Langmuir–Submesoscale Interactions: Descriptive Analysis of Multiscale Frontal Spindown Simulations

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ABSTRACT

The interactions between boundary layer turbulence, including Langmuir turbulence, and submesoscale processes in the oceanic mixed layer are described using large-eddy simulations of the spindown of a temperature front in the presence of submesoscale eddies, winds, and waves. The simulations solve the surface-wave-averaged Boussinesq equations with Stokes drift wave forcing at a resolution that is sufficiently fine to capture small-scale Langmuir turbulence. A simulation without Stokes drift forcing is also performed for comparison. Spatial and spectral properties of temperature, velocity, and vorticity fields are described, and these fields are scale decomposed in order to examine multiscale fluxes of momentum and buoyancy. Buoyancy flux results indicate that Langmuir turbulence counters the restratifying effects of submesoscale eddies, leading to small-scale vertical transport and mixing that is 4 times greater than in the simulations without Stokes drift forcing. The observed fluxes are also shown to be in good agreement with results from an asymptotic analysis of the surface-wave-averaged, or Craik–Leibovich, equations. Regions of potential instability in the flow are identified using Richardson and Rossby numbers, and it is found that mixed gravitational/symmetric instabilities are nearly twice as prevalent when Langmuir turbulence is present, in contrast to simulations without Stokes drift forcing, which are dominated by symmetric instabilities. Mixed layer depth calculations based on potential vorticity and temperature show that the mixed layer is up to 2 times deeper in the presence of Langmuir turbulence. Differences between measures of the mixed layer depth based on potential vorticity and temperature are smaller in the simulations with Stokes drift forcing, indicating a reduced incidence of symmetric instabilities in the presence of Langmuir turbulence.

1. Introduction

Recent studies have indicated that submesoscale turbulent processes can have a significant impact on the dynamics and structure of the oceanic mixed layer...
These processes affect the transfer of momentum and chemical species at the air–sea interface, thereby impacting the global carbon cycle and climate. The present study examines a broad range of scales below the mesoscale cutoff (~10 km), with an emphasis on the interactions between quasigeostrophic, two-dimensional, hydrostatic features in the submesoscale range (roughly 1–10 km) and three-dimensional, non-hydrostatic turbulence at small scales (1 km and below).

Over the last several decades, numerous studies of submesoscale fronts and instabilities have been performed. Recent research has uncovered the important effects of large vertical velocities and restratification associated with submesoscale eddies on upper-ocean structure and dynamics (Spall 1997; Haine and Marshall 1998; Ferrari and Rudnick 2000; Thomas 2005; Mahadevan and Tandon 2006; Capet et al. 2008b; Klein and Lapeyre 2009). These eddies are typically 1–10 km in horizontal dimension and span the mixed layer depth, which is $O(100 \text{ m})$. They may penetrate into the thermocline below and are typified by their large Rossby numbers, which gives them substantially larger vertical velocities than is typical of larger mesoscale eddies. The potential energy of fronts is stored in horizontal density gradients, from which energy may be extracted through slumping by eddies into vertical gradients. As a result of this slumping, a positive submesoscale vertical temperature flux is created since warmer water is transported toward the surface and cooler water is transported to greater depths, thereby reinforcing vertical stratification (Boccaletti et al. 2007; Fox-Kemper et al. 2008).

It has long been known that turbulent processes at even smaller scales are also important in the mixing and dynamics of the upper ocean. These processes are three-dimensional and nonhydrostatic (see the review by Sullivan and McWilliams 2010) and include wave-driven Langmuir turbulence, which consists of disordered collections of counterrotating Langmuir cells. These cells create convergence zones at the ocean surface where foam, plankton, and other debris collect in long “windrows” (Langmuir 1938). The length of the cells may extend up to 1 km, while their width is much smaller (10–100 m). Langmuir cells are thought to originate from the tilting of vertical vorticity anomalies into the horizontal by the Stokes drift shear (Thorpe 2004); this instability mechanism was first described by Craik and Leibovich (1976) and Craik (1977). A number of observational (Weller and Price 1988; Smith 1992; D’Asaro and Dairiki 1997) and numerical (Skyllingstad and Denbo 1995; McWilliams et al. 1997; Noh et al. 2004; Sullivan et al. 2004; Li et al. 2005; Polton and Belcher 2007; Sullivan et al. 2007; Harcourt and D’Asaro 2008; Grant and Belcher 2009; Van Roekel et al. 2012a) studies of Langmuir turbulence provide insight and motivation for the present work.

Although submesoscale processes and Langmuir turbulence have each received considerable attention individually, the interactions between these phenomena have yet to be examined in detail. This is due, in large part, to the enormous range of spatial and temporal scales that must be resolved in order to simultaneously capture both Langmuir turbulence and submesoscale eddies. To meet this challenge, Malecha et al. (2013) have proposed an asymptotically motivated multiscale numerical algorithm to simulate Langmuir cell dynamics at ocean submesoscales. In the present study, we instead use large-eddy simulation (LES)—where an attempt is made to resolve all dynamically active scales—in order to address the following important questions:

(i) To what extent does vertical mixing by Langmuir turbulence affect restratification by submesoscale eddies, and what are the resulting changes to the mixed layer depth and mixed layer instabilities?

(ii) Do submesoscale processes affect the intensity, orientation, and other properties of smaller-scale Langmuir cells?

(iii) How do small-scale processes influence submesoscale ocean features?

(iv) What are the relative contributions of Langmuir turbulence and submesoscale eddies in transporting momentum and tracers across the oceanic mixed layer?

The specific physical setting for our investigation is the evolution (or spindown) of an initially uniform large-scale temperature front. The simulations are performed by numerically solving the wave-averaged Boussinesq (WAB) equations, which are commonly used in studies of Langmuir turbulence and account for the effects of surface waves through additional Stokes drift forcing terms (Craik and Leibovich 1976; Gjaja and Holm 1996; Holm 1996; McWilliams et al. 2004). The simulations span scales ranging from 20 km to 5 m in horizontal directions and from 1 to 160 m in the vertical direction. The immense scale range encompassed by these simulations allows submesoscale features and Langmuir turbulence to be simultaneously captured, thereby permitting an examination of the interactions between these two widely separated flow regimes. To specifically isolate the effects of Langmuir turbulence on submesoscale features, we also perform simulations without Stokes drift forcing for comparison. It is important to note that the setup of these simulations does not require a priori that there be strong interactions between Langmuir turbulence and submesoscale eddies. It is easy...
to imagine scenarios such as frontogenesis where strong multiscale interactions across this scale range are mandatory in order to ensure energy dissipation and to promote a cascade of energy. Thus, the interactions examined in the present study correspond to the “weak” interaction limit.

A primary focus of this study is the characterization of instabilities associated with Langmuir turbulence and submesoscale features. Symmetric instabilities, in particular, have been recognized as an important potential linkage between the submesoscale and turbulent nonhydrostatic scales (Thomas and Lee 2005; Taylor and Ferrari 2010; Thomas and Taylor 2010; Thomas et al. 2013). Similar to Craik–Leibovich instabilities that lead to Langmuir turbulence, symmetric instabilities derive their energy from shear; in the symmetric case, the relevant shear is the geostrophic shear of fronts associated with horizontal buoyancy gradients and the thermal wind. In the simulations performed here, both Stokes drift shear and geostrophic shear are present, and it will be shown that their relative importance is a key factor in determining the types of interaction between boundary layer mixing and submesoscale fronts. Other linkages between these scales have been studied in more idealized settings (Malecha et al. 2013) and will be explored using LES in future papers.

In the present study, a descriptive analysis of numerically generated spatial fields, statistics, spectra, multiscale fluxes, instability maps, and mixed layer depths is presented. Scale decompositions are carried out using two-dimensional filtering at each depth, and transport due to submesoscale eddies and boundary layer (e.g., Langmuir) turbulence are separated using multiscale fluxes of momentum and temperature; the latter quantity is linearly related in the simulations to the density and buoyancy. These multiscale fluxes are compared to results from an asymptotic analysis of the WAB equations. Maps of instabilities are calculated based on the potential vorticity and Richardson and Rossby numbers. The identification criteria for each instability have been outlined by Thomas et al. (2013) and allow us to identify regions that tend to be dominated by symmetric, inertial, or gravitational instabilities. We also measure the mixed layer depth using definitions based on temperature and potential vorticity. Concurrent analyses of Stokes and Stokes-free simulations allow us to isolate the effects of Langmuir turbulence on vertical transport, flow instabilities, and mixed layer depth.

2. Numerical simulations

The numerical simulations performed in the present study solve the WAB equations given by (Craik and Leibovich 1976; Gjaja and Holm 1996; Holm 1996; McWilliams et al. 2004)

$$\frac{du}{dt} + (\omega + f) \times u_L = -V \left( p \frac{1}{2} |u_L|^2 \right) + b \frac{d^2}{dz}, \quad (1)$$

$$\frac{db}{dt} + u_L \cdot V b = 0, \quad \text{and}$$

$$V \cdot u = 0, \quad (3)$$

where $u$ is the Eulerian velocity averaged over surface gravity waves, $\omega = V \times u$ is the Eulerian vorticity, $f$ is the Coriolis parameter, $p$ is the pressure, $b$ is the buoyancy, $u_L = u + u_s$ is the Lagrangian velocity, and $u_s$ is the Stokes drift velocity induced by surface gravity waves. These equations are a wave-filtered version of the Boussinesq Euler equations and, as is typical for Boussinesq systems, the buoyancy is proportional to the negative density as $b = -g \rho / \rho_0$, where $g$ is the gravitational acceleration, $\rho$ is the density, and $\rho_0$ is a reference density. The density is given by the linear equation of state

$$\rho = \rho_0 [1 + \beta_T (\theta_0 - \theta)], \quad (4)$$

where $\theta$ is the temperature, $\beta_T$ is the (presumed constant) thermal expansion coefficient, and salinity effects have been neglected. Parameter values used to obtain $p$ and $b$ are $g = 9.81 \text{ m s}^{-2}$, $\theta_0 = 290.16 \text{ K}$, $\rho_0 = 1000 \text{ kg m}^{-3}$, and $\beta_T = 2 \times 10^{-4} \text{ K}^{-1}$. The Coriolis parameter, $f = 0.7 \times 10^{-4} \text{ s}^{-1} \text{ z}$, is strictly in the vertical direction for simplicity. These and other parameters used in the simulations are summarized in Table 1.

