Centrifugal Instability and Mixing in the California Undercurrent

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ABSTRACT

A regional numerical study of the California Current System near Monterey Bay, California, is conducted using both hydrostatic and nonhydrostatic models. Frequent sighting of strong anticyclones (Cuddies) have occurred in the area, and previous studies have identified Monterey Bay as an apparent region of strong unbalanced flow generation. Here, by means of a downscaling exercise, a domain just downstream of Point Sur is analyzed and argued to be a preferred site of diapycnal mixing. The scenario suggested by the simulations involves the generation of negative relative vorticity in a bottom boundary layer of the California Undercurrent on the continental shelf break. At Point Sur, the current separates from the coast and moves into deep waters where it rapidly develops finite-amplitude instabilities. These manifest as isopycnal overturnings, but in contrast to the normal Kelvin–Helmholtz paradigm for mixing, this study argues that the instability is primarily centrifugal. The evidence for this comes from comparisons of the model with linear results for ageostrophic instabilities. Mixing increases background potential energy. The authors argue the regional potential energy generation near Point Sur in the upper few hundred meters is comparable to that found in open-ocean regions of strong diapycnal mixing, either by abyssal tides and lee waves near topography. This study computes diapycnal fluxes and estimates turbulent diffusivities to argue mixing by centrifugal instability is characterized by diffusivities \(O(10^{-4})\) m\(^2\) s\(^{-1}\), although the potential for contamination by explicit diffusivities exists.

1. Introduction

The California Current System is composed of several elements, including the California Undercurrent (CUC), which is the focus of this work. The CUC consists of a poleward current running along the California continental slope, nominally at depths of 50–400 m. It transports warm and relatively salty waters to the North Pacific Subpolar Gyre and is a significant contributor to regional hydrographic properties. The CUC has been measured several times now and is characterized by speeds of 0.1–0.3 m s\(^{-1}\). It is also important in eastern ocean basin variability, acting as the apparent source of intense, subsurface, submesoscale anticyclones known as “Cuddies” (Simpson and Lynn 1990; Garfield et al. 1999) and contributes to the formation of the eastern Pacific “jets” and “squirts” (Davis 1985; Flament et al. 1985; Kosro and Huyer 1986).

The topography of the California coast consists of a relatively narrow shelf of a few tens of kilometers followed by a steep drop with slopes as great as 25%. The continental slope is “rugged” with numerous deep canyons and ridges along the break. A bathymetry of the Monterey Bay appears in Fig. 1 that illustrates the above points. Also, canyons, promontories, ridges, and slopes are common features of continental boundaries, suggesting our analysis here will apply to several locations where the mean flows are in the direction of topographic waves.

We are drawn to the CUC given an interest in the possible role of steep topography as a control on open-ocean dynamics and mixing. Indeed, it has long been postulated that the boundaries play a key role in the mixing of tracers...
[Armi (1978) is an early reference; see also Munk and Wunsch (1998) and Ferrari and Wunsch (2009)] and their role in dissipation has recently gained considerable attention (Dewar and Hogg 2010; Zhai et al. 2010; Wright et al. 2012; Molemaker et al. 2015, hereinafter M15). The CUC, situated on steep slopes and with a moderate and persistent undercurrent flowing in the direction of Kelvin and topographic waves, is an ideal place to conduct detailed studies of balanced flow–topographic interactions in geophysically relevant settings.1

Such was the motivation behind M15, who conducted a downscaling study of the CUC, ending with an embedded simulation at a horizontal resolution of 166 m centered on the Monterey Bay Canyon. The study was primarily numerical and used the Regional Ocean Modeling System (ROMS) model (Shchepetkin and McWilliams 2005; and references therein). It was argued in M15 that the interaction between the turbulent bottom boundary layer upstream of Point Sur and the CUC generated extreme and large negative relative vorticities well in excess of the local Coriolis parameter. Thus, the absolute vorticity in the boundary layer was of a sign opposite to that in the interior where the Coriolis parameter dominated. Stratification in the boundary layer was nonetheless found to be stable. There were at best weak indications of flow arrest in the model, as characterizes the so-called slippery bottom boundary layers of MacCready and Rhines (1991, 1993) and Trowbridge and Lentz (1991). There are several reasons for this; primary causes include the steep slopes and highly variable nature of the CUC. In any case, the potential vorticity (PV) along isopycnals and on geopotentials changed sign in the region but for the reasons of absolute vorticity sign change, rather than unstable stratification. As pointed out by Hoskins (1974), a change in the sign of potential vorticity is a sufficient condition for instability in an unbounded fluid.

It was further argued in M15 that the CUC along the coast experiences frequent separations from the bottom, mostly at points of strong topographic curvature. At such locations, the change in the direction of bathymetric curves appears to be too severe for the current to successfully negotiate in a balanced way. There are several such promontories on the eastern Pacific continental slope, a representative one being Point Sur just upstream of Monterey Bay (see Fig. 1). CUC separation at Point Sur was a routine event in M15 and resulted in the injection of dynamically unstable but stably stratified fluid into the open ocean.

2. Objectives of the present study

The simulations in M15 were obtained using a hydrostatic model (ROMS) that could only partially comment on the regional flow development. Accordingly, the focus of the present study is to examine the detailed regional flow dynamics through the application of a nonhydrostatic model, the MITgcm (Marshall et al. 1997). The strategy adopted here is to use the flow solutions generated by ROMS to set the boundary conditions of an embedded implementation of the MITgcm. Choosing the southern boundary of the embedded domain to coincide with the apparent location of the CUC separation, the inflow is well described by the hydrostatic dynamics of the ROMS model. The size of the domain was chosen to capture the area in which unstable potential vorticities were predicted by the ROMS solution. Model resolution was set to 55 m in the horizontal and 5 m in the vertical throughout the jet. The role of nonhydrostatic dynamics is further investigated through comparisons of the results with comparable hydrostatic implementations of the MITgcm, all other aspects of the model deployment held fixed.

