A Mechanism of Ice-Band Pattern Formation Caused by Resonant Interaction between Sea Ice and Internal Waves: A Theory

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(Manuscript received 7 August 2014, in final form 30 October 2015)

ABSTRACT

Ice bands are frequently observed over marginal ice zones in polar seas. A typical ice-band pattern has a regular spacing of about 10 km and extends over 100 km in the marginal ice zone. Further, the long axis of an ice band lies to the left (right) with respect to the wind direction in the Northern (Southern) Hemisphere. Here, the study shows that the resonance between ice-band pattern propagation and internal inertia–gravity waves below the sea ice well explains the ice-band pattern formation. Internal waves are generated by the difference between the stress on the open water and the stress on ice-covered water. This in turn reinforces the formation of an ice-band pattern with a regular band spacing. Specifically, the authors have found the following: 1) A band spacing on the order of 10 km is selected by the resonance condition in which the ice-band pattern propagation speed coincides with the phase speed of internal inertia–gravity waves. 2) The ice bands tend to develop favorably when the wind direction and the band propagation direction are nearly parallel. The velocity acceleration caused by the periodic differential stress associated with the ice bands, driven by the wind parallel to the band propagation direction, is important. The wind direction may turn to the left (right) slightly in the Northern (Southern) Hemisphere as a result of the Coriolis force acting on ice. Satellite images confirmed that the band spacing of the ice-band pattern in the polar seas is consistent with this theory.

1. Introduction

A plumelike structure of sea ice often appears in the marginal ice zone (MIZ) in polar oceans (Fig. 1). This organized structure of sea ice is called an ice band. Formation of ice bands causes increases in the area of open water in MIZ, and rapid melting may be promoted. Therefore, it has been suggested that the seasonal evolution of the MIZ is likely influenced by ice-band pattern formation (e.g., Martin et al. 1983).

Ice bands show a variety of band spacing from $O(1)$ km to $O(10)$ km, where the distance from the center of one ice band to the adjacent one is defined as the band spacing. In this paper, we will discuss ice bands with relatively large band spacing (about 10 km), which have been observed from satellite, as shown in Fig. 1. Muench and Charnell (1977) presented basic characteristics of such large-scale ice bands, where the band spacing ranges between 6 and 12 km and the spacing becomes larger as the wind speed increases. The band pattern often spreads over the MIZ extending more than 100 km. Further, the long axis of an ice band tends to lie to the left (right) of the wind direction at 40°–90° in the Northern (Southern) Hemisphere (Muench and Charnell 1977; Wadhams 1983, 2000; Ishida and Ohshima 2009), where the long axis is defined as the band orientation that is perpendicular to the band spacing (see Fig. 2). The satellite images in Fig. 1 also exhibit this feature.

Mechanisms of ice-band formation have been discussed since the 1980s. McPhee (1979, 1982, 1983) suggested that an ice edge band may be separated from the top of the ice pack by the reduction of drag between sea ice and ocean owing to intense buoyancy flux by melting when the ice edge moves into warm water. Wadhams (1983) suggested that inhomogeneity of the wave...
radiation stress in the fetch-limited open waters of various spacing would produce ice bands. The wave radiation stress is concentrated on floes at the downwind end of the open water, and it accelerates them toward the neighboring floes farther downwind. These cumulated ice floes may form the band structure. Wadhams (1983) applied this theory to ice bands with a band spacing of $1 \text{ km}$, although this mechanism may work for larger-scale ice bands.

Mechanisms of relatively large-scale ice-band formation with a band spacing of about $10 \text{ km}$, which is our main subject, have been discussed from a point of view of a coupling with internal inertial–gravity waves. A steady wind may generate ice bands by generating internal lee waves behind an ice edge, where the vertical velocity is caused by the convergence/divergence of the Ekman transport as a result of differential stress between the open water and ice-covered water (Sjørberg and Mork 1985; Fujisaki and Oey 2011, hereinafter referred to as FO11). The internal waves are generated when the ice edge speed $U$ is larger than the baroclinic phase speed $c_I$; here, $c_I = \sqrt{g' h_1}$ for a reduced-gravity, 1.5-layer ocean, where $g'$ and $h_1$ denote the reduced-gravity and upper-layer thickness, respectively (Muench et al. 1983). This lee-wave model is equivalent to that for an oceanic response to a moving storm (e.g., Gill 1982).

In this paper, based on the internal inertia–gravity wave model, we will discuss basic ice-band characteristics such as

1) what determines the ice-band spacing; and
2) is there any favorable wind direction for ice-band formation?

We will discuss these ice-band problems from a point of view of a pattern formation as a result of resonant interaction between the sea ice drift and the internal inertia–gravity waves. We hypothesized that at resonance, in which the phase speed of the internal inertia–gravity waves coincides with the ice-band speed, the internal waves that produce ice bands are in turn reinforced by the generated ice bands so that a positive feedback works. Therefore, an organized structure of the band pattern emerges from an infinitesimal initial sea ice field as a result of instability; overall, the energy source for the instability is attributed to a uniform wind
over the ice bands. The present problem is different from the previous ice-band problem that involves internal inertia–gravity waves. In the previous models, ice bands may form as a result of internal lee waves behind the ice edge where an ice concentration changes rapidly. In the present model, on the other hand, a band pattern can grow from infinitesimal disturbances that are randomly distributed in the ice concentration field.

This paper is organized as follows: In section 2, an ice–ocean coupled model is formulated using a hydrostatic, 1.5-layer model. In section 3, the above two questions are discussed by obtaining a plane wave solution in the vicinity of resonance. We present the discussion in section 4, where ice-band observations from satellites, nonhydrostatic effects, and effects of earth rotation are discussed. Further considerations on sea ice convergence by variations in ice concentration and by wind forcings are also included. Finally, we summarize the results in section 5.

2. Formulation

a. Sea ice model

We consider equations of freely drifting ice motion driven by steady wind. Since the adjusting time scale is much shorter than the inertial period that is a typical time scale of this problem, we assume to neglect the time derivative term (Leppävirta 2005). The momentum equations of the sea ice drift may be written as follows:

\[ \rho_i f k \times u_i = -\tau_{iw} + \tau_{ai}, \tag{1} \]

where \( u_i \) is the ice drift velocity, \( \rho_i \) is the ice density, \( f \) is the Coriolis parameter, \( d \) is the ice thickness, and \( \tau_{ai} \) and \( \tau_{iw} \) are air–ice and ice–water stresses, respectively (Fig. 2). The variable \( k \) is the unit vector directing upward. Internal stress is neglected because we only considered a free drifting case. The sea surface gradient is not involved in (1) because we do not consider a background geostrophic flow.