The WAB equations in (1)–(3) differ from the standard Boussinesq Euler equations by the presence of the Lagrangian velocity $u_L$, in the advective, Coriolis, and modified pressure terms. The time-independent Stokes drift velocity required to obtain $u_L$ is specified in the simulations by

$$u_s(z) = u_s(z) [\cos(\theta_0) x + \sin(\theta_0) y], \quad (5)$$

where $u_s(z)$ is a vertical depth profile that is constant at all horizontal ($x$–$y$) locations and $\theta_0$ is the angle made by the Stokes drift velocity with respect to the $x$ axis. In the present simulations, $u_s(z)$ is taken from the empirical Donelan spectrum (Donelan et al. 1985; Webb and Fox-Kemper 2011) and decays superexponentially from the surface. The strength and depth dependence of the Stokes drift are based on fully developed waves appropriate to the wind [the wave age is 1.2 and the turbulent Langmuir number is near 0.3; see Alves et al. (2003), Hanley et al. (2010), Van Roekel et al. (2012a), and Table 1], as specified by the surface wind stress.
Table 1. Parameters used in the simulations with (Stokes) and without (No Stokes) Stokes drift forcing. The simulations with Stokes drift forcing are characterized by the prevalence of both Langmuir and shear turbulence at small scales, while the simulations without Stokes drift forcing are characterized only by shear turbulence. In both simulations, the surface temperature cooling is initially $-5 \text{ W m}^{-2}$ but is turned off after approximately 10 days.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Stokes (Stokes)</th>
<th>No Stokes (No Stokes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of turbulence</td>
<td>Langmuir and Shear</td>
<td>Shear</td>
</tr>
<tr>
<td>Physical domain size, $L_x \times L_y \times L_z$</td>
<td>$20 \text{ km} \times 20 \text{ km} \times 160 \text{ m}$</td>
<td>$20 \text{ km} \times 20 \text{ km} \times 160 \text{ m}$</td>
</tr>
<tr>
<td>Computational grid size, $N_x \times N_y \times N_z$</td>
<td>$4096 \times 4096 \times 128$</td>
<td>$4125 \times 4125 \times 128$</td>
</tr>
<tr>
<td>Grid resolution, $\Delta_x \times \Delta_y \times \Delta_z$</td>
<td>$4.9 \text{ m} \times 4.9 \text{ m} \times 1.25 \text{ m}$</td>
<td>$4.8 \text{ m} \times 4.8 \text{ m} \times 1.25 \text{ m}$</td>
</tr>
<tr>
<td>Reference temperature, $\theta_0$</td>
<td>$290.16 \text{ K}$</td>
<td>$290.16 \text{ K}$</td>
</tr>
<tr>
<td>Reference density, $\rho_0$</td>
<td>$1000 \text{ kg m}^{-3}$</td>
<td>$1000 \text{ kg m}^{-3}$</td>
</tr>
<tr>
<td>Coriolis parameter, $f$</td>
<td>$0.729 \times 10^{-4} \text{ s}^{-1} \hat{z}$</td>
<td>$0.729 \times 10^{-4} \text{ s}^{-1} \hat{z}$</td>
</tr>
<tr>
<td>Initial mixed layer depth, $H_{ML,0}$</td>
<td>$50 \text{ m}$</td>
<td>$50 \text{ m}$</td>
</tr>
<tr>
<td>Friction velocity, $u^*$</td>
<td>$5.46 \times 10^{-3} \text{ m s}^{-1}$</td>
<td>$5.46 \times 10^{-3} \text{ m s}^{-1}$</td>
</tr>
<tr>
<td>Surface wind stress magnitude, $\tau$</td>
<td>$0.025 \text{ N m}^{-2}$</td>
<td>$0.025 \text{ N m}^{-2}$</td>
</tr>
<tr>
<td>Surface wind angle, $\theta_w$</td>
<td>$30^\circ$</td>
<td>$30^\circ$</td>
</tr>
<tr>
<td>Initial surface temperature cooling, $\overline{w'\theta'}(z = 0)$</td>
<td>$-5 \text{ W m}^{-2}$</td>
<td>$-5 \text{ W m}^{-2}$</td>
</tr>
<tr>
<td>Horizontal buoyancy gradient, $M^2 =</td>
<td>\partial b/\partial y</td>
<td>$</td>
</tr>
<tr>
<td>Thermocline stratification, $N^2 = \partial b/\partial z$</td>
<td>$5.3 \times 10^{-9} \text{ s}^{-2}$</td>
<td>$5.3 \times 10^{-9} \text{ s}^{-2}$</td>
</tr>
<tr>
<td>Surface stokes drift, $u_s (z = 0 \text{ m})$</td>
<td>$0.063 \text{ m s}^{-1}$</td>
<td>$0 \text{ m s}^{-1}$</td>
</tr>
<tr>
<td>Stokes drift angle, $\theta_s$</td>
<td>$30^\circ$</td>
<td>$-$</td>
</tr>
<tr>
<td>Turbulent Langmuir number, $L_{a_{t}}$</td>
<td>$\sqrt{u^*/u_s(0)}$</td>
<td>$0.29$</td>
</tr>
</tbody>
</table>

$$
\tau = \rho_0 u^* \left[ \cos(\theta_w) \hat{x} + \sin(\theta_w) \hat{y} \right],
$$

where $u^*$ is the friction velocity and $\theta_w$ is the horizontal wind direction (see Table 1).

Two simulations are contrasted in the present study; they are otherwise identical runs with and without Stokes drift forcing $u_s$. The simulation without Stokes drift forcing solves (1)–(3) with $u_s = 0$, while the simulation with Stokes forcing uses (5) for $u_s$. The same wind stress is used in both simulations, and the wind forcing corresponds to a modest wind speed of about $5 \text{ m s}^{-1}$. Both Langmuir and boundary layer shear turbulence are present in the simulation with Stokes drift forcing, but in the simulation without Stokes forcing, small-scale motions are dominated by shear turbulence alone. In the following, the simulation with Stokes drift forcing and Langmuir turbulence is termed the “Stokes” simulation and the simulation without this forcing is termed the “no-Stokes” simulation.

The National Center for Atmospheric Research (NCAR) LES model (Moeng 1984) is used to solve (1)–(3); this code is described at length in McWilliams et al. (1997) and Sullivan et al. (2007). Spatial derivatives are calculated spectrally in the horizontal ($x$ and $y$) directions and using either second-order (for $u$) or third-order (for tracers such as $\theta$ or $b$) finite differences in the vertical ($z$) direction. Periodic boundary conditions are used for $u$ and $\theta$ in the horizontal directions. The vertical velocity is zero at both the top and bottom boundaries, and horizontal velocities are determined by wind stress and stress-free conditions, respectively, on the top and bottom boundaries. Initially, there is temperature cooling at the surface at the rate $-5 \text{ W m}^{-2}$ in order to induce convective overturning; after the simulations have progressed for approximately 10 virtual days, the cooling is turned off. All of the analysis presented here is carried out at times near day 12. A fractional step method with third-order Runge–Kutta scheme and constant Courant number is used for time advancement. The pressure field is calculated from a Poisson equation by requiring that the flow remain divergence-free, and the subgrid-scale stresses are obtained using a two-equation approach that takes into account $u_s$ (Deardorff 1980; Moeng 1984; Sullivan et al. 1994). The simulations were performed on $O(10^3)$ computational cores on the Kraken and Nautilus supercomputers managed by the National Institute for Computational Sciences and on the Janus supercomputer managed by the University of Colorado Boulder and NCAR.

An initial two-front, or warm filament, temperature configuration is chosen where warmer water is initialized over a $10 \text{ km}$ section at the center of the domain (see Fig. 1). For simplicity, this temperature anomaly occurs only within the mixed layer and does not affect the initial mixed layer depth. This warmer section creates two fronts in the domain and a corresponding geostrophic velocity field is initialized. The winds and waves make an angle of $\theta_w = \theta_s = 30^\circ$ with respect to the fronts aligned with the $x$ axis, and the resulting Ekman transport is directed to the right of the wind, that is, obliquely across the fronts. In the Stokes drift simulation, this forcing is expected to create Langmuir cells oriented near $30^\circ$ in open water away from the temperature fronts (Van Roekel et al. 2012a). Near the fronts,
however, the Langmuir cell alignment may be disturbed. The two temperature fronts are labeled “stable” and “unstable” in Fig. 1 since the Ekman buoyancy flux (EBF) carries warm water over colder water along one front, thereby leading to stabilization of the front, whereas the other front is destabilized as more dense, colder water is carried over less dense, warmer water. As a result of the EBF, strong submesoscale eddy growth is expected along the unstable front (Mahadevan et al. 2010). The simulations are initialized with a uniform mixed layer depth of \( H_{ML,0} = 50 \) m, with constant stratification at greater depths.