We have analyzed these solutions with a view toward tracer mixing and balanced flow dissipation. Classical results argue that the primary energy source for flow instabilities in a stably stratified fluid with a negative vertical absolute vorticity is the lateral shear, as opposed to the vertical, geostrophic shear. We will adopt the terminology of Thomas et al. (2013), referring to the former scenario as “centrifugal instability (CI),” to

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1 The interactions of unbalanced flows and topography have recently been reviewed by MacKinnon et al. (2013).
differentiate it from “symmetric instability (SI)” arising when potential vorticity changes sign because of vertical shear. Direct numerical evidence of isopycnal inversions as part of the finite-amplitude character of the instability is provided along with support for the proposition that the fundamental instability is centrifugal. Conversions from kinetic to mean potential energy at the depths of the CUC are argued to average $O(0.5) \text{mW m}^{-2}$ with local maxima reaching 3.4 mW m$^{-2}$. The rates computed here are comparable to those estimated in M15.

Modern global energetics budgets routinely estimate the flux of kinetic energy to small scales needed to maintain the observed stratification at roughly 6 mW m$^{-2}$. Of this, 1.0 mW m$^{-2}$ is thought to create the potential energy stored in the mean stratification. This comparison argues that centrifugal instability in the near-surface Monterey Bay is an important, and perhaps dominant, contributor to the regional mixing climatology. The area around Monterey Bay is typical of the broader eastern North Pacific continental slope, and the CUC is found from the Baja Peninsula to the southern border of Alaska. This prompts the speculation that comparable mixing is found broadly along the North American west coast. We also note comparable poleward currents and topography are found off western South America (Brockmann et al. 1980). Last, the mixing occurring here is because of a different mechanism than the Kelvin–Helmholtz shear-driven overturning normally associated with internal wave breaking.

3. Model description

The domain appearing in Fig. 1 is the smallest embedded domain described in M15. That domain was inside of successively finer regional models with resolutions of 4, 1.5, and 0.5 km; the reader is referred to M15 for technical details. Also shown in Fig. 1 is the border of the embedded model used in the present study. The most important aspect of the current domain is that it was situated downstream of the separated CUC throughout the computational period. We perform both hydrostatic and nonhydrostatic calculations in this domain with all parameters between them identical save for the vertical momentum equation. In the analysis of the numerical results appearing later, we will discuss the nonhydrostatic results unless stated otherwise.

The resolution of the resulting domain was 55 m in both horizontal dimensions. Boundary data for the embedded model were obtained by straightforward interpolation of the numerical results in M15 and applied every 3 h. A relaxation toward the boundary values was imposed on the outer 20 grid points with time scales ranging from 3 h at the boundary to 2 days about 0.5 km inside the boundary. The finescale domain was modeled as an $f$ plane, with a local Coriolis parameter of $f = 0.864 \times 10^{-4} \text{s}^{-1}$, characteristic of the latitude of Monterey Bay (36.4°N). The same topography as used in M15, interpolated to the finer grid, was employed here. The vertical resolution of the nonhydrostatic model was uniformly 5 m from a depth of roughly 100 m to more than 600 m. The thickest shallow-water grid cell was 10 m and in the deeper water was 150 m. The poleward flow of the CUC is entirely contained between the depths of 50 and 450 m, both in observations and in all of the above-mentioned models.

The present hydrostatic and nonhydrostatic simulations used a simple relaxation with a time scale of 3 h to the interpolated SST and sea surface salinity fields computed as part of the M15 solutions. Vertical diffusivities and viscosities were set to $10^{-4} \text{m}^2 \text{s}^{-1}$ and horizontal values to $1.5 \text{m}^2 \text{s}^{-1}$. Model time steps were 15 s, and the model was run for 17 days, corresponding to roughly 22 inertial periods. Output of all dynamical variables was archived at intervals of 30 min for the first 6.25 days and at intervals of 90 min thereafter. The first 18 h were not used in any of the following analysis. Dynamical variables at every time step were archived at 20 individual model points within the domain over the first 6.25 days as an aid in assessing temporal aliasing. Most of our quantitative analysis comes from the higher sampled period, as it was found the time scales of the instabilities were very short and evidence of temporal aliasing was found in the 90-min data. The latter did prove useful for qualitative purposes.

As a proof of concept, a comparison is given in Fig. 2 of the absolute vertical vorticity at 127.5 m taken directly from the ROMS solution of M15 and from the present downscaled exercise. The ROMS data have been interpolated to match the resolution of the nonhydrostatic run, and the times of the plots are identical. The evolution of the vorticity is that it is introduced at the boundaries from the specified conditions, but once inside the model interior evolves according to the model dynamics. The regional mean flow is to the northwest at $\sim$0.2 m s$^{-1}$ and advects the vorticity downstream. It is clear that there is value in the downscaling procedure. The vorticity has been introduced with little distortion and its evolution is clearly better behaved, consistent with resolved explicit dynamics. We mention that the smoothness of the solution shown here is sensitive to the subgrid-scale parameterization that, in turn, is Laplacian diffusion and viscosity at horizontal values of $1.5 \text{m}^2 \text{s}^{-1}$ and vertical values of $10^{-4} \text{m}^2 \text{s}^{-1}$. Experiments at smaller viscosities/diffusivities returned noisier results with structure closer to the grid scale.
4. Theoretical analysis