We first define the coordinate frame. We let \( x \) be a propagation direction of the ice-band pattern, and let \( y \) be the orientation of the long axis of the band that is normal to the \( x \) axis. This coordinate frame allows us to assume that variables and parameters are independent of the long axis direction of an ice band, that is, \( \partial / \partial y = 0 \). The quantities \( \tau_{ai} \) and \( \tau_{iw} \) may be written as

\[ \tau_{iw} = |\tau_{iw}|(i \cos \theta_i + j \sin \theta_i), \tag{2} \]
\[ \tau_{ai} = |\tau_{ai}|(i \cos \theta_a + j \sin \theta_a), \tag{3} \]

where \( i, j \) are unit vectors of the \( x \) and \( y \) axis, respectively. The term \( \theta_i \) is the angle between the \( x \) axis and the ice–water stress direction, while \( \theta_a \) is the angle between the \( x \) axis and the air–ice (air–water) stress direction (Fig. 2). Note that the angles \( \theta_i \) and \( \theta_a \) are not defined with respect to the ice drift direction but defined with respect to the band propagation direction. Equation (1) implies that \( \theta_i \) does not coincide with \( \theta_a \) because the Coriolis force acts on sea ice (e.g., Leppävirta 2005; Nakayama et al. 2012).

To include the effects of the Coriolis force, the turning angle \( \delta \theta \) between the air–ice stress direction and the ice–water stress direction is defined as

\[ \delta \theta = \theta_a - \theta_i. \tag{4} \]

Later in this paper, the growth rate is determined as a function of \( \theta_a \), with \( \delta \theta \) as a known parameter.

The contribution of the Coriolis force is generally small and hence (1) yields

\[ |\tau_{ai}| \simeq |\tau_{iw}|, \quad \text{with} \quad \delta \theta \ll 1. \tag{5} \]

Equation (5) means that the balance between the ice–water stress and the air–ice stress is the dominant relation in the ice drift model (Leppävirta 2005). Here, \( u_i \) is divided into steady motion and perturbations as follows:

\[ u_i = \bar{U}_i + u_i', \]

where \( \bar{U}_i \) denotes the mean ice drift, and \( u_i' \) denotes the perturbation that represents the band pattern. We note that an angle between \( \bar{U}_i \) and \( \tau_{ai} \) exists, which is about 20°, although it depends on properties in the turbulent boundary layer in the upper ocean. This angle is set to be \( \theta_u \). The ice-band pattern propagates with the speed

\[ \bar{U}_i = |\bar{U}_i| \cos(\theta_u - \theta_a) \]

that corresponds to the \( x \) component of \( \bar{U}_i \) (see Fig. 2).

As we will see later, since the band pattern propagation is coupled with the internal inertia–gravity wave, \( \bar{U}_i \) determines the character of the coupled system whether it has a wave solution or a decaying solution (e.g., FO11; Gill 1982).

Finally, the equation for the ice concentration may be written as follows:

\[ \frac{\partial A}{\partial t} + \frac{\partial}{\partial x} (u_i A) = 0, \tag{6} \]

where \( \partial / \partial y = 0 \) in this coordinate frame, and \( u_i \) is the \( x \) component of \( u_i \). The ice concentration is divided into the mean and perturbation such that

\[ A = \bar{A} + A', \]

and we have, in a linearized form,

\[ \frac{\partial A'}{\partial t} + \bar{U}_i \frac{\partial A'}{\partial x} + \bar{A} \frac{\partial u_i'}{\partial x} = 0, \tag{7} \]

where \( u_i' \) is the \( x \) component of \( u_i' \).
b. Ocean model

Next, we considered a 1.5-layer ocean model driven by the surface stress. The 1.5-layer model is a layer model with the hydrostatic approximation, in which the lower layer is assumed to be so deep that the lower-layer motion is at rest. Since the polar oceans typically have a well-mixed upper layer underlain by a strong pycnocline, the 1.5-layer model would be suited for modeling the upper polar oceans (e.g., Muench 1983).

We formulate the oceanic motion in the upper layer with a thickness of \( h_1 \) as in Fig. 3. The upper-layer velocity \( \mathbf{u}_1 \), which is vertically averaged, is driven by the stress \( \mathbf{\tau} \). The variables are divided into mean and perturbation such that

\[
\begin{align*}
\mathbf{u}_1 &= \overline{\mathbf{u}}_1 + \mathbf{u}'_1, \\
h_1 &= h_1 - \zeta', \quad \text{and} \\
\mathbf{\tau} &= \mathbf{\tau} + \mathbf{\tau}' ,
\end{align*}
\]

where the bar and primes denote the mean and perturbation. Here, \( \mathbf{\tau} \) yields \( \overline{\rho} \overline{\mathbf{u}}_1 = \mathbf{k} \times \mathbf{\tau}/\rho_w f \), which is the mean Ekman transport. The mean current should have vertical structure associated with the Ekman layer. We do not pursue the Ekman layer structure here but assume that the layer is thin enough not to reach the density interface so that it does not affect the internal wave propagation.

Next, the momentum and continuity equations for perturbations yield

\[
\begin{align*}
\frac{\partial \mathbf{u}'_1}{\partial t} + \mathbf{f} \times \mathbf{u}'_1 &= g' \nabla \zeta' + \frac{1}{\rho_w} \mathbf{\tau}' , \quad \text{and} \quad (8) \\
\overline{h}_1 \nabla \cdot \mathbf{u}'_1 - \frac{\partial \zeta'}{\partial t} &= 0 , \quad (9)
\end{align*}
\]

where \( \rho_w \) is the water density, and \( g' \) is the reduced gravity. Then, the linearized shallow-water equation is written as follows:

\[
\frac{\partial}{\partial t} \left[ \left( \frac{\partial^2}{\partial t^2} + f^2 \right) \zeta' \right] - c_t^2 \nabla^2 \zeta' = \frac{1}{\rho_w} \left( \frac{\partial}{\partial t} \mathbf{\tau}' + f \mathbf{k} \cdot \nabla \times \mathbf{\tau}' \right) , \quad (10)
\]

where we recall that \( c_t = \sqrt{g' \overline{h}_1} \).