The parameters used in the simulations (summarized in Table 1) were determined by a number of constraints and requirements, as well as through the use of numerous single-scale (submesoscale) Massachusetts Institute of Technology general circulation model (MITgcm; Adcroft et al. 2014) simulations, as described in Van Roekel et al. (2012b). Two requirements are of fundamental concern in choosing these parameters: (i) there should be a sufficient number of submesoscale eddies in the domain in order to obtain reasonable statistics and (ii) eddies along the unstable front should be able to grow for sufficient time prior to interfering with the stable front (this is a concern because of the periodicity of the boundary conditions). It has been shown using linear stability analysis (Stone 1966; Boccaletti et al. 2007) that the length and time scales, denoted \( L_s \) and \( \tau_s \), respectively, associated with the fastest growing eddies are given by

\[
L_s = \frac{2\pi M^2 H_{ML,0}}{f^2} \sqrt{\frac{1 + Ri}{5/2}} \quad \text{and} \quad \tau_s = \sqrt{\frac{54}{5} \frac{1 + Ri}{f}},
\]

(7)

where \( Ri = N^2 \theta^2 / M^4 \) is the Richardson number, \( N^2 \) is the vertical stratification, \( M^2 \) is the magnitude of the horizontal buoyancy gradient, and \( f = ||\mathbf{f}|| \) is the magnitude of the Coriolis parameter. Using single-scale MITgcm simulations as a guide, the parameter values \( N^2 = 5.3 \times 10^{-9} \text{ s}^{-2}, \quad M^2 = 2.1 \times 10^{-8} \text{ s}^{-2}, \quad \) and \( u^* = 5.46 \times 10^{-3} \text{ m s}^{-1} \) (see Table 1) were found to give an \( L_s \) that is roughly \( 1/20 \) the domain size, thereby providing a sufficient number of eddies for a statistical analysis. These values also allow eddies on the unstable front to grow for roughly 30\( \tau_s \) over a 15-day period without interfering with the stable front. With these values of \( N^2 \) and \( M^2 \), the scales of the fastest growing eddies are given from (7) as \( L_s \approx 800 \) m and \( \tau_s \approx 0.5 \) days, and the corresponding Richardson number is \( Ri = 0.06 \). The simulation parameters were additionally constrained such that the expected rate of eddy restratification (Fox-Kemper et al. 2008) matches the rate of vertical mixing by Langmuir turbulence (McWilliams and Sullivan 2000). The resulting EBF, which is a good measure of the impact of Ekman-driven convection and restratification (Thomas et al. 2013), is given by

\[
|\text{EBF}| = \left| \frac{\tau \times \mathbf{f}}{\rho_0 f^2}, \mathbf{v}_{h}b \right| = \frac{u^* M^2 \cos \theta_w}{f} \approx 7.5 \times 10^{-9} \text{ m}^2 \text{ s}^{-3} = 16 \text{ W m}^{-2},
\]

(8)

where \( \mathbf{v}_{h}b = (\partial b/\partial x)\mathbf{x} + (\partial b/\partial y)\mathbf{y} \) is the horizontal buoyancy gradient, which reduces to simply \( \mathbf{v}_{h}b = (\partial b/\partial y)\mathbf{y} \) for the initial frontal configuration shown in Fig. 1. The magnitude of the horizontal flux is given as \( |\mathbf{v}_{h}b| = M^2 \). From (8), the EBF is thus strong at both fronts when compared to the initial surface cooling of \( -5 \) W m\(^{-2} \) applied in the simulations.

Note that we choose to have the vertical buoyancy flux due to Langmuir turbulence rival the submesoscale EBF, which is in rough agreement with diagnostic estimates (Fox-Kemper et al. 2011; Belcher et al. 2012). This equivalency will be evaluated using multiscale buoyancy flux diagnostics in section 4. These combined constraints place the simulations in the weak wind and waves regime, thus enforcing a strong scale separation between
the length scales associated with submesoscale eddies (0.8 km) and Langmuir turbulence (10 m). Changes to these parameters will tend to unbalance the stratifying and destratifying fluxes, as is the case in other similar scenarios (Haney et al. 2012).

It should be noted that the configuration examined here is similar to that studied by Özgökmen et al. (2011) and Skyllingstad and Samelson (2012). The present simulations are distinguished, however, by the presence of Stokes drift forcing. This leads to the scale separation between Langmuir turbulence and submesoscale features, as well as a focus on interactions across that scale gap. This is in contrast to a cascade from large to small scales as emphasized in prior studies. We also choose the wind and waves to arrive obliquely at an angle of 30° with respect to the front (McWilliams and Fox-Kemper 2013). The downfront and upfront components of the wind lead to the creation of both stable and unstable fronts due to the EBF, and the cross-front component should exhibit an anti-Stokes effect. Although the simulations were initialized in thermal wind balance, some inertial oscillations occur throughout the run as the flow adjusts to the wave and wind conditions (McWilliams and Fox-Kemper 2013). All diagnoses were analyzed over a complete inertial cycle to ensure that conclusions drawn are not due only to the phase of inertial oscillations.

3. Spindown of the submesoscale front

In the following, we examine temperature, velocity, and vorticity fields after the spindown of the submesoscale front has progressed for roughly 12 days. In our simulation configuration, the temperature field and other flow variables are homogeneous only along the x direction. As a result, we make extensive use of the x average, denoted by \( \langle \phi \rangle_x \), where \( \phi \) is an arbitrary variable and the average is generally a function of \( y \), \( z \), and time \( t \). In the following, fluctuating quantities are defined with respect to the x average as \( \phi' = \phi - \langle \phi \rangle_x \). Averages in both horizontal directions are denoted \( \langle \phi \rangle_{xy} \) and are generally a function of only \( z \) and \( t \).

a. Temperature

Figures 2a and 2d show that the submesoscale evolution of \( \theta \) is qualitatively similar for both the Stokes and no-Stokes cases. At the unstable front labeled in Fig. 1, the Ekman flow carries colder, more dense water over warmer, less dense fluid. This causes an instability to develop, and submesoscale eddies form along the front and grow in size with increasing time. Identification of the instabilities associated with the formation of these eddies is discussed in more detail in section 5. Figures 2a and 2d show that no large-scale eddies form along the stable temperature front where the EBF transports less dense fluid over more dense fluid.

Despite the similarity in the overall evolution of the temperature fields, there are small differences between the Stokes and no-Stokes cases. Comparison of Figs. 2a and 2d indicates that submesoscale eddies are slightly less able to horizontally spread temperature anomalies in the Stokes case. This can be seen by noting that there is less fingering and filamentation after 12 days in the Stokes \( \theta \) field as compared to the no-Stokes field. The bottom panels in Figs. 2a and 2d also show that the x-averaged horizontal (y) gradients of \( \theta \) are slightly larger in the Stokes case, particularly near the stable front. This difference can be attributed to changes in large-scale horizontal transport in the Stokes case, suggesting that differences in boundary layer turbulence between the two cases play a role in determining the eddy strength. Power spectra of \( \theta \) in Fig. 3 similarly indicate that there is slightly greater small- and intermediate-scale energy content in the no-Stokes \( \theta \) fields than in the Stokes fields; this difference is most pronounced at \( z = -25 \) m in Fig. 3b.

b. Velocity

The top panels of Figs. 2b and 2e show the x component of the horizontal velocity field \( u \) near the surface for the Stokes and no-Stokes cases. The velocity \( u \) is typically larger in magnitude for the no-Stokes case, even at locations away from the unstable eddy region. Larger velocities are also evident from the y–z profiles of \( \langle u'^2 \rangle_x \) in the bottom panels of Figs. 2b and 2e. For \( z > -10 \) m, \( \langle u'^2 \rangle_x \) is larger for all \( y \) in the case without Stokes forcing. The locations of large velocities in the no-Stokes case align with cold filaments whose velocities add to the direction of the Ekman spiral. In the case with Stokes forcing, the \( u \) velocity field is not as large in magnitude, with the submesoscale geostrophic shear more clearly visible than the wind-driven shear. This distinction is indicative of a suppression of large-scale \( u \) velocities by Langmuir turbulence. Direct effects of Stokes drift on submesoscale fronts and filaments may also play a role (McWilliams and Fox-Kemper 2013). The bottom panels of Figs. 2b and 2e further show that the magnitude of \( u \) remains larger for the no-Stokes case than for...
the Stokes case within the unstable eddy region at depths approaching the base of the mixed layer.

The vertical velocity $w$ fields in the top panels of Figs. 2c and 2f show that $w$ in the simulation with Stokes forcing is more spatially homogeneous than in the no-Stokes simulation. In the Stokes case, $w$ is concentrated at the smallest scales of motion and there are only weak signatures of submesoscale eddies at the depth shown. This independence from submesoscale processes is also reflected in the $y$–$z$ profiles of $(w^2)_x$, shown in the bottom panels of Figs. 2c and 2f; there are strong vertical velocities near the surface in the Stokes case but substantially weaker velocities in the no-Stokes case. For the Stokes case, $(w^2)_x$ peaks near $z = -4$ m and has a magnitude that is approximately 4 times greater than the peak value in the no-Stokes case, which occurs at $z = -17$ m. The strengthened vertical velocities and shallower peak in the Stokes case are due to the presence of near-surface Langmuir turbulence, resulting in increased vertical mixing.