In an insightful paper, Hoskins (1974) demonstrated a number of useful aspects of the instabilities of vertically and horizontally sheared, stratified flow. Assuming a flow $V(x, z)$ uniform in the meridional direction and with constant lateral and vertical shears, he showed the normal-mode dispersion relation governing nonhydrostatic disturbances in the zonal, vertical plane is given by

$$s^2 = N^2 \sin^2 \phi - 2s^2 \sin \phi \cos \phi + f \zeta \cos^2 \phi,$$  \hspace{1cm} (1)

where $s$ is the (possibly complex) frequency of the disturbance tilted at an angle $\phi$ with respect to the horizontal. The quantity $\zeta = f + V_x$ denotes the vertical component of the absolute vorticity of the flow and $N^2 = B_z$ denotes the usual buoyancy frequency. Vertical shear is related to the lateral gradient of buoyancy by thermal wind, that is, $s^2 = fV_z = B_x$. The maximum growth rate as a function of $\phi$ occurs for

$$\tan 2\phi = \frac{2V_z f}{N^2} \left[ 1 - \frac{f}{N^2 (f + V_x)} \right]$$  \hspace{1cm} (2)

from which it can be shown that $\sigma$ is purely imaginary if the Ertel potential vorticity

$$q = (f + V_x)N^2 - fV_z^2$$  \hspace{1cm} (3)

is negative. The final contribution to $q$ in the above is associated with the so-called twisting terms and represents the correction to the vertical component of relative vorticity needed to compute relative vorticity on a density surface. Ertel potential vorticity can be negative for an unstable density distribution consistent with gravitational instability. Stable density gradients can also be associated with negative potential vorticities for either strong vertical or lateral velocity shears. The case where the vertical shear destabilizes potential vorticity will be referred to here as symmetric instability (see also Thomas et al. 2013). Recent studies argue that symmetric instability has been observed in both the Kurono (D’Asaro et al. 2011) and the Gulf Stream (Thomas et al. 2013). The classical paradigm for potential vorticity destabilization by horizontal shear is usually taken to be a barotropic fluid, although unstable normal modes also occur for stratified fluids, as seen in (1). For the CUC, vertical shear is normally less than 10$f$, while outside of the surface mixed layer $(f^2/N^2) \ll 1$. We thus expect from (2) that $\phi \ll 1$ for unstable modes in the CUC. This implies the vertical wavenumber of an instability will be much larger than the horizontal wavenumber or that the instability will resemble a stack of thin overturnings.

For convenience, we will write the horizontal shear as $V_x = -\alpha f$ ($\alpha > 1$ for supercritical shears) and the vertical shear as $V_z = \beta f$. Expanding the right-hand side of (2) for $(\beta^2/N^2) \ll 1$ shows that the angle $\phi$ is slightly smaller than the isopycnal slope.

2 It is also possible to extend Hoskins’ analysis to include both components of the Coriolis parameter (Wirth and Barnier 2008). The result is comparable to the classical result and for the subsurface, centrifugal instability is only modestly affected by the other Coriolis component.
\[
\phi = \frac{B_x}{B_z^2} \left[ 1 + \frac{f^2}{B_z^2} (1 - \alpha) \right],
\]

provided \(\alpha > 1\), that is, that the flow meets the conditions for centrifugal instability. The fastest growing modes cross isopycnals from more buoyant to less buoyant fluid.

Taking the analysis in Hoskins (1974) further, an equation for perturbation kinetic energy is formed from the nonhydrostatic equations of motion linearized about the mean flow:

\[
(u^2 + v^2 + w^2)_t = -\nabla \cdot (\rho u) - uwV_x - vwV_z + wb,
\]

where \((u, v, w) = \mathbf{u}\) denotes the perturbation velocities, \(b\) is the perturbation buoyancy, and meridional variations are assumed to vanish. The linearized buoyancy equation can be used to eliminate the kinetic–potential energy exchange \(wb\), yielding the full energy equation

\[
(K + P)_t = -\nabla \cdot (\rho u) - uwV_x - vwV_z - ub\frac{B_x}{B_z},
\]

where \(K\) is the kinetic energy and \(P\) is the potential energy, \(P = [\beta^2/(2N^2)]\), illustrating that the sources of perturbation energy are lateral shear (HS), vertical shear (VS), and sloping buoyancy surfaces (BU). Following Hoskins (1974), the perturbation quantities can be expressed in terms of a streamfunction \(\psi(x, z)\) in the vertical-zonal plane. Integrating (5) in time and with a little algebra, the source terms can be computed in terms of the mean flow parameters:

\[
\begin{align*}
\text{HS} &= -\Gamma uwV_x = \cos\phi[\beta \sin\phi - \cos\phi(1 - \alpha)]\alpha \\
\text{VS} &= -\Gamma wV_x = \sin\phi[\beta \sin\phi - \cos\phi(1 - \alpha)]\beta \\
\text{BU} &= -\Gamma ub\frac{B_x}{B_z} = \beta \cos\phi \left( \frac{\beta^2}{N^2} \cos\phi - \sin\phi \right),
\end{align*}
\]

where \(\Gamma = |\sigma|/(f^2k^2|\psi_i|^2 e^{2\sigma|i|})\) is a normalizing factor, \(k\) is the wavenumber amplitude, and \(\psi_i\) is the imaginary part of the streamfunction. In the above, shears have again been written as \(V_x = -af, V_z = \beta f\), so that \(\alpha > 1\) denotes an absolute vertical vorticity of sign opposite to the Coriolis parameter. The frequency \(\sigma\) is guaranteed to be purely imaginary, and thus nonpropagating, provided \(f\sigma < 0\).

Equation (6) can be used to compare the roles of the various perturbation energy sources as a function of the mean flow parameters \(\alpha\), \(\beta\), and \(N^2\). Stratifications in the CUC region are \(N^2 < O(10^{-4}) \text{s}^{-2}\), and for mid-latitudes, typical values for \(\beta\) run between 0 (no shear) and 10 (strongly sheared). The CUC is described roughly by \(\beta\) between 5 and 10. For such parameters, the linearly unstable growth rates in (1) are typically several hours. Figure 3 shows a contour plot of the ratio \(\text{VS}/\text{HS}\) computed at the most unstable mode for vertical shears between \(\beta = 0\) and 10 and nondimensional potential vorticities between \(-3 < q/(fN^2) < 0\). Only values for lateral shears with \(\alpha > 1\) are contoured. The contours are not at uniform intervals in order to show the structure of the ratio, which is everywhere in this parameter regime smaller than 1. In fact, the ratio is often significantly less than this, implying that lateral shears strong enough to reverse the sign of the absolute vorticity dominate the energy budget of the instability.

