyan). The square of the tidal velocity at the Luzon straits smoothed by 24 h and lagged by two days is shown for comparison.](unauthenticated|downloaded 07/17/23 07:36 AM UTC)
are resolved by the high-frequency sampling at 0300 and 2000 UTC on day 125 of 2007. Strong dissipation is observed near the seafloor, associated with the onshore phase of the near-bottom baroclinic currents rather than the solitary waves.

b. Decomposition of signals

Our goal is to understand energy fluxes over the shelf-break region. To calculate these it is helpful to decompose the signals both in frequency and depth. We use the usual methods for these decompositions; first vertical means are removed, so that \( u(z, t) = u(z, t) - \langle u(z, t) \rangle_z \). The perturbation signals at both moorings are very much concentrated at the diurnal and semidiurnal bands \((D_1 \text{ and } D_2; \text{ Figs. 8, 9})\), so these are isolated with second-order Butterworth filters with cutoff frequencies for \( \Delta f(D_1) = [0.71-1.4] \) cycles per day and \( \Delta f(D_2) = [1.43-3.3] \) cycles per day (indicated in Fig. 8). The \( D_2 \) bandpass picks up some of the \( 2D_1 \) harmonic, so some of the nonlinearity of the tide is included in the filtering.

It is also worth noting that the large sampling interval of the MMP mooring MP1 means that it misses some high-frequency energy (Figs. 8, 10). Some of this high-frequency energy is contained in solitary waves that pass through the region at a semidiurnal interval (Ramp et al. 2004; Alford et al. 2010). The sampling from the MMP does not capture these signals very clearly. Regardless, the kinetic energy observed by each method is not very different when averaged over a day (Fig. 10, thin lines). If we assume energy flux is proportional to energy density, as is the case for linearly propagating waves, the MMP measurements are capturing almost 80% of the total flux.

Finally, it is useful to decompose the signals into vertical modes. Vertical modes are formed by separation of variables of the equations of motion, isolating the vertical structure of internal waves.

\[
\frac{d}{dz} \left( \frac{1}{N^2} \frac{d\Psi}{dz} \right) + \frac{1}{c_e^2} \Psi(z) = 0. \tag{2}
\]

This equation has imposed boundary conditions at the sea surface and seafloor of \( d\Psi / dz(z = 0) = d\Psi / dz(z = -H) = 0 \) and therefore has orthogonal solutions that decompose the flow into discrete vertical modes (Kundu 1990). These modes will be modified in the presence of mean shear, a complication we ignore here because mean flows are relatively weak.

For nonrotating waves, the possible discrete values of \( c_e \) are the group and phase speed for each mode. In a rotating ocean, the horizontally propagating signals are Poincaré waves and the phase and group speeds are...
\[ c_p = \frac{\omega}{(\omega^2 - f^2)^{1/2}} c_e \quad \text{and} \]
\[ c_g = \frac{(\omega^2 - f^2)^{1/2}}{\omega} c_e, \]

with the group speed slower than the phase speed. Each of these modes has a horizontal wavenumber derived from the dispersion relation as

\[ k_x = \frac{(\omega^2 - f^2)^{1/2}}{c_e}. \]

The eigenvalues \( c_e \) and vertical structure \( \Psi(z) \) are determined numerically from observed profiles of \( N(z) \) and projected onto the perturbation signals,

\[ u_1(t) = u(z, t)[\Psi_1(z)]^{-1}, \]

which yields the time series of the mode-1 horizontal velocity perturbation. The projection and the temporal filtering are both linear operations on the data, so they can be carried out independently.

There is a relatively large caveat in decomposing flows into vertical modes over sloping topography, because technically the decomposition is only valid on a flat-bottom ocean. The problem can be appreciated by considering the shape of vertical modes in the deep basin (Fig. 6, black lines), at MP1 (gray lines), and on the plateau (thick gray line). Clearly, signals identified as mode 1 in the deep basin are modified in space as they propagate onto the plateau. Of particular note is the comparison of mode 2 in the basin and mode 1 on the plateau; if the transition to the plateau is abrupt enough, a mode-2 wave in the basin may strongly excite a mode-1 response on the plateau. As we see below, this scattering of the incoming energy from deep modes into shallow modes can significantly complicate the interpretation of signals at any of our observation sites.