For the no-Stokes case, the top panel of Fig. 2f shows that there are submesoscale regions where vertical velocities are reduced, and this suppression of the vertical velocity approximately spans the mixed layer. Comparing with Fig. 2e for the $u$ velocity, the $w \approx 0$ regions correspond to $u > 0$ features, while $w < 0$ regions lie between these features. The prevalence of these regions in the no-Stokes case—but not the Stokes case—indicates that there is a suppression of small-scale $w$ in the presence of submesoscale eddies, but that Langmuir
turbulence overwhelms these effects. The $y-z$ profiles of $w^2$ in the bottom panels of Figs. 2c and 2f confirm these conclusions, since it can be seen that there are slightly weaker $w$ intensities below the eddy region. This suppression is most evident for the no-Stokes case in the bottom panel of Fig. 2f, although a weak suppression is seen even for the Stokes case in Fig. 2c at sufficient depth, indicating that submesoscale eddies generated in the present simulations compete with small-scale vertical mixing. This is, in turn, indicative of the restratifying effects associated with submesoscale eddies.

The small-scale spatial suppression of $w$ is shown in more detail by the 1 km$^2$ fields in Fig. 4. The locations of these fields are given by the black boxes in Figs. 2c and 2f and have been chosen to show the small-scale structure both outside and within the unstable eddy region. Comparison of the Stokes and no-Stokes cases shows that, because of the Stokes drift forcing and additional energy supplied by Stokes shear, there is substantially greater small-scale structure in the simulation with Stokes forcing at all locations; the small-scale structure shown in Figs. 4a and 4b is characteristic of Langmuir turbulence. The greatest changes to small-scale turbulence in the Stokes simulations occur near steep temperature gradients where strong vertical velocities are observed. Figure 4b shows such a region where a band of intense negative velocity is approximately aligned with a locally steep temperature front. By contrast, Fig. 4d shows that there are relatively large regions of small $w$ magnitude in the no-Stokes simulation, consistent with results for the full domains shown in Figs. 2c and 2f. The existence of these regions indicates that small-scale turbulence is suppressed by submesoscale eddies, although the mechanism of this suppression is not yet fully understood. It should be noted that even the no-Stokes case possesses coherent small-scale structures with large horizontal vorticity, although these structures are characteristic of shear turbulence in the no-Stokes case while structures in the Stokes case are characteristic of Langmuir turbulence.

The small-scale structure of $w$ in both simulation cases can be examined more directly using the two-dimensional vertical velocity spectra shown in Fig. 5, which are given as functions of the horizontal wave-numbers $k_x$ and $k_y$. Near the surface, there is substantially greater small-scale energy content in the Stokes simulation than in the no-Stokes simulation. In the Stokes case, the two-dimensional spectrum is largest along an angle of approximately $30^\circ$, consistent with the direction of the wind and waves, $q_w = q_s = 30^\circ$. For the no-Stokes case, by contrast, the near-surface spectrum is largest along a smaller angle (roughly $15^\circ$), corresponding to the direction of the surface Ekman flow. These different angles are due to differences in the small-scale dynamics and structure of each case; for the Stokes case, the small scales are dominated by Langmuir cells aligned with the direction of the Stokes forcing, while for the no-Stokes case, the small scales are dominated by shear from the Ekman flow. By $z = -25$ m, Fig. 5 shows that the energy spectra in the two sets of simulations are similar and that both spectra become increasingly isotropic, with nearly equal energies in the $x$ and $y$ directions. Once again, as the depth increases, the effect of Stokes drift forcing becomes weaker and surface winds and waves have an increasingly smaller effect for both the Stokes and
no-Stokes cases. This results in a close correspondence of the spectra with the Ekman flow direction for both the Stokes and no-Stokes cases.

The energy content in each of the three velocity components is compared as a function of the wave-number magnitude \( k \) using the circularly integrated spectra in Fig. 3. In both simulation cases, the flow at all depths is essentially two-dimensional at large scales with an approximately \( k^{-2} \) cascade decay rate. This quasi-2D cascade is associated with submesoscale eddies where \( u \) and \( v \) velocities are similar and vertical velocities are substantially smaller. Integration of the spectra in Fig. 3 from scales of 20 km to 800 m shows that there are over three orders of magnitude more energy in the horizontal large-scale eddies than in the vertical eddies for both simulation cases at both depths shown in Fig. 3; this is indicative of substantial anisotropy in large-scale flow motions. This 2D cascade can be seen in Fig. 3 to extend to scales of approximately \( k = 2\pi/(400 \text{ m}) \), and a quasi-3D cascade associated with boundary layer...
turbulence begins at approximately $k = 2\pi/(100\,\text{m})$. Near the surface, the flow remains anisotropic at small scales for both simulation cases, but at sufficient depth (as shown in Fig. 3b) the small-scale flow is essentially isotropic. It should be noted that the vertical velocity ($w$) spectrum continues to rise for the Stokes case until the gridscale cutoff; this indicates that the small-scale resolution is marginal, but greater spatial resolution is currently infeasible with the available computational resources.

Figure 3 shows that near the surface there is substantially more $u$ energy at all scales in the no-Stokes case than in the Stokes case. Integration over small wavenumbers shows that there is only 44% as much $u$ energy in large-scale eddies for the Stokes case compared to the no-Stokes case. This difference lessens with depth, and by $z = -25\,\text{m}$ the large-scale $u$ energy in the Stokes case is 61% of that in the no-Stokes case. Near the surface, these differences are also observed in small-scale eddies, but by $z = -25\,\text{m}$ the small-scale energy in $u$ is similar for both simulation cases. Compared to the differences observed in the $u$ energy, the spectra for $v$ in the two simulation cases are more similar at all scales, although large-scale eddies continue to be stronger in the no-Stokes case. In the Stokes case, these eddies contain 63% and 69%, respectively, of the $v$ energy in the no-Stokes cases at the two depths shown in Fig. 3. These results indicate that large-scale eddies are weaker in the Stokes case than in the no-Stokes case. Note that Langmuir turbulence is very effective at transferring mean horizontal momentum downward. Since $u$ is greatest near the surface, this could be the physical mechanism for the reduction in $u$ in the Stokes case. There is also no expectation that there should be equal amounts of energy in the two runs, given that Stokes production $\langle u'w' \rangle_{x}\,du/dz$ is a new source of energy in the Stokes case, in addition to different large-scale injection of wind energy. Finally, differences in small-scale production due to $\langle u'w' \rangle_{x}$ and $\langle w'\theta' \rangle_{x}$ will affect the large-scale versus small-scale distribution of energy.

Even more pronounced differences between the two simulation cases are observed in the vertical velocity $w$ spectra. Figure 3 shows that at large and intermediate scales, there is greater $w$ energy content in the no-Stokes case than in the Stokes case, consistent with Fig. 2. Near the surface, the $w$ energy of large-scale eddies in the Stokes case is only 26% of that observed in the no-Stokes case; by $z = -25\,\text{m}$, this difference becomes substantially weaker and there is 63% as much large-scale vertical energy in the Stokes case as compared to the no-Stokes case. The differences in large-scale vertical velocities indicate that Langmuir turbulence has the effect of weakening—or at least competing with—submesoscale vertical transport. It will be shown in section 4 from an analysis of the flux $\langle w'\theta' \rangle_{x}$ that despite the reduction in submesoscale energy for the Stokes case, the restratification associated with large-scale motions is in fact stronger. It will be seen, however, that small-scale destratification also increases in the Stokes case and competes with restratification by larger-scale eddies, resulting in a net reduction in restratification for the Stokes case. For wavenumbers greater than $k \approx 2\pi/(400\,\text{m})$, the $w$ energy in the Stokes simulations is larger than in the no-Stokes simulations. This difference decreases with depth, and by $z = -25\,\text{m}$, spectra for both simulation cases are nearly identical at all scales. The overall appearance of the spectra in Fig. 3 is suggestive of a double cascade of energy with a weak spectral gap near $k \approx 2\pi/(400\,\text{m})$, although higher spatial resolution is required in order to more accurately determine the slope of the second inertial range at small scales.

c. Vorticity

The relevant relative vorticity for the WAB equations is the Eulerian one (Craig and Leibovich 1976; Holm 1996), $\boldsymbol{\omega} = \nabla \times \mathbf{u}$, while advection, tilting, and stretching of vorticity are accomplished by the Lagrangian velocity and its shear. Figure 6 shows fields of the horizontal
vorticity magnitude $\omega_H = (\omega_x^2 + \omega_y^2)^{1/2}$ normalized by $f$, as well as the vertical vorticity $\omega_z/f$, where $f = |\mathbf{f}|$. Note that $\mathrm{Ro} = (\mathbf{\omega}_a)/f$ is the vertical Rossby number, where $(\mathbf{\omega}_a)_z$, $\omega_y$, and $\omega_z$ are the components of the absolute vorticity $\mathbf{\omega}_a = \nabla \times \mathbf{u} + \mathbf{f}$ (recall that $\mathbf{f}$ is strictly in the vertical direction; see Table 1). Figure 6a shows that, consistent with the fields of $u$ and $w$ in Fig. 2, the field of $\omega_H$ is only weakly affected by submesoscale eddies in the Stokes simulation; within the unstable eddy region, there are small increases in $\omega_H$. For the no-Stokes case in Fig. 6c, however, $\omega_H$ indicates rapid overturning (with $\omega_H/f = 50$) within the eddy region and the magnitude of $\omega_H$ is larger overall than in the Stokes case, even for locations away from submesoscale features. This increased magnitude is somewhat surprising given that intense coherent Langmuir cells are present near the surface in the Stokes case. Recall that the formulation for $\omega_H$ used in Fig. 6 does not include the Stokes shear. In the absence of forces to balance the terms in the momentum equation that depend on Stokes drift, an anti-Stokes Eulerian flow often arises (McWilliams and Fox-Kemper 2013) such that $\mathbf{u} = -\mathbf{u}_s$. In this case, the anti-Stokes flow...
can acquire a large horizontal vorticity in opposing the Stokes shear. This anti-Stokes shear does not appear to dominate the other sources of $\omega_H$ in Fig. 6.