Figure 4 plots HS, VS, and BU for the two transects through the \([\beta, q/(fN^2)]\) plane at \(q/(fN^2) = -1.5\) and \(\beta = 7.5\) appearing in Fig. 3. Both are values characteristic of the separated CUC. Also in those plots are curves of \(\alpha\), showing the regions over which the lateral shear is supercritical. The entire range for the former plot consists of supercritical lateral shears, and dominance of the lateral shear-driven instability power source is evident. Note also that BU, the direct release of potential energy, is negligible over the range. This is consistent with the result that the angle of the fastest growing mode relative to that
of the isopycnals is very small. The lateral shears are supercritical over most of the domain in the latter plot, but drop below $\alpha = 1$ near the instability threshold. Again, BU is negligible everywhere. The key result is the general dominance of the horizontal, shear-driven power sources for $\alpha > 1$.

These plots illustrate the nature of symmetric and centrifugal instabilities. Both are instabilities based in potential vorticity dynamics but have clear and fundamental dynamical distinctions. Once $\alpha$ grows beyond one, that is, once the absolute vorticity changes sign, energy release to the instability is dominantly from the horizontal shear. We will use this as a signature of CI, as compared to SI, in the following diagnostics.

## 5. Background potential energy

To accurately quantify mixing requires an extension of background potential energy as outlined in Winters et al. (1995, hereinafter W95). The derivation in W95 assumed a linear equation of state and must be generalized as outlined below to apply to a seawater equation of state.

According to Young (2010), a consistent local potential energy per unit mass for a seawater equation of state is given by the so-called dynamic enthalpy:

$$h(\Theta, S_A, z) = \int_0^z b(\Theta, S_A, \lambda) \, d\lambda, \quad (7)$$

where $\Theta, S_A$ are the tracers Conservative Temperature and Absolute Salinity; $b = [-g(\rho - \rho_w)]/\rho_w$ is buoyancy; and where in the performance of the integral, the tracer values are held fixed at their values at depth $z$. As argued in Young (2010), energy conservation for the Boussinesq equations requires that density be evaluated using static pressures, indicated symbolically in (7) by $z$. Available potential energy is that energy stored in the slopes of isopycnals and is measured relative to the potential energy of a hypothetical background state in which the fluid is adiabatically leveled. Referring to the leveled depths by $z^*$, the form of (7) for the background potential energy (BPE) becomes

$$h(\Theta, S_A, z^*) = \int_{z^*}^0 b(\Theta, S_A, \lambda) \, d\lambda. \quad (8)$$

We have used an iterative sorting procedure to find $z^*$ that mirrors the technique used in W95. First, we select a subdomain of the region (here, we worked with the interior domain 50 grid points away from the boundaries and from the surface to 500 m, in this way avoiding boundary contamination and focusing on the depth range of the instability). The fluid buoyancy at any one time in this domain is sorted from highest to lowest, moving parcels vertically in the process. The sorted buoyancies are then recomputed using the equation of state evaluated at the new static pressures. In contrast, the absolute temperatures and salinities are

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3 Young refers to $h$ as dynamic enthalpy instead of potential energy for technical reasons. We will adopt both terms for $h$ in this paper.
unchanged, in keeping with the search for an adiabatically leveled state. The recomputing normally results in a statically unstable profile, so the fluid is sorted again and the procedure is repeated until the locally pressure-referenced profile is everywhere stable. It is our experience that several tens of iterations are required to converge. This is not the only way to adiabatically level a fluid given a pressure-dependent equation of state but is a rational choice and if used consistently will yield a defensible statement of potential energy creation.

Again, following W95, all variables, including $z^*$, are considered functions of initial location. The question of the local contribution to background potential energy by diabatic processes is addressed by evaluating

$$h_t[\Theta(x, t), S_A(x, t), z^*(x, t)] = -z^*_t b(\Theta, S_A, z^*) + h_\Theta \Theta_t + h_{S_A} S_A,$$  \hspace{1cm} (9)

where the time derivatives of the tracers and of $z^*$ are taken at the unleveled fluid location $x$. The total contribution to background potential energy is given by the volume integration of (9) over the leveling domain.

The evolution equations for the tracers consist of advective and diffusive parts; the latter is the only contribution that contributes irreversibly to the potential energy field. Retaining it, the diabatic background potential energy change is

$$z^*_t(x, t) = \frac{1}{A'} \int_{V'} z^*_t(x', t) \delta[z^*(x', t) - z^*(x, t)] \, dx' - z^*_t(x, t) \frac{1}{A'} \int_{V'} \delta[z^*(x', t) - z^*(x, t)] \, dx',$$  \hspace{1cm} (13)

where

$$\delta(x) = 0; \quad x < 0, x > 0;$$

$$\int_{-\infty}^{\infty} \delta(x) \, dx = 1$$  \hspace{1cm} (14)

is recognized as the usual Dirac delta function. If the first right-hand side quantity in (10) is now area integrated over the leveling domain, there results

$$\int_V b(x, y, z^*) z^*_t(x, t) \, dx$$

$$= \frac{1}{A} \int_{V'} z^*_t(x', t) \delta[z^*(x', t) - z^*(x, t)] \, dx' \, dx$$

$$- \frac{1}{A} \int_{V'} b^* z^*_t(x, t) \int_{V'} \delta[z^*(x', t) - z^*(x, t)] \, dx' \, dx.$$

The order of integration can be reversed in the first right-hand side integral. The delta function picks out all leveled depths equal to a fixed value. Buoyancy when evaluated at those leveled depths then becomes a constant independent of location $x$. Equation (15) is thus equivalent to

$$\int_V b(x, y, z^*) z^*_t(x, t) \, dx$$

$$= \frac{1}{A} \int_{V'} b^* z^*_t(x', t) \int_{V'} \delta[z^*(x', t) - z^*(x, t)] \, dx' \, dx$$

$$- \frac{1}{A} \int_{V'} b^* z^*_t(x, t) \int_{V'} \delta[z^*(x', t) - z^*(x, t)] \, dx' \, dx,$$  \hspace{1cm} (16)

which clearly vanishes.