c. Net fluxes

The shoreward energy flux can be evaluated at MP1 by removing the depth mean of \( u \) and \( p \) to get \( u'(z, t) \) and \( p'(z, t) \), where \( p' = \int_0^z b \, dz \) and \( b = -(g/\rho_0)(\rho - \bar{\rho}) \) is the buoyancy anomaly from a time mean (Kunze et al. 2002; Nash et al. 2005). Over a flat bottom, the baroclinic energy flux is given by

\[ F = \int_{-H}^0 u' p' \, dz, \]
a calculation that is approximately correct over a sloping bottom as well (Kelly et al. 2010). Usually we are interested in this quantity averaged over a tidal period or a number of tidal periods, so we average for four $D_2$ tidal periods for the results shown here. We decompose the perturbations into frequency windows and vertical modes to isolate the various components of the energy fluxes (Fig. 11).

The internal wave fluxes impacting the continental slope at MP1 are relatively complicated (Fig. 11). In general, the fluxes are on slope; mode 1 dominates, but mode 2 is also important, particularly during diurnal spring tides when the two fluxes are approximately equivalent. Most of the energy is contained in the diurnal and semidiurnal tides (dashed line in Fig. 11c and Table 1), with diurnal mode 1 dominating. The semidiurnal tide is almost all mode 1 and is relatively constant through the mooring deployment, with slight peaks out of phase with the diurnal spring tides (Fig. 11c, solid line). The decomposition is summarized in Table 1.

The energy flux farther onshore on Dongsha Plateau can be estimated from the Doppler mooring LR1 at 580-m depth (Figs. 12, 9). The energy here is mostly mode 1, though the diurnal component has the energy inverted, with more mode 3 than modes 2 or 1 (Fig. 9). Much of the energy is either diurnal or semidiurnal; however, higher harmonics also play a large role, indicative of the nonlinear effects on the plateau. To calculate an energy flux, we assume that the energy is all propagating shoreward and therefore approximate the tidal-period mean flux as $F_n = c_{n\omega}E_n = c_nK\omega(\omega^2 - f^2)^{1/2}/(\omega^2 + f^2)$ for each mode $n$ and bandpassed frequency $\omega$, where $K$ is the local

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**Fig. 8.** Spectra of mean velocity between 144 and 224 m on MP1; comparison between the Doppler and the MMP. The main peaks are well captured by the coarser sampling of the MMP, but the high-frequency nonlinear harmonics are underestimated. Gray shading indicates the bandpass frequencies used in the paper.

**Fig. 9.** Kinetic energy spectra at the plateau mooring LR1. Component vertical-mode spectra are shown as gray lines.

**Fig. 10.** Comparison of the velocity signal from the MMP and Doppler. The mean velocity between 144 and 224 m was squared as a rough proxy for energy flux. One-day running means of both signals are shown as thin lines. Based on this, the coarser sampling of the MMP may underestimate energy flux by approximately 20%.

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kinetic energy evaluated from the Doppler and \( c_e \) is the linear eigenspeed (Chang et al. 2006).

As noted below, this will overpredict the net energy flux propagating shoreward because reflected waves will reduce the net flux but enhance the energy density. In addition, any noninternal wave energy will not be filtered out in the same way that it is if we can use both pressure and velocity perturbations. Therefore, this estimate is a crude upper bound. If we do not do any frequency filtering, we still get very large values averaging almost 20 kW m\(^{-1}\), which is not very realistic and is not presented. If we isolate the diurnal and semidiurnal components, we get values that are higher than observed at the deeper mooring (Fig. 12, Table 1), particularly for \( D_2 \), so we consider this calculation an upper bound on the possible fluxes.

d. Evidence for reflected energy at the continental slope

Here we attempt to estimate how much of the energy that impacts the continental slope is reflected back into the basin by comparing the low-passed energy flux \( F \) to the energy density \( E \) for horizontal plane mode-1 and mode-2 waves. For a horizontally progressive wave, \( F = c_e E \) if averaged over a number of periods. If there is a horizontally standing wave (100% reflection), the net flux will be zero, \( F = F_o - F_o \), but there will be standing energy. In the absence of rotation, the standing energy is simply

![Fig. 11. Decomposition of tidal fluxes at MP1 (continental slope). Positive fluxes are on slope (i.e., propagating from east to west): (a) all frequencies decomposed into vertical modes, (b) bandpassed at \( D_1 \) [gray shading is total flux from (a)], and (c) bandpassed at \( D_2 \).](image)

![Fig. 12. Energy flux estimated from standing energy at onshore mooring LR1. Note that the deployment only overlaps with MP1 from day 115 to when it was pulled from the water on day 128.](image)

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**Table 1.** MP1 and LR1 modal and frequency mean energy fluxes for a 28-day period; positive is shoreward. Energy flux for LR1 is estimated from energy density and is an upper bound.