The vertical vorticity $\omega_z/f$ does not directly include the Stokes shear, and Figs. 6b and 6d show that the magnitudes of $\omega_z/f$ are similar in both the Stokes and no-Stokes cases and are also similar to other submesoscale simulations and observations (e.g., Capet et al. 2008a; Shcherbina et al. 2013). For both cases, however, $|\omega_z/f|$ is largest within the eddy region near the unstable front along the fronts and filaments created near typical submesoscale eddies. The prevalence of increased $|\omega_z/f|$ seems to be somewhat greater in the no-Stokes case, although substantial variations are also seen for the Stokes case. This indicates that Langmuir turbulence (which is associated with vorticity primarily in the x and y directions) exerts a weak, but systematic, magnitude reduction on the vertical vorticity associated with submesoscale eddies. As depth increases toward the bottom of the mixed layer (not shown here), the horizontal vorticity magnitude decreases for both cases while the vertical vorticity is largely unchanged.

The most probable angle of the horizontal vorticity across the front can be examined quantitatively using probability density functions (pdfs) of $\alpha = \tan^{-1}(\omega_x/\omega_y)$, as shown in Fig. 7. The pdfs are calculated as functions of $y$ and each pdf is normalized so that the integral of the pdf with respect to its argument is unity. The pdfs in Fig. 7a show that, near the surface in the Stokes case, there are peaks in the distributions near $\partial_x = 30^\circ$ and $-180^\circ + 30^\circ$. These peak locations are consistent with the existence of counterrotating Langmuir cells that are aligned with the direction of the wind and waves. The Langmuir cells create small-scale upwelling and downwelling regions at the surface, resulting in windrows of $w$ that are oriented at $30^\circ$; the angled spectra in Fig. 5 also indicate the most probable direction of the windrows, since these features dominate $w$ on small scales. At $z = -25$ m, Fig. 7b shows that the pdfs of $\alpha$ become broader for the Stokes case and the most probable $\alpha$ shifts to a value close to $90^\circ$, which is the direction of the thermal wind shear in the initial temperature configuration. The resulting pdfs are similar to those shown in Fig. 7d for the no-Stokes case and are consistent with the similarity of the large-scale and mid-depth two-dimensional flow and spectra in Figs. 2, 3, and 5. For both the Stokes and no-Stokes cases, variations in the pdfs across fronts (in the $y$ direction) are greater at $z = -25$ m than near the surface, as indicated by the decreases in preferential vorticity angle near the stable fronts at $y = 18-20$ km in Figs. 7b and 7d.

Near the surface in the no-Stokes case, Fig. 7c shows that the most probable value of $\alpha$ is close to $90^\circ + 15^\circ$. Although the observed angle is relatively constant across the front (the $y$ direction) near the surface, Fig. 7d shows that at $z = -25$ m there are significant variations in the pdf across the unstable front in the no-Stokes case. In particular, the most probable value of $\alpha$ decreases to $\alpha \sim 0^\circ$ within the eddy region. These variations are also seen in Fig. 6c and are the signatures of submesoscale eddies on the relatively weak vortices associated with shear turbulence.

As discussed in Van Rockel et al. (2012a), the angle of the vorticity in the oceanic mixed layer can be related to the angle of the Lagrangian shear, $\alpha_L = \tan^{-1}(\langle du_L/\partial z \rangle_x)/\langle du_L/\partial y \rangle_z)$. This angle is obtained from an analysis of the vortex stretching term in the transport equation for $\omega_y$. The pdf in Fig. 7a shows that $\alpha_L$ is in good agreement with the observed angles at the surface in the Stokes case, where $\alpha_L$ is calculated using the x-averaged Lagrangian shear $\langle du_L/\partial z \rangle_x$; the Lagrangian shear is the main agent in the stretching and tilting terms and $\alpha_L \approx \partial_x = 30^\circ$ because of the dominance of the Stokes shear in the Stokes simulation case. At increasing depth, however, the preferential vorticity angle varies as the Stokes shear decreases in magnitude and the flow becomes dominated by the shear induced by the Ekman spiral. Figure 8 shows variations in the pdfs with
increasing depth; at the surface, the Stokes pdfs have peaks at \(-180^\circ + 30^\circ\) and \(30^\circ\), but as the depth increases, the pdfs increasingly approach a single peak at \(0^\circ\), which may be associated with the overturning of the unstable front.

In the no-Stokes case, the vorticity field near the surface behaves similarly to many other boundary layer flows where Langmuir turbulence is absent. In particular, the horizontal velocity field is dominated by the Ekman flow direction, which is close to \(15^\circ\), and the vorticity is preferentially oriented at \(90^\circ + 15^\circ\), as shown in Fig. 7c. It is significant that there is a peak at only one location in the no-Stokes case, whereas there are two peaks in the Stokes case. In the Stokes case, the vorticity field near the surface is dominated by counterrotating Langmuir cells, and thus the angles \(-180^\circ + 30^\circ\) and \(30^\circ\) occur with nearly equal probability. In the no-Stokes case, the orientation of the vorticity is determined by the flow direction, and there is thus a single peak at \(90^\circ + 15^\circ\). As the depth increases, frontal overturning dominates and the pdfs of \(\alpha\) become increasingly similar for both of the simulation cases.

4. Multiscale fluxes

To separate submesoscale and Langmuir-scale motions, we perform two-dimensional spectral decompositions with a cutoff at 400 m. Features smaller than this scale are assumed to be characteristic of 3D boundary layer (i.e., shear or Langmuir) turbulence and features larger than this scale are assumed to be characteristic of 2D submesoscale eddies. Figure 3 shows the separation between the low-pass (submesoscale) and high-pass (Langmuir scale) wavenumber ranges; low-pass-filtered fields are denoted by the subscript \(h\) and high-pass-filtered fields are denoted by the subscript \(l\).

Scale decompositions are performed using horizontal fast Fourier transforms (FFTs) and filtering. Despite the inhomogeneity in the \(y\) direction created by the temperature front, we use 2D FFTs at each depth and time to examine spectral properties of the flow fields. Based on the resulting spectra, we carry out 2D circularly symmetric filtering. Low-pass filtering is achieved by retaining only modes \(k < k_c\), where \(k_c = 2\pi/(400\text{ m})\) is the cutoff wavenumber, and high-pass filtering is achieved by retaining only modes \(k > k_c\).

We examine decompositions of the momentum and temperature fluxes as a function of depth (\(z\)) and location across the front (\(y\)). Fluxes are calculated using fields filtered either at large or small scales (e.g., \(\langle u'_i w'_j \rangle_x\) and \(\langle u'_i w'_j \rangle_y\)). Given the 2D nature of the filtering and the one-dimensional \(x\) averaging used to obtain the fluxes, the mixed-scale fluxes (e.g., \(\langle u'_i w'_j \rangle_{xy}\)) are not exactly zero.

There is little coherence, however, in these fluxes and they are subsequently not considered here. We also average the fluxes over the unstable eddy region (see Fig. 1) and compare the resulting depth profiles of \(\langle u'_i u'_j \rangle_{xy}\) and \(\langle u'_i \theta' \rangle_{xy}\) with results from an asymptotic analysis.

a. Momentum fluxes

Figure 9 shows total \(\langle u' w' \rangle_x\), low-pass \(\langle u'_i w'_j \rangle_x\), and high-pass \(\langle u'_i w'_j \rangle_x\), horizontal momentum fluxes for the Stokes and no-Stokes simulations. These fluxes represent transport of horizontal velocity \(u\) by vertical velocity \(w\). Figures 9a and 9d show that the total flux \(\langle u' w' \rangle_x\) is strongly negative near the surface, indicating that large \(u\) is transported to greater depths while small \(u\) is transported toward the surface. Moreover, the flux magnitude is significantly
greater for the Stokes case than the no-Stokes case. This strong downward flux of $u$ provides a possible explanation for the observed smaller values of $u$ in the Stokes case shown in Fig. 2b. That is, in the presence of Langmuir turbulence, wind-driven momentum is rapidly redistributed across the mixed layer, while in the absence of Langmuir turbulence it can accumulate near the surface.

The low-pass fluxes in Figs. 9b and 9e show that there is relatively little horizontal momentum flux associated with submesoscale eddies. The total momentum flux $\langle u'w' \rangle_z$ thus occurs primarily at small scales in both the Stokes and no-Stokes cases, as shown in Figs. 9c and 9f. The small-scale fluxes are, however, nearly twice as large in the Stokes case than in the no-Stokes case because of the increased vertical mixing associated with Langmuir turbulence.

The remaining momentum flux components $\langle u'u'_y \rangle_{xy}$ are shown as a function of depth $z$ in Fig. 10. Figures 10a, 10b, and 10d show that for fluxes involving only $u$ or $v$, there are comparable contributions from the low-pass and high-pass fluxes to the total flux near the surface. Below the surface, the horizontal high-pass fluxes are negligible. For fluxes involving the vertical velocity $w$ most of the total flux is associated with small-scale motions at all depths, and there are substantial differences between the Stokes and no-Stokes cases. In particular, the magnitudes of the vertical fluxes in the Stokes case are always significantly greater than the magnitudes of the no-Stokes fluxes because of enhanced vertical transport by Langmuir turbulence.

**b. Temperature (buoyancy) fluxes**

Figure 11 shows total $\langle w'\theta' \rangle_z$, low-pass $\langle w'\theta'_l \rangle_z$, and high-pass $\langle w'\theta'_h \rangle_z$, vertical temperature (or buoyancy) fluxes for the Stokes and no-Stokes cases. These fluxes represent vertical transport of temperature, which is a primary reservoir of potential energy. By contrast to the momentum flux in Fig. 9, the total temperature fluxes in Figs. 11a and 11d have significant amplitudes down to the base of the mixed layer. For both simulation cases, the total temperature flux is primarily positive within the unstable eddy region, which is indicative of a restratifying effect on the mixed layer as warmer water is transported upward and colder water is transported to greater depth. Although this restratifying effect is present in both the Stokes and no-Stokes cases, the total flux $\langle w'\theta' \rangle_z$ is roughly 40% weaker in the Stokes case.

This weakening of the total restratification in the Stokes case can be understood in more detail by considering the decomposition of the temperature flux into low- and high-pass parts. Figures 11b and 11e show that much of the very deep and generally positive temperature flux occurs at large scales and is thus associated with submesoscale eddies. At small scales, Figs. 11c and 11f show that $\langle w'\theta'_l \rangle_z$ is negative in the Stokes case, but only weakly negative in the no-Stokes case. This indicates that Langmuir turbulence transports warmer, near-surface water to greater depths and carries colder, deeper water toward the surface, thereby competing with the restratifying effects of submesoscale eddies, adding potential energy, and reducing the total flux $\langle w'\theta' \rangle_z$ shown in Fig. 11a. The small-scale flux for the no-Stokes case is 4 times weaker than that for the Stokes case and thus provides little competition for the restratifying effects of submesoscale eddies; this is why the total flux shown in Fig. 11d for the no-Stokes case is more strongly positive than for the Stokes case.

The depth profiles of $\langle w'\theta' \rangle_{xy}$ and $\langle w'\theta' \rangle_{xy}$ in Figs. 10g,h show that nearly all of the horizontal buoyancy flux occurs at large scales in both the Stokes and no-Stokes cases. These fluxes are roughly a tenth of the EBF in the frontal regions. Figure 10i shows that most of the vertical flux $\langle w'\theta' \rangle_{xy}$ also occurs at large scales in the no-Stokes case, but for the Stokes case $\langle w'\theta'_h \rangle_{xy}$ is negative and partially balances the strongly positive submesoscale
The small “kinks” in the total and high-pass vertical temperature fluxes near $z = -30$ m are due to the matching between subgrid-scale models from Moeng (1984) and Sullivan et al. (1994). It should be noted that the submesoscale flux in Fig. 10i is actually larger in the Stokes case than in the no-Stokes case, but the small-scale flux is also enhanced in the Stokes case such that the total flux—and, hence, the total restratifying effect—is weaker in the presence of Langmuir turbulence.

c. Asymptotic analysis

We conclude this section by showing that the LES results for low- and high-pass fluxes are consistent with a multiscale interpretation of the WAB, or Craik–Leibovich, equations first put forth by Malecha et al. (2013) in the idealized setting of downwind-invariant flows. In this interpretation, the submesoscale dynamics dominantly evolve according to the hydrostatic WAB equations augmented by divergences of finescale momentum and buoyancy fluxes, while the finescale dynamics dominantly follow the nonhydrostatic, nonrotating WAB equations with a local environment set by submesoscale eddies. As argued by Malecha et al. (2013), the aspect ratio $A$ (i.e., the ratio of depth to horizontal scales) of the submesoscale motions is the crucial small parameter for the multiscale decomposition. This demonstration is significant because it suggests that the multiscale modeling framework proposed by Malecha et al. (2013) is appropriate and that 3D extensions are, in fact, warranted. In this context, recall that the simulations described here aim to resolve all dynamically active scales from meters...
to tens of kilometers, requiring immense computational resources. By contrast, multiscale models can be used to comparatively rapidly explore parameter space for the purposes of identifying interesting dynamical regimes, informing the design of future LES process studies, aiding in the interpretation of the physics, and ultimately guiding the development of parameterizations with only rare reevaluation by the expensive multiscale simulations such as those described herein.

The multiscale interpretation of the mixed layer flow field, as well as the low- and high-pass filtering used to calculate the multiscale fluxes shown in Figs. 9–11, is suggestive of a separation of temporal and horizontal spatial scales. This implies that spatial and temporal derivatives can be separated using a multiple-scale decomposition as

\[
\begin{align*}
\partial_x &\to \partial_x + A \partial_X, \\
\partial_y &\to \partial_y + A \partial_Y, \\
\partial_z &\to \partial_z + A \partial_T,
\end{align*}
\]

where \(x, y,\) and \(t\) denote the relatively fast spatial and temporal scales associated with small-scale dynamics and \(X, Y,\) and \(T\) denote the slow scales associated with submesoscale dynamics. The separations in (9) can be combined with an associated decomposition of the Eulerian fluid variables, namely, \(u_i = \pi_i + u''_i\) where \(u''_i = 0,\) and similarly for other fields. Here the averaging operator can be taken as a suitable average over fast spatial and temporal scales along homogeneous directions in the flow, and thus the average effectively acts as a low-pass filter that can be used in the multiscale asymptotics; the wave-filtered equations themselves already imply one such average over surface gravity waves. In the asymptotic analysis, large-scale vertical velocities are substantially weaker than large-scale horizontal velocities. Small-scale velocities, by contrast, are assumed to be approximately equal in magnitude in all three directions and to have magnitudes that rival the horizontal velocities associated with large-scale motions. Moreover, the temperature and, hence, buoyancy anomalies are assumed to be substantially weaker at small scales than at large scales and vertical variations in the large-scale buoyancy field are weak within the mixed layer. These results can be expressed using the multiple-scale framework as

\[
\begin{align*}
w &= O(A[\pi, \tau]), \\
b'' &= O(A\bar{b}) \quad \text{and} \quad \partial\bar{b}/\partial z &= O(A\bar{b}/H_{ML}),
\end{align*}
\]

where \(H_{ML}\) is the mixed-layer depth and \(A\) is a small parameter that quantifies the observed separation of small and large scales, as expressed in (9). For simplicity, vortices induced by finescale Langmuir or shear turbulence are assumed to be isotropic in cross section. These scaling relationships imply that the quadratic fluxes shown in Figs. 9–11 should have the forms

\[
\begin{align*}
\overline{ww} &= \overline{w}\overline{w} + \overline{w''w''}, \\
\overline{wb} &= \overline{w}\overline{b} + \overline{w''b''}, \\
\overline{bb} &= \overline{b}\overline{b} + \overline{b''b''}, \quad \text{and} \quad \overline{w''b''} &= A(\overline{w'b'} + \overline{w''b''}).
\end{align*}
\]

These results are consistent with the depth profiles shown in Fig. 10. From the relations in (11), the presence of a vertical velocity \(w\) in the fluxes introduces a factor \(A\) in front of the large-scale component. Since \(A\) is a small parameter, this means that such fluxes are dominated by small-scale contributions, in agreement with the weak
Table 2. Criteria used for identification of symmetric, gravitational, and inertial instabilities (Thomas et al. 2013), where $\phi_{RI} = \tan^{-1}(-Ri^{-1})$ and $\phi_{Ro} = \tan^{-1}(-Ro - 1)$. It is assumed that $q < 0$ in all of the unstable cases and that $q > 0$ corresponds to stable regions.

<table>
<thead>
<tr>
<th>Instability $(q &lt; 0)$</th>
<th>Anticyclonic vorticity</th>
<th>Cyclonic vorticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stable (S)</td>
<td>$\phi_{Ro} &lt; \phi_{RI} \leq 0$</td>
<td>$\phi_{Ro} &lt; \phi_{RI} \leq 0$</td>
</tr>
<tr>
<td>Inertial/symmetric instability (ISI)</td>
<td>$-\pi/4 &lt; \phi_{RI} \leq \phi_{Ro}$</td>
<td>$-\pi/2 &lt; \phi_{RI} \leq \phi_{Ro}$</td>
</tr>
<tr>
<td>Symmetric instability (SI)</td>
<td>$-\pi/2 &lt; \phi_{RI} \leq -\pi/4$</td>
<td>$-\pi/2 &lt; \phi_{RI} \leq -\pi/4$</td>
</tr>
<tr>
<td>Symmetric/gravitational instability (SI/G)</td>
<td>$-3\pi/4 &lt; \phi_{RI} \leq -\pi/2$</td>
<td>$-3\pi/4 &lt; \phi_{RI} \leq -\pi/2$</td>
</tr>
<tr>
<td>Gravitational Instability (G)</td>
<td>$-\pi \leq \phi_{RI} \leq -3\pi/4$</td>
<td>$-\pi \leq \phi_{RI} \leq -3\pi/4$</td>
</tr>
</tbody>
</table>

large-scale fluxes shown in Figs. 10c, 10e, and 10f. The small-scale contribution to the buoyancy fluxes is weaker than the large-scale contribution, except for the vertical flux, where the velocity $w$ introduces an additional factor of $A$ that renders the small- and large-scale fluxes comparable in magnitude. Once again, these scalings are consistent with results in Figs. 10g–i. It should be noted that Stokes drift forcing and Langmuir turbulence have not explicitly been addressed in this analysis, but the relations in (11) are nevertheless a useful starting point for future multiscale analyses spanning the range from submesoscale eddies to Langmuir turbulence. Similar to Malecha et al. (2013), such analyses hold the prospect of producing coupled equation sets that capture the balanced dynamics occurring at both large and small scales, as well as the relevant interactions amongst them.