In summary then, the complete diabatic change in background potential energy is effectively
The underlined horizontal derivatives are on a surface of constant $z^*$ (denoted by the symbol $\nabla z^*$) and cancel.\footnote{In some ways, this is the definition of a constant $z^*$ surface.} The sorting procedure we use insures that the vertical buoyancy profile derivatives in (22) are positive definite, and hence the first terms on the right are always greater than zero.\footnote{A comparison between (22) and its form for a linear state equation as in W95 appears in the appendix.} Note that the remaining terms are not necessarily positive. Nonetheless, the numerical values of (22) are dominated (at least in the present evaluation) by the first quantity, showing that the effect of mixing is to increase the background potential energy. The quantity in (22) is the local mixing contribution to BPE, and a volume integration of it over the leveling domain yields the total diabatic contribution of BPE.

Equation (22) corresponds to the diabatic contribution to the turbulent potential to kinetic energy conversion in the full equation for dynamic enthalpy:
\[
\frac{dh}{dt} = -wb + h^o = -w'b_{\text{diab}} - wb_{\text{reversible}} + h^o
\]
\[
= (h^o)_{\text{diab,local}} - wb_{\text{reversible}} + h^o \quad \text{(23)}
\]

where \(h^o\) denotes molecular diffusive effects that in the present case are neglected.

The results in (22) and (23) will be used later.

6. Numerical analysis

The linear instability analysis of section 4 provides a context within which to analyze the numerical results, although the mean flow structure in the model is not as simple as the two-dimensional, uniformly sheared, analytical model. We have therefore subjected the numerical results to a series of tests to determine the relevance of symmetric/centrifugal instability and clarify the dynamics.

In all the analysis of the numerical results reported here, we examined a subdomain of the full domain that was 50 points inside the open boundaries in an attempt to minimize any boundary contamination.

We begin by categorizing model potential vorticity. The key for the preceding instability analysis was negative potential vorticity in combination with a stable buoyancy gradient. This characterizes our numerical results, as evidenced by the scatterplots of potential vorticity and buoyancy frequency in Fig. 5 (upper) and of potential vorticity and normalized absolute vorticity \((\nu_x - u_y + f)/f\) in Fig. 5 (lower). The time of the plot corresponds to 5.75 days into the simulation. Potential vorticities of both signs are present in both panels. The upper panel illustrates the majority of negative PVs are associated with stable stratification (sector CI, SI), although a small number of points exhibit unstable density profiles (sector convection, left). The model shows similar distributions at other times, consistent with the fluid being dynamically unstable while statically stable. (bottom) PV (x axis) vs absolute vorticity divided by the local Coriolis parameter \((\nu_x - u_y + f)/f\). The SI, CI stable sector is the most populated but is followed by the CI and SI unstable sector characterized by negative PV and absolute vorticity.
Thus, the conditions needed for symmetric and/or centrifugal instability are present in our calculation and clearly extend well into the regime where centrifugal instability is possible.

Last, we qualitatively characterize the vertical structure of the supercritical relative vorticities in Fig. 7. The upper panel presents the fractional area of the domain away from the boundaries on average covered by supercritical vorticities from the last 5.5 days of the run. There is a rapid growth in the presence of strong negative relative vorticity moving down from 50 m and peaking sharply around 100 m. This is followed by a small drop in occurrence from 100 to 200 m, beyond which there is a noisy, but decidedly monotonic, decay. No supercritical conditions were observed for depths greater than about 450 m. The lower panel shows the average value of the supercritical vorticities. This is a conditional average based on the presence at a given location and time of relative vorticity lower than \( -f \) and should be related to the growth rate of the instability as a function of depth. Again, the signal strengthens moving down from 50 m and peaks slightly below 100 m. The global extremum appears around 175 m, followed by a persistent, noisy decay to 450 m.

To facilitate comparison with the instability analysis, we have defined a mean model geostrophic flow by differentiating the hydrostatic pressure averaged over the duration of the run, after discarding the first 18 model hours because of contamination by the initiation shock. A plot of the mean geostrophic speed at a depth of 152.5 m appears in Fig. 8. The current at this and the surrounding few hundred meters depths is oriented \(-40^\circ\) counterclockwise from north and is nearly unidirectional. The flow does, however, exhibit some degree of downstream variation. A typical current speed in the flow is 0.2 m s\(^{-1}\), implying fluid parcels remain in the domain for something like 12 h, which is comparable to the \( e \)-folding time scale estimated by the linear instability analysis.

Writing the full velocity as this geostrophic flow and a perturbation and linearizing the horizontal momentum equations in the perturbation yields

\[
\begin{align*}
\partial_t u + U_x u + u U_x + V_y + w U_z - f v &= -p_x \\
\partial_t v + U_x v + u V_x + V_y + w V_z + f u &= -p_y,
\end{align*}
\]

(24)
where \((U, V)\) represent the geostrophic flow, \((u, v, w)\) are the velocity corrections, and the geostrophic balance has been removed. Note that the vertical velocity in (24) is the total vertical velocity as the mean geostrophic flow is purely horizontal. Forming the kinetic energy of the perturbation from (24) identifies

\[
SE = -uu \cdot VU - uv \cdot VV
\]  

(25)

as the sources for perturbation energy (SE) provided by the mean velocity shear. The quantities in (25) are analogs of the sources in (4) but reflect the slightly more complicated flow geometry.

The quantities in (25) appear in Fig. 9. They have been averaged in time over the last 5.5 days of the run and horizontally over a region centered on the jet. The velocities have been rotated to along- and across-stream coordinates, \(y\) and \(x\) respectively, as defined by the flow at 150 m seen in Fig. 8. The orientation of the mean flow is largely insensitive to depth throughout the 50–400-m range. There are four contributors to the horizontal shear source corresponding to combinations of perturbation momentum fluxes and mean flow gradients and two for the vertical source. All of them are compared individually in Fig. 9.