<table>
<thead>
<tr>
<th></th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Mode 3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP1</td>
<td>3.1</td>
<td>2.9</td>
<td>7.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.6</td>
<td>0.3</td>
<td>3.9</td>
<td></td>
</tr>
<tr>
<td>Higher</td>
<td>0.4</td>
<td>0.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tot</td>
<td>5.6</td>
<td>3.6</td>
<td>12.4</td>
<td></td>
</tr>
<tr>
<td>LR1</td>
<td>4.6</td>
<td>11.9</td>
<td>&lt;15.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.9</td>
<td>1.1</td>
<td>&lt;3.0</td>
<td></td>
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<tr>
<td>Higher</td>
<td>1.9</td>
<td>0.4</td>
<td>&lt;3.0</td>
<td></td>
</tr>
<tr>
<td>Tot</td>
<td>&lt;8.4</td>
<td>&lt;13.4</td>
<td>&lt;21.1</td>
<td></td>
</tr>
</tbody>
</table>
Rotation adds an along-flux dependence to the energy density (Nash et al. 2004; Alford and Zhao 2007; Martini et al. 2007) that requires some care to derive. If $F_o$ is the incoming flux and $F_r$ is the reflected flux, then the energy density is

$$ E(x) = \frac{1}{C_g} \left[ F_o + |F_r| - 2 \frac{f^2}{\omega_0^2} |F_o F_r|^{1/2} \cos(2kx) \right], $$

where $x$ is the distance from the reflection point and $k$ is the horizontal wavelength of the internal wave. If $F_r = -\alpha F_o$, then the observed energy flux is $F = F_o(1 - \alpha)$ and we can write

$$ \frac{F}{E(x)c_g} = \frac{1 - \alpha}{1 + \alpha - 2 \frac{f^2}{\omega_0^2} \alpha^{1/2} \cos(2kx)}. $$

Choosing exactly how far MP1 is from the loosely defined reflection point. Here we use $x = 30$ km, and note that a $\pm 10$ km change in this has an approximately 20% error to the reflection estimate.

This estimate is relatively crude, because $E$ will include any noise in the measurements whereas $F$ tends to filter out uncorrelated noise between the density and velocity measurements. However, the vertical-mode fits and the frequency filtering serve to reduce noise. It also should be borne in mind that this analysis does not distinguish between reflected energy and energy that is radiated offshore from barotropic conversion at the topographic break; we estimate barotropic conversion below.

These caveats in mind, it appears that there is considerable reflection of the incoming internal tide energy at this site (Fig. 13a), with the flux-weighted average $1/3$ of the incoming average for the mode-1 diurnal signal (Fig. 13c, dashed line). In contrast, the semidiurnal energy flux has weak reflection (Fig. 13i), consistent with the subcriticality of the slope to the semidiurnal...
tidal frequencies. Interestingly, the mode-2 diurnal reflection has only a weak reflection (Fig. 13f), a surprising feature that we reconcile below.

e. Dissipation on the continental slope

There are very large overturns observed at both MP1 (Fig. 4), and during the shipboard time series (Fig. 7). Overturns of greater than 100 m thick were observed at both sites, with temperature differences exceeding 0.1°C. These overturns are concentrated near the seafloor, and phase locked to the tide, reminiscent of similar features seen at Hawaii (Levine and Boyd 2006; Aucan et al. 2006; Klymak et al. 2008) and on the Oregon Slope (Nash et al. 2007). The strongest dissipation at both locations is associated with the on-slope flow, where the oncoming tide forms a nonlinear shock near the seafloor. Sometimes there are overturning phenomena concentrated at the seafloor associated with off-slope flow (i.e., day 141.5, Fig. 5b).