The stress vector on the sea surface may be written using the air–water stress vector \( \mathbf{\tau}_{aw} \), the ice–water stress vector \( \mathbf{\tau}_{iw} \), and the sea ice concentration \( A \) (0 \( \leq A \leq 1 \)) as follows (cf. Hakkinen 1986; FO11):

\[
\mathbf{\tau} = (1 - A) \mathbf{\tau}_{aw} + A \mathbf{\tau}_{iw} , \quad \mathbf{\tau}' = -A' \mathbf{\tau}_{aw} + A' \mathbf{\tau}_{iw} , \quad (11)
\]

where \( \mathbf{\tau}_{aw} \) is denoted as

\[
\mathbf{\tau}_{aw} = |\mathbf{\tau}_{aw}| (i \cos \theta_a + j \sin \theta_a) . \quad (12)
\]

The perturbation in \( \mathbf{\tau} \) comes from \( A' \), provided that the wind is given and constant; \( \mathbf{\tau}_{iw} \) is then constant and so is \( \mathbf{\tau}_{iw} \) through (1) as the Coriolis term is small.

If (2), (4), (5), and (12) are substituted into (11), the perturbed stress \( \mathbf{\tau}' \) on the water may be written as

\[
\mathbf{\tau}' = i[A' (\Delta \tau \cos \theta_a + \sin \delta \theta \tau_{aw}^r)] + j[A' (\Delta \tau \sin \theta_a - \sin \delta \theta \tau_{aw}^r)] , \quad (13)
\]

where \( \Delta \tau = (|\mathbf{\tau}_{aw}| \cos \theta_a - |\mathbf{\tau}_{aw}|) \) is the stress difference. Since we set the \( x \) axis as the propagation direction of the ice band, we assume that the band structure, or the surface stress in (13), is independent of \( y \) as in the section 2a. Equation (13) is substituted into (10), and we obtain, considering \( \partial / \partial y = 0 \),

\[
\frac{\partial}{\partial t} \left[ \left( \frac{\partial^2}{\partial t^2} + f^2 \right) \zeta' \right] - c_t^2 \nabla^2 \zeta' = \frac{1}{\rho_w} \left[ \frac{\partial^2 A'}{\partial t \partial x} (\Delta \tau \cos \theta_a + \sin \delta \theta \tau_{aw}^r) \right. \\
+ f \frac{\partial A'}{\partial x} (\Delta \tau \sin \theta_a - \sin \delta \theta \tau_{aw}^r) \right] , \quad (14)
\]

where (14) is hyperbolic in character when \( \overline{U}_i > c_t \) because \( A' \) propagates with the speed \( \overline{U}_i \). The forcing term with \( \Delta \tau \) in the right-hand side (RHS) of (14) generates the vertical velocity by the differential stress between the open water and the ice-covered water. The third term in the RHS of (14) with \( f \) is associated with the Ekman divergence driven by the \( y \) component of the wind (Fig. 4b), which was discussed in previous studies (e.g., Sjørberg and Mork 1985; FO11). In addition, the first two terms, representing the stress divergence, are important because the ice-band pattern that is coupled with the internal inertia–gravity wave has a frequency...
higher than \( f \). This term represents the vertical velocity caused by the convergence/divergence of water transport associated with velocity acceleration driven by the \( x \) component of the wind (Fig. 4a). Other forcing terms on the RHS of (14) are associated with the turning angle \( \delta \theta \) between \( \tau_{sw} \) and \( \tau_{iw} \) (Fig. 4c). Since the Ekman transport also has a turning angle that is \( \delta \theta \), this term contributes to the divergence of the Ekman transport and generates the vertical motion (Fig. A1 in appendix A).

Finally, we assume that \( u'_i \) is advected by \( u'_i \) because ice drifts freely and no perturbed stress should act on the ice–ocean interface. This gives \( u'_i = u'_i \). Then, (7) is rewritten by using the continuity equation (9) such that

\[
\frac{\partial A'}{\partial t} + \frac{\partial A'}{\partial x} + \frac{A}{h_i} \frac{\partial c'}{\partial t} = 0. \tag{15}
\]

This equation indicates that the fluctuations associated with the internal wave contribute to the time evolution of the sea ice concentration through promoting convergence/divergence of sea ice by the internal wave velocity. More generally, \( u'_i \) may be generated as a result of variability in the ice concentration, floe size, and draft.
and so on, because \( \tau_{ai} \) and \( \tau_{mi} \) are complicated functions of these parameters (Steele et al. 1989). Further, the wave radiation stress should also be included in the ice motion equation (1). These effects are significant and will be discussed in section 4d.

Consequently, the two equations with respect to \( \xi' \) and \( A' \) are derived as in (14) and (15). In the next section, an eigenvalue problem will be discussed using these equations.

3. Theory and results

a. Scaling

In this section, we attempt to explain the characteristic band spacing and the favorable wind direction by formulating an eigenvalue problem. First, the variables are nondimensionalized as follows:

\[
\begin{align*}
\tau_{ai} &= T \tau_{ai}^*, \\
\xi' &= Z \xi'^*, \\
\tau' &= T \tau_{ai}^* \tilde{h}_{i1} = \tilde{c}_1 = (f L C)^2, \\
U_i &= f L U_i^*,
\end{align*}
\]

where the asterisks denote nondimensional variables. The relationship between the internal wave amplitude scale \( Z \) and wind stress scale \( T \) can be evaluated by the balance between the RHS and the left-hand side (LHS) of (14):

\[
f^3 Z = \frac{f T}{\rho_a L}.
\]

If typical values of \( T \approx 10^{-1} \text{N m}^{-1} \), \( f \approx 10^{-4} \text{s}^{-1} \), \( \rho_a \approx 10^{3} \text{kg m}^{-3} \), and \( L \approx 10^{3} \text{m} \), \( Z \) may be evaluated as 1 m.

Then, (16) is nondimensionalized as follows:

\[
\begin{align*}
\frac{\partial}{\partial t^*} \left[ \frac{\partial^2}{\partial x^*} + 1 \right] \xi'^* - \epsilon^2 \frac{\partial^2 \xi'^*}{\partial x^*} &= \frac{\partial^2 A'}{\partial t^*^2} \frac{\partial^2 x_{\text{ai}}}{\partial x^*} + \tau_{ai}^* \sin \delta_d \\
&+ \frac{\partial A'}{\partial x^*} \left( \frac{\partial}{\partial x^*} \xi'^* - \tau_{ai}^* \sin \delta_d \right),
\end{align*}
\]

where \( \tau' = |\tau| \cos \theta_a \) and \( \tau'' = |\tau| \sin \theta_a \) are used, and \( \delta_d \) is defined as \( \Delta \tau / |\tau_{ai}| \). Furthermore, (17) is nondimensionalized as

\[
\frac{\partial A'}{\partial t^*} + \frac{U_i}{T} \frac{\partial A'}{\partial x^*} + \epsilon \frac{\partial \xi'^*}{\partial t^*} = 0,
\]

where \( \epsilon = \frac{\Delta Z \tilde{h}_{i1}}{h_i} \). Therefore, the strength of the coupling is characterized by the ratio between the upper-layer thickness and the internal wave amplitude. In the rest of this section, asterisks will be omitted from the nondimensional variables for simplicity.