5. Regions of potential instability

Instabilities smaller than mixed layer eddies can arise when the Ertel potential vorticity $q$, which is defined as

$$q = \omega_a \cdot \nabla b,$$  \hspace{1cm} (12)

has the opposite sign to the Coriolis parameter, $f = |f|$ (viz., when $q_f < 0$) (Thomas et al. 2013; Li et al. 2012), where $\omega_a$ is the absolute vorticity; ongoing work generalizes these criteria to cases including Stokes forcing (S. Haney 2013, personal communication). Depending on the values of the Richardson and Rossby numbers, the flow in regions where $q_f < 0$ can be locally stable, inertially unstable, symmetrically unstable, gravitationally unstable, or of mixed instability. Identification of these instabilities can be carried out using the modified Richardson and Rossby parameters, which are given as (Thomas et al. 2013)

$$\phi_{RI} = \tan^{-1}(-Ri^{-1}) \text{ and } \phi_{Ro} = \tan^{-1}(-Ro - 1),$$  \hspace{1cm} (13)

where $Ri$ is the shear Richardson number given by

$$Ri = \frac{N^2}{(\partial u_L / \partial z)^2}, \text{ where } N^2 = -g \frac{\partial (\rho/\rho_o)}{\partial z},$$  \hspace{1cm} (14)

and $Ro$ is the vertical Rossby number given by

$$Ro = \frac{\omega_a}{f} \cdot \frac{z}{1}.$$  \hspace{1cm} (15)

Note that the Lagrangian shear appears in the denominator of (14) rather than the usual Eulerian shear (Holm 1996; S. Haney 2013, personal communication). The arctangent forms of the modified Richardson and Rossby numbers in (13) are used primarily as a matter of convenience; arctangent varies most strongly for small values of its argument, corresponding to the values of $\text{Ri}^{-1}$ and $Ro$ near which stability conditions change, and asymptotes to $\pm \pi/2$ for (the somewhat less interesting) large magnitudes of $\text{Ri}^{-1}$ and $Ro$.

The conditions necessary for each of the various instabilities to occur have been outlined by Thomas et al. (2013) in terms of $\phi_{RI}$ and $\phi_{Ro}$; the resulting criteria are summarized in Table 2 for anticyclonic and cyclonic vorticity. Since $f > 0$, the flow can only be unstable when $q < 0$, and when $q \equiv 0$ the flow is assumed to be stable to small-scale disturbances. For $q < 0$ and $Ri > 0$, the flow is either symmetrically or inertially unstable, or stable, with the cutoffs between these three states determined by the relative magnitudes of $Ri$ and $Ro$. When $q < 0$ and $Ri < 0$, the flow is either gravitationally unstable or mixed symmetrically/gravitationally unstable. The mixed inertial/symmetric instability only arises for anticyclonic vorticity.

In the following, we consider fields of $q$, $\phi_{RI}$, and $\phi_{Ro}$ in the Stokes and no-Stokes cases prior to constructing instability maps based on the conditions summarized in Table 2. One goal of our ongoing work is to determine if these conditions are exact or only approximate in the presence of Stokes drift; a first step in this direction has been provided by the 2D investigation of Li et al. (2012), which suggests that the conditions are approximate and that a hybrid Langmuir–symmetric mode can dominate the instability in certain parameter regimes when Stokes drift effects are incorporated. Nevertheless, we continue to employ the (only slightly modified) stability criteria given by Thomas et al. (2013) as a reasonable indicator of likely sites of unstable dynamics.
Potential vorticity

Figures 12a and 12e show fields of $q$ both in $x$–$y$ and $x$-averaged $y$–$z$ planes for the two simulation cases. Away from the eddy region, $q$ is homogeneous and fine-scaled for both cases. Within the eddy region, $q$ has a tendency to be negative, most likely due to Ekman overturning (Thomas et al. 2013), although $q$ is more strongly negative in the no-Stokes case than in the Stokes case. In particular, the bottom panels of Figs. 12a and 12e show that, on average, $q$ is strongly negative near the surface within the eddy region for the no-Stokes case and that the region of negative $q$ extends to greater depths than in the Stokes case. Near the eddy-free stable temperature front, Figs. 12a and 12e show that $q$ is positive near the surface for both simulation cases, indicative of a stable flow. Once again, however, the positive magnitude of $q$ is greater in the eddy-free region for the no-Stokes case than for the Stokes case. Combined with the behavior of $q$ near the unstable eddy region, it is clear that Langmuir turbulence is generally associated with a decrease in the magnitude of $q$ in the mixed layer.

It should be noted that, at sufficient depth, the $x$ average of $q$ becomes positive for all $y$. It is interesting to note that the depth at which $q > 0$ in both simulation cases is shallowest beneath the eddy region, which is also the region where $q < 0$ near the surface. This indicates that submesoscale eddies are associated with unstable flow near the surface, but increase the stability of the flow at greater depths.
b. Richardson and Rossby numbers

Figures 12b and 12f show maps of $\phi_{RI}$ for the Stokes and no-Stokes cases, respectively. Outside of the eddy region, $\phi_{RI}$ is generally close to $-\pi/2$, corresponding to the cutoff between symmetric and gravitational instabilities (see Table 2). Within the eddy region, however, $\phi_{RI}$ becomes increasingly less negative, corresponding to increasing dominance of symmetric instability. These regions of increased $\phi_{RI}$ are more pronounced in the no-Stokes case than in the Stokes case, as shown by comparison of Figs. 12b and 12f. The bottom panels of Figs. 12b and 12f show that $\phi_{RI}$ is negative and close to $-\pi/2$, near the surface, but increases with depth. The bottom panels show that the critical value $\phi_{RI} = \tan^{-1}(-0.25^{-1})$ occurs at shallower depths in the no-Stokes case, indicative of a shallower mixed layer region.

Figures 12c and 12g show $\phi_{Ro}$ for the two simulation cases. For the no-Stokes case in Fig. 12g, there are pronounced regions of less negative $\phi_{Ro}$ within the eddy region, indicative of larger $Ro$. These regions of increased $\phi_{Ro}$ correspond to an increase in the likelihood of obtaining symmetric instabilities, since symmetric instabilities require, at the very least, that $\phi_{Ro} > -\pi/2$. The bottom panels of Figs. 12c and 12g show that $\phi_{Ro}$ is close to $-\pi/2$ at the surface, but increases weakly with increasing depth. This increase is slightly stronger in the no-Stokes case, where $\phi_{Ro} > -\pi/2$ occurs at shallower depths than in the Stokes case. Comparison with Fig. 12f shows that the increase in $\phi_{Ro}$ roughly coincides with the increase in $\phi_{RI}$ beneath the eddy region. Since the increases in $\phi_{Ro}$ are stronger in the no-Stokes case than in the Stokes case, we may expect symmetric instabilities to play a more dominant role in the no-Stokes flow evolution.

c. Instability maps

Figures 12d and 12h show maps of mixed layer instabilities using the criteria summarized in Table 2. The instability criteria are initially applied at every point using the fields of $q$, $\phi_{RI}$, and $\phi_{Ro}$ shown in Figs. 12a–c, 12e, and 12f. To remove the effects of localized changes in stability due to grid-scale turbulent variations, the pointwise instability maps are processed so that the (in)stability at each point is obtained from the most probable instability condition in a 100-m square section centered on the point of interest, provided that at least a third of the points in the square section are unstable. This procedure results in an effective coarse-graining of the instability fields, thereby avoiding potentially unphysical transitions in the instability mechanism at adjacent grid points.

Figures 12d and 12h show that the unstable eddy region is dominated by symmetric instabilities, particularly for the no-Stokes case; 47% of the points in this region (extending from approximately $y = 7$ km to $y = 12$ km) are symmetrically unstable, while only 14% of the points are of mixed symmetric–gravitational instability (the remaining points are stable). Symmetric instabilities are associated with submesoscale eddies and are subsequently less pronounced for the Stokes case where Langmuir turbulence competes with the effects of larger-scale eddies, as shown in Fig. 12d. For the Stokes case, mixed symmetric–gravitational instabilities frequently occur away from the eddy region and are also more prevalent within the unstable eddy region than in the no-Stokes case. Within this region for the Stokes case, symmetric instability is present at 33% of the points and mixed symmetric–gravitational instabilities are present at 27% of the points. The increased prevalence of mixed symmetric–gravitational instabilities in the Stokes case is associated with small-scale mixing by Langmuir turbulence. Some of the observed differences may also be due to Langmuir turbulence affecting the temporal development of the instabilities. Overall, comparison of the two simulation cases indicates that simulations of the ocean surface layer without Stokes drift forcing are likely to overestimate the importance of symmetric instabilities.

The bottom panels in Figs. 12d and 12h show that, on average, instabilities for both simulation cases are found primarily near the surface and within the eddy region. The concentration of instabilities near the surface is due to the fact that $q$ is only negative on average at the surface, as shown in the bottom panels of Figs. 12a and 12e. By $z = -20$ m, the bottom panels of Figs. 12d and 12h indicate that the mean flow is stable everywhere. It should be noted, however, that the flow is stable only in the along-channel mean; the instantaneous fields in the top panels of Figs. 12d and 12h show that there can be substantial variations in the local instability mechanism across the domain due to the eddy meanders.

6. Mixed layer depth

The depth of the mixed layer, which we denote $H_{ML}$, provides a sensitive measure of the competition between Langmuir turbulence and submesoscale eddies. It also serves as an indicator of the results of certain classes of instability. For example, Taylor and Ferrari (2010) argue that the depths of homogenization of temperature and potential vorticity indicate the aftereffects of convective and symmetric instability, respectively. It is anticipated that Langmuir turbulence generally acts to increase
$H_{ML}$ through vertical mixing, while submesoscale eddies generally reduce $H_{ML}$ through restratification due to slumping of horizontal density gradients. The maps of $\phi_{Ri}$ in Figs. 12b and 12f indicate that $H_{ML}$ may be shallower in the no-Stokes case than in the Stokes case where Langmuir turbulence is present, and in the following we further examine the variation of $H_{ML}$ using the temperature $\theta$ and potential vorticity $q$.

**a. Mixed layer depth fields**

Figure 13 shows the local variation of $H_{ML}$ using criteria based on the temperature $\theta$ and potential vorticity $q$. The depth based on $\theta$ is calculated at each $x, y$ location as the value of $z$, denoted $H_{\theta}(x, y)$, at which $\Delta \theta > (\Delta \theta)_c$ for all $z \leq H_{\theta}(x, y)$, where $\Delta \theta = [\theta(x, y) - \theta(x, y, z)]$ and we use $(\Delta \theta)_c = 2 \times 10^{-3}$ K in constructing Fig. 13. The criterion based on $q$ is formulated in a similar fashion; at each $x, y$ location, $H_q(x, y)$ is chosen as the value of $z$ where $q(x, y, z) > q_c$ for all $z \leq H_q(x, y)$, with $q_c = 8 \times 10^{-11}$ s$^{-3}$. Prior to calculating $H_q$, the $q$ field is coarse-grained over 200 m$^2$ horizontal regions in order to avoid large and unphysical differences in $H_q$ at adjacent grid points. The specific values of $(\Delta \theta)_c$ and $q_c$ are chosen to give similar mixed layer depths outside of the unstable eddy region.

Figure 13 shows that in the simulation cases, $H_\theta$ and $H_q$ have their shallowest values near the unstable front. The shallowest values of $H_\theta$ and $H_q$ occur on the cold side of the front (near the blue contour lines). These trends are stronger for the no-Stokes case than for the Stokes case, and the no-Stokes case has a shallower mixed layer depth at nearly all locations.

The shallower $H_\theta$ and $H_q$ in the no-Stokes case indicates that Langmuir turbulence is effective in creating a deeper mixed layer. The mechanism of this deepening cannot, however, be directly inferred from Fig. 13. Two mechanisms are possible: either Langmuir turbulence increases vertical mixing and directly creates a deeper mixed layer, or Langmuir turbulence weakens the much larger submesoscale eddies, thereby weakening restratification and indirectly creating a deeper mixed layer. Many results of this study do in fact indicate that submesoscale eddies are weaker in the Stokes case than in the no-Stokes case, thereby lending support to the latter explanation. Figures 13d and 13h show that the locations of large-scale eddies roughly correspond to the shallower mixed layer regions in Figs. 13a, 13b, 13e, and 13f, indicating that submesoscale eddies are directly associated with restratification of the mixed layer. These
eddy's are weaker in the Stokes case, providing a direct indication that submesoscale eddies are weakened by Langmuir turbulence.

However, Fig. 10i argues for the opposite conclusion and is a more direct representation of restratification and mixing. This figure shows that large-scale restratification and mixing are both stronger in the Stokes case than in the no-Stokes case. Only the net result of these two competing processes is weaker in the Stokes case, and thus stronger mixing coexists with stronger restratification. This result is similar to the balance between stronger eddies and stronger Ekman buoyancy fluxes seen by Mahadevan et al. (2010) and may also be connected to the results from the study by Li et al. (2012), where it was found that Langmuir turbulence may actually enhance vertical restratification.

Finally, comparison of Figs. 13a and 13c with Figs. 13b and 13f shows that the spatial variations of \( H_\theta \) and \( H_q \) are similar for both simulation cases. The fields in Figs. 13c and 13g reveal generally larger differences between \( H_q \) and \( H_\theta \) in the no-Stokes case than in the Stokes case, with \( H_q \) typically deeper than \( H_\theta \). The argument advanced by Taylor and Ferrari (2010) suggests that the greater differences between \( H_q \) and \( H_\theta \) observed in the no-Stokes case are consistent with a dominance of symmetric instability over convective instability, as is also suggested by the instability maps in Fig. 12.

b. Average mixed layer depth

Figure 14 shows measures of the \( x \)-averaged \( H_{ML} \) as a function of \( y \) based on \( \theta \) and \( q \). The mixed layer depths are determined from the quantities \( \langle \Delta \theta \rangle_c/\langle \Delta \theta \rangle_x \) and \( \langle q \rangle_c/\langle q \rangle_x \), where \( \langle \Delta \theta \rangle_c = 1 \times 10^{-3} \) K and \( \langle q \rangle_c = 4 \times 10^{-11} \) s \(^{-3} \). Once again, \( \langle \Delta \theta \rangle_c \) and \( \langle q \rangle_c \) were chosen to give similar mixed layer depths outside of the unstable eddy region (close to \( -50 \) m, as shown in Fig. 14). The values of \( z \) at which \( \langle \Delta \theta \rangle_c/\langle \Delta \theta \rangle_x \) and \( \langle q \rangle_c/\langle q \rangle_x \) are both approximately one gives \( H_{ML} \).

Figure 14 shows that the mixed layer depth is shallowest within the eddy region near the unstable front for both simulation cases and both depth measures, indicative of restratification by submesoscale eddies. Consistent with Fig. 13, the mixed layer depth within this region is shallower for the no-Stokes case than for the Stokes case. For the temperature-based mixed layer depth, \( H_{ML} = -30 \) m in the Stokes case and \( H_{ML} = -14 \) m in the no-Stokes case, while for the potential vorticity depth, \( H_{ML} = -44 \) m and \( H_{ML} = -40 \) m in the Stokes and no-Stokes cases, respectively. Based on the temperature, the mixed layer depth is thus more than doubled in the presence of Langmuir turbulence. This is again indicative of stronger mixing when Langmuir turbulence is present, despite potentially compensating changes to submesoscale eddy restratification.

7. Conclusions

In this study, large eddy simulations of submesoscale frontal spindown are described, with particular emphasis on multiscale interactions between submesoscale eddies and boundary layer Langmuir or shear turbulence. The present results indicate that large scales affect small scales primarily through symmetry breaking due to Stokes drift, Ekman flow, and geostrophic flow orientation. There is also detectable suppression of small-scale mixing where submesoscale features shoal the mixed layer base. The effects of small scales on larger scales are also apparent. The Stokes simulation has much stronger vertical mixing of momentum and temperature, particularly at small scales, which diffuses and slows the large-scale geostrophic shear. Furthermore, the enhanced momentum mixing results in greatly reduced vertical shear and near-surface \( u \) in the Stokes as...
compared to the no-Stokes case. The prevalence of symmetric instabilities in the mixed layer is reduced by nearly a third when Langmuir turbulence is present. Similarly, the mixed layer depths based on temperature and potential vorticity are approximately 100% and 10% deeper, respectively, in the Stokes case, also due to enhanced Langmuir mixing. Interestingly, increased mixing by Langmuir turbulence preferentially deepens the convective (temperature) mixed layer depth, while only slightly deepening the potential vorticity mixed layer depth, consistent with the changes to the stability criteria from symmetric to gravitational instabilities. There are indications that submesoscale eddies and large-scale motions are weakened in the presence of Langmuir turbulence, but the present study suggests that the primary effect of Langmuir turbulence is to enhance small-scale vertical mixing and transport, thereby competing with the restratifying effects of larger-scale eddies.

The demonstrated two-way connections between large and small scales suggest that particular care must be exercised in studies purporting to isolate submesoscale or boundary layer turbulence. In particular, the effects of these two scale ranges may not be separable. Nor is it the case that boundary layer mixing is predominantly the tail of a cascade from larger submesoscale eddies. Here the prevalence and effects of symmetric instability were greatly reduced when Stokes forcing was added, confirming that large-scale flow and Ekman-driven overturning are not the only mechanisms for generation of small-scale turbulence.

Overall, many questions are raised by the descriptive approach taken here. Perhaps the most important question concerns how representative these simulations are of realistic ocean mixed layers. Since submesoscale fronts can spin down via submesoscale eddies alone in this configuration, as they do in Boccaletti et al. (2007) and Fox-Kemper et al. (2008), there is no requirement for a forward cascade or equilibration mechanism. It is likely that in quasi-persistent fronts, such as those studied by Thomas et al. (2013) and D’Asaro et al. (2011), greater coupling is required between large and small scales. These simulations are very likely in the weak coupling regime, owing to the weak winds, mixing, and fronts chosen to satisfy the numerical constraints in this scenario. It should also be noted that both Li et al. (2012) and the present study have found evidence that submesoscale vertical transport is in fact strengthened by Langmuir turbulence, and additional work is required to determine the dynamical origins of this effect.

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