The time series of the depth-integrated dissipation at MP1 reveals a pronounced spring–neap cycle (Fig. 4d). This cycle correlates with the incoming tidal $D_1$ flux consistent with a power law $D \sim F_{D_1}^{3/2}$ (Fig. 14a). It does not scale well with the $D_2$ tidal flux (and is in fact slightly inversely scaled, no doubt because the $D_2$ and $D_1$ spring–neap cycles are out of phase during these observations) and only scales well with the total tidal flux when $D_1$ dominates (Figs. 14b,c). This implies the $D_1$ internal tide in the local mixing on the continental slope. The scaling $F_{D_1}^{3/2}$ was chosen to be consistent with the observations (Levine and Boyd 2006) and theory (Klymak et al. 2010b) that indicates dissipation scales with the barotropic tidal forcing as $D \sim U_0^3$. In this case the tidal forcing is the incoming flux $F$, which will tend to scale as $U_0^3$ (St. Laurent and Garrett 2002; Llewellyn Smith and Young 2003).

The vertically integrated average dissipation rate at the shipboard occupation at 700 m was 0.105 W m$^{-2}$, which is about twice the dissipation rate on yearday 125 at MP1 (Fig. 4d). The stratification is such that $N^2$ is 4 times higher at the shallower depth, so the stratification-scaled energy sink is a factor of 2 higher at the deeper site. This indicates that a dissipation scheme based on a simple halo of stratification-scaled dissipation will not be adequate to describe dissipation at this site.

The mean vertically averaged dissipation at MP1 for the spring–neap cycle from day 125 to day 139 is $7 \times 10^{-2}$ W m$^{-2}$. Given MP1’s high dissipation signal, we suspect this represents an upper bound for the slope region. If we multiply by the width of the slope, approximately 40 km,
we get 3 kW m$^{-1}$ of local dissipation. It is certainly a large dissipation and on the order of the energy fluxes impacting the slope. Without a more general turbulence survey, we are not able to constrain this value further.

4. Linear theory for reflection

The observations above indicate that there is considerably more internal tide energy at the MP1 mooring than a progressive wave interpretation of $F = c_e E$ would indicate, and, if we were to take the observations at face value, perhaps 33% of the incoming mode-1 $D_1$ energy reflects back into the larger basin and 67% transmits up the slope (Fig. 13). Conversely, very little mode-2 energy is reflected. Here we investigate if this proportion is reasonable by considering a simple linear model of tidal reflection.

To solve this problem we follow the geometric approach suggested by St. Laurent et al. (2003) for baroclinic-to-barotropic tidal conversion.$^1$ For supercritical topography, the continental slope is approximated as a step change from the basin depth $H$ and to a shelf depth (or plateau depth in our case) $h_s$ (Fig. 16). The flow is decomposed into an

\[ \text{FIG. 16. Velocity snapshots of linear model for (left) mode-1 and (right) mode-2 incoming waves, with } F_1 = 2F_2. \text{ Depth and velocity are WKB scaled, } h_s/H = 0.5. \text{ Energy fluxes are indicated with text inside the domains.} \]
incident wave with velocities \( u_i \), a reflected wave \( u_r \), and a transmitted wave \( u_t \). At the shelf break, the velocities match so that \( u_t = u_i + u_r \) for \( z > -h \), and \( 0 = u_t + u_r \) for \( -H < z < -h \), and \( w_i = w_t + w_r \). Following St. Laurent et al. (2003), we make a Wentzel–Kramers–Brillouin (WKB) coordinate transform to a constant-stratification ocean. The coordinate transform from real to WKB-stretched coordinates is normalized so that \( \tilde{H} = H \), and the shelf depth \( \tilde{h}_s = \int_0^h (N/N_0) \, dz \).

The flow is decomposed into vertical modes,

\[
\begin{align*}
\tilde{u}_i &= \Re \left\{ \sum_{n=1}^{N} \cos \left( \frac{n \pi \tilde{z}}{\tilde{H}} \right) c_n \exp[i(k_n x - \omega t)] \right\}, \\
\tilde{u}_r &= \Re \left\{ \sum_{n=1}^{N} \cos \left( \frac{n \pi \tilde{z}}{\tilde{H}} \right) a_n \exp[i(k_n x - \omega t)] \right\}, \quad \text{and} \\
\tilde{u}_t &= \Re \left\{ \sum_{n=1}^{N} \cos \left( \frac{n \pi \tilde{z}}{\tilde{h}_s} \right) b_n \exp[i(k_n x - \omega t)] \right\}
\end{align*}
\]

where \( k_n \) and \( k_n' \) are the horizontal wavelengths in the deep basin and shelf, respectively, and the incoming wave is represented as a sum of incoming internal modes rather than a barotropic flow; a similar modal decomposition holds for the vertical velocities. The coefficients are complex because we must allow for the incoming modes to have different phases with respect to each other when they reach the continental shelf.

We arrive at a pair of coupled equations

\[
e_n + a_n = A_{mn} b_n \quad \text{and} \quad b_n = B_{nl} (a_l - c_l),
\]

which can easily be solved for \( a_n \),

\[
a_n = (A_{mn} B_{nl} - I_{ml})^{-1} (A_{mn} B_{nl} - I_{ml}) c_l,
\]

and for \( b_n \),

\[
b_n = B_{nl} (a_l - c_l),
\]

where \( A_{mn} \) and \( B_{nl} \) are given by St. Laurent et al. (2003),

\[
A_{mn} = \frac{2m \gamma(-1)^n \sin m \pi \gamma}{\pi[m^2 \gamma^2 - n^2]} \quad \text{and} \quad B_{nl} = \frac{2n(-1)^n \sin n \pi \gamma}{\pi[n^2 - \tilde{g}^2 \gamma^2]},
\]

where \( \gamma = \tilde{h}_s / \tilde{H} \) and \( I_{ml} \) is the identity matrix (this notation departs slightly from St. Laurent et al. 2003, who use \( \delta = 1 - \gamma \)).

We want the energy flux averaged over a tidal period in dimensional units,

\[
F_n = \int_{-H}^0 \langle u_n p_n \rangle \, dz,
\]

\[
= \int_{-H}^0 \langle \tilde{u}_n \tilde{p}_n \rangle \frac{N}{N_0} \, dz,
\]

\[
= \int_{-H}^0 \langle \tilde{u}_n^2 \rangle \frac{g(\omega) N}{m(n) N_0} \, dz,
\]

\[
= \frac{g(\omega)}{m(n)} \int_{-\tilde{H}}^{\tilde{h}_s} \langle \tilde{u}_n^2 \rangle \, d\tilde{z}.
\]

Here, we have used the WKB-stretched values for \( u = (N/N_0)^{1/2} \tilde{u} \) and \( d\tilde{z} = (N_0/N) \, dz \). The function

\[
g(\omega) = \rho \left[ (N_0^2 - \omega^2)(\omega^2 - \tilde{g}^2) \right]^{1/2}
\]

converts from \( \tilde{p}_n \) to \( \tilde{u}_n \) when divided by the WKB vertical wavenumber: \( m(n) = n \pi / \tilde{H} \). This yields

\[
F_n^r = \frac{g(\omega)}{m(n)} \left| a_n^2 \right| \int_{-\tilde{H}}^0 \cos^2 \left( \frac{n \pi \tilde{z}}{\tilde{H}} \right) \, d\tilde{z}
\]

\[
= \tilde{H} \frac{g(\omega)}{m(n)} \left| a_n^2 \right| \frac{1}{2}.
\]

In the shallow water, the energy flux is similarly derived,

\[
F_n^r = \frac{g(\omega)}{m(n)} \left| b_n^2 \right| \frac{1}{2}.
\]

FIG. 17. Fraction of energy reflected from continental slope as a function of shelf depth (in WKB coordinates) for monochromatic incoming waves.

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where \( m(n) = n\pi h_s \) are the shallow-water vertical-mode numbers.

a. Application to South China Sea

If the incoming wave field is largely composed of a single mode, the response of the Dongsha Plateau is relatively straightforward. The fraction of energy reflected back into the basin is a function of the depth of the continental shelf compared to the vertical-mode shapes. The response for mode-1 and mode-2 incoming waves shows that a substantial amount of energy can be reflected back into the basin (Fig. 16), with a standing pattern in the shape of beams that have the vertical extent of the plateau depth. This results in a largely red mode-number spectrum of the reflected energy, analogous to the spectra for barotropic conversion (St. Laurent et al. 2003). The fraction of energy reflected depends on the depth of the plateau, with more reflection for shallower shelves (Fig. 17) and a slight structure in the mode-2 response because of the midwater maximum in the velocity structure. For a mode-1-only incoming flux, a very large fraction of the energy is reflected for a WKB shelf of \( h_s/H \approx 0.5 \) (appropriate to a WKB-normalized SCS), with \( F_r \approx 0.75 F_o \).

The actual situation in SCS is more complicated, with both mode-1 and mode-2 energy impacting Dongsha Plateau (Fig. 13). In this case, the phase of the incoming waves relative to each other strongly impacts the response. If the waves are in phase at the plateau break such that modes 1 and 2 are positively reinforced at the sea surface, there is very strong transmission onto the plateau and relatively weak reflection (Fig. 18, left). If they are out of phase, strong incoming velocities are felt at the bottom and almost all of the energy is reflected, with almost none transmitted onto the plateau (Fig. 18, right).

It is noteworthy that a linear model would have such an asymmetric response. What happens is that the response to an incoming wave is phase locked with the forcing, either perfectly in phase or \( 180^\circ \) out of phase. If the mode-1 and mode-2 incident waves are out of phase, their reflections will also be out of phase, and this can lead
to positive reinforcement or cancellation of the modes in the response. The waves are linear, but the energy flux is inherently nonlinear.

There is a continuum between these two extremes depending on the exact phase (and amplitude) of the incoming waves (Fig. 19). Interestingly, the amount of mode-2 reflected energy is very low if the waves are nearly in phase (i.e., phase $< 50^\circ$ or $> 310^\circ$), because the mode-2 wave reflected from the incident mode-2 wave has the opposite phase as the one reflected from the incident mode-1 wave. This compares favorably with the observation above that there is little energy reflected from mode 2 at the MP1 mooring (Fig. 13, middle), indicating that the incoming mode-1 and mode-2 waves may be nearly in phase at the topographic break.

It is difficult to say a priori what the phase should be between mode 1 and mode 2 at the topographic break without knowing the exact location where they were generated.

However, we can make an iterative estimate by considering the observed phase at MP1. The observed phase difference at MP1 between mode 1 and mode 2 is almost exactly $180^\circ$ (Fig. 20). If we assume the mode-1 signal is composed of an incident wave and a reflected wave with 33% of the incident energy flux and 0% of the mode 2, then we can trace back the expected phase at MP1, $x = 32$ km offshore of the reflection point (Fig. 2),

$$u_1 = a\{\exp(-ik_1x) + \sqrt{0.33}\ \exp(i k_1 x)\} e^{i\Delta\phi}$$  \hspace{1cm} (27)

$$u_2 = b\{\exp(-i(k_2 x - \Delta\phi))\} e^{i\Delta\phi}.$$  \hspace{1cm} (28)

The phase difference between mode 1 and mode 2 at the topographic break is $\Delta\phi \approx 320^\circ$, though slight variations in the distance of the slope break can change this phase substantially (Fig. 21).

This phase difference between the mode-1 and mode-2 signals at MP1 and the results of the linear model therefore indicate that the fraction of mode-1 energy reflected would be 40% (Fig. 19b) and the fraction of mode-2 energy reflected would be 5%. This is in rough agreement

![Fig. 19](image1.png)

**Fig. 19.** Dependence of fraction of energy reflected on phase between mode-1 and mode-2 waves. The mode-2 wave has half the flux of the mode-1 wave as pictured in Fig. 18. (a) Total fraction reflected as a fraction of energy incident. Thick line is total, and thin lines are cumulative fractions in mode 1 and in modes 1 and 2. (b) Fraction of mode 1 and mode 2 reflected compared to the incident mode-1 and mode-2 energy. The vertical dashed lines are the approximate phase we think is valid at Dongsha Plateau.

![Fig. 20](image2.png)

**Fig. 20.** Coherence and phase difference between mode-1 and mode-2 east–west velocity at MP1. The diurnal tide (gray box) has a phase difference of almost exactly $180^\circ$. 
with the observations (Fig. 13), though the predicted reflection for both modes is higher than observed. This fraction is close enough to justify the iterative approach to determining the phase.

If the incoming mode-1 and mode-2 internal tides are 320° out of phase at the reflection point, then we can trace eastward in the basin to find possible points of generation. At a single knife-edge ridge, mode 1 and mode 2 are either in phase or 180° out of phase; for ridges that are the height of those in the Luzon Strait, we would expect the modes to be out of phase. Tracing back (Fig. 22), we find only three locations where the mode-1 or mode-2 internal tides are in or out of phase. One location just west of 119°E is unlikely to be the source of a strong internal tide, leaving the shorter ridge at 120.6°E where the internal tides are predicted to be in phase and 20 km to the east of the taller ridge at 122°E. This second potential generation point is the correct phase to indicate simple knife-edge generation at this location. However, the generation point does not lie exactly at the ridge crest, perhaps because this estimate is very crude; perhaps because of three dimensionality; or perhaps because the reflections, both from the continental slope and from the smaller ridge, complicate the phasing of the internal tide generation. Finally, mode 2 has the potential to be refracted more than mode 1 (Rainville and Pinkel 2006), perhaps changing the phase relation by the time the waves reach the continental slope. The phase response of the complete system will be the subject of further research.

The linear theory is valid for supercritical topography ($\beta > \frac{dh}{dx}$). This approximation appears to be valid for knife-edge generation problems (Di Lorenzo et al. 2006; Klymak et al. 2010b), so we anticipate that it is valid for the scattering problem, though verification will need to wait for appropriate numerical studies. Most of the slope is supercritical to the diurnal tides, but it is subcritical to the semidiurnal tides (Fig. 2). For the $D_2$ tides, we expect most of the energy to propagate onshore, again as indicated by the observations (Fig. 13).

5. Discussion

a. Slope criticality

There is a near-critical region in the continental slope with respect to the $D_1$ internal tide, which prompted the location of the ship time series. It is interesting to consider how much of the slope is near critical by comparing the bottom slope to the slope of internal waves of either $D_1$ or $D_2$ frequency (Figs. 23a,b). Doing this with the readily available Smith–Sandwell database (Smith and Sandwell 1997) yields poor results, but a high-resolution database from National Taiwan University indicates that there are wide regions of the slope that are near critical to $D_1$ internal waves. There are narrow regions of near criticality for $D_2$ waves, but they are not as prevalent as for the dominant $D_1$ tide.

The hypothesis put forward by Cacchione et al. (2002) is that the shelf breaks of continental slopes are eroded...
by tidal flows over them until they reach the critical condition. For the slope we are considering, there are diurnal internal tides impacting the slope from the east-southeast, so the hypothesis would be that the slope facing in these directions would be more near critical than slopes facing in other directions. We test this by sweeping through the wave directions, and the normalized distribution of slope criticality (Fig. 24) indicates that the southeast is more likely to have near-critical slopes than slopes facing in the other directions. The histograms are diffuse and this observation could have geological reasons, but the observation is indicative.

There is much less semidiurnal energy on this continental slope, partly because the semidiurnal internal tides generated at Luzon Strait appear to disintegrate into solitons (Farmer et al. 2009; Alford et al. 2010). Although the solitons have considerable energy and energy flux, it is largely at higher frequencies and there is little opportunity for interactions with the seafloor that might cause localized erosion at the continental break. Certainly, in the shipboard observations, there was little evidence of turbulence near the seafloor associated with the solitons (Fig. 7).

The continental slope of the South China Sea is a passive margin and composed of a very thick layer of sediments (>1 km; Nissen et al. 1995), so the mechanism proposed by Cacchione et al. (2002) is feasible insofar as the internal-wave-induced stress does not need to act on bedrock. However, being a passive margin, there are geological reasons that the slope is more gentle here than at an active margin. In their geological history, the islands of Taiwan and Luzon moved to the east-southeast from this continental slope, so the fact that the gentler slopes are in the direction of rifting is perhaps coincidental.

However, Luzon Strait itself is more near critical to $D_1$ tides (Fig. 25), and this is the “active” margin for this basin. The $D_1$ criticality for a basin that is very resonant to the $D_1$ tides and has a very strong $D_1$ internal tide is indicative that the Cacchione et al. (2002) hypothesis is applicable here.

b. Discussion of the energy balance terms

The rough energy budget inferred from the observations is that there is approximately 7 kW m$^{-1}$ of low-mode $D_1$ energy approaching the continental slope (Fig. 26). Of that, maybe one-third of the mode-1 energy reflects (1.5 kW m$^{-1}$), whereas most of the mode-2 energy

![Fig. 23. (a) Slope criticality for $D_1$ tides. (b) Slope criticality for $D_2$ tides. Bathymetry is a 1.8-km-resolution product from National Taiwan University. Older versions of Smith–Sandwell (i.e., 8.2 and previous) do not show as coherent near-critical regions.](image)

![Fig. 24. Probability density functions for criticality for waves coming from a spread of directions. The slopes with the largest near-critical areas are found for waves coming from the southeast, broadly consistent with an internal tide source near the Luzon Strait.](image)
propagates onshore. The sum of the mode-1 and mode-2 energy at the plateau mooring LR1 balances is approximately 6.5 kW m\(^{-2}\) propagating onshore.