b. Plane wave solution

Next, the relationship between the perturbations of the internal waves and those of sea ice concentration is investigated by use of a plane wave solution:

\[
\left( \begin{array}{c} \xi' \\ A' \end{array} \right) = \left( \begin{array}{c} \tilde{ \xi} \\ \tilde{A} \end{array} \right) e^{(k_x - x')/a},
\]

where \( k \) is the wavenumber, and \( \omega \) is the frequency. By substituting (18) into (16) and (17), we obtain a characteristic equation such that

\[
\begin{align*}
0 &= \omega^2 - \left( 1 + c_i^2 k^2 \right) \left( \omega - U_i k \right) - c_i [i \omega (\delta_d \tau'_{ai}^* + \sin \delta \theta \tau'_{ai}^*) + k (\delta_d \tau'_{ai}^* - \sin \delta \theta \tau'_{ai}^*)].
\end{align*}
\]

We assume that \( \epsilon \) is a small parameter so that (19) is solved using a perturbation expansion. Then, we see that \( \omega^2 - \left( 1 + c_i^2 k^2 \right) = 0 \), and the first term of LHS denotes the dispersion relationship of the internal inertia–gravity wave. Further, \( \omega = U_i k \) represents the speed of the ice-band pattern on the \( k - \omega \) plane. These dispersion relationships are shown in Fig. 5.

c. Perturbation expansion in the vicinity of the resonance condition

The dispersion relationships are expanded in the vicinity of the resonance point in Fig. 5 such that

\[
\begin{align*}
\omega &= \omega_0 + \epsilon^{1/2} \omega_1 + \cdots, \quad \text{and} \\
k &= k_0 + \epsilon^{1/2} k_1 + \cdots.
\end{align*}
\]

Here, \( \omega_0 \) and \( k_0 \) represent the frequency and wavenumber of the exact resonance that occurs at the intersection of the two dispersion curves. Then, \( k \) and \( \omega \) are substituted into (19) and arranged up to the \( O(\epsilon) \) terms as follows:

\[
\begin{align*}
\{ [\omega_0^2 - (1 + c_i^2 k_0^2)] + 2 \epsilon^{1/2} (\omega_0 \omega_1 + c_i^2 k_1 k_1) \} [\omega_0 - U_i k_0] \\
+ \epsilon^{1/2} [\omega_0^2 - U_i k_1] = c_i [\omega_0 k_0 (\delta_d \tau'_{ai}^* + \sin \delta \theta \tau'_{ai}^*) \\
- k_0 (\delta_d \tau'_{ai}^* - \sin \delta \theta \tau'_{ai}^*)]
\end{align*}
\]

1) \( O(\epsilon^0) \) solution

The two equations of \( O(\epsilon^0) \) should hold at the same time at the intersection point \( (\omega = \omega_0, k = k_0) \) in Fig. 5:

\[
\begin{align*}
\omega_0^2 - (1 + c_i^2 k_0^2) &= 0, \quad \text{and} \\
\omega_0 - U_i k_0 &= 0.
\end{align*}
\]

Therefore, the characteristic scale of the ice band may be chosen when (20) and (21) hold simultaneously. The band spacing \( \lambda \) may be calculated as follows:
or we obtain

$$l = \frac{2\pi}{k_0} = 2\pi(U_i/c_I)^2/c^2_{1/2}. \quad (23)$$

The relationship between $U_i/c_I$ and the nondimensional wavenumber $k_0$ is shown in Fig. 6.

Equation (23) indicates that if the sea ice drift speed becomes large, the band scale tends to become large. This is consistent with results by Muench and Charnell (1977). Once $U_i/c_I$ becomes less than unity, resonance does not take place because (14) becomes parabolic as long as the 1.5-layer model is used. We noted, however, that higher modes of the internal waves can exist in a continuously stratified ocean, and therefore (14) remains hyperbolic in character through these higher-mode internal waves with slower phase speeds as the frequency or $U_i$ is larger than $f$ (e.g., FO11).

**Figure 7** displays nondimensional band spacings in terms of the nondimensional sea ice drift speed. The dashed line in this figure represents the nondimensional inertial frequency $\omega = 2\pi U_i/\lambda$ with $\omega = 1$. Three curves denote band spacings for various baroclinic phase speeds $c_I$. Figure 7 indicates that in the case of a relatively large $U_i$, the difference of the band spacings with respect to different $c_I$ is small even if stratification varies. Further, $\omega_0$ asymptotes to $f$ for the large $U_i$.

### 2) $O(\epsilon^{1/2})$ Solution

Next, $\omega_1$, the $O(\epsilon^{1/2})$ solution in terms of $\omega$, is considered; $O(\epsilon)$ of the characteristic equation may reduce to

$$2\omega_0(\omega - c_I k_1)^2 = [i\omega_0 k_1(\delta d r_{ai}^x + \sin\delta\theta r_{ai}^y) - k_0(\delta d r_{ai}^y - \sin\delta\theta r_{ai}^x)]. \quad (24)$$

Therefore, we obtain

$$\omega_1 = \pm \left(\frac{k_0 G}{2}\right)^{1/2} e^{i(\phi/2)} + c_I k_1 \quad \text{or} \quad \omega_1 = \pm \left(\frac{k_0 G}{2}\right)^{1/2} \left(\cos\frac{\phi}{2} + i\sin\frac{\phi}{2}\right) + c_I k_1, \quad (25)$$

where

$$G = [(1/\omega_0^2)(\delta d r_{ai}^x - \sin\delta\theta r_{ai}^y)^2 + (\delta d r_{ai}^y + \sin\delta\theta r_{ai}^x)^2]^{1/2}, \quad \text{and}$$
where $f$ is defined for $2\pi/2, f, p/2$. The growth rate of the ice band $n$ is given by the imaginary part of (25), which is written as follows:

$$n = \text{Im}(v_1) = \frac{C_{18}k_0G}{C_{19}^{1/2}} \sin\left(\frac{f}{2}\right) \phi > 0,$$

(26)

where the growing solution with $\nu > 0$ in (25) is chosen. For the up-ice-wind case where $\phi = \pi/2$, which was considered in Sjørberg and Mork (1985) and FO11, $G^2 = |\tau_{a1}|^2(\delta^2/\omega_0^2 + \sin^2\delta\theta)$ and $\tan\phi = -(\sin\delta\theta)\omega_0/\delta$. Hence, the exponential growth occurs for nonzero $\delta\theta$ according to (26).

