Vortex Stability in a Large-Scale Internal Wave Shear

Anne-Marie E. G. Brunner-Suzuki and Miles A. Sundermeyer
University of Massachusetts—Dartmouth, Dartmouth, Massachusetts

M.-Pascale Lelong
NorthWest Research Associates, Redmond, Washington

(Manuscript received 9 August 2011, in final form 8 May 2012)

Abstract

The effect of a large-scale internal wave on a multipolar compound vortex was simulated numerically using a 3D Boussinesq pseudospectral model. A suite of simulations tested the effect of a background internal wave of various strengths, including a simulation with only a vortex. Without the background wave, the vortex remained apparently stable for many hundreds of inertial periods but then split into two dipoles. With increasing background wave amplitude, and hence shear, dipole splitting occurred earlier and was less symmetric in space. Theoretical considerations suggest that the vortex alone undergoes a self-induced mixed barotropic–baroclinic instability. For a vortex plus background wave, kinetic energy spectra showed that the internal wave supplied energy for the dipole splitting. In this case, it was found that the presence of the wave hastened the time to instability by increasing the initial perturbation to the vortex. Results suggest that the stability and fate of submesoscale vortices in the ocean may be significantly modified by the presence of large-scale internal waves. This could in turn have a significant effect on the exchange of energy between the submesoscale and both larger and smaller scales.

1. Introduction

Lateral mixing, often parameterized by lateral diffusivity, can be enhanced by a variety of processes. Among these is the geostrophic adjustment of multiple and sporadic patches of well-mixed water (Sundermeyer et al. 2005). Geostrophic adjustment transforms mixed patches into vortices (McWilliams 1988) that then stir the surrounding fluid. When vortices and internal waves coexist, the vortices are subject to internal wave shear and strain. Once the shear exceeds a certain threshold, the vortex is thought to be torn apart and ceases to exist (e.g., Brickman and Ruddick 1990).

In this paper, we explore the stability of submesoscale coherent vortices. We begin with a review of the physical oceanic context of these processes. Internal wave breaking can lead to shear instability and overturning. Surrounding fluid is mixed, energy is converted into potential energy (e.g., Birch and Sundermeyer 2011), scalar quantities are dissipated, and a patch of relatively well mixed fluid is generated (e.g., Alford and Pinkel 2000). Such diapycnal mixing has been observed episodically in time and space (e.g., Gregg et al. 1986). However, episodic wave breaking is only one of many possible mechanisms that may generate mixed patches. Other mechanisms include baroclinic instability, deep convection or hydrothermal plumes, meddy–seamount interactions and boundary layer interactions (e.g., Send and Marshall 1995; Helfrich and Battisti 1991; Kunze and Sanford 1993; Thorpe 2005). Submesoscale mixed patches have been observed on scales of 1–10 m in the vertical and 100–1000 m in the horizontal: for example, by Sundermeyer et al. (2005) on the New England shelf, Haury et al. (1979) on Stellwagen Bank, Alford and Pinkel (2000) off the coast of California, and Gregg et al. (1986) in the California Current. During the North Atlantic Tracer Release Experiment (NATRE), scales of inferred mixed patches were larger, 10–50 m vertically (Polzin et al. 2003).

Once a mixed patch has formed, the fluid spreads laterally outward under the pressure gradient and an anticyclone is spun up by the Coriolis force. Above and
below the anticyclone, fluid converges, forming two cyclones (e.g., McWilliams 1988). Such a multipolar compound vortex has been referred to as an “S vortex” (Morel and McWilliams 1997), as it is dominated by stretching; a “sandwich vortex,” as the anticyclone is sandwiched between two cyclones; and as pancake eddy, or blini, due to the flat aspect ratio. After the initial adjustment, in the absence of outside influences, the S vortex reaches a steady rotational state, which lasts until the vortex is either diffused away or an instability breaks it apart.

Multiple sporadically occurring mixed patches and their adjustment and equilibrium phases are thought to increase lateral diffusivity. Observed diffusivity can be explained in terms of horizontal stirring by the vortices using scaling arguments (Sundermeyer et al. 2005). Alternatively, Polzin et al. (2003) inferred vortical mode spectra, from which they estimated contributions of shear dispersion and vortical mode stirring to lateral diffusivity, and concluded that their results could also explain observations. As we will show in the present study, these findings could be affected by the presence of background internal waves. In general, internal waves can exist anywhere in the stratified ocean (Garrett and Munk 1972). Breaking internal waves can potentially generate such S vortices. Consequently, internal waves, mixed patches, and potentially S vortices may interact at a variety of scales.

Breaking internal waves and submesoscale stirring are among the many processes that modulate biogeochemical budgets (Levy 2008), nutrient and pollutant distribution (Thorpe 2005), and phytoplankton patchiness (Martin 2003). These properties in turn affect the spatial distribution of higher trophic levels. Productivity can be influenced by (sub)mesoscale dynamics through their influence on stratification; see, for instance, Flierl and McGillicuddy (2002) for the impact of submesoscale physics on the biological dynamics. These physical and biological processes are at subgrid scale for global circulation models, and thus the particular submesoscale dynamics discussed here are not included in such models. Parameterizations for this type of process do not yet exist.

To better understand the problem, several numerical studies have examined the stability of single monopolar vortices. Kloosterziel and Carnevale (1999) studied “shielded” vortices consisting of a vortex core and a shielding ring of opposing vorticity. Once the horizontal vorticity profile exceeded a critical steepness, an azimuthal instability developed such that the shield broke into satellite vortices and transformed into a tripole. With increasing steepness, dipole splitting can occur (Kloosterziel and Carnevale 1999; Higgins et al. 2002; Beckers et al. 2003).

The stability of vortices is also influenced by their aspect ratio via the Burger number (Bu) such that smaller Bu implies a more stable vortex and a smaller fastest growing mode (e.g., Flierl 1988; Helfrich and Send 1988). The Burger number is defined as $Bu = (h^2 \Delta N^2) / (L^2 f^2) = R_D^2 / L^2$, with $L$ being the radius of the vortex, $h$ its half height, $R_D = \Delta N h / f$ the Rossby radius of deformation, and $\Delta N$ the difference in buoyancy frequency between the vortex and the background stratification. Note, here our definition of $R_D$ is based on the dimensions of a single submesoscale vortex and, hence, differs from the classic oceanic mesoscale $R_D$, which is typically $O(50 \text{ km})$. Consequently, for the vortices of interest here, $Bu \leq 1$ (i.e., our vortex is wide relative to its $R_D$ and hence is dominated by rotation), despite $L$ being seemingly small relative to the mesoscale. Numerous studies have shown the importance of Bu for the adjustment and stability of single vortices (Griffiths and Linden 1981; Helfrich and Send 1988; Hopfinger and van Heijst 1993; Flor and van Heijst 1996; Lelng and Sundermeyer 2005), and we include some simulations varying the aspect ratio of the vortex. However, a detailed exploration of Bu dependence is left for future study.

More relevant to the present work is that the evolution and stability of a vortex can also be altered by the presence of a background shear and/or strain. Vortices in a strain field can become elliptical (Ruddick 1987; Mariotti et al. 1994; Kunnen et al. 2002). As the vortex distorts, stripping can occur such that filaments of weak vorticity around the vortex edge are transported away from the vortex core, enhancing gradients at the edge. If the external shear exceeds that of the vortex beyond a critical threshold, vortex splitting can occur (Mariotti et al. 1994). Laboratory and numerical experiments of an anticyclonic lens by Brickman and Ruddick (1990) showed that, when subjected to a large-scale horizontal strain above $0.1f$, the lens narrowed, thinned, and became oriented along the major axis of strain. The effect of a vertical shear on a heton was simulated by Vandermeer et al. (2002). Somewhat in contrast to the above, they found that a vertical shear inhibited the vertical separation of hetons with radius larger than $0.5R_D$ (Bu $\leq 4$). This could indicate a strong vertical coherence of S vortices.

The present study builds on previous work of monopolar shielded vortices by focusing on an S vortex originating from a density mixed patch. Such vortices have been studied numerically by Lelng and Sundermeyer (2005) and in the laboratory by Stuart et al. (2011). However, in those studies, vortex stability was not considered. Owing to its vertical variations of meridional potential vorticity gradient (e.g., Morel and McWilliams 1997), an S vortex could be subject to baroclinic
instability, while its horizontal vorticity shield could make it subject to barotropic instability (e.g., Kloosterziel and Carnevale 1999) or a combination of both. Send and Marshall (1995) found that baroclinic instability can cause a deep convection plume to break apart into multiple eddies after two Eady time scales. Flierl (1988) further showed that geostrophic baroclinic vortices with radii larger than $2R_{D}$ ($Bu \geq 0.25$) could also be subject to baroclinic instability.

What happens to a geostrophically adjusted mixed patch in a large-scale temporally varying internal wave shear and strain field? We know that vortices and internal waves can and do interact when on similar scales (Lelong and Riley 1991; Riley and Lelong 2000), enabling the transfer of energy and the induction of instabilities via triad interactions. Lelong and Sundermeyer (2005) found that the interaction between a geostrophically adjusting mixed patch and high frequency waves generated in the process remain small. However, this can be different for large-scale near-inertial waves. In what follows, we examine the effect of a large-scale near-inertial wave on an S vortex. We focus on waves that are themselves convectively stable for many hundreds of inertial periods. We do not address the generation mechanism of the mixed patches or the details of their geostrophic adjustment. We show that, if the wave’s amplitude is small, its effect on the vortex is negligible. However, with increasing amplitude, the wave modifies the evolution of the S-vortex structure.

The remainder of this paper is organized as follows. After introducing the numerical model, analytical techniques and theoretical considerations to describe and quantify our simulations are given. Then we describe the evolution of a vortex in a weak and strong internal wave shear. Three criteria are used to characterize the development of vortex instability and examine instability growth times over a wide range of internal wave amplitudes. Kinetic energy spectra are used to shed further light on the details of the breakup. Finally, we provide a discussion of the results in the context of other studies, highlighting our most significant findings, followed by summary and conclusions.

2. Methods

a. Model equations and setup

Numerical experiments were conducted using the model of Winters et al. (2004) with some modifications by Sundermeyer and Lelong (2005). This pseudospectral model solves the nonlinear, three-dimensional (3D) Boussinesq equations with f-plane approximation and advection–diffusion equations for density:

\[
\frac{D\mathbf{u}}{Dt} + f \mathbf{z} \times \mathbf{u} = -\frac{1}{\rho_0} \nabla P - \frac{g}{\rho_0} \mathbf{z} + \nu_6 \nabla^6 \mathbf{u},
\]

\[
\frac{D\rho'}{Dt} = -w \frac{D\bar{p}}{Dz} + \kappa_6 \nabla^6 (\rho' + p),
\]

\[
\mathbf{V} \cdot \mathbf{u} = 0.
\]

Here $\mathbf{u} = (u, v, w)$ is the velocity vector, $P$ is the perturbation pressure, $\rho'$ is the density anomaly, $\bar{p}$ is a linear background density, $\rho_0 = 1024 \text{ kg m}^{-3}$ is a reference density, and $\nu_6$ and $\kappa_6$ are hyperviscosity and hyper-diffusion coefficients normalized by the maximum non-dimensional wavenumber, $k_{max}$ and $m_{max}$. To minimize aliasing while maintaining numerical accuracy, 1/6th of the wavenumbers were truncated following Patterson and Orszag (1971). This normalization prevents buildup of small-scale enstrophy caused by finite grid resolution without dynamically affecting the scales of interest. The resulting diffusivities used in our model are $\kappa_{6,H} = \nu_{6,H} = 1 \times 10^{3}(k_{max}/2)^6 \text{ m}^6 \text{ s}^{-1} = 48 \text{ m}^6 \text{ s}^{-1}$ in the horizontal and $\kappa_{6,V} = \nu_{6,V} = 1 \times 10^3(m_{max})^6 \text{ m}^6 \text{ s}^{-1} = 1.85 \times 10^{-10} \text{ m}^6 \text{ s}^{-1}$ in the vertical. This corresponds to approximately inviscid conditions while maintaining numerical stability.

For time stepping, a third-order Adams–Bashforth scheme was used. Linear terms were computed in spectral space, while nonlinear terms were computed in physical space. The model is triply periodic with a domain size of $5 \text{ km} \times 5 \text{ km} \times 12.5 \text{ m}$ ($L_x, L_y, L_z$). This allowed us to model typical submesoscale mixed patch sizes such as observed during the Coastal Mixing and Optics Experiment (Sundermeyer et al. 2005), where mixed patch half height and half width are approximately 1/30th of the domain size; namely, $h = 1.25 \text{ m}$ and $L = 500 \text{ m}$. The model time step was $\Delta t = 15 \text{ s}$ to accommodate high-frequency internal waves generated during the geostrophic adjustment of the vortex. These high frequency waves act on very different time scales than the background large-scale internal wave, which was on the order of the inertial frequency.

b. N/f scaling

Internal waves, mixed patches, and S vortices can be linked via their associated aspect ratio, $h/L \approx L_z/L_x \approx N/f$. Mixed patches with $N/f \approx 200$ were inferred from observations by Sundermeyer et al. (2005) on the New England shelf, and this ratio was chosen in our model. These length and time scales span several orders of magnitude and require high spatial and temporal resolutions to be modeled. To bring these scales together and hence increase computational efficiency, we employed $N/f$ scaling, as described by Lelong and Dunkerton.
shear owing to their flat aspect ratio. On the New England shelf values of 0.0065 s\(^{-1}\) were observed after storm events (Sundermeyer 1998; MacKinnon and Gregg 2005). Our largest rms vertical internal wave shear was 0.0036 s\(^{-1}\).

A Gaussian-shaped mixed patch was superimposed on the internal wave at the center of the domain (\(x_0, y_0, z_0\)) by imposing a vertical diffusivity \(\kappa_z\) for 100 time steps:

\[
\kappa_z(x,y,z,t) = \frac{1}{\Delta t} \frac{\Delta N^2 h^2}{N^2/4} \times \exp \left\{-\frac{1}{2} \left( \frac{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}{L^2/2 + h^2/2} \right) \right\}.
\]

For an initial linear density profile, the resulting density profile of the mixed patch is then

\[
\rho = \frac{\Delta N^2}{N^2} \frac{dp}{dz} \times \exp \left\{-\frac{1}{2} \left( \frac{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}{L^2/2 + h^2/2} \right) \right\},
\]

as described by Lelong and Sundermeyer (2005). For an initial condition of a background low mode internal wave, given by (4)–(7), the result is modified somewhat from this form. In all of our simulations, the mixed patch was completely mixed; that is, \(\Delta N/N = 1\) at its center.

After the initialization of wave and mixed patch, within the first few inertial periods \(T_i\), the mixed patch spun up an S vortex. Following spinup, typical minimum and maximum local Rossby number of the vortex, defined as \(\mathrm{Ro} = \frac{\zeta}{f} = (dv/dx - du/dy)/f\), were \(\mathrm{Ro} = -0.13\) for the anticyclone and 0.06 for the cyclones. In all simulations, unless otherwise noted, the final dimensions of the associated anticyclone were approximately \(L = 500 \text{ m} = 2R_D\) and \(h = 1.25 \text{ m}\) (or \(\text{Bu} = 0.25\)). Thus, the vortex is wide relative to \(R_D\) and is dominated by rotation. A more detailed description of the geostrophic adjustment process can be found in Lelong and Sundermeyer (2005).

In a suite of 15 simulations the amplitude of the background internal wave was increased, and consequently its velocity and the background shear and strain relative to the vortex were enhanced. Specifically, we varied the wave amplitude \(a\) over several orders of magnitude from \(8 \times 10^{-7}\) m for the flattest wave to \(2.4\) m for the steepest wave. The associated vertical rms shear ranged from \(10^{-9}\) to \(3.6 \times 10^{-3}\) s\(^{-1}\). Also included is a run with only a vortex and no initialized background
internal wave. The largest amplitude wave had velocities approximately 10 times that of the adjusted vortex, or a maximum wave velocity of 10 cm s$^{-1}$.

d. Analysis

An analytical stability analysis would be rather difficult given the three-dimensional multipolar structure of our vortices. In lieu of such analysis, as a first step to determine the nature of the instability of our base case vortex (i.e., whether it is fundamentally a barotropic or baroclinic instability), we examined the development of the main azimuthal instability modes of the vortex using spectral analysis. Specifically we focused on the development of the mean flow (mode 0) and the fastest growing mode (mode 2). Mode 1 represents a radial displacement of the center of the vortex relative to the center of the azimuthal analysis and, hence, for our purposes has no dynamical significance. For the modal analysis we computed kinetic energy (KE) and available potential energy (APE) at every grid point and examined the radially averaged azimuthal fast Fourier transform of horizontal cross sections through the anticyclone’s core. A running mean of $47T_i$ was applied to minimize higher frequency variations. The fastest growing instability is observed to be two dimensional; hence, it is sufficient to look at azimuthal modes without the vertical structure. Helfrich and Send (1988) and Carton and Legras (1994) used a similar technique to describe the development of azimuthal modes. Following their diagnostic approach, we shall show that our vortices undergo a mixed baroclinic–barotropic instability.

Considering our vortex instability further, for each simulation three different criteria were employed to measure when the instability reached finite amplitude. First, we fit an ellipse to the negative relative vorticity $(-\zeta)$ on a horizontal slice at middepth and recorded when the ellipticity, defined as the ratio of the minor to major axis, increased by 20%. Second, we measured when the anticyclone and cyclone maximum $|Ro|$ exceeded 25% of a mean reference value. The reference value was computed by averaging relative vorticity over $2T_i - 4T_i$ early in the model runs. Third, the onset of dipole splitting was estimated as the time when the contour equaling 25% of the anticyclone’s core $Ro$ split from a single closed contour to two closed contours. The 25% of the anticyclone core $Ro$ was based upon the initial value of the S vortex. While there is some flexibility in the definition of “significant” increases in ellipticity and $|Ro|$, the values chosen here allowed easy incorporation of all simulations, even in cases when the instability developed rather quickly. As wave amplitude increased, however, shearing and straining by the wave caused the error associated with our various measures of instability growth to increase. To quantify errors, standard deviations of ellipticity and $|Ro|$ were computed for the first few inertial periods following adjustment before the respective threshold criteria were reached. A different error arose from the time between saved model output fields: the final errors for ellipticity and $|Ro|$ are based upon whichever error was larger. Errors for the break in vorticity contour lines are based entirely on the time between model output fields.

Horizontal KE spectra were computed to show the general effect of a large-scale background internal wave on a vortex. Spectra were first integrated over all vertical wavenumbers and then sorted by horizontal wavenumber, yielding isotropic horizontal spectra. Time evolutions of these spectra were compared for three simulations: a wave only, a vortex only, and a wave plus a vortex. Within these, we further contrasted the evolution of three different wavenumber bands: (i) wavenumber 1, corresponding to the background internal wave; (ii) the sum of intermediate wavenumbers 2–6, which contained $\sim 88\%$ of the vortex energy; and (iii) the sum of the remaining higher wavenumbers.

Finally, we related vortex instability growth times in our simulations to two theoretical scalings. As will be discussed in section 3a, analysis of azimuthal modes of KE and APE suggests our vortices undergo a mixed barotropic–baroclinic instability. Relating our results to classic instability problems, we might therefore expect the barotropic instability growth time to scale with the horizontal shear; for example,

$$T_{BT} = C \frac{1}{\partial u/\partial y},$$

where $C$ is a scale factor typically much larger than 1. Helfrich and Send (1988) used such scaling to examine the instability of quasigeostrophic hetons. In their case, the horizontal shear was defined in terms of the Bickley jet (McWilliams 2008) such that $\partial u/\partial y = u^*/L$, with $u(y) = u^* \text{sech}^2(y/L)$, and the scale factor $C = (2\pi)/0.1$. Applying a similar approach to our simulations, we estimated $u^*$, $L$, and hence $\partial u/\partial y$, by fitting $u(y)$ to the velocity profile of our adjusted anticyclone at middepth. Note, since increasing internal wave amplitude caused our horizontal vortex velocity profile to be increasingly obscured by the wave, we did not estimate $T_{BT}$ for our largest wave amplitude runs.

Alternatively, relating our results to classic baroclinic instability scaling, we might expect the baroclinic instability growth time to scale with the vertical shear; that is,

$$T_{BC} = C' \frac{1}{\partial u/\partial z}$$

in which $C'$ is again a scale factor typically much larger than 1. In the classic Eady problem, $\partial u/\partial z$ is the shear associated with thermal wind balance, and $C'$ is related to the aspect ratio of the geostrophic shear; for example, $C' = (1/0.3)(N/f)$ (Vallis 2006). Send and Marshall (1995) compared the Eady growth time to the time it takes for a deep-convection mixed patch to break up. In our case, we wish to relate the more general scaling given by (11) to the time it takes for a compound S vortex to split into dipoles.

An important point regarding the above scalings is that the time it takes an instability to reach finite amplitude depends on both the linear growth rate of the instability and the magnitude of the initial perturbation, $A_i$; that is, 

$$A = A_i e^{\tau/T},$$

(12)

where $A$ is the amplitude of the instability at time $t$ and $T$ is the $e$-folding time of the instability, as given, for example, by (10) or (11). For our problem, we can estimate the linear growth rate for the vortex-only simulations from the aforementioned azimuthal instability analysis. For vortex only/weak wave forcing, high frequency internal waves generated during the initial geostrophic adjustment likely provide the initial perturbation, independent of the low-mode background wave. However, above a certain wave amplitude, as we shall show, our results suggest that the background wave increasingly dominates in setting the magnitude of the initial perturbation.

Finally, we note that in general (10) and (11) can be applied even when there is curvature to the flow, provided the curvature is small. This was true in our case, as the cyclostrophic term ($v^2/\rho f$) in the momentum equations remained much smaller (i.e., 30–50 times) than the Coriolis $(\beta f)$ and pressure gradient ($\rho_0^2 \partial P/\partial r$) terms (i.e., $Ro < 1$), as is typical for the geostrophic regime characteristic of our vortices. The same three terms can be compared as Rossby and Burger numbers, $Ro^2$, $Ro$, and $Bu$, by dividing by $L f^2$. Also noteworthy is that the vertical scale of our vortex was 10 times smaller than the background wave, which spanned the domain; thus, the background shear was always large scale compared to the vortex. Meanwhile, in all but one of our simulations, the vertical vortex shear ($0.0014 \text{ s}^{-1}$) was larger than the vertical wave shear ($10^{-9}$ to $3.6 \times 10^{-3} \text{ s}^{-1}$). Horizontal vortex and wave shears were approximately 330 times weaker, respectively.

3. Results

a. Vortex only

Numerical experiments simulated the effect of a background internal wave field on the stability of an S vortex generated by the relaxation of a diapycnal mixing event. Generally speaking, the resulting vortex was sheared and strained by the wave until the vortex split into a pair of dipoles. For runs with greater wave amplitude and shear, the instability grew faster, and the vortex split earlier. To understand the nature of the vortex instability and its dependence on the background internal wave, we begin by examining the instability and breakup for the case of a vortex only (no/weak wave). We then contrast two simulations: a weak wave case, defined as a vertical wave shear $< 10^{-4} \text{ s}^{-1}$, and a strong wave case with a vertical shear $\geq 10^{-4} \text{ s}^{-1}$. For a vertical shear $\geq 10^{-4} \text{ s}^{-1}$, we show that the wave clearly affects the development of the instability.

The evolution and breakup for the case of a vortex only (contrasted with the case of a strong wave) is shown in Fig. 1. For the vortex-only case, after initial geostrophic adjustment, the vortex reaches an apparently stable phase during which relative vorticity, velocity, and density remain approximately steady for several hundreds of inertial periods. An anticyclone is surrounded at middepth by a ring of shear with opposing relative vorticity (a “shield”) and two smaller cyclones above and below (Fig. 1a, 45$\text{Ti}$). The instability first manifests as asymmetries in the shield, which becomes enhanced in some areas and reduced in others. Two satellite vortices develop, and the anticyclone becomes elliptical, consistent with an azimuthal mode-2 instability dominating the growth of the instability (Fig. 1a, 240$\text{Ti}$; e.g., Higgins et al. 2002). By 280$\text{Ti}$, the anticyclone is S shaped; it is elongated and thinned in the horizontal and stretched vertically, similar to simulations of a single lens in a horizontal shear field (e.g., Brickman and Ruddick 1990; Kloosterziel and Carnevale 1999; Higgins et al. 2002). Gradually, the shield and the two cyclones merge and encase the anticyclone, similar to simulations of a shielded anticyclone without rotation by Beckers et al. (2003, their Fig. 11). The encasing relative vorticity reshapes into two tall cyclones on either side of the anticyclone. These cyclones not only spin the anticyclone around its axis but also strain and pull it until, at 320$\text{Ti}$, the anticyclone’s core splits. Two dipoles emerge, self-propelling in opposite directions.

Additional details of the breakup are revealed by vertical and horizontal sections of $Ro$ (Fig. 2). In the vortex-only scenario, the anticyclonic core’s relative vorticity is approximately twice the magnitude of the cyclonic shield’s relative vorticity. High frequency waves generated during the adjustment are visible in the background. Following the initial apparently stable phase, at 240$\text{Ti}$, the anticyclone thins and its peripheral relative vorticity gradients sharpen (Fig. 2a); the cyclonic relative vorticity caps the anticyclone (Fig. 2b). By 280$\text{Ti}$,
satellite vortices result in filamentation and vortex stripping of the anticyclone as $|Ro|$ increases. After $320T_i$, the anticyclonic core splits into two dipoles whose width is half the original mixed patch ($\approx 1R_D$). Meanwhile, $|Ro|$ increases further while APE decreases, suggesting an energy transfer from APE to KE and possibly from the internal wave field as well.

Considering azimuthal modes of KE and APE, we can quantify the instability of the S vortex. The fastest growing mode for both KE and APE is mode 2, although mode 0 contains the most energy (Fig. 3). Recall that mode 1 is a displacement mode and, hence, has no dynamical significance in the present context. Relating the time series of modes 0 and 2 to Carton and Legras (1994), we can identify several different phases: I) the geostrophic adjustment (between $0T_i$ and $8T_i$); II) a ramping of APE ($8T_i$–$56T_i$); III) the decay of KE and APE in all modes (only modes 0–4 shown) due to model diffusion and viscosity ($56T_i$–$137T_i$); IV) a linear growth of the most unstable mode, here mode 2 ($137T_i$–$156T_i$); V) nonlinear amplification, in which higher modes (e.g., modes 3 and 4) also grow ($156T_i$–$267T_i$); and VI) dipole splitting ($\approx 300T_i$) after which APE significantly decreases. Noteworthy in the above is that, during the linear growth phase IV) mode-0 KE and APE decay 50% and 70%, respectively, faster compared to the previous phase III. This accelerated loss of mean KE and APE, together with the mode-2 energy gain over the same period, indicates a mixed baroclinic–barotropic instability. Last, we note that the latter portion of the nonlinear amplification phase revealed by azimuthal instability analysis also corresponds approximately to the time scales indicated by our three instability metrics (symbols in Fig. 3; see section 3c).

b. Vortex plus strong wave

With increasing wave amplitude, the wave increasingly hastens the vortex instability. An example of a strong wave case, to contrast the weak wave case, is shown in Fig. 1b. When the wave and vortex velocities are comparable, again initially an S vortex emerges. However, after only $2T_i$ (Fig. 1b) and thus still during the geostrophic adjustment phase, the vertical axis of the S vortex tilts, possibly reflecting a stronger initial perturbation $A_i$ by the wave and/or the simultaneous development of a “tilting instability” (Flierl 1988). At this point, the shield is already asymmetric. By $11T_i$, the anticyclone is S shaped, spiraling over several depth layers (Fig. 1b). Similar to the weak wave scenario, shield and cyclones merge and encase the anticyclone. Dipole splitting occurs 10–20 times faster in the strong wave scenario. After the initial dipole splitting, the wave continues to modify the evolution of the vortices. The dipoles often collide, sharing (or splitting) part of their
relative vorticity, sometimes generating additional dipole pairs. In viscous simulations these dipoles eventually diffuse away.

Considering horizontal and vertical slices of Ro (Figs. 2c,d, respectively), the mode-1 background wave is evident. The wave’s presence alters Ro at the location of the vortex, and the anticyclonic core shifts off center after only $27T_i$. By $11T_i$, the anticyclonic core shows two extrema, that is, the start of dipole splitting, and the vertical axis is tilted. By $18T_i$, both anticyclonic and cyclonic Ro increase, similar to the weak wave simulation, but with larger values. By $30T_i$, two dipoles propagate at $90^\circ$ away from the center, spinning faster not only compared to the original S vortex but also compared to the weak wave dipoles. The cores of the resulting dipoles also lay on different depths, a configuration that Morel and McWilliams (1997) called a “tilted dipole.” Because of the vertical tilt, however, it is unclear if both dipoles have the same size and strength; either is possible in our simulations.

c. Instability growth rates

We next examine the time scale required for the vortex instability in our various simulations to reach finite amplitude per our three metrics described in section 2. Considering first the vortex only/weak wave case, recall that the vortex is apparently stable for hundreds of inertial periods. Ellipticity is thus approximately constant for $200T_i$ (Fig. 4a) before it eventually grows and reaches its threshold criterion. Shortly thereafter, ellipticity

---

**Fig. 2.** An expansion of horizontal and vertical slices of Ro = $\xi/f$ through the center of the domain for (a),(b) a vortex only and (c),(d) a vortex plus a strong wave. Green ellipse and magenta contours in (a) and (c) are used to compute parameters in Fig. 4. The color bar applies to all plots. Time stamps correspond to snapshots in Fig. 1.
becomes unbounded as the dipoles separate and propagate away from one another. Such rapid growth of ellipticity during the breakup is characteristic of all weak wave simulations once the instability has grown to finite amplitude. An analogous unbounded ellipticity (as well as an increase in $|\text{Ro}|$) was found for anticyclonic lenses in a horizontal shear field by Brickman and Ruddick (1990). By $304T_i$, following the rapid growth phase of ellipticity, the break in vorticity contour is observed (third criterion). Meanwhile, for times $< 5T_i$, $|\text{Ro}|$ varies during the initial geostrophic adjustment, as APE (slumping vortex) is exchanged with KE (outward spreading) (Fig. 4b). Thereafter, $|\text{Ro}|$ fluctuates around a constant value with a small downward trend, possibly due to hyperdiffusivity. By $254T_i$, however, the second criterion is reached ($|\text{Ro}|$ threshold). Noteworthy is the slightly earlier peak in $+\text{Ro}$ compared to $-\text{Ro}$, indicating the time when the shield and cyclones start to merge to form the positive satellite vortex cores. By $278T_i$, $|\text{Ro}|$ associated with the anticyclone reaches a minimum before relaxing to a value larger than the original reference value.

By contrast, in the strong wave simulation, the ellipticity criterion was reached after only $7T_i$ (Fig. 4c). Compared to the weak wave case, where ellipticity grows to approximately 100, in the strong wave case ellipticity never exceeds 15. Two considerations can explain the difference. First, the ellipticity was computed from a horizontal slice at middepth and, therefore, grossly underestimates the extent of the anticyclone, which in the strong wave case is actually tilted vertically (Fig. 1b). Second, following dipole splitting, the growth of ellipticity is limited as the internal wave background shear field modifies the propagation of the dipoles, keeping them closer together. Meanwhile, by $8T_i$, the $|\text{Ro}|$ criterion is met. Then $|\text{Ro}|$ continues to grow until, at $14T_i$, the S vortex splits into dipoles (third criterion). Note that the initial $|\text{Ro}|$ is the same for all simulations since each run is initialized with the same mixed patch. However, the subsequent gain in relative vorticity is greater for larger background wave amplitude such that the maximum and minimum $|\text{Ro}|$ are greater (0.5 versus 0.2 in Figs. 4c,d). In other words, the stronger the background internal wave, the faster the rotation of the resulting dipoles.

d. Evolution of spectra

The temporal evolution of isotropic KE spectra is contrasted in four different simulations (Fig. 5): (i) a run with only the mode-1 wave, (ii) a short ($20T_i$), and (iii) a long ($400T_i$) run with only a mixed patch, and (iv) a run initialized with the wave from (i) and the mixed patch from (ii). In (i), over 99% of KE is contained in wave-number 1 (Fig. 5a): that is, the background wave. Over time, a fraction of the KE spreads into higher modes; however, this fraction is barely above the noise level associated with machine precision. In (ii), the initial geostrophic adjustment involves higher wavenumbers
(Fig. 5b) and is followed by a nearly steady state of the vortex spectrum (Fig. 6, inset). Here 88% of the vortex KE is contained in wavenumbers 2–6 (henceforth intermediate wavenumbers), while 11% are found at wavenumber 1. Scenario (iii) expands (ii) in time: during the long nearly stationary phase (first 200 $T_I$ in Figs. 5c and 6, main panel), intermediate wavenumbers lose KE at a rate of $5.9 \times 10^{-12}$ J m$^{-3}$ per $T_I$. This modest KE loss is less than expected from hyperdiffusion alone and could thus indicate that the growing instability (first visible in intermediate and then higher wavenumbers) counteracts hyperdiffusive losses. After reaching a maximum at the time of dipole splitting (312 $T_I$), KE across the entire spectrum subsequently decreases but remains greater at all wavenumbers compared to the initial state (Fig. 6). This overall gain in KE is accompanied by a drop in potential energy (PE) (not shown).

Finally, in scenario (iv), when both wave and vortex are present, wavenumber 1 represents the sum of both background wave and vortex energy (Fig. 5d). Here, within the first 20 $T_I$ KE at intermediate wavenumbers increases tenfold more than in 400 $T_I$ of scenario (iii) (Fig. 7), obscuring the initial geostrophic adjustment. Higher wavenumbers gain KE as well but less so. Dipole splitting occurs at 14 $T_I$ after which the highest wavenumbers lose energy, while intermediate wavenumbers gain energy. Overall, more energy is transferred when both wave and vortex are present, despite wavenumber 1 containing a mix of wave and vortex KE. Presumably, the loss in wavenumber 1 (attributed to the internal wave) would be even greater if we separated the wave from the vortex KE. We conclude that the vortex is drawing energy from the internal wave, enabling and enhancing the growth of the instability.

e. Comparison with instability theory

Putting theory and our numerical simulations together, we now examine the full suite of 15 simulations, spanning six orders of magnitude of wave amplitudes. Figure 8 shows the time of vortex instability per our
three instability metrics as a function of vertical wave shear. In all simulations, the ellipticity criterion is met first, followed by the increase in $|\text{Ro}|$, and then the break in vorticity contour. The exact timing varies somewhat between the criteria. However, the general behavior of the three metrics agrees to within approximately a factor of 2 across the more than two orders of magnitude range spanned by our simulations. This suggests that these metrics are sufficient to quantify the range of times to breakup.

The most striking feature in Fig. 8 is the marked transition that occurs at wave shear of approximately $10^{-4}$ s$^{-1}$, indicating a transition from an instability independent of background wave amplitude to one that depends strongly on wave amplitude. We interpret the transition between these two regimes as follows: In the weak wave regime, that is, wave shear $<10^{-4}$ s$^{-1}$, vortex instability is self-induced in the sense that the time to reach finite amplitude is determined only by the vortex (not the wave) shear, per either (10) or (11). Similarly, the initial perturbation for the instability, per (12), is set by factors that depend only on the vortex, for example, numerical noise or, more likely, high-frequency internal waves generated during the mixed patch adjustment, not the background wave. Since the instability of our vortex is a mixed barotropic–baroclinic instability, we consider here both barotropic and baroclinic instability scalings described in section 2. Assuming barotropic instability scaling given by (10), we find our results for vortex only/weak wave are consistent with a scale factor $C = 100$. Alternatively, using the baroclinic scaling given by (11), we obtain a scale factor $C' = 33 344$.

Noteworthy here is that instability scalings such as (10) and (11) are formally based on linear theory. By contrast, the three metrics depicted in Fig. 8 measure when the instability has reached finite amplitude (Fig. 3; e.g., Carton and Legras 1994). Strictly speaking, the linear growth time and the time to breakup are different. Nevertheless, numerous studies of mesoscale as well as mixed layer eddies have used linear instability theory together with a scale factor to quantify finite amplitude instability (e.g., Fox-Kemper et al. 2008; Fox-Kemper and Ferrari 2008).

Considering the strong wave forcing regime (Fig. 8, wave shear $>10^{-4}$ s$^{-1}$), we find that the time to finite amplitude depends strongly on the amplitude (or equivalently shear) of the background wave such that the stronger the wave, the faster the instability manifests. While it is tempting to interpret this inverse dependence on the wave shear as indicating a faster instability growth rate, recall that it is the vortex and not the wave itself that is going unstable. Thus, estimating an instability growth time of the vortex based on the wave shear may not be appropriate. Also, recall that in our
case the large-scale shear is that associated with the background wave and not a thermal wind shear. Nevertheless, drawing the analogy between the instability of a large-scale baroclinic shear and our large-scale internal wave, if we take the vertical shear to be that of the background wave in (11), we would obtain a value of the scale factor: $C' = 400$. Consistent with this idea that the wave drives the vortex instability are our findings that the wave supplies energy to the vortex (Fig. 7), allowing it to go unstable sooner, and that the $|\text{Ro}|$ of the resulting dipoles increased with increasing background wave shear (Fig. 4).

As an alternative to the above, for the strong wave regime we can view the background internal wave as setting the initial perturbation for the vortex instability such that the large-scale shear associated with the wave drives an azimuthal mode-2 deformation of the vortex that increases with increasing wave shear. In this case, we can think of $A_i$ in (12) as proportional to the background wave amplitude or, equivalently, the wave shear. Thus, again in Fig. 8, the larger the background wave, and hence $A_i$, the sooner the instability manifests and the shorter the time to breakup. In this case, formally we expect the instability growth time to scale inversely with the logarithm of the vertical wave shear. Using Eq. (12) and assuming $T = T_{BT}$ and $A_i = B\partial u/\partial z$, where $\partial u/\partial z$ is the vertical wave shear, we can fit the results in Fig. 8 by eye for large shear cases, yielding $B = 250$. Regardless as to whether the wave shear is viewed as setting the instability growth rate or the size of the initial perturbation, the main result is the same: in the weak wave regime (Fig. 8, left) the S vortex goes unstable independent of the background internal wave, while in the strong wave regime (Fig. 8, right) the time of instability decreases markedly with increasing wave shear or amplitude.

![Fig. 6](image1.png)

**FIG. 6.** Time series of change in kinetic energy ($\Delta KE$) for different wavenumber summations of the vortex-only run. The insert highlights the first $20T_i$ oscillating during the geostrophic adjustment. Note the rapid growth in intermediate wavenumbers (dotted line) as well as in both mode 1 and higher wavenumbers as the vortex goes unstable ($-250T_i$). Gray shading indicates when dipoles have split and ellipticity of the original vortex is ill defined.

![Fig. 7](image2.png)

**FIG. 7.** Similar to Fig. 6 but for wave plus vortex run. Note the shorter time axis and larger $\Delta KE$ range (two orders of magnitude). Visible vortex instability develops almost immediately with dipole splitting occurring by $t = 14T_i$.

![Fig. 8](image3.png)

**FIG. 8.** Vertical internal wave shear vs time scale of instability for 15 simulations spanning over six orders of magnitude of wave shear. Diamonds represent $|\text{Ro}|$ (closed symbols) and ellipticity (open symbols) criteria; gray squares mark the break in vorticity contours. Corresponding scaling for weak wave shear [i.e., based on horizontal vortex shear per (10) as $T_{BT} = 100\partial u/\partial y$] and strong wave shear [i.e., based on vertical wave shear per (12) with $A_i = 250\partial u/\partial z$] are indicated by bold lines.
4. Discussion

The S vortices originating from mixed patches in a rotating linearly stratified environment are found to be stable for many hundreds of inertial periods. Eventually, an azimuthal mode-2 instability develops, resulting in dipole splitting. Similar dipole splitting has been observed numerically (Ruddick 1987; Helfrich and Send 1988; Brickman and Ruddick 1990; Kloosterziel and Carnevale 1999; Beckers et al. 2003) and in the laboratory (e.g., Griffiths and Linden 1981; Brickman and Ruddick 1990; Helfrich and Battisti 1991; Higgins et al. 2002; Beckers et al. 2003). When set against a background internal wave, as internal wave amplitude/shear increases, the vortex instability goes from being dominated by self-induced shear to being dependent on the internal wave shear. Surprising here is how simply our observations scale with the inverse of the wave shear/amplitude in cases of strong wave forcing (Fig. 8), despite the periodicity of the shear. The internal wave varies much faster (tenfold or more) than the instability growth rate.

While the focus of this study is on mode-2 instabilities, the specific structural details of a Gaussian S vortex make it susceptible to higher mode instabilities. The steepness parameter of the relative vorticity profile at middepth in our simulations was $\approx 4.5$, and, hence, larger than the critical steepness estimated by Beckers et al. (2003) for dipole splitting. Also, our results compare well with Carton and Legras (1994), who predicted that, for two-dimensional monopolar vortices with steepness between 1.85 and 6, azimuthal mode 2 would be the fastest growing mode. A shallower vorticity profile means a smaller vorticity ratio between the shield and core. This will lead to weaker satellite vortices, making dipole splitting less likely. A key result here is that the vortex itself is unstable, scaling with Eqs. (10) or (11) evaluated at the vortex center. Numerical results of a shielded vortex by Beckers et al. (2003) showed barotropic rather than baroclinic instability that was most pronounced at middepth. Potentially due to the multipolar nature of the S vortex, in our case the instability was clearly mixed baroclinic–barotropic, emphasizing the importance of the surrounding cyclones in the development of the instability.

Flierl (1988) analyzed instability of a variety of circular geostrophic vortices on an $f$ plane with stratification. He computed unstable modes depending on the width of a potential vorticity shield. When relating our S vortex to his findings and treating the S vortex as three independent vortices, we would not expect baroclinic or barotropic instability to grow—even if hyperdiffusivity eventually eroded the vorticity shield. However, considering the vertical potential vorticity distribution, the presence of two zero crossings makes the S vortex vulnerable to baroclinic instability (Helfrich and Send 1988; Morel and McWilliams 1997). In the context of Flierl, our depth-dependent potential vorticity profile suggests modes 2 and 3, and potentially even mode 1, could be unstable (the tilting instability). Such tilting of the vertical vortex axis was observed for the strong wave runs (Fig. 2), possibly excited by the internal wave. Although, the instability affects the position of the vertical axis of the S vortex and, although there are small differences in timing for stability criteria for cyclones and anticyclones (see Fig. 4), our results show that the cyclones and anticyclone act almost in unison. The vertical structure of the vortex remains coherent until the satellite vortices are formed. This is consistent with findings of Vandermeirsh et al. (2002), who showed that a vertical background shear can inhibit a vertical splitting of a heton.

Once the S vortex breaks apart, the newly shaped dipoles are longer lived than the original S vortex. A similar observation was made by Morel and McWilliams (1997), who focused on the pathways of tripoles, S vortices, and the subsequent dipoles. Furthermore, dipoles observed in our simulations are smaller in diameter than the original S vortex and, hence, are possibly more stable. While the core $|Ro|$ increases with increasing background internal wave shear, a reduction of horizontal extent $L$ also occurs (see Figs. 4 and 2). These characteristics of the newly shaped dipoles all suggest that the dipoles reach a stable state suitable to withstand the background wave field. In the ocean, the longevity of our dipoles would still be limited by diffusion. The presence of a weak wave speeds up the breakup process slightly but does not affect the basic characteristics of the instability.

Although the main focus of this study is submesoscale mixed patches of aspect ratios resembling findings of Sundermeyer et al. (2005), we also performed a few simulations in which we varied the vortex aspect ratio and, hence, $Bu$ from the base case value of $Bu = 0.25$. Specifically, we conducted three simulations with varying wave amplitude and $Bu = 1$ (i.e., halving $L$ while holding $h$ and hence $R_h$ constant) and one simulation with $Bu = 0.2$. For the $Bu = 1$ cases, for wave shear $\approx 3 \times 10^{-4} \text{ s}^{-1}$, the S vortex converted into a stable tripole at times similar to the contour criterion in the $Bu = 0.25$ case. For larger wave shear, dipole splitting was observed. This transition to wave-induced dipole splitting appeared to correspond to an enhanced separation between the cyclone and anticyclone cores, as suggested by Flor and van Heijst (1996). These results again emphasize the two different regimes. For $Bu = 0.2$, $L$ was increased. Here, the formation of a triangular vortex was
observed, indicating that the most unstable mode increased from 2 to 3. Spanning a broader range of $Bu$ would quickly expand the parameter space to be explored and is left for future study.

Laboratory studies by Griffiths and Linden (1981) showed that monopolar vortices were unstable to baroclinic instability if $Bu < 0.04$; the authors also point out that the critical $Bu$ can be larger if the ratio of vortex height to water depth is $\leq 0.2$. This is different from our observation, where S vortices went unstable even for $Bu = 1$. Also, we showed that our vortices underwent a mixed baroclinic–barotropic instability, which might also alter the dependence on $Bu$. Saunders (1973) found in laboratory experiments that surface bound vortices were unstable to baroclinic instability for $Bu < 1.8$. Their critical $Bu$ seems to agree with our findings. Flor and van Heijst (1996) found that the mode of instability increased with increasing $L/h$ ratio. Similarly, Hopfinger and van Heijst (1993) observed that with increasing Froude number $F = L^2/R_D^2 = 1/Bu$, or decreasing $Bu$, the most unstable mode increased for two-layer hetons. These results are again consistent with our findings. Theoretical considerations by Helfrich and Send (1988) and Flierl (1988) showed that the larger $Bu$, the less likely dipole splitting, consistent with our observation of a stable tripole rather than dipole splitting for $Bu = 1$. Also, Lelong and Sundermeyer (2005) found in numerical simulations of S vortices that, with increasing $Bu$, the ring of positive vorticity surrounding the anticyclone widened due to changes in the geostrophic adjustment. Again, a wider and flatter vorticity profile suggests a more stable vortex (Carton and Legras 1994). With increasing $Bu$, the geostrophically adjusted mixed patch is increasingly dominated by APE until rotation is no longer important (Lelong and Sundermeyer 2005).

We also compared our temporally varying internal wave shear to simulations with a constant vertical shear. Here the model was initialized with a sinusoidal density field, $\rho' = a_x \cos(ly)$ and velocity computed via thermal wind balance, $u = -a_y l g z/(\rho_0 f) \sin(ly)$, where $l = 2\pi/L_y$. Boundary conditions were changed to free slip in the vertical but kept periodic in the horizontal. This created two opposing surface bound jets with a constant vertical shear in the $x$–$z$ plane. The jet’s shear was matched to the rms vertical shear of the internal waves in the primary simulations. Two simulations with (i) $a_x = 0.002 \text{ m s}^{-1}$ and (ii) $a_x = 0.02 \text{ m s}^{-1}$ resulted in vertical jet shears of $-1.3 \times 10^{-4} \text{ s}^{-1}$ and $-1.3 \times 10^{-3} \text{ s}^{-1}$, respectively, where (ii) was twice as strong as the strongest wave case (see Fig. 8 for comparison). The ellipticity criterion for these runs was reached at similar times as the corresponding internal wave simulations (viz., $24.4T_i$ and $6T_i$ for the jet versus $23T_i$ and $77T_i$ for the internal wave). This indicates similar linear growth rates. For the weaker jet, a pair of dipoles was visible after $80T_i$, again at times similar to the corresponding internal wave simulation. Details such as the position of the dipoles following the instability process differed, probably owing to differences in the nonlinear amplification of the instability. For the stronger jet, however, no dipole splitting was observed—even for many of the growth times $T_{BC}$ predicted by (11). Instead, the S vortex was sheared apart. After only $3T_i$, the two cyclones were sheared away from the anticyclone. Once the anticyclone became elliptical, by $7T_i$, the resulting filaments were quickly transported away, resulting in the gradual decay of the anticyclone. Thus, in this stronger jet case, although the linear instability growth times were comparable, the resulting fate of an S vortex is rather different for the case of a very strong jet versus strong internal wave shear.

A limitation of the present study is the inviscid condition. A suite of simulations with approximately realistic values of viscosity ($2.5 \times 10^{-5} \text{ m}^2 \text{s}^{-1}$ after $f$ scaling to preserve the Ekman number) introduced a finite diffusion time scale. For weak wave cases, after about $20T_i$, both the vortex and wave had eroded. However, for strong wave cases, dipole splitting still scaled well with (11), albeit slightly later than in the inviscid simulations. This is consistent with the conclusions of Beckers et al. (2003), who suggest that lateral diffusion could decrease the horizontal vorticity profile of the vortex and, hence, make it more stable. In viscid conditions, the instability evolution was qualitatively more symmetric than in the inviscid simulations with similar wave shear. This could indicate a damping of the tilting instability. Meanwhile, in simulations where the wave was forced (to avoid its erosion), the vortex diffused without going visibly unstable when the associated instability growth time was longer than the viscous time scale (i.e., $\approx 100T_i$).

Finally, while submesoscale vortices were the primary motivation for the simulations presented here, scaling of our results enables us to relate our findings to mesoscale phenomena such as meddies (anticyclonic lenses of Mediterranean origin). These are commonly thought to be monopolar, but a discussion of the literature by Morel and McWilliams (1997) reveals that some meddies could actually be compound S vortices. Direct comparison of our wave-forced simulations is difficult, as the background internal wave field would require amplitudes larger than the typical vertical scale of a meddy ($\approx 400 \text{ m}$, Thorpe 2005) and thus larger than the largest known internal waves. Hence, we do not expect to find internal waves of the required scales in the ocean. However, there are other sources of vertical shear such
as major currents with depth scales of 100–1000 m, making the results presented here more generally applicable.

5. Summary and conclusions

The evolution of a submesoscale compound S vortex in the presence of a large-scale internal wave field was analyzed in the context of instability growth times of the vortex. Numerical simulations were initialized with a plane mode-1 internal wave, plus a mixed fluid patch, from which the S vortex originated. In a suite of simulations, internal wave amplitude, and the associated internal wave vertical shear, was varied. A weak internal wave shear did not affect the basic characteristics of the instability, suggesting that the vortex behaved independently of the background wave. In this case, the time for the instability to reach finite amplitude scaled inversely with the shear of the vortex (Fig. 8). Small deviations from this scaling were observed, possibly due to minor enhancement of instability growth rates by the wave. With increasing internal wave strength, dipole splitting occurred progressively earlier and the development of the instability became more asymmetric, both horizontally and vertically. Moreover, the vortex instability went from self-induced to externally excited such that the initial perturbation was being dominated by the large-scale background wave, reducing the time to dipole splitting with increasing background wave amplitude. In cases of strong wave shear, part of the energy for the dipole splitting was supplied by the wave, while a smaller part of the energy was from potential energy and vortex stretching. For strong internal waves, the time to dipole splitting scaled inversely with the vertical wave shear, per (11). When viewing the wave as setting the initial perturbation to the fastest growing instability mode of the vortex, the time to dipole splitting scales inversely with the log of the vertical wave shear, per (12).

Understanding the impact of an internal wave on a single S vortex is a first step in understanding the impact of a wave on a field of submesoscale eddies. Observing such interactions in the real ocean is difficult owing to the myriad processes acting on similar length and time scales. However, understanding the basic (potentially coexisting) processes can aid in the interpretation of field data. In the ocean, multiple mixed patches can be found in close proximity. With increasing proximity, vortices can interact and potentially cascade to larger scales (Sundermeyer and Lelong 2005). Understanding the impact of a large background wave on a single vortex will aid in understanding the potential for such a cascade. Here, we found that the presence of a large-scale internal wave can, even under viscous conditions, result in dipole splitting of a single S vortex. The resulting dipoles have a reduced (50%) horizontal extent, a slightly increased vertical extent, and contain more kinetic energy than the original vortex. This could indicate a transfer to smaller scales and, hence, an interruption of the previously observed (Sundermeyer and Lelong 2005) inverse energy cascade. Conversely, however, the propagation of dipoles away from their place of origin could also facilitate interactions between dipoles and other vortices in a field of vortical mode eddies, thus enhancing an inverse cascade. Whether these interactions are limited by background viscosity and diffusivity is an open question; our results suggest that the vortex instability itself does not depend strongly on viscosity and diffusion. Further research is currently being conducted by the authors to address some of these questions.

Acknowledgments. This work was supported by the National Science Foundation under Grants OCE 0351892 and OCE 0623193 and by the Office of Naval Research under Grant N00014-09-1-0194. We would also like to thank Joshua Jacobs at the University of Washington for his comments and his help with some of the methodology of the modal analysis. Four anonymous reviewers also provided valuable input and suggestions.

REFERENCES