This picture is complicated by the high-mode energy flux inferred at LR1 (Table 1), though that energy could be subject to considerable noise or balanced by unresolved high-mode energy at MP1. It is also complicated by the presence of significant dissipation at both MP1 and the shipboard site, the simple extrapolation of which may lead us to expect as much as a 3 kW m\(^{-2}\) energy loss.

Similarly, there is the potential for local generation of diurnal energy (Fig. 15) of similar magnitude to the fraction of mode-1 energy we estimate as being reflected. We do not have any way to improve our knowledge of this system from just one mooring offshore. If the phase between the incoming waves and those generated locally were well separated, it might be possible to distinguish the relative energy in each, but that requires better knowledge of the geometry of the system than we have here.

Our budget for the semidiurnal tide is not as well constrained, with 3.5 kW m\(^{-2}\) observed offshore at MP1 but 7.5 kW m\(^{-2}\) observed onshore at LR1. This discrepancy is unlikely due to local generation because the \(D_2\) tides are quite weak here. Again, it is possible that the energy flux is simply being overestimated at LR1 because we are using the energy density multiplied by group speed rather than a proper estimate of flux.

Even for \(D_1\), the energy budget presented above is very schematic and would require more data and modeling to make it more robust. The first limitation is that we have made vertical model decompositions. This is necessary for getting a reflection coefficient because we require the group speed of a progressive wave to compare against. We plan to test the robustness of this assumption in a series of numerical experiments where it is possible to separate eastward-propagating signals from westward-propagating signals using complex demodulation. To do the same thing with real data, a pair of moorings spaced significantly apart so that the phase lags could be determined would be necessary. For this experiment, another mooring farther east in the basin would have helped.

**Fig. 25.** Criticality in Luzon Strait. Again, the strait is more near critical to \(D_1\) than \(D_2\) tides.
Similarly, we are hampered by our assumption of a two-dimensional wave. In the real ocean, the scattering from the three-dimensional shelf break will lead to out of phase signals that will increase $E$ but not be readily apparent in the energy flux.

Finally, the fraction of energy reflected is difficult to determine, even for the simple linear step model. If all the incoming energy is mode 1, the answer is unambiguous. If the incoming energy has a broader modal content, the fraction of reflected energy becomes a very complicated function of the modal content, its phase, and the depth of the continental shelf. Evaluating this further is beyond the scope of this work simply because we do not have the appropriate data to compare with.

In terms of where the dissipation of low-mode tidal energy occurs, these observations and the linear model indicate that continental shelves have the potential to dissipate a large fraction of the low-mode energy. However, depending on the relative phase of the modes in the incoming tide, the theoretical results here indicate that, for many supercritical slopes, most of the energy may be reflected. It should be noted that our WKB-scaled plateau depth was exactly half the WKB water depth in the South China Sea, whereas most continental shelves are shallower, $h_w/H \approx 0.33$ (S. Kelly 2010, personal communication). For $F_2/F_1 = 0.5$, almost all the energy would be reflected, with less dependence on the phasing of the incoming waves (Fig. 27). The dependence becomes more complicated for other values of $F_2/F_1$ with the result weighted toward the single-mode values (Fig. 17).

Of great interest then is what happens to the fraction of energy that reflects to the deep basin. If it radiates all the way back to Luzon Strait, then a lossy standing wave will set up, and this will fundamentally alter the generation mechanisms at the strait. If the dissipation occurs before the reflected energy makes it that far east, the question is, what mechanisms drive the dissipation?

Finally, the observations here are not so well constrained that we have adequately answered how “efficient” the reflection/transmission problem is at this locale. The large dissipations at MP1 and the shipboard survey indicate that a large fraction of the incoming energy may dissipate on the slope. Similar observations made on the Oregon continental slope indicate that there are many near-critical regions where local hydraulics and bores form (Nash et al. 2007), indicating that local roughness may be an important factor in dissipating the tide. Determining the fraction lost more precisely will need to be the subject of future research.

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