Now, we will discuss the question of whether or not there are any favorable wind directions for ice-band formation. If $\delta\theta = 0$, both $G$ and $\sin(\phi/2)$ in $\nu$ are maximum when $\theta_a = 0$ because $\omega_0 > 1$ for the internal inertia–gravity waves. In other words, the growth rate is maximum when the band propagates in the direction that is parallel to the wind or when the long axis of the band lies in the right angle with respect to the wind. In this case, the convergence/divergence in the ocean transport is not owing to the Ekman transport but owing to the stress divergence term, that is, $\partial/\partial r \cdot \tau$ in (10). This term is not necessarily small relative to the Ekman term because $\omega_0 > 1$. Figure 8 depicts the relationship between the ice band and the internal wave when mutual enhancement occurs. An important aspect in Fig. 8 is that the stress divergence term causes upwelling where ice concentration is small. The upwelling amplifies the internal waves that propagate along with the ice band. Then, the velocity field associated with the internal wave in turn aggregates ice and enhances the ice-band formation as in Fig. 8. This is similar to a kinematic explanation of hydrodynamic instability such as the Kelvin–Helmholtz instability (e.g., Baines and Mitsudera 1994). Note that the convergence/divergence in the Ekman transport, caused by the $y$-component wind, produces downwelling in the front side of the ice band. This downwelling does not induce mutual enhancement (see appendix B for more discussion), although it produces a lee-wave train behind the ice edge as discussed in Sjørberg and Mork (1985) and FO11.

When $\delta\theta \neq 0$, we may obtain the maximum growth rate when

$$\tan\theta_a = \frac{\sin\delta\theta}{\delta}.$$

(27)

This implies $\tan\theta_a > 0$ in the Northern Hemisphere because the turning angle $\delta\theta$ is positive there. This means that the ice bands tend to develop favorably when the wind blows slightly to the left with respect to the propagation direction of the ice-band pattern. By substituting (27) into (26), we obtained the maximum growth rate $\nu_{\text{max}}$ as
\[ \nu_{\text{max}} = \frac{(k_0 \tau_{\text{ai}})^{1/2} (\delta_a^2 + \sin^2 \delta \theta)^{1/4}}{2} \]

On the other hand, \( \nu \) becomes zero when \( \phi = 0 \). Therefore, a stable state is found when
\[ \tan \theta_a = -\frac{\delta_d}{\sin \delta \theta} \]

This implies that the ice band hardly develops when the wind leans to the right with respect to the propagation direction of the ice-band pattern. A similar result was obtained in FO11, although they considered melting under ice as an explanation.

Finally, the growth rate \( \nu \) in (26) is plotted in Fig. 9 in terms of wind directions \( \theta_a \), where the parameter values are \((\omega_0, k_0) = (1.516, 8.33 \times 10^{-1})\), \( |\tau_{\text{ai}}| = 1.0 \), and \( \delta_d = \Delta \tau/|\tau_{\text{ai}}| = 0.3 \). As for \( \delta \theta = 0 \), \( \nu \) becomes maximum when the ice band travels parallel to the wind direction. Nonzero but small \( \delta \theta \) modifies \( \theta_a \) in a consistent manner to the observation.

d. Numerical results of coupled development

In this subsection, we try to verify the ice-band pattern formation using the ice–ocean coupled numerical model with simple settings. The present theory implies that if there is a random sea ice concentration field initially, the band patterns may emerge gradually, having the most favorable band-axis direction with respect to the imposed wind.

We use an ice–ocean coupled model that is the same as that of FO11. The ocean model is based on the Princeton Ocean Model, which employs the primitive equations with hydrostatic as well as Boussinesq approximations (Mellor et al. 2002). The ice model uses the elastic–viscous–plastic rheology (Hunke and Dukowicz 1997) with ice collision parameterization (Sagawa 2007). The model domain is \( 160 \text{km} \times 220 \text{km} \), with a horizontal resolution of 250 m. The depth of the ocean is set to be 150 m, having the Bering Sea shelf break in mind, with a vertical resolution of 5 m. Figure 10 shows the initial density stratification, which approximates a two-layer model ocean. The stratification after 7 days is also depicted in Fig. 10, which indicates that the upper mixed layer further develops during the evolution of ice bands. This configuration is dynamically equivalent to the 1.5-layer model for the first baroclinic mode. In this experiment, the water temperature is set at \(-1.8^\circ\text{C}\), and all thermal fluxes are removed. That is, thermodynamic effects are eliminated because we want to investigate whether the ice bands are formed by dynamical effects only. The initial sea ice field is set with a random sea ice concentration (Fig. 11a). Mean sea ice concentration is 0.5 because we consider a free drift condition without internal stress. The ice thickness \( d = 0.5 \text{ m} \). The lateral boundary condition of this domain is periodic.

The homogeneous wind starts blowing over the initial sea ice field. As a result, the ice-band patterns emerge all over the domain as in Fig. 11b. The ice bands in this simulation exhibit a regular, 10-km-scale band spacing (Fig. 11c), consistent with those of ice bands seen in the satellite images (e.g., Figs. 1, 13). Further, wind leans to the left with respect to the band propagation direction.

Figure 12b shows the vertical section of the vertical velocity along the black line in Fig. 11c. Mean amplitude of vertical velocity in a unit volume (1 m\(^3\)) grows exponentially (Fig. 12c), indicating mutual intensification between the ice bands and the internal waves. After 120 h, the growth of the vertical velocity slows down a little, indicating that the ice band grows sufficiently so that ice concentration is close to one in the ice band. Sea ice concentration on day 7 along this section is also drawn in Fig. 12a. Comparing Fig. 12a with Fig. 12b, we found that the phase relationship between sea ice and internal waves is the same as that in the theoretical model (Fig. 8), in such a way that the upwelling vertical velocity is seen in front of each ice-band promoting convergence in ice motion. Therefore, the numerical results in Figs. 11 and 12 represent the ice-band pattern formation by resonant interaction between sea ice and internal waves. This implies the importance of the stress divergence term in (10) for the mutual enhancement. As for the role of \( \delta \theta \), Fig. 11c shows that \( \delta \theta \approx 2^\circ \). This yields \( \theta_a \approx 3.5^\circ \) for \( \delta_d = \cos \delta \theta - (C_{D\text{aw}}/C_{D\text{ai}}) \approx 0.5 \), where the air–water drag coefficient \( C_{D\text{aw}} = 1.5 \times 10^{-3} \).
and the air–ice drag coefficient $C_{Dai} = 3.0 \times 10^{-3}$ in the model. This $\theta_a$ is too small to explain the simulated angle fully, although the turning direction is consistent.

The numerical results are apparently different from those of FO11 although the numerical model used is the same. In particular, FO11 suggested that an ice band and a lee-wave train hardly occur for the wind with $\theta_a = 0$. This difference probably comes from the initial configuration of the sea ice. In FO11 they configured the ice-band modeling with a Heaviside function–type initial ice concentration, where $A = 0$ in front of the ice edge, and $A = 0.8$ behind the ice edge, in contrast to our case in which the mean of $A$ is 0.5 in order for sea ice to drift freely at least initially. Because $A = 0.8$ in FO11 is large enough for internal stress to become effective, sea ice does not react directly to the sinusoidal flow field of internal waves but acts as a low-pass filter. As a result, the vertical velocity caused by the stress divergence becomes much smaller than that of our case. Indeed, a similar experiment to FO11 with the initial ice edge of the Heaviside function form, but for $A = 0.5$ behind the ice edge, shows the band formation for $\theta_a = 0$ (figure not shown).

e. Brief summary

Based on the theoretical and numerical modeling results, we have found a new unstable mode caused by resonant interaction between the internal inertia–gravity waves and sea ice drift. Results are summarized briefly as follows:

1) The ice-band scale is selected by the resonance condition in which the ice-band speed coincides with the phase speed of internal inertia–gravity waves.

![FIG. 10. Stratification in the numerical experiments: Dashed line denotes the initial stratification, where a potential density profile is given by $0.15 \times \tanh[0.03(z - 50)] + 1026.3$ (kg m$^{-3}$), with $z$ being the depth. Solid line denotes the stratification on day 7 after the wind starts blowing. Ocean temperature is constant and set at $-1.8^\circ$C so that melting does not occur.](image1.png)

![FIG. 11. Ice-band pattern formation: (a) Initial random sea ice concentration field on horizontal plane, where mean sea ice concentration is set at 0.5. Red vector denotes the wind vector. Wind velocity is $U_a = (U_a, V_a) = (15.0, 7.5)$ m s$^{-1}$. (b) Ice-band patterns on day 3.5 on horizontal plane. (c) Ice-band patterns on day 7 on horizontal plane. Black line denotes the location representing the vertical sections of Figs. 12a and 12b. Red vector denotes $\tau_{ai}$ and blue vector denotes $\tau_{iw}$, where $|\tau_{ai}| = 10.5 \times 10^{-3}$ kg m$^{-1}$ s$^{-2}$ and $|\tau_{iw}| = 9.63 \times 10^{-3}$ kg m$^{-1}$ s$^{-2}$, respectively.](image2.png)
2) The ice bands tend to develop favorably when the wind direction and the band propagation direction are nearly parallel (or the long axis of the band is almost perpendicular to the wind). The velocity acceleration caused by the wind parallel to the band propagation direction is important. The wind direction may turn to the left slightly in the Northern Hemisphere for the maximum growth rate, as seen in the numerical results. The Coriolis force acting on ice may yield the angle, consistent with the numerical results, although it is small compared with the simulated angle.

Considering these features, we presented a schematic view of ice-band formation by the resonant interaction in Fig. 8. The band spacing is the same as the wavelengths of the internal lee waves produced by a moving ice edge (e.g., Sjøberg and Mork 1985; FO11). This correspondence arises because the wavelength is determined when the ice bands and the internal waves are stationary with each other in both the present model and in the previous lee-wave models. On the other hand, favorable wind directions are determined by the resonant growth condition in which the ice bands and the
internal waves have a certain phase relationship that causes a positive feedback (Fig. 8). This was not considered in the previous lee-wave models.

4. Discussion

a. Ice-band pattern formation in real ocean

The discussion so far implies that once instability occurs as a result of the positive feedback between the ice band and internal waves, the ice-band pattern emerges from the random initial ice concentration field. In this subsection, we present an example of the ice-band pattern formation seen in satellite images in which ice bands emerges over a broad area.

Figure 13 shows a satellite image of the Sea of Okhotsk off Sakhalin Island on 26 March 2003. Although there were ice packs of various scales over the sea ice area, there were no major ice-band patterns on this day. Then, a low pressure system passed through this area from 27 to 30 March (Fig. 13b). After that the ice-band patterns were clearly seen (on 2 April) in Fig. 13c, with a spread of over 300 km. The wind vectors from 1 day earlier (on 1 April)
are superimposed on the ice-band pattern because the ice-band development should have occurred during the passage of the low pressure system. This implies that the ice-band pattern emerged within a week. Instability modes due to resonance are likely responsible as they are capable of generating such a large-scale ice-band pattern that emerged within a short period of time.

Figure 13 shows that the ice-band spacings are about 5–10 km. The band spacings are similar to those in the Greenland Sea and Southern Ocean shown in Fig. 1 as well as those of the Bering Sea in Muench and Charnell (1977). Here, we evaluated the band spacing based on (23). Figure 13d shows that the typical sea ice drift speed was about 0.2 m s\(^{-1}\) in this ice-band region on 2 April 2003. Note that in the rest of the discussion, variables are given in dimensional forms; asterisks are restored for denoting nondimensional variables. Equation (23) is rewritten with the dimensional variables such that

\[
\lambda = \frac{2\pi}{k_0} = \frac{2\pi (U_i^2 - c_i^2)^{1/2}}{f}. \tag{28}
\]

The maximum band spacing \(\lambda_{\text{max}}\) is thus obtained for \(c_i \to 0\) with (28). By substituting \(U_i = 0.2\) m s\(^{-1}\) and \(f \approx 10^{-4}\) s\(^{-1}\) into (28) with \(c_i = 0\), we obtain

\[
\lambda_{\text{max}} = 2\pi \frac{U_i}{f} \approx 12\text{ km}. \tag{29}
\]

Since \(c_i\) is a nonzero parameter, the band spacing \(\lambda\) must be smaller than \(\lambda_{\text{max}}\). Hence, (28) implies the band spacing of \(\lambda \approx 5\text{–}10\) km, consistent with those bands found in the satellite image of Fig. 13c.

We also examine the relation between the band spacing and the wind speed over the Bering Sea shelf discussed by Muench and Charnell (1977, see their Fig. 4). Since the higher vertical-mode waves for continuous stratification may develop over the shallow Bering Sea shelf, \(c_i\) of a 1.5-layer model in (28) is replaced by eigenvalues of higher vertical modes, which are slower than \(c_i\) (e.g., Sjøberg and Mork 1985; FO11). The dashed line in Fig. 14 denotes the relationship \(\lambda_{\text{max}} = 2\pi U_i/f\) as in (27), where \(U_i\) is scaled by 0.02\(|U_a|\), where \(|U_a|\) is the wind speed according to Muench and Charnell (1977). The internal waves can be generated in the shaded part of Fig. 14 because (10) becomes hyperbolic in this regime. Almost all of the observed band spacings in the Bering Sea are present in the shaded area, implying that these ice-band formations are explained reasonably by the resonance discussed in this study. Note that we implicitly assume a mixed layer–like stratification as in Fig. 10. The stratification may give profound effects on stress (e.g., Mellor et al. 1986; McPhee 1979, 1982, 1983), which is a future study.

Next, the growth rate is evaluated. Supposing \(\epsilon^* = AZ/|U_1| = 0.05\), a dimensional growth rate \(\nu_d\) may be evaluated as

\[
\nu_d = \epsilon^{1/2} \nu_{\text{max}}^* f \approx 0.41\text{ (day}^{-1}\text{)},
\]

where \(\nu_{\text{max}}^*\) becomes ~0.21 from Fig. 9 and the asterisks denote nondimensional variables. Thus, ice bands may grow within a few days. This time scale is consistent with that for the band growth seen in Fig. 13.

The resonance condition may be modified by the presence of the background flow. If the flow is purely barotropic, then the resonance condition is basically unchanged because both internal waves and ice bands are advected by the uniform flow. If the flow is baroclinic, however, the internal wave phase speed may be modified by a vertically sheared flow, and hence the resonance condition can be affected. Coupling between ice bands and internal waves in a vertically sheared flow is an important subject for the quantitative applicability of this theory to reality.

b. Nonhydrostatic effects

The present theory is based on the hydrostatic approximation. Therefore, the hydrostatic approximation
is likely to be valid for this band scale. Then, the next question is what is expected if the hydrostatic approximation is relaxed so that nonhydrostatic effects are involved?

The resonant condition between sea ice drift and internal waves is again considered. In a nonhydrostatic case, the dispersion relation of the internal inertia–gravity wave becomes

$$\omega = \left(\frac{N^2 k_x^2 + f^2 k_z^2}{k_x^2 - k_z^2}\right)^{1/2}, \quad (30)$$

where \( N \) denotes the Brunt–Väisälä frequency, and \( k_x \) and \( k_z \) are the horizontal and vertical wavenumbers, respectively. Although continuous stratification is incorporated here, the discussion with a 1.5-layer model is qualitatively the same.

The dispersion relation is displayed in Fig. 15 by using \( N = 30 f \), which is likely a typical value of \( N \) in polar seas; we estimated \( N \approx 60 f \) in the Greenland Sea from McPhee et al. (1987) and \( N \approx 20f-30f \) in the Sea of Okhotsk from Fukamachi et al. (2004), Mizuta et al. (2004), and Ohshima et al. (2001), whereas \( N \approx 15f \) in the Southern Ocean from Rintoul and Bullister (1999) and Wong and Riser (2011). The line \( \omega = (N/k_z)k_x \), which is a long-wave limit (\( k_x \to 0 \)) of the internal gravity waves on a nonrotating frame (i.e., \( f = 0 \)), is also plotted in Fig. 15. The dispersion curve of the nonhydrostatic internal inertia–gravity waves (Fig. 15) has an inflection point because \( \omega \to f \) as \( k_x/k_z \to 0 \), while \( \omega \to N \) as \( k_x/k_z \to \infty \). Therefore, this dispersion curve gives an additional intersection (or a resonance point) with the line \( \omega/k_x = \bar{U}_i \) at the high wavenumber regime beyond this inflection point. In other words, the ice bands with narrowband spacings may be generated under the nonhydrostatic condition. This also suggests that ice-band formation occurs even when \( \bar{U}_i/c_i \ll 1 \) in Fig. 6, in which the ice-band formation did not occur in the hydrostatic approximation.

Based on the above argument, the ice-band formation could appear at the high wavenumber. For example, a number of 2–3-km ice bands were observed in the Southern Ocean (Ishida and Ohshima 2009), although these narrow bands could be generated by various other factors such as melting processes in the boundary layer below ice (McPhee 1983; FO11) and wave radiation stress (Wadhams 1983).

c. Nonrotational system

If Earth’s rotation is not present so that \( f = 0 \), the dispersion relationship in (22) may be rewritten as follows:

$$\omega_0 = c_i k_0. \quad (31)$$

In this case, the resonance point is obtained only when \( k_0 \) and \( \omega_0 \) are zero. That is, the band spacing becomes infinity. This implies that the ice band with 10-km-scale band spacing is unlikely to be generated without the rotation of the earth.

d. Considerations on variability in the ice drift speed depending on ice concentration and wind stress variations

So far, we assumed that \( \bar{u}_i \) is caused by the perturbed flow field associated with the internal gravity waves. However, \( \bar{u}_i \) may be generated in the ice layer alone because \( \bar{u}_i \) is a complicated function of sea ice conditions such as the ice concentration \( A \), the floe size \( L_f \), and ice draft \( D \) (Steele et al. 1989), that is, \( \bar{u}_i = \bar{u}_i(\bar{A}, L_f, D,...) \). Thus, we have \( \bar{u}_i \) in terms of variations in these parameters such that

$$\bar{u}_i = \bar{u}_i + \left(\frac{\partial \bar{u}_i}{\partial \bar{A}}\right) A' + \left(\frac{\partial \bar{u}_i}{\partial L_f}\right) L_f' + \left(\frac{\partial \bar{u}_i}{\partial D}\right) D' + \cdots,$$

where we recall that the prime on variables denotes the perturbation. Here, we suppose that \( A \) varies for given \( L_f \) and \( D \). Therefore, the equation for the ice concentration [(15)] may be written as follows:

$$\frac{\partial A'}{\partial t} + \left[U_i + \left(\frac{\partial \bar{u}_i}{\partial \bar{A}}\right) (\bar{A} + 2A')\right] \frac{\partial A'}{\partial \bar{A}} + \frac{\bar{A}}{\bar{h}_i} \frac{\partial \bar{c}'}{\partial t} = 0,$$

\hspace{1cm} (32)

where the \( (\partial^2 u_i/\partial A^2) \) term is neglected because \( u_i \) tends to increase linearly with the ice concentration for small \( L_f \) (see Fig. 6 of Steele et al. 1989). In (32), we assume a
wave solution that propagates in the $x$ direction as in the previous sections and hence variables and parameters are independent of $y$. Equation (32) implies that the ice-band propagation speed, given in the bracket, is modified in a similar manner to the nonlinear term in the Burgers equation. We suppose that the floe size is small in MIZ. According to Steele et al. (1989; see, e.g., their Fig. 7), $(\partial u_i/\partial A)$ in (32) tends to be positive for almost all $A$ provided $L_f < 100$ m. This implies that the ice band tends to steepen forward, and hence the banding structure would be enhanced by convergence/divergence in ice motion caused by variable ice concentration (Fig. 16). This is consistent with satellite observations in which bands appear to be thin compared with their spacing (see, e.g., Fig. 1). We thus conjecture that the ice-band pattern that emerges as a result of the resonant interaction may set a regular spacing of the 10-km scale even if the initial ice field is random, and then the pattern is enhanced further as a result of the concentration-dependent ice speed.

We also assumed that the wind is uniform in time and space in order to extract the self-organized band pattern emerging from a homogeneous background field. In reality, the wind has various structures associated with the atmospheric boundary layer. The perturbation in the wind stress can force the organization of sea ice patterns. For example, streaky surface winds with roll-like cells, such as those in Fig. 1 visualized by clouds, may cause an ice-band pattern by the convergence/divergence of the Ekman drift of sea ice. In this case, however, the long axis of the band should be parallel along the downwind direction (e.g., Muench and Charnell 1977). Apparently, effects of direct wind forcing on ice can be seen in a numerical result in Fig. 11b (on day 3.5), showing streaks that are parallel to the wind. Interaction between the atmospheric boundary layer and the ice bands requires further study.

5. Conclusions

In this paper, we discussed the formation mechanisms of ice-band patterns from a point of view of the resonant interaction using an ice–ocean coupled system. The following two main questions have been discussed: 1) What determines the ice-band scale? 2) Is there a favorable wind direction with respect to ice-band formation? To answer these questions, we formulated a 1.5-layer ocean model coupled with a sea ice model.

We have found that resonant interaction between internal inertia–gravity waves and sea ice pattern propagation well explains the formation of the ice bands with 10-km-scale band spacing. In this coupled system, the internal waves are generated as a result of the difference between the stress on the open ocean and the stress on the ice-covered ocean as well as the turning angle between these two stresses. The generated internal waves in turn affect the evolution of the sea ice concentration so that the ice band and internal waves are mutually interacting with each other. Assuming a plane wave solution, we obtained a characteristic equation that describes instability representing a positive feedback between the ice-band formation and internal wave growth. The coupling strength $\epsilon$ is characterized by a ratio between the surface layer thickness and the typical amplitude of internal waves.

In this paper, we assumed that $\epsilon^{1/2}$ is a small parameter and we expanded the characteristic equation with respect to $\epsilon^{1/2}$. The $O(\epsilon^0)$ solution represents both the phase speed of the internal waves and the ice drift speed in the direction of ice-band propagation. Resonance occurs when these speeds coincide with each other, and it is this condition that determines the band spacing of 10 km. The $O(\epsilon^{1/2})$ solution represents the growth rate as well as modification to the phase speed. We have found that the growth rate is maximum when the band propagation direction is nearly parallel to the wind. The wind for the maximum growth may turn to the left slightly in the Northern Hemisphere, owing to the Coriolis force acting on the ice.

In the present study, we did not deal with thermodynamics because we focused on the dynamical process of band pattern formation. However, thermodynamics should also be important for the formation and melting of the ice bands. For example, intense surface buoyancy flux by rapid melting may reduce the frictional coupling as ice drifts over relatively warm water so as to increase ice speed, and then separation of a band may result (McPhee 1979, 1982, 1983; FO1 1). Therefore, the thermodynamic process associated with ice-band formation should be considered in future studies.

In this study, we show a mechanism of how the ice-band pattern, whose spacing is regular, is organized even from a random ice field in the MIZ. Streaky winds with roll-like cells in the atmospheric boundary layer may also initiate generating perturbations in the MIZ. Once a regular pattern forms, inhomogeneous ice drift velocity that is dependent upon various ice conditions (Steele et al. 1989),
as well as the radiation stress by wind waves (Wadhams 1983), may enhance the ice-band pattern further. In the future, we will evaluate these contributions to the evolution of the MIZ because the ice-band formation and melting is a ubiquitous phenomenon.

Acknowledgments. We thank Drs. A. Manome, T. Toyota, N. Kimura, J. Ukita, M. Leppävirta, T. Nakamura, T. Nakanowatari, and S. Aoki for comments and discussion. Support from Pan-Okhotsk Research Center members is greatly appreciated. We thank four reviewers for comments that helped to improve the manuscript. Numerical results were obtained with the Pan-Okhotsk Information System. Satellite images from the Kitami Institute of Technology and National Aeronautics and Space Administration (NASA) were used to analyze the data. Ice drift data were produced by Dr. N. Kimura, which is greatly appreciated. This work was supported by KAKENHI 22106010 and 26247074.

APPENDIX A

Generation of Vertical Motion Associated with $\delta \theta$

Let us consider an up-ice-wind parallel to the band’s long-axis direction. The stress $\tau_{aw}$ generates the Ekman transport in the off-ice direction. On the other hand, $\tau_{iw}$ generates the Ekman transport that turns to the right by $\delta \theta$ from the off-ice direction. Therefore, the off-ice component of the Ekman transport becomes smaller by the factor of $\sin(\delta \theta)$. This can cause divergence (convergence) in front of (behind) the ice band as in Fig. A1. This schematic diagram represents the effect of the fourth terms of the right-hand side of (14).

APPENDIX B

Ice–Ocean Coupling in Case the Wind is Parallel to the Long-Axis Direction of an Ice Band

An along-ice wind in the positive y direction causes the Ekman transport in the positive x direction. Therefore, $\Delta \tau$ causes the downwelling (upwelling) in front of (behind) each ice band as in Fig. 4b. This is not a favorable phase relationship between the internal wave and the ice band for mutual enhancement. For example, if the internal waves are in a favorable location for the ice band to aggregate (see Fig. B1a), the downwelling (upwelling) in front of (behind) each ice band, generated by $\Delta \tau$, tends to reduce the amplitude of the internal wave. Nevertheless, a neutral wave mode is possible in case the vertical velocity associated with the internal wave and that associated with the ice band are in phase (Fig. B1b). This implies that the growth rate is zero for the along-ice wind in the positive y direction if $\delta \theta = 0$.  

Fig. A1. Schematic diagram of the generation of vertical motion by the turning angle between stress on the ice-covered water and stress on the open water. This schematic diagram represents the effect of the fourth term of the right-hand side of (10).
REFERENCES


FