Coupled Sea Ice–Ocean-State Estimation in the Labrador Sea and Baffin Bay

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ABSTRACT
Sea ice variability in the Labrador Sea is of climatic interest because of its relationship to deep convection, mode-water formation, and the North Atlantic atmospheric circulation. Historically, quantifying the relationship between sea ice and ocean variability has been difficult because of in situ observation paucity and technical challenges associated with synthesizing observations with numerical models. Here the relationship between ice and ocean variability is explored by analyzing new estimates of the ocean–ice state in the northwest North Atlantic. The estimates are syntheses of in situ and satellite hydrographic and ice data with a regional \( \frac{1}{3} \) coupled ocean–sea ice model. The synthesis of sea ice data is achieved with an improved adjoint of a thermodynamic ice model. Model and data are made consistent, in a least squares sense, by iteratively adjusting control variables, including ocean initial and lateral boundary conditions and the atmospheric state, to minimize an uncertainty-weighted model–data misfit cost function. The utility of the state estimate is demonstrated in an analysis of energy and buoyancy budgets in the marginal ice zone (MIZ). In mid-March the system achieves a state of quasi-equilibrium during which net ice growth and melt approaches zero; newly formed ice diverges from coastal areas and converges via wind and ocean forcing in the MIZ. The convergence of ice mass in the MIZ is ablated primarily by turbulent ocean–ice enthalpy fluxes. The primary source of the enthalpy required for sustained MIZ ice ablation is the sensible heat reservoir of the subtropical-origin subsurface waters.

1. Introduction

At its wintertime peak, sea ice in the Labrador Sea and Baffin Bay constitutes approximately 20% of the total seasonal ice area in the Northern Hemisphere and 27% of its observed interannual variability (the area bounded by the maximum and minimum wintertime extents). Besides contributing significantly to Earth’s cryosphere variability on the whole, sea ice variability in the Labrador Sea has been hypothesized to have far-reaching impact on the climate through its influence on deep convection and atmospheric circulation patterns. Sea ice insulates the ocean, modifying air–sea buoyancy fluxes and therefore the propensity for deep convective overturning (Visbeck et al. 1995; Bitz et al. 2005; Griffies et al. 2009). In the Labrador Sea where surface buoyancy losses can be intense and the interior stratification weak, convective overturning has been observed to 2300 m (Lazier et al. 2002). Such deep convective mixing is required for the formation and distribution of Labrador Seawater, an important water mass in the North Atlantic Ocean (Pickart et al. 2002).

Ice cover variability in the Labrador Sea may alter large-scale meteorological patterns on seasonal and longer time scales. By modifying zonal surface temperature gradients along the North Atlantic storm track, ice extent anomalies modify the genesis, intensity, and trajectory of synoptic weather systems in the North Atlantic and Europe (Deser et al. 2004; Magnusdottir et al. 2004; Strong and Magnusdottir 2010). As sea ice anomalies often persist on time scales longer than synoptic events, understanding the factors contributing to their variability may improve short-term and seasonal weather forecasts, predictions of future high-latitude climate, and our understanding of past climate.

The interannual variability of wintertime ice extent occurs mainly in the northern Labrador Sea above the
continental slope between the 1000- and 3000-m isobaths, a region stretching between eastern Davis Strait and the Hamilton Bank on the Labrador Shelf. The boundary current system around the slope, hereafter the Northern Slope, consists of the northward-flowing West Greenland Current, its westward bifurcation, and the southward-flowing Labrador Current. The northward and southward components transport fresh, cold waters of Arctic and Nordic origins above the Greenland and Labrador Shelves, respectively, and warm salty waters of subtropical origin above the continental slope. Waters above the Northern Slope are hydrographically complex, owing to the progressive modification of these distinct water classes via eddy and convective mixing and air-sea heat and buoyancy fluxes (Prater 2002; Cuny et al. 2002). Observations also indicate significant hydrographic variability on annual and interannual time scales driven by factors both local (e.g., synoptic air–sea flux variability) and remote (e.g., the propagation of salinity and temperature anomalies around the subpolar gyre) (Yashayaev 2007; Myers et al. 2007, 2009).

This study explores the problem of how the complex hydrographic variability in the northern Labrador Sea modulates the maximum wintertime seasonal sea ice extent by analyzing three-dimensional time-varying reconstructions of the coupled sea ice–ocean state.

Basic questions about the relative importance of factors influencing ice expansion remaining unanswered include the following.

1) What are the relative roles of sea ice advection versus thermodynamic growth and melt?
2) Do sea ice–ocean exchanges contribute significantly to the total wintertime ocean surface buoyancy budget?
3) How much of the observed ice extent can be explained by purely one-dimensional thermodynamic processes and can foreknowledge of the upper-ocean hydrography in the Labrador Sea be used to predict the maximum sea ice extent?
4) What are the important sea ice–ocean feedbacks operating during the expansion of ice cover and how does hydrographic variability affect the operation of these feedbacks?

This study lays the foundation for answering these questions by creating and analyzing a one-year reconstruction of the coupled sea ice–ocean state in the Labrador Sea and Baffin Bay in 1996/97. The reconstruction (hereafter state estimate or StE) is a least squares fit of a coupled sea ice–ocean model to a set of hydrographic and sea ice observations and their uncertainties. The purpose of this paper is threefold: to describe the approach, to provide a summary description of the inferred StE, and to investigate questions 1) and 2) through an initial analysis of ice mass and ice–ocean buoyancy flux budgets from the state estimate. The analysis also demonstrates the power of a StE that is consistent with observations, compared to observation-only or model-only based studies. Questions 3) and 4) are investigated in a companion manuscript (Fenty and Heimbach 2013, hereafter FH13).

The structure of the paper is as follows. Section 2 reviews the annual progression of the coupled sea ice–ocean state. The state estimation methodology is presented in section 3 with a focus on the novel aspect of the work presented here: the coupled nature of the sea ice–ocean-state estimate. A brief overview of the estimated state is given in section 4. The ice mass, energy, and buoyancy budgets of the coupled system are examined in section 5. The results are summarized in section 6.

2. The sea ice annual cycle and its variability

The annual cycle of ice extent and a sense of its interannual variability are summarized in Fig. 1. The range of variability follows the median ice extent; variability increases between October and February and falls between April and August.

The annual cycle begins in mid-September with a small amount of residual ice surviving the summer melt in the straits connecting Baffin Bay with the central Arctic. Multiyear ice and upper ocean water from the Arctic are advected through these straits into the Baffin Bay. The Arctic-origin water, Arctic Water (AW) (wintertime $\theta \sim -1.8^\circ C, S \sim 34.5$), is the source of the well-stratified near-surface layer of 100–300 m found across Baffin Bay (Tang et al. 2004).
expansion of ice across Baffin Bay, Davis Strait, and the northern Labrador Shelf—areas with similar upper-ocean hydrographic properties, a shallow AW layer, and meteorological forcing, westerly and northwesterly winds advecting cold, dry continental air.

The distribution of AW across the Baffin Bay is associated with the Baffin Island Current (BIC), the primary advective pathway connecting the northern passages of Baffin Bay to Davis Strait (Cuny et al. 2005). At the Labrador Coast, the BIC becomes the surface component of Labrador Current (LC), which flows above the shelf and shelfbreak. To aid the discussion, Fig. 2 depicts a schematic of the basic structure of the upper-ocean circulation in the study domain.

The December ice edge coincides with the climatological position of the thermohaline front (THF) separating the cold fresh AW of Baffin Bay with the warmer and saltier subtropical-origin waters found in the northern Labrador Sea and southeastern Baffin Bay. A sense of the climatological location and hydrographic conditions associated with the THF is provided in Fig. 3. Using an ocean $T$ and $S$ climatology for December, the THF is identified with the 1026.6 kg m$^{-3}$ isopycnal outcropping, the 0.75°C isotherm, or the 33.25 isohaline at 15 m.

Between December and February, major ice cover developments occur in waters across the THF. These waters are mainly sourced by the West Greenland Current (WGC), a two-component extension of the subpolar gyre boundary current system. The first component, AW, is found above the shelf to the shelfbreak (Cuny et al. 2002). A warmer and saltier component, Irminger Water (IW), extends beyond the shelfbreak with temperature and salinity maxima ($\theta \approx 4.3^\circ$C, $S \approx 34.9$) between 200–500 m (Fratantoni and Pickart 2007; Pickart et al. 2002). The subtropical-origin IW is entrained into the boundary current system in the Irminger Sea (Bower et al. 2002).

The WGC flows poleward until the separation of the 3000-m isobath whereupon it bifurcates westward—the WGC extension. The circulation above the northern Labrador Sea over which the WGC extension passes, the Northern Slope (NS), is complex owing to an energetic mesoscale eddy field (Cuny et al. 2002). Eddies, many of which form through various instability processes along the boundary current, advect and mix AW and IW from the WGC across the NS toward the basin interior (Eden and Böning 2002; Prater 2002; Lilly et al. 2003; Katsman et al. 2004; Straneo 2006; Fratantoni and Pickart 2007). When IW on the WGC extension reaches the Labrador shelf, the LC gains a qualitative resemblance to the WGC. Quantitatively, IW on the LC is cooler and fresher due to its aforementioned modification during its NS transit.

Typically, the maximum ice extent occurs by late February or early March. In the northern Labrador Sea, the marginal ice zone (MIZ), the narrow boundary region separating consolidated pack ice and the open ocean, is found along or across the THF above the NS. To the west, wind and ocean stresses drive ice along the inner Labrador Shelf to Newfoundland (Prinsenberg and Peterson 1992). Around the time when the sea ice reaches its maximum extent, an approximate steady state is achieved with respect to MIZ location, ice concentrations, and ice thickness. In reality, synoptic wind and ocean forcing can easily drive the small (<10 km radius), diffuse ice floes by tens of kilometers a day thus making any steady-state MIZ necessarily approximate (McPhee et al. 1987; Morison et al. 1987; Greenan and...
Prinsenberg 1998). Behind the MIZ, ice concentrations approach 100% and thickness growth rates slow. We term the approximate steady state achieved by the ice around this time as the sea ice quasi equilibrium or the quasi-equilibrium state.

The quasi-equilibrium state includes a nonzero mean ice drift induced by surface currents and winds. A quasi-stationary MIZ location during a period of nonzero ice drift implies an approximate balance between advective ice mass convergence and thermodynamic ice mass divergence (i.e., melting) in the MIZ. A sustained ocean sensible heat flux convergence into the MIZ is expected to provide the required melt energy for such a balance (Yao and Ikeda 1990; Bitz et al. 2005). After approximately the vernal equinox, the melting ice retreats in roughly the reverse order of expansion.

In section 5 we begin to address the question of what sets and maintains the maximum sea ice extent position in the Labrador Sea through a detailed budget analysis of the coupled sea ice–ocean StE.
3. Coupled sea ice–ocean-state estimation

Three one-year coupled sea ice–ocean StEs for the Labrador Sea and Baffin Bay were constructed following the methodology developed by the consortium for Estimating the Circulation and Climate of the Ocean (ECCO). These StEs are dynamical reconstructions of the three-dimensional time-evolving ice–ocean system, which freely evolve from a set of initial and boundary conditions according to the physics and thermodynamics encoded in a general circulation model. Synthesizing observations with the numerical model requires optimizing a set of independent variables or “controls” (initial and boundary conditions in this study) such that the full trajectory of the model state, which includes the controls, is made consistent with a set of observations and their uncertainties in a least squares sense (Wunsch 2006; Wunsch and Heimbach 2007).

Formally, the weighted least squares model–data misfit cost function $J$ can be written,

$$
J = \sum_{t=1}^{t_f} \{y(t) - E(t)x(t)\}^T R(t)^{-1} \{y(t) - E(t)x(t)\} 
+ [x_0 - x(0)]^T P(0)^{-1} [x_0 - x(0)] + \sum_{t=0}^{t_f-1} u(t)^T Q(t)^{-1} u(t).
$$

(1)

Here, $y(t)$ is a vector of observations at time $t$, $x(t)$ is the model-state vector, and $E(t)$ is a matrix that maps the model-state space to the observation space. The model steps forward the state $x(t) = L[x(t - \Delta t)]$ via the operator $L$, with time step $\Delta t$. The error covariance matrices for the observations, initial conditions, and model control variables, are given by $R(t)$, $P(0)$, and $Q(t)$, respectively.

Typically, $R(t)^{-1}$, $P(t)^{-1}$, and $Q(t)^{-1}$ are interpreted as weights with each matrix term determining the relative contribution of model–data misfit and initial condition and control variable adjustment to the cost function. If no covariance structure is assumed, $R(t)$ reduces to a diagonal matrix filled with the individual error variance elements $\sigma(t)^2$,

$$
R(t) = \text{diag} (\sigma(t)^2_1, \sigma(t)^2_2, \ldots, \sigma(t)^2_n).
$$

(2)

We proceed by providing details of each ingredient required to generate the StEs.

a. Ocean–sea ice model and its adjoint

The ocean is simulated with a regional configuration of the 23 $z$-level Massachusetts Institute of Technology General Circulation Model (MITgcm) of Marshall et al. (1997a,b) with boundaries delineated in Fig. 2. The horizontal resolution ranges from 32.3 km $\times$ 33.8 km in the southern Labrador Sea to 32.8 km $\times$ 17.3 km in the northern Baffin Bay. Unresolved tracer stirring and mixing and the extraction of potential energy from density gradients is parameterized with the closures of Gent and McWilliams (1990) and Redi (1982) as implemented by Griffies (1998). The KPP nonlocal boundary layer mixing scheme of Large et al. (1994) parameterizes unresolved vertical mixing from buoyancy loss–driven free convection and forced convection from shear instabilities. At these resolutions, KPP reasonably parameterizes vertical mixing under partially ice-covered ocean conditions (Losch et al. 2006).

The sea ice model consists of separate thermodynamical and dynamical components that are coupled to the ocean model to solve prognostic equations for ice area fraction (concentration), mean snow and ice thickness, and ice velocity. The fundamental assumptions of the dynamical component are described in Hibler (1979) and Hibler (1980). The original ice dynamics implementation in the MITgcm closely followed Zhang and Hibler (1997), but has since evolved (Menemenlis et al. 2005; Losch et al. 2010). The thermodynamic ice model is a variation of the 0-layer formulation of Semtner (1976). Its main features and parameters are summarized in appendix A and Table 1.

The MITgcm is suited for adjoint-based state estimation because it has been adapted to processing by automatic differentiation tools throughout its development. The adjoint of the coupled sea ice–ocean model was generated with the Transformation of Algorithms in FORTRAN (TAF) tool of Giering et al. (2005). Following previous state estimation work using the MITgcm, the adjoint model approximates the nonlinear KPP and Gent–McWilliams subgrid-scale parameterizations and the sea ice dynamics submodel to avoid numerical instabilities (Heimbach et al. 2005; Gebbie et al. 2006; Mazlof et al. 2010).

An early version of the sea ice adjoint model was applied to study sensitivities of ice export through the Canadian Arctic Archipelago (Heimbach et al. 2010). The sea ice model has since been improved with the goal of generating robust gradients useful for optimization. These improvements include updated parameterizations of sea ice–ocean turbulent heat fluxes, latent sea ice–atmosphere heat fluxes, and ice areal expansion and thickening. The improved sea ice thermodynamics submodel is a novel technological contribution of this work. A full description is beyond the scope of this paper and is deferred to a forthcoming manuscript.
Table 1. Ocean and sea ice model parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ocean model parameters</strong></td>
<td></td>
</tr>
<tr>
<td>Horizontal resolution</td>
<td>32 × 32 km–32 × 14 km</td>
</tr>
<tr>
<td>Vertical resolution</td>
<td>500 m</td>
</tr>
<tr>
<td>Ocean/sea ice grid points</td>
<td>102 × 42 (horizontal) 23 (depth)</td>
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<tr>
<td>Time step</td>
<td>1 h</td>
</tr>
<tr>
<td>Ocean lateral open boundary forcing period</td>
<td>6 h</td>
</tr>
<tr>
<td>Horizontal harmonic viscosity</td>
<td>6.5 × 10⁻²–1.5 × 10⁴ m² s⁻¹</td>
</tr>
<tr>
<td>Horizontal biharmonic viscosity</td>
<td>2.8 × 10⁻¹–1.6 × 10⁻²3 m² s⁻¹</td>
</tr>
<tr>
<td>Vertical viscosity</td>
<td>50 m² s⁻¹</td>
</tr>
<tr>
<td>Vertical tracer diffusivity</td>
<td>1.0 × 10⁻³ m² s⁻¹</td>
</tr>
<tr>
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<td>1.0 × 10⁻⁵ m² s⁻¹</td>
</tr>
<tr>
<td>Thermal conductivity (snow, ice)</td>
<td>0.31 W m⁻³ K⁻¹, 2.17 W m⁻³ K⁻¹</td>
</tr>
<tr>
<td>Seawater Latent heat of fusion</td>
<td>3.34 × 10³ J kg⁻¹</td>
</tr>
<tr>
<td>Density (ice, snow)</td>
<td>910 k m⁻³, 330 kg m⁻³</td>
</tr>
<tr>
<td>Seawater freezing temperature</td>
<td>−1.96°C</td>
</tr>
<tr>
<td>Sea ice albedo (dry, wet)</td>
<td>0.75, 0.66</td>
</tr>
<tr>
<td>Snow albedo (dry, wet)</td>
<td>0.85, 0.70</td>
</tr>
<tr>
<td>Sea ice longwave emissivity</td>
<td>0.97</td>
</tr>
<tr>
<td>Sea ice shortwave extinction coefficient</td>
<td>5.0</td>
</tr>
<tr>
<td>Sea ice shortwave penetration coefficient</td>
<td>0.3, 0.0</td>
</tr>
<tr>
<td>Air–ice sensible heat transfer coefficient</td>
<td>2.28</td>
</tr>
<tr>
<td>Air–ice latent heat transfer coefficient</td>
<td>6.45</td>
</tr>
<tr>
<td>Sea ice bulk salinity</td>
<td>5.0</td>
</tr>
<tr>
<td>Drag coefficient (ice–ocean, ice–air)</td>
<td>8.5 × 10⁻³, 1.0 × 10⁻³</td>
</tr>
<tr>
<td>Hibler ice strength parameter (P*)</td>
<td>5.0 × 10⁰ N m⁻²</td>
</tr>
<tr>
<td>Hibler ice strength parameter (C)</td>
<td>20</td>
</tr>
<tr>
<td>Hibler yield curve eccentricity ratio (e)</td>
<td>2</td>
</tr>
</tbody>
</table>

**Sea ice model parameters**

- Thermal conductivity (snow, ice) = 0.31 W m⁻³ K⁻¹, 2.17 W m⁻³ K⁻¹
- Seawater Latent heat of fusion = 3.34 × 10³ J kg⁻¹
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- Hibler ice strength parameter (P*) = 5.0 × 10⁰ N m⁻²
- Hibler ice strength parameter (C) = 20
- Hibler yield curve eccentricity ratio (e) = 2

b. Surface and lateral boundary conditions

Air–sea fluxes are calculated using the ocean surface state and a prescribed atmospheric state from the National Centers for Environmental Prediction (NCEP)–National Center for Atmospheric Research (NCAR) reanalysis (Kalnay et al. 1996). The fluxes are computed with the bulk parameterization of Large and Yeager (2004) and a parameterization for the melting of falling snow of Sathiyamoorthy and Moore (2002). Atmospheric fluxes across each model grid cell are determined by adding the proportionate contributions of fluxes across their ice-free and ice-covered interfaces.

Dirichlet boundary conditions of temperature, salinity, and velocity are prescribed at each time step along the lateral ocean open boundaries following Ayoub (2006). The first-guess initial and lateral boundary conditions are derived using a form of the two-level nested multiscale method described by Gebbie et al. (2006), using the 1° × 1° global ECCO product version 2 iteration 199 (Wunsch 2006). The present configuration possesses northern, eastern, and southern open boundaries. Non-normal velocities on the open boundary are set as zero. Spurious circulations parallel to the open boundaries are damped by means of a viscous sponge layer applied across the first three grid cells adjacent to the boundary.

Sea ice inflow is assumed zero and ice advected out of the domain across an open boundary is instantly removed. We assume that neglecting ice inflow is reasonable because the annual inflow of ice into the domain (~200 km³ yr⁻¹) is likely quite small compared to estimates of total ice production (1300–2100 km³ yr⁻¹) (Aagaard and Carmack 1989; Kwok 2005; Prinsenberg and Hamilton 2005). Nevertheless, the importance of ice inflow in the domain is unknown and must be investigated in future work with high-resolution models that resolve ice transports around Greenland and through the narrow straits connecting Baffin Bay with the central Arctic.

c. Observations and associated uncertainties

In situ hydrographic observations are drawn from CTDs, autonomous profiling floats, and expendable bathythermographs (XBT). These data are compiled from the Hydrobase 2 of R. Curry (2001, unpublished manuscript) and the Global Temperature and Salinity Profile Program from the National Oceanographic Data Center Operational Oceanography Group (2006) and include AR7W World Ocean Circulation Experiment (WOCE)-line cruises and measurements from the 1996/97 Labrador Sea Experiment. The spatial and depth distributions of in situ data for each StE period is shown in Fig. 4. The 1996/97 period is particularly well suited for state estimation because of its unusual abundance of in situ measurements. A more detailed discussion about how the number of in situ observations affects the StEs is given in Fenty (2010).

The scientific evaluation of the StE requires knowledge of the prescribed observation and control uncertainties. These uncertainties are described in appendix B.

Sea ice concentration data are taken from the Comiso (2008) product, hereafter CM2008, provided by the National Snow and Ice Data Center. Satellite measurements are converted to ice concentrations using the Comiso and Nishio (2008) “bootstrap algorithm” to produce a daily-averaged product on a 25 × 25 km grid. The CM2008 product was chosen because of 1) its comparatively small errors in regions with seasonal sea ice (Comiso et al. 1997), 2) a simple uncertainty formulation based on a single instrument design (SSM/I passive microwave radiometers) and geophysical transfer algorithm, and 3)
extensive quality control that removes coastal and atmospheric contamination.

Sea ice concentration data uncertainties are given in appendix C.

d. Control variables and their uncertainties

The synthesis of model and data is accomplished via adjustments to the first-guess initial ocean state, the time-dependent open boundary conditions, and the atmospheric boundary conditions. Each control variable is assigned a corresponding a priori uncertainty. These uncertainties are used to penalize adjustments of the first-guess variables.

The solution is deemed acceptable when the residual model–data misfits and the control variable adjustments normalized by their prior uncertainties are consistent with a $\chi^2$ distribution with a value close to 1.

FIG. 4. Oceanographic in situ measurements for each one-year StE (August 1–July 31) by (a)–(c) location and (d)–(f) aggregate number as a function of depth. CTD casts (blue), XBTs (red), autonomous profiling floats (black).
Adjustments to the first-guess ocean $T$ and $S$ initial and lateral boundary are penalized by assuming that the uncertainties of the ECCO fields are consistent with the global estimate of $T$ and $S$ variance of Forget and Wunsch (2007, hereafter FW2007—see appendix B for details).

The prescription of atmospheric-state control variable uncertainties is one of the most important considerations in ocean–sea ice–state estimation, and is discussed in some detail in appendix D.

A demonstration of the consistency of the atmospheric control variable adjustments in the optimized solution is presented in Fig. 5.

The initial ice state is not included as a control variable because little ice remains in the domain following the summer melt. Initial sea ice conditions are prescribed using concentrations from CM2008 and thicknesses estimated from a separate model run.

The complete numerical model state at any given time, $\mathbf{x}(t)$, consists of $2.5\times10^5$ distinct elements representing the ocean and sea ice variables: ocean temperature and salinity, ocean and sea ice velocity, sea surface height, sea ice concentration, and ice and snow thickness. Each model run spans one year, 8684 1-h time steps, for a total of $2.2\times10^9$ elements in each StE. The model control vector, $\mathbf{u}(t)$, consists of $8.9\times10^8$ elements. Finally, the data vector, $\mathbf{y}(t)$, contains $2.2\times10^6$ elements, split approximately equally between ocean and sea ice data.

e. Suitability of observations to constrain the state estimate

To draw credible inferences about basin-scale sea ice–ocean interaction, the StE must synthesize a sufficient number of hydrographic observations that constrain the reproduction of sea ice in the model. However, hydrographic measurements near sea ice are some of the hardest to acquire (Morison et al. 1987; McPhee 2008). Moreover, proportionally more ship-based measurements are taken during ice-free seasons. Fortunately, the situation is improving with data from the new ice-tethered profilers (ITPs) becoming available, for example, Krishfield et al. (2008); Proshutinsky et al. (2009); Toole et al. (2010); Timmermans et al. (2011).

Many in situ hydrographic measurements in the Labrador Sea sample the mesoscale eddy field (Lilly and Rhines 2002; Lilly et al. 2003; Hátún et al. 2007). For the purposes of ocean-state estimation, these data are sparse and noisy. Nevertheless, several factors favor the adequacy of the available hydrographic data to sufficiently constrain the reproduction of sea ice in the model: the general cyclonic tendency of the basin, the long observed residence time of water mass properties, and their abundance.


The 1996/97 StE incorporates 6.3 and 2.2 times more in situ discrete $T$ and $S$ measurements than the 1992/93 and 2003/04 StEs, respectively. Most of the
quantitative analysis will therefore focus on the 1996/97 reconstruction as it is expected to provide the most robust inferences about the coupled sea ice–ocean system.

We review the reconstruction of sea ice cover, mean wintertime ice drift, wintertime ocean mixed layer depths, and ocean circulation. The iterative optimization of the model–data misfit cost function is successful in producing a StE of acceptably consistency with the observations in approximately 30 iterations. By way of example, Fig. 6 shows integral values of cost misfits as a function of the optimization iteration. Cost functions are grouped according to instruments and normalized to provide a mean cost value per entry. More detailed misfit diagnostics, including spatial patterns, are presented in Fenty (2010).

a. Sea ice cover

We begin with a depiction in Fig. 7 of the evolution of the ice cover in the StE, and a spatially integrated summary in Fig. 8. Total sea ice extent decreases from August (the first estimated month) until mid-September (the melting of residual ice from the previous annual cycle) leaving a small amount of multiyear ice in northern Baffin Bay. The expansion of ice cover proceeds rapidly until December (the initial Baffin Bay freeze-up) and then slowly until the end of January (the expansion of ice along the Labrador Shelf). A second period of expansion occurs between early February and mid-March (the expansion of ice across the THF in the NS). The sea ice quasi-equilibrium period is identified as the four week period, centered in mid-March, during
which the aggregate ice area is within 4% (0.05 \times \times 10^6 \text{ km}^2) of its maximum (1.31 \times \times 10^6 \text{ km}^2).

The StE reproduces the observed ice concentration patterns and domain-integrals. In particular, the timing of ice advance and retreat, and the position of the ice edge are both accurately represented. The time series of observed and estimated total ice area show good agreement, recreating the major transitions in the ice pack: initial Baffin Bay freeze-up, extension along the Labrador Coast and NS, and the near linear contraction associated with seasonal melt. The fidelity of the ice concentration evolution in the StE exceeds all known extant, dynamically consistent or otherwise, reproductions in the literature for this domain and time period.

The integrated ice area in the StE exhibits less high-frequency (daily) variability than found in the data. Most daily variability in the data occurs in the vicinity of the MIZ where the observations are assigned large uncertainties due to large measurement and model representation errors (see appendix C). When the specified data uncertainties are considered, the model is quantitatively consistent with the data despite the high-frequency model–data misfit.

b. January–March mean ice drift

The estimated mean January–March (JFM) 1997 ice velocity and mean sea level pressure from the NCEP–NCAR reanalysis is presented in Fig. 9.1 To clarify the basin-scale sense of drift, only velocity vectors where ice concentration exceeds 35% for at least 10 days are shown. This cutoff mainly filters out high velocity (>50 cm s\(^{-1}\)) short-lived ice in the MIZ.

The mean JFM ice drift indicates a close relationship between surface geostrophic winds and mean ice drift. Following isobars, a weak (\(\pm 5\) cm s\(^{-1}\)) cyclonic ice flow in the northern Baffin Bay joins the faster (7.5 cm s\(^{-1}\)) south–southeastward flowing pack. Passing through the Davis Strait, the ice accelerates with faster velocities near the center of the Strait (12.5 cm s\(^{-1}\)) than to either side (5 cm s\(^{-1}\)). Over the NS, the ice velocities have a nonzero component normal to the ice edge. There is a significant velocity gradient across the Labrador Shelf ranging from 5 cm s\(^{-1}\) inshore to greater than 50 cm s\(^{-1}\) in the MIZ of the LC.

The patterns and magnitudes of ice drift in the StE agree well with the available, albeit noncontemporaneous, in situ observations on the LC (Peterson and Prinsenberg 1989; Prinsenberg and Peterson 1992; Tang and Yao 1992) and estimates from satellite data north of Davis Strait (Kwok and Cunningham 2008).

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1 The mean surface pressure field provides a sense of the geostrophic wind stresses that drive the ice during this period.
c. Wintertime mixed layer depth

The representation of mixed layer depth (MLD) in the StE is important to describe and compare with observations because well-reconstructed MLDs provide an indication of skill in reproducing observed ocean stratification, heat content, and air–sea heat fluxes. Mixed layer depths are determined by identifying the depth at which surface-referenced potential density exceeds the minimum potential density below 45 m by 13 g m$^{-3}$. The MLD determination algorithm was found to be robust for profiles in the region.

Estimated MLDs (StE-MLDs) are first compared with MLDs from in situ $T$ and $S$ profiles at the same locations and times for a several week period in February and March, 1997, and shown in Fig. 10. Convection to 400 m is found in the StE-MLDs near the Hamilton Bank. Shallower StE-MLDs are found to the north (400–600 m) seaward of the MIZ and to the east (0–200 m). StE-MLDs near the eastern open boundary (57°N, 46°W) are very deep (≥800 m). StE-MLDs are shallow within 30–60 km of the MIZ (100–200 m) and deepen seaward (e.g., 60°N, 57°W).

Overall, StE-MLDs agree well with data, especially in the observation-abundant western Labrador Sea. Larger model–data misfits are found to the east and northeast of the deep convection site near Hamilton Bank. Immediately surrounding the site of deep convection, observed MLDs are 300–700 m while StE-MLDs are somewhat shallower, 100–400 m.

In the east (toward the open boundary near 46°W, 57°N) and northeast (on the WGC near 48°W, 60°N and downstream near 55°W, 61°N), StE-MLDs are deeper than observed. Too-deep MLDs seaward of the WGC and in northeast corner of the LS are very likely related to the model’s lack of energetic mesoscale eddies, eddies which advect fresh buoyant AW from the boundary currents to the LS interior. While the model does transport AW from the WGC to the interior, the model’s coarse resolution does not permit the advected AW to maintain its distinct properties in eddies as observed. With less well-stratified near-surface waters in the northeast LS, air–sea heat losses cause deeper mixed layers than observed. One likely consequence of the mixed layer bias in the StE is excessive subsurface IW ventilation to the upper ocean, warmer seawater temperatures near the MIZ, and subsequent excessive melting of sea ice. Indeed, excessive melting may be the cause of the negative bias in total ice area noted in Fig. 8c.

Too-shallow MLDs in the StE’s central LS were traced back to adjustments to the atmospheric control variables (not shown). To reduce a model misfit against SST data in which model SSTs were too cold, adjustments to the atmospheric control variables were made to reduce air–sea heat loss. With less surface cooling and associated buoyancy loss, convective mixed layers in the central basin were unable to deepen to their observed values. Despite these discrepancies, the MLD model–data misfits in the northern and western Labrador Sea near the ice edge are well reproduced, suggesting that the StE is useful for the examination of sea ice–ocean processes.
d. Ocean circulation

The circulation of the model domain (not shown) is driven by a combination of local wind and buoyancy forcing and from the inflows from the lateral open boundaries (Böning et al. 2006). As the first-guess lateral ocean open boundary conditions come from the coarser, $1^\circ \times 1^\circ$ ECCO global ocean-state estimate, the velocities and widths of the boundary currents entering the domain are generally weaker and broader than observed (e.g., Holliday et al. 2009) but consistent with models of similar resolution (Treguier et al. 2005). Interestingly, the adjusted open boundary velocities of the StE (not shown) are not significantly different from their first-guess values. Therefore, we may conclude that on the time scales considered here, the first-guess ECCO initial open boundary velocities and the adjusted atmospheric forcing drive an ocean circulation, which is sufficient to bring the StE into consistency with the hydrographic and sea ice data used to constrain the solution.

Despite the model–data consistency of the StE, we conducted a sensitivity experiment to evaluate how the StE responded to stronger open boundary inflows. We repeated the simulation of the StE using the final control variable adjustments with the exception of the open boundary velocities. The StE open boundary velocities were replaced with velocities extracted from the 18 km ECCO eddy-permitting global ocean-state estimate of Menemenlis et al. (2008). Using the more vigorous ECCO2 inflows, the maximum Labrador Sea barotropic transport doubles to 24 Sv ($1 \text{ Sv} = 10^6 \text{ m}^3 \text{ s}^{-1}$), with $\sim$23 Sv from the WGC and $\sim$1 Sv from the northern open boundaries, and the boundary currents become narrower and modestly faster ($+15\%–20\%$). Outside of the boundary currents, the upper ocean circulation (above 150 m) was little changed.

The annual cycle of sea ice expansion and contraction was found to be insensitive to the ECCO2 inflows with the exception of the location of the quasi-equilibrium MIZ. With the more vigorous inflow, the maximum wintertime MIZ does not extend quite as far south as the original-state estimate. Nevertheless, the impact on the MIZ location is small compared to the typical interannual MIZ location variability. We conclude that although more vigorous open boundary inflows do make for a more realistic barotropic circulation in the domain, the impact on the sea ice in the marginal ice zone over the time scales considered is comparatively minor.

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2 It must be noted that, although the ECCO2 model solution has more realistic velocities, its misfit with respect to in situ ocean and sea ice data is larger than our StE.
5. Sea ice and ocean property and flux budgets

The budgets of mass, energy, and buoyancy in the sea ice–ocean system provide insight to critical balances and imbalances. During sea ice quasi equilibrium, the system is hypothesized to be characterized by several balance conditions. Understanding these balances provides clues about the imbalances that drive the sea ice to its winntertime maximum state.

a. Mass budget

The tendencies of ice mass and concentration are functionally related in the model. They highlight the relative importance of thermodynamic and advective processes for maintaining sea ice cover in a given location. The ice mass budget also provides insight into the processes relevant for the development and maintenance of the quasi-equilibrium sea ice state.

As ice density is assumed constant in the model, one may interchangeably speak of the budgets of ice mass, mean thickness, and volume. Of these, it is convenient to choose mean ice thickness, \( h \). To distinguish between the thermodynamic and advective contributions to the ice thickness budget, the ice thickness tendency term is decomposed into the contributions from thermodynamical growth (and melt) and advective convergence. The decomposed tendency equation is

\[
\frac{\partial h}{\partial t} = \left( \frac{\partial h}{\partial t} \right)_{\text{thermo}} - \mathbf{v} \cdot (\mathbf{u}_{\text{ice}} h),
\]

where \( \mathbf{u}_{\text{ice}} \) is the ice velocity and the two terms on the rhs describe thermodynamical growth (or melt) and advective convergence, respectively. In Eq. (3), the advective term excludes subgrid-scale mechanical processes that rearrange ice between thickness categories while leaving the mean ice thickness unchanged.

The interpretation of thickness tendencies is greatly simplified by considering the periods when the ice edge is relatively stable. Here we consider the quasi-equilibrium period 21 February–20 March 1997.

The thickness tendencies during quasi equilibrium are shown in Fig. 11. Along the coasts the dominant patterns are of thermodynamic growth (and melt) and advective divergence. Such a pattern is expected as climatologically west- and northwesterly winds drive ice offshore, thinning the ice and exposing open water. Thinner ice and greater open water fractions in turn permit high thermodynamic

![Fig. 11. Sea ice thickness growth rate tendencies averaged during the quasi-equilibrium period 21 Feb–20 Mar 1997. Thickness tendencies are presented as (a) thermodynamic, (b) advective terms, and (c) their sum. Solid lines denote each the maximum sea ice extent during quasi-equilibrium.](image-url)
growth rates. In the central Baffin Bay, both advective and thermodynamical ice thickness tendencies approach zero. Despite nonzero ice drift in the central Baffin Bay, there is little advective contribution to the ice thickness tendencies because of the very small ice thickness gradients in the mean direction of drift. In the Labrador Sea MIZ, thermodynamic melt (maximum 6 cm day\(^{-1}\)) is almost entirely offset by advective convergence (same maximum).

In quasi equilibrium, regions of thermodynamic growth (melt) are coincident with regions of advective divergence (convergence) of comparable magnitudes. The locations of high thermodynamic melt, generally along the MIZ, are distant from the main locations of high thermodynamic growth. As a first important conclusion, the MIZ location in quasi-equilibrium is sustained through a process of distant ice production, lateral transport, and ultimate destruction via thermodynamic melt.

b. Sea ice enthalpy budget

The sea ice enthalpy budget examined here analyzes fluxes across a system defined by an uppermost ocean model grid cell and the sea ice found within it. For brevity, we only consider the budget in and around the MIZ during quasi equilibrium. The convergence of ice mass to the MIZ is associated with a negative enthalpy flux of 100–300 W m\(^{-2}\). Negative net air–sea fluxes are found with the same magnitude in the open water fraction of the MIZ. Behind the MIZ, where the open water fraction approaches zero, net air–sea fluxes become vanishingly small. Negative net air–sea ice fluxes are found in and around the MIZ, but are quite small, generally less than 10 W m\(^{-2}\).

The negative enthalpy fluxes in the MIZ are approximately balanced by positive radiative and advective ocean sensible heat fluxes. During quasi-equilibrium, downwelling short- and longwave radiation in the MIZ contribute approximately 50 W m\(^{-2}\) and 150 W m\(^{-2}\), respectively. Vertical and lateral advective ocean sensible heat fluxes together provide the remaining energy to the MIZ, up to 500 W m\(^{-2}\).

c. Sea ice buoyancy budget

Melting of ice in the MIZ modifies seawater buoyancy through the release of low-salinity ice meltwater (increasing buoyancy) and the reduction of sensible heat (decreasing buoyancy). During thermodynamic ice formation, seawater buoyancy is mainly reduced through the removal of low-salinity water. Because of the nonlinear seawater equation of state, the actual net buoyancy flux realized by the ocean following ice growth or melt is a function of the enthalpy and salinity of both the ocean and ice.

The role of ocean surface buoyancy fluxes associated with ice processes in setting the maximum wintertime sea ice extent is the focus of FH13. Here we quantify the contribution of sea ice processes to the ocean surface buoyancy flux budget during sea ice quasi equilibrium.

1) Buoyancy flux definitions

The time rate of change of seawater surface buoyancy \(B\) is written

\[
\frac{\partial B}{\partial t} = \frac{g}{\rho_0} \left( \frac{\partial \rho_{sw}}{\partial T} \right)_{T,S} \left( \frac{\partial Q_{\text{flux}}}{\partial T} \right)_{T,S} \left( \frac{\partial \rho_{sw}}{\partial S} \right)_{T,S} \rho_0 S_{\text{flux}},
\]

where \(\rho_{sw}\) is seawater density, \(\rho_0\) is a reference seawater density, \(T\) is seawater temperature, \(Q_{\text{flux}}\) is the net surface heat flux that results in a temperature change, \(S\) is salinity, \(c_{sw}\) is seawater heat capacity, and \(S_{\text{flux}}\) is the net surface salinity flux.

The meaning of \(Q_{\text{flux}}\) is subtle in the presence of sea ice. When seawater is at its freezing point, net surface heat flux divergence may trigger sea ice growth without an accompanying change in temperature, implying no ocean surface buoyancy flux from the first term on the lhs of Eq. (4). In our model, sea ice growth may be preceded by a reduction in seawater temperature. Hence, only the fraction of total \(Q_{\text{flux}}\) that changes seawater temperature is considered in Eq. (4).

It is useful to decompose the total surface buoyancy flux into contributions from sea ice and non-sea ice processes

\[
\left( \frac{\partial B}{\partial t} \right)_{\text{total}} = \left( \frac{\partial B}{\partial t} \right)_{\text{sea ice}} + \left( \frac{\partial B}{\partial t} \right)_{\text{non-sea ice}}.
\]

The \(Q_{\text{flux}}\) and \(S_{\text{flux}}\) terms in Eq. (4) for sea ice processes correspond to the ocean sensible heat loss associated with sea ice melt and the addition or removal of freshwater from the ocean during ice melt or growth, respectively.

2) Buoyancy fluxes in the quasi-equilibrium state

Ocean surface buoyancy fluxes during the sea ice quasi-equilibrium period are separated by type [temperature versus salinity changes, Eq. (4)] and source [sea ice–related processes versus all processes, Eq. (5)] and presented in Fig. 12.

Seawater buoyancy loss from the reduction of seawater temperature due to turbulent ocean-ice sensible heat fluxes (Fig. 12a) is mainly confined to the MIZ and NS. This is expected as the MIZ is a region where advective ocean enthalpy flux convergence permits sustained ice melt. Far behind the MIZ, temperatures beneath the ice.
FIG. 12. Ocean surface buoyancy fluxes from sea ice and from all ocean surface processes during the quasi-equilibrium state. Buoyancy fluxes from sea ice–related fluxes of (a) ocean enthalpy and (b) salt and their sum (c). Buoyancy fluxes from all surface fluxes of (d) enthalpy and (e) salt and (f) their sum.
approach the seawater freezing point which suppresses ocean-ice heat fluxes, driving $Q_{\text{flux}}$ to zero.

In contrast, surface buoyancy fluxes due to sea ice–related surface salinity fluxes (Fig. 12b) are found across the entire domain. Sea ice buoyancy fluxes are large and positive in the MIZ due to meltwater release. Large negative buoyancy fluxes are found adjacent to the shorelines, closely following the patterns of thermodynamic ice growth. Small negative buoyancy fluxes are also noted in the central Baffin Bay, indicating sustained, albeit slow, thermodynamic ice thickening and salt release. In quasi-equilibrium, sea ice salt-related fluxes dominate the sea ice buoyancy budget (Fig. 12c).

Total surface buoyancy fluxes due to changes in seawater temperature reveal the intense wintertime air–sea heat fluxes in the Labrador Sea (Fig. 12d). Compared to the effect of buoyancy loss from air–sea heat fluxes, the contribution from ocean–ice heat fluxes is small. Conversely, the total ocean surface salinity flux (Fig. 12e) is dominated by sea ice processes. Small negative ocean surface salt-related buoyancy fluxes are found seaward of the MIZ, possibly reflecting the entrainment and subsequent vertical redistribution of saltier subsurface waters in the mixed layer.

The sum of all enthalpy and salinity-related buoyancy fluxes (Fig. 12f) reveals a sharp contrast along the sea ice margin where intense positive buoyancy fluxes (sea ice meltwater release) are found adjacent negative buoyancy fluxes (open water air–sea heat loss) on the same order of magnitude. The MIZ is evidently a location where net ocean surface buoyancy fluxes are approximately balanced during a sea ice quasi-equilibrium state.

We note that the spatial patterns of sea ice thickness growth rate tendencies, the sea ice enthalpy budget, and ocean surface buoyancy fluxes found in the 1996/97 StE are qualitatively identical to those found in the 1992/93 and 2003/04 StEs (not shown). The main difference between each StE is not in the nature of the balance conditions achieved between the sea ice, ocean, and atmosphere during quasi equilibrium, but only in the ultimate ice edge location. Insight into the development of the quasi-equilibrium sea ice state given the significant differences in the sea ice edge location is the focus of FH13.

6. Summary and discussion

This work demonstrates the feasibility of the adjoint method in reconstructing the coupled sea ice–ocean state in a region of climatic importance. We synthesized in situ and satellite-based hydrographic and sea ice data with a modern coupled sea ice–ocean general circulation model. The approach taken was to create and analyze a dynamically consistent three-dimensional time-varying reconstruction of the ocean–sea ice system during the sea ice annual cycle of 1996/97. The statistics of the residual model versus data misfits, as well as the distribution of adjusted control variables indicate a formally acceptable solution within prior uncertainties.

The inferred sea ice mass balance during the quasi-equilibrium period of 1996/97 is between thermodynamic production and mass divergence along the shoreline, advective transport across Baffin Bay and Labrador Shelf, and ultimately thermodynamic melt and advective convergence in the MIZ of the Labrador Sea. Ocean–ice heat flux convergence is sufficient to melt all converging in the MIZ during quasi equilibrium. The dominant energy source that sustains the ocean–ice heat flux convergence is oceanic heat transport to the mixed layer via lateral and vertical transport. In contrast, the contributions from long- and shortwave radiative convergences are relatively minor.

Finally, the contribution of ocean surface buoyancy fluxes from ice-related processes was quantified and compared with total ocean surface buoyancy fluxes. Closure of property budgets by the state estimate is critical to enable accurate balance calculations. Positive buoyancy fluxes from ice melt are of the same order as negative buoyancy fluxes from air-sea heat fluxes near the MIZ which implies that ocean surface buoyancy fluxes must be balanced near the ice edge.

We focus on the 1996/97 reconstruction here because our solution is constrained by an unusual abundance of in situ hydrographic data available. Two additional reconstructions were created for years with fewer in situ data, corresponding to extreme ice extent anomalies: positive, 1992/93, and negative, 2003/04. In each reconstruction, we find qualitatively similar patterns of sea ice thickness growth rate tendencies and ocean surface buoyancy fluxes (Fenty 2010).

Specifically, we find that sea ice advective convergence is approximately balanced by thermodynamic divergence (melt) in the MIZ during the period of maximum wintertime extent. In each reconstruction, the ocean surface buoyancy fluxes within and just behind the MIZs are dominated by the positive contribution from sea ice meltwater release. In the ice-free open ocean beyond the MIZ, the basin’s characteristically intense air-sea heat fluxes generate negative ocean surface buoyancy fluxes of comparable magnitudes. Hence, in all cases the maximum sea ice edge coincides with a region of approximately zero net ocean surface buoyancy flux.

As one might expect, the development of the mixed layer on either side of the MIZ is profoundly different because of the opposite tendencies of ocean surface buoyancy forcing. In the open ocean, negative buoyancy forcing triggers convective deepening and the subsequent
entainment and ventilation of warm, salty Irminger Waters. In and behind the MIZ where sea ice melting is significant, the positive buoyancy forcing associated with meltwater release leads to a shoaling of the mixed layer and isolation of the subsurface Irminger Water via the development of a freshwater “cap,” which greatly increases the upper ocean stratification.

The main differences between the reconstructions are related to the spatial distributions of the patterns described above and not to the patterns themselves. For example, the wintertime maximum MIZ location for the 2003/04 state estimate is very close to the climatological Thermohaline Front as identified in Figs. 3c,d. Consequently, there is virtually no ice melt from ice–ocean heat fluxes behind the MIZ because of the initially high stratification and cold temperatures of the Arctic Waters found there. In contrast, the quasi-equilibrium MIZ location in the 1992/93 state estimate is found far south of the Thermohaline Front, in waters with initially weaker stratifications and higher temperatures. Despite having a MIZ far to the south, that reproduction shows the same basic pattern of sea ice advective convergence balanced by thermodynamic melt during quasi-equilibrium. As both the severity of wintertime atmospheric conditions and the initial hydrographic stratifications differ in each year, large differences are also noted in the maximum mixed layer depths in each reconstruction. However, the general pattern of shoaled mixed layers inshore of the MIZ and deeper mixed layers entraining Irminger Waters offshore of the MIZ are found in each case. Hence, we believe that the general patterns and quantitative magnitudes of the sea ice mass and ocean surface buoyancy budgets reported here are robust.

Our findings support some of the existing hypotheses on the role played by sea ice advection in the Labrador Sea, mainly that advection is essential for establishing the ice mass balance during quasi-equilibrium. A deeper understanding of the role of ocean-sea ice buoyancy feedbacks requires further investigation into the time evolution of the sea ice quasi-equilibrium state, the focus of FH13.

Modeling the Labrador Sea presents serious technical challenges. It is difficult to predict how our StE solutions would change if our model was capable of explicitly resolving important unresolved processes and circulation features, such as meso- and submesoscale eddies, sea ice brine plumes, narrow energetic boundary currents, and boundary current recirculations. Given these model shortcomings, care should be taken in applying the StE for certain types of analyses, such as mixed layer restratification processes or exchanges between the boundary current and basin interior. Undoubtedly, synthesizing these observations with more sophisticated models will alter details of future StEs. Nevertheless, to the extent that the prior uncertainty estimates used here are scientifically credible, any alternative solution inferred from the same available data and targeting the same space and time scales will likely prove indistinguishable from the one constructed here, which successfully interpolates the available data.

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APPENDIX A

Sea Ice Thermodynamic Model

The thermodynamic sea ice model is based on the zero-layer approximation of Semtner (1976) with several extensions summarized in the following. Vertical conductive heat flux through the ice is treated as a simple one-dimensional heat diffusion problem for a two-layer system (ice plus snow), each with different conductivities following Parkinson and Washington (1979). Thermal diffusion is assumed to establish a steady state with respect to conductive heat fluxes on time scales shorter than the model time step—a reasonable assumption (Feltham et al. 2006).

Snow and ice albedos are functions of surface type (ice or snow) and surface conditions (melting or frozen). To capture the increased surface albedo associated with standing water in melt ponds, melting ice is assigned the lowest albedo. Turbulent air–ice fluxes are calculated using standard bulk aerodynamic formulæ without corrections for variations in atmospheric boundary layer stability or air density. Latent heat fluxes between the atmosphere and ice surface use the empirical saturation vapor pressure parameterization of Marti and Mauersberger (1993). Shortwave radiation is assumed to penetrate only bare, snow-free ice and attenuates as it is absorbed. The conversion of snow to ice via flooding is modeled following Winton (2000).

The parameterization of the subgrid-scale turbulent ocean–ice sensible heat flux across the ice–ocean
Table 1.

interface $F_{oi}$ is given by an empirical parameterization given by McPhee (1992),

$$F_{oi} = c_p \rho_{sw} \Delta T \Delta T = c_p \rho_{sw} \Delta T \Delta T,$$

where $c_p$ and $\rho_{sw}$ are seawater heat capacity and density, respectively, $T$ and $T_f$ are the far-field ocean temperature and seawater freezing points, respectively, $u_w$ is the friction velocity beneath ice, and $St$ is the observationally inferred Stanton number. At each model time step, a fraction of the available seawater enthalpy, $\xi = \max[c_p \rho_{sw} \Delta T \Delta T, 0]$ is used to melt ice. New ice formation in open water is calculated as the residual between the potential ice production by air-sea heat fluxes, $F_{oa}$, and the potential ice melt from turbulent ocean-ice fluxes:

$$\left(\frac{\partial h}{\partial t}\right)_{oa} = (\rho_i L_i)^{-1} (F_{oa} + F_{oi}),$$

where $\rho_i$ and $L_i$ and sea ice density and seawater latent heat of fusion, respectively. Thus, net open water ice production may occur when $T \geq T_f$. The parameterizations of ice area expansion and contraction associated with thermodynamic growth and melt follow Hibler (1979).

For completeness, the parameter choices of the coupled ocean–sea ice model configuration are listed in Table 1.

APPENDIX B

Hydrographic Uncertainties

Uncertainties in the in situ hydrographic data are assumed to be dominated by model representation error as the model resolution excludes the reproduction of mesoscale eddies and observed boundary current fluctuations. The ocean model representation error estimate is derived from the three-dimensional global estimate of $T$ and $S$ variability of FW2007. To the extent that $T$ and $S$ variance from features that cannot be resolved by the model is the limiting factor for $T$ and $S$ model–data misfit, FW2007 can be used to provide a reasonable estimate of the model representation error. With FW2007, we assume that our model achieves consistency with the hydrographic data when the statistics of the model–data misfit are consistent with the region’s observed $T$ and $S$ variance. One limitation of using the FW2007 approach is that where few in situ are available, such as beneath the seasonal ice zone and in the boundary currents, the observed $T$ and $S$ variance may be underestimated. Consequently, the $T$ and $S$ variance estimates of FW2007 are probably underestimated in the domain.

Deviations of the StE from sea surface temperature (SST) observations and a $T$ and $S$ climatology are penalized in the cost function. SST data in ice-free open water are taken from the daily 0.25° dataset of Reynolds et al. (2007). The $T$ and $S$ climatology is a combination of the 1° World Ocean Atlas 2001 of Stephens et al. (2001) and Boyer et al. (2001) and the 0.5° Gouretski and Koltermann (2004) climatologies following Wunsch and Heimbach (2007). Representation error is again assumed to be the main factor limiting the achievable consistency between the model and both the SST and climatology data. Thus, we use FW2007 to estimate their uncertainty.

APPENDIX C

Sea Ice Concentration Uncertainties

Uncertainties for the sea ice concentration data must account for both measurement and model representation errors. Instrument and geophysical transfer algorithm errors have been estimated to introduce ice concentration data uncertainties of between 5% and 15% (Comiso 2008). Sea ice model representation errors are expected because of model limitations, mainly the absence of landfast ice and mesoscale eddies. Without landfast ice, tensile stresses cause ice to separate from the coast rather than at the landfast ice edge, an edge which can be tens of kilometers offshore (Tremblay and Hakakian 2006). The model’s spurious divergence of ice from the coast causes nearshore ice concentrations to be lower than observed. The transport of ice by mesoscale eddies acts to diffuse ice into the open ocean thereby broadening the MIZ (Zhang et al. 1999; Ikeda 1991). Eddies behind the MIZ cause mechanical ice convergence which thickens ice while reducing its areal fraction (Holland 2001). The model’s lack of eddies is therefore expected to give a narrower MIZ and higher sea ice concentrations behind the MIZ than observed.

Taking these errors into account, ice concentration uncertainty, $\sigma_{ic}(x,y,t)$, is formulated as location-dependent baseline, $\lambda(x,y)$, modified by a factor, $\alpha(y)$, a function of the observed ice concentration, $\gamma_{ic}(x,y,t)$, as

$$\sigma_{ic}(x,y,t) = \lambda(x,y)\alpha[\gamma_{ic}(x,y,t)].$$

The baseline location-dependent uncertainties are a combination of estimated ice concentration measurement errors and the assumed higher model representation errors near coasts.
\[ \lambda(x, y) = \begin{cases} 15\% & \text{if } \leq 50 \text{ km from coastline}, \\ 10\% & \text{if } > 50 \text{ km from coastline}. \end{cases} \]

The expectation of higher errors in the low-concentration MIZ and lower errors where no ice is reported is incorporated into the specification of \( \alpha \)

\[ \alpha(\gamma_{ic}) = \begin{cases} 0.85 & \text{if } \gamma_{ic} = 0, \\ 1.20 & \text{if } 0 < \gamma_{ic} < 0.15, \\ 1.10 & \text{if } 0.15 \leq \gamma_{ic} \leq 0.25, \\ 1.00 & \text{if } 0.25 < \gamma_{ic}. \end{cases} \]  

(6)

With \( \lambda \) and \( \alpha \) thus defined, the spatially and temporally varying sea ice uncertainties range from 8.5\% to 18\%.

**APPENDIX D**

**Atmospheric Control Uncertainties**

The evolution of seasonal sea ice in coupled sea–ocean models is sensitive to the choice of atmospheric forcing fields (Curry et al. 2002). The specification of a credible atmospheric state is therefore an important component of coupled sea ice–ocean-state estimation. In ocean-state estimation, the temporally and spatially varying atmospheric state is formally part of the solution. Consistency of the state estimate requires an estimated atmospheric state that is consistent with its prior specified uncertainties.

Adjustment of the atmospheric control variables cause the atmospheric state to deviate from its first-guess values. Acceptability of these deviations is specified with the uncertainties of the first-guess atmospheric state. The more uncertain a first-guess atmospheric state, the greater the allowable adjustments to that state. Hence, the prior uncertainties of the atmospheric control variables limit the magnitude of their adjustment in the optimization.

In this work, atmospheric control variable adjustments are penalized in the cost function using reanalysis error estimates. Even though a formal error analysis is not provided with the product, estimates of reanalysis uncertainties are available. The main sources of reanalysis error relevant here are the paucity of high latitude meteorological sources and reanalysis model representation errors (e.g., Renfrew et al. 2002; Moore and Renfrew 2002).

The atmospheric-state control variable contribution to the cost function, \( \mathbf{u}_a(t) \), is decomposed into two terms representing a time-averaged component, \( \bar{\mathbf{u}}_a \), and a time-varying residual, \( \mathbf{u}'_a(t) \)

\[ J_{\text{atmospheric controls}} = \sum_{t=0}^{t_f-1} \mathbf{u}'_a(t)^T \mathbf{Q}'_a^{-1} \mathbf{u}'_a(t) + \bar{\mathbf{u}}_a^T \bar{\mathbf{Q}}_a^{-1} \bar{\mathbf{u}}_a. \]

Thus, \( \bar{\mathbf{u}}_a \) corrects reanalysis biases while \( \mathbf{u}'_a(t) \) corrects its time-varying errors.

Reanalysis error estimates inform our specification of the uncertainties assigned to our atmospheric-state control variables.

The specification of \( \mathbf{Q}'_a \) and \( \bar{\mathbf{Q}}_a \) assumes time-invariant error covariances without spatial structure—an assumption which is almost certainly wrong. Recalling Eq. (2), \( \mathbf{Q}'_u \) and \( \bar{\mathbf{Q}}_u \) can be written

\[ \mathbf{Q}'_u = \text{diag}(\sigma^2_{u_t}, \sigma^2_{u_q}, \sigma^2_{u_{lw}}, \sigma^2_{u_{sw}}, \sigma^2_{u_p}), \quad \text{and} \]

\[ \bar{\mathbf{Q}}_u = \text{diag}(\sigma^2_{\bar{u}_t}, \sigma^2_{\bar{u}_q}, \sigma^2_{\bar{u}_{lw}}, \sigma^2_{\bar{u}_{sw}}, \sigma^2_{\bar{u}_p}). \]

With subscripts identifying the individual variables on the atmospheric control vector: \( u \), wind speed (10 m); \( t \), surface air temperature (2 m); \( q \), specific humidity (2 m); \( lw \) and \( sw \), long- and shortwave downwelling radiation (surface), respectively; \( p \), precipitation rate (surface). The uncertainties used in this work are presented in Table D1. A more detailed discussion of their sources can be found in (Fenty 2010, chapter 2, p. 88ff)

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Comiso, J., cited 2008: *Bootstrap Sea Ice Concentrations from Nimbus-7 SMMR and DMSP SSM/I-SSMIS* [01 August 1992

**TABLE D1. Atmospheric-state control variable adjustment uncertainties.**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>( \sigma'_u )</th>
<th>( \sigma''_u )</th>
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<td>15</td>
</tr>
<tr>
<td>Precipitation</td>
<td>mm day(^{-1})</td>
<td>1.5</td>
<td>1.5</td>
</tr>
</tbody>
</table>
MAY 2013

FENNY AND HEIMBACH

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