The sign of the various horizontal members varies depending on which it is, but clearly the biggest contribution, by far, is that due to the momentum flux in the direction of the cross-stream mean shear. Further, this term is positive, consistent with a release of energy from the mean flow. The vertical shear components are dominated by the downstream shear contributions, as expected. Note, however, that the amplitude of the vertical shear components is considerably smaller, by roughly an order of magnitude, than the horizontal shear contribution for the depth ranges from ~50 to ~300 m. At deeper levels, all the components decay to small values as might have been anticipated from the absence of supercritical potential vorticities. The net horizontal and vertical contributions are compared in Fig. 10 and again emphasize the dominance of the horizontal processes. This is a signature of centrifugal instability.

We interpret the above evidence to support the hypothesis that the flow becomes unstable dominantly to centrifugal instability. The numerical calculation of course proceeds into the nonlinear regime and invites diagnoses of the finite-amplitude behavior. Our suspicion is that the instability promotes mixing, and we ask if evidence of diapycnal exchange appears.

Unstable density gradients, evident in Fig. 5, indicate overturning; this is also seen in Fig. 11, which is a time series of potential density at the marked point in Fig. 8 directly downstream of the separated current and in a location where negative vertical absolute vorticities frequently appear. The data for this plot have been archived at every model time step and show an interval from 3.5 to 5.75 days of the integration. Note that several ~25-m
overturning events appear in the depth range from 75 to 100 m and at depths greater than 150 m. It is also evident that the activity is independent of surface convection. At least three events occur in the near-surface region over this interval, arguing that overturning frequency is comparable to and perhaps slightly higher than once per day. Our 17-day simulation supported this estimate.

The obvious presence of overturning in this plot motivates us to consider more quantitative measures. These will be computed relative to a time-averaged buoyancy field obtained from the last 5.5 days of the 6.25-day experiment.

The time-averaged buoyancy field is governed by

$$\overline{b} + \mathbf{u} \cdot \nabla \overline{b} = \overline{H},$$

where $\overline{b}$ is averaged buoyancy, and $\overline{H}$ represents the averaged effects of both adiabatic and diabatic buoyancy fluxes:

$$\overline{H} = -\nabla \cdot \mathbf{u} \overline{b_{ad}} - \nabla \cdot \mathbf{u} \overline{b_{di}} + \text{(molecular)}.$$  

We are primarily interested in the contribution of the diabatic flux divergence, also known as entrainment. Molecular effects will be ignored throughout. Division of (26) by $\overline{b}$ yields

$$-\nabla \cdot \mathbf{u} \overline{b_{z}} + \overline{w} = \overline{\mathcal{H}} \overline{z_{b}} = \overline{\sigma} - \nabla \cdot \mathbf{u} \overline{b_{z}} \overline{z_{b}},$$

where horizontal derivatives are now taken along the mean buoyancy surfaces and the quantity $\overline{z_{b}}$ represents the associated “thickness” field. The $\overline{b}$ surfaces are almost horizontal, suggesting that the entrainment is well approximated by

$$\overline{e_{z}} \approx -\overline{\overline{w} \overline{b_{z}}_{\text{di}}},$$  

which is how entrainment will be estimated in the following.

Equations (22) and (23) provide formulas from which mean entrainment and turbulent work on the mean stratification can be computed. We have evaluated these formulas using the model tracer output every 30 min. The data were fed to the MATLAB package EOSlib that returned the temperature and salinity contraction coefficients, and these were differentiated in $\Theta, S$ space in order to compute the integrals. A plot of the mean diapycnal flux at a depth of roughly 100 m appears in the upper panel of Fig. 12, from which it is seen that the northeast region of the separated jet is distinct in this measure relative to the other more quiescent areas to the south and west. The amplitudes of the diapycnal velocity are large, with extrema around $10^{-5}$ m s$^{-1}$ and thus an order of magnitude greater than the vertical velocities normally associated with Ekman pumping. Entrainments this large and of both signs are seen in the jet and have lateral scales of a few kilometers. Here, as in the vorticity plot, the smoothness of the entrainment estimate is controlled by the viscosities and diffusivities. In an experiment with horizontal and lateral viscosities/diffusivities reduced by factors of 5 and 2, respectively, enhanced entrainments in the jet were found, but the structure was considerably noisier. The
same quantity computed from the hydrostatic run appears in the lower panel of Fig. 12, and the amplitudes and distributions of the diapycnal flux are seen to be similar.

Figures 13a–e present measures of the regional water mass conversions, that is, the spatially averaged $\overline{\varepsilon_d}$ values as a function of depth over the northeast and southwest domains appearing in Fig. 12 (lower). The three depths indicated by the dotted black lines in Fig. 13a are referred to in Figs. 13b–e. The same plot for the hydrostatic case is virtually indistinguishable.

The surface 50–100-m range is characterized by strong upward entrainment in the jet and weak entrainment in the southwest (see Figs. 13a–e). In fact, with the exception of 100 m, southwest entrainment is generally weak. At this same depth, the jet entrainment is at its extreme and negative. A nonzero signal in the mean between the depths of 120–180 and 220–400 m is not obvious; however, at all depths, the jet region is considerably more active. This appears in Figs. 13b–e where time series of entrainments at the depths indicated in Fig. 13a are plotted. The near-surface entrainments (Figs. 13b–e) argue the distinction between the regions is persistent throughout the experiment. Downward, rightward sloping lines in Fig. 13a indicate regions of mass accumulation within density surfaces, and this seems to describe the 80–120-m zone. There is the suggestion that the jet at 200 m is preferentially entraining upward. The depth interval between has no clear mean signal, but at all those depths, the entrainment time series (not shown) look like Fig. 13c but are more centered about zero. Considerable mixing occurs in that range, but there is no preferred direction. The overall entrainment sense suggests the depths from 100 to 200 m are regions of mass loss from density layers. The deepest times series, 252 m, again favors the jet for activity but is weaker in the mean. In summary, we argue entrainment provides strong evidence in support of direct mixing in the jet where centrifugal instability has been identified.

It is interesting that the hydrostatic model apparently does an excellent job of capturing this behavior. The ROMS model employs no explicit diffusivity, relying instead on their advective scheme discretization error that resembles a hyperdiffusivity and a KPP parameterization in the event of an unstable buoyancy profile. Both the hydrostatic and nonhydrostatic models generate considerable small-scale structure and inversions, and the subgrid schemes in both act to remove that variance. This is a likely explanation for the quantitative skill that ROMS shows in mixing. It is also encouraging as it shows hydrostatic models with KPP can be usefully employed in nonhydrostatic regimes and lessens the need for the overhead associated with nonhydrostatic dynamics. Of course, these are very finescale solutions, and it is unlikely that ocean general circulation models will run with anything close to them in the near future, so parameterization will still be required. Elevating diffusivities in regions of negative absolute vorticity may prove useful for this purpose.

Diapycnal entrainment requires energy as work is performed against gravity to elevate dense waters and deepen light waters. The result is an elevation of the center of mass, or equivalently an increase in mean potential energy, so we now discuss estimates of the work against the background stratification.

Instantaneous and time-averaged plots of turbulent production $-w'\overline{b_{\text{diab}}}$, as computed by (22) at a depth of 100 m, appear in Fig. 14. The upper plot is from day 6 of the integration and the lower is an average over the last 5.5 days. The instantaneous amplitudes routinely are $O(10^{-7})$ W kg$^{-1}$ in the jet and are considerably smaller.
elsewhere as might have been expected from the entrainment. The net positive value for the production also clearly differentiates the process from other adiabatic instabilities, such as classical baroclinic instability, in which the typical exchange is from potential to kinetic. The model has in it an explicit, subgrid-scale diapycnal diffusivity with the relatively large (from an oceanographic perspective) value of $10^{-2} \text{m}^2 \text{s}^{-1}$.

We show in Fig. 15 plots of the mean potential energy (PE) production in the northeast and southwest model sectors unaccounted for by the background diffusion. This was obtained by multiplying the regional mean buoyancy frequency by the explicit diffusivity and subtracting it from the model PE production. A small residual is found in the more quiescent southwest region, whereas the jet region consistently exhibits more PE production than the background production. This is consistent with an energy source available for mixing in the jet, which we

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6 The negative values in the southwest are indicative of the scatter in the calculation caused by the choice of reference density profile. The comparison seen there is stable when differing choices are made.
interpret as the effect of CI. PE production in the jet also exhibits the same character as entrainment, in that the activity in the jet is always considerably higher than that in the southwest. The net, residual PE production in the jet, $10^{29} W k g^{-2}$, is smaller than but comparable to the $10^{29} W k g^{-2}$ background potential energy generation rate inferred by Ledwell et al. (2000). This assumes a 20% efficiency acting on the ($\sim 0.5 \times 10^{-4} m^2 s^{-1}$) dissipation levels they observed. As a cautionary note, our resolution of 55 m is also quite fine by general circulation model standards but still relatively coarse with regards to the turbulence that will drive the mixing. Our cell Reynolds numbers are

$$Re = \frac{UL}{\nu} = \frac{(0.3 \text{ m s}^{-1})(55 \text{ m})}{(1.5 \text{ m}^2 \text{ s}^{-1})} = O(10),$$

and so our solutions are too strongly damped. This might cause our PE generation estimates to be too conservative.

Standard parameterization practice is to equate the diabatic vertical buoyancy flux to an “eddy diffusivity” coefficient via

$$\overline{w' B'}_\text{diab} = -K_v N^2. \quad (30)$$

The mean buoyancy frequencies in the upper few hundred meters of the CUC average $\sim 4 \times 10^{-5} s^{-2}$, with stronger stratification above 100 m and decreasing values below. The turbulent diffusivities $K_v(z)$ computed according to (30) for the northeast and southwest regions in Fig. 12 appear in Fig. 16. Persistently larger diffusivities are found in the jet at all depths where the flow meets the critical conditions for centrifugal instability. The distinction in the mean values is somewhat modest, reflecting the episodic nature of the mixing and our somewhat overdamped mechanics. A time series of the diffusivity from the two regions appears in Fig. 16b, and an average over 1.5 h appears in Fig. 16c. Both are from depths of 200 m. In both measures the jet region clearly shows a much more vigorous region for mixing. Ledwell et al. (2000) inferred diffusivities of $3 \times 10^{-4} m^2 s^{-1}$ in the abyssal ocean. Our diffusivities are smaller (i.e., $\sim 0.5 \times 10^{-4} m^2 s^{-1}$), but we are considering an area of much stronger stratification, and instantaneous diffusivity estimates grow to $10^{-3} m^2 s^{-1}$. Our estimates are also likely to be conservative.

It is commonly accepted that energy is consumed at a rate of roughly 2 TW in the maintenance of the observed ocean stratification (St. Laurent and Simmons 2006). Normalizing by the surface area of the deep ocean, this is roughly $6 m W m^{-2}$. Estimates of the efficiency at which mechanical turbulence mixes buoyancy implies mixing in the ocean generates potential energy at a rate of about $1 m W m^{-2}$. Integrating our turbulent production rates (removing the potential energy production that would occur because of the background diffusivity) from 100 to 500 m yields rates of $0.5 m W m^{-2}$ that compares well with the open-ocean estimates. It should be noted, however, that the global estimate applies to the total energy consumed by mixing throughout the water column, whereas our calculations obtain only to the upper few hundred meters.

The cross-shelf length scale of the process is roughly 10 km. Assuming that potential energy is generated at the above rates over most of the eastern continental coast of the Pacific Ocean ($58^\circ N$ to $52^\circ S$, $\sim 10^7 m$), roughly

$$(10^{-3} W m^{-2})(10^4 m)(10^7 m) = 10^8 W$$

is stored as potential energy. This is a modest contribution to the roughly $3.5 \times 10^{11} W$ global potential.
energy requirement and suggests the mechanism is of more regional significance than global significance. On the other hand, we are examining a very near-surface phenomenon. It is also possible ocean mixing reflects the combination of several differing mechanisms, of which this is one.

The depths being relatively shallow also imply that the mixing occurs on near-surface densities, that is, on isopycnals where the wind-driven circulation is found. While the levels of computed mixing compare to estimates from the open ocean, they are not off scale. They nonetheless focus attention on lateral topography as a preferred location for ocean mixing. This idea has appeared in the literature for a few decades; we begin to assign dynamics to it.

7. Summary

We have here argued that current interactions with topography on the eastern coast of the Pacific Ocean generate conditions conducive to loss of energy from the balanced flow and the efficient generation of ocean mixing. The poleward flow of the California Undercurrent naturally results in anticyclonic vorticity that can locally overwhelm the Coriolis parameter. At major topographic features, like Point Sur just south of Monterey Bay, the current can separate from the coast and enter the ocean interior. At such points away from the apparently stabilizing influence of the topography, a suite of instabilities become available to the flow. We have presented here through an analysis of a regionally downscaled nonhydrostatic calculation evidence that centrifugal instability dominates the evolution. The evidence emerges primarily from an analysis of the energy source terms of the linearized equations. Theoretical instability calculations show that with the onset of negative absolute vorticity, a switch in the primary instability source occurs, moving from a release from the vertical shear to a release...
from the lateral shear. This clear indication of centrifugal instability arises in our calculations, along with the other hallmarks of centrifugal instability, that is, negative potential vorticity coupled to stable stratification and negative absolute vertical vorticity.

We further argue the regional evolution results in subsurface mixing and conversion of water between density classes. The regional depths at the locations where centrifugal instability is expected exhibit zones of mass accumulation and loss, consistent with an elevated level of entrainment. The resulting waters are a mixture of the warm, salty CUC waters and that of the surrounding environs, a product that is like the core of Cuddies. The end state of the mixing processes studied here should be a state stable to SI/CI, and one expects more classical upscale cascades to regularize the mixed waters into submesoscale eddies. We argue this as a formation mechanism for Cuddies. Lindborg in a series of papers, with perhaps the best known being Lindborg (2006), studied forward cascades of energy into the balanced flow of a stratified fluid.

Lindborg focused on unbalanced variability in stratified fluids leading to shear instability. We argue the primary mechanism at work here is centrifugal instability, itself caused by frictionally generated negative absolute vorticities occurring in the bottom boundary layer, leading eventually to overturns. The CI emerges from an almost two-dimensional flow, the balanced flow of a stratified fluid.

Last, an analysis of potential energy suggests a conservative estimate of the flux of energy into the background stratification is $O(0.5) \text{mW m}^{-2}$ because of the centrifugally driven overturning. This single, upper-ocean mechanism is thus comparable to the global average mixing rate thought to characterize the full water column and is associated with an increase of mixing equivalent to diffusivities of $\sim 10^{-4} \text{m}^2 \text{s}^{-1}$. We remark our explicit vertical diffusivity is comparable to this and might contaminate our effective diffusivity estimate, although the difference between the turbulent and quiescent regions is clear. In as much as the combination of effects needed to establish the instability is characteristic of the entire western seaboard of North America, we speculate that comparable physics and rates are regionally commonplace. In summary, we argue the classical idea that mixing is a topographically mediated process is supported by our study and further that centrifugal instability plays a central role.

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### APPENDIX

**A Comparison of the Power Provided to the Background Potential Energy according to Linear and Nonlinear Equations of State**

Equation (22) expresses the power provided to background potential energy for a nonlinear equation of state. It differs from the formula in W95 for a linear state equation, and it is useful to make the differences explicit. The easy distinctions are found in the second derivation of the buoyancy that is by definition absent from linear equations. A more interesting distinction is to be found in the first right-hand side quantity in (22). That term by itself is

$$\{h_{ij}(\Theta(x,t), S_A(x,t), z^*(x,t))\}_{\text{diab,local}} \approx V z^* \cdot K \cdot V z^* \left( b_{\Theta}^* \Theta_{z^*} + b_{S_A}^* S_{A,z^*} \right), \quad (A1)$$

where we have removed the quantities that cancel. The sorted buoyancy profile is a monotonic function of the leveled depths, so the $z^*$ gradients can be expressed as

$$V z^* = z_{b}^* \cdot V b^* = \frac{V b^*}{b_{z^*}^*}, \quad (A2)$$

where the superscript $^*$ denotes the buoyancy at $(x, y, z)$ evaluated using the sorted depth for the purposes of this derivative. Thus, (A1) becomes

$$V z^* \cdot K \cdot V z^* \left( b_{\Theta}^* \Theta_{z^*} + b_{S_A}^* S_{A,z^*} \right)$$

$$= V b^* \cdot K \cdot V b^* \frac{\left( b_{\Theta}^* \Theta_{z^*} + b_{S_A}^* S_{A,z^*} \right)}{b_{z^*}^*}^2. \quad (A3)$$

For an incompressible fluid, the numerator of the fraction simplifies to $b_{z^*}^*$, and the resulting formula is identical to (17) in W95. Indeed, we have verified our implementation against the Winters formula using a test problem with a linear equation of state. It is important to retain compressibility when computing stability for seawater, however, and this contributes to the denominator in (A3) but not to the numerator. In fact, compression will increase the size of the vertical derivative in the denominator relative to those in the numerator. This provides a reduction of the mixing contribution to
background potential energy for nonlinear equations of states relative to linear ones. The physical reason for this is fluid moving vertically can change buoyancy through purely adiabatic mechanisms, and these contribute to the total background change of potential energy. This change should not be counted when computing the irreversible mixing input.

When computing the local diabatic potential energy change in this paper, we have adopted (A3), using a smoothed form for the vertical derivative in the denominator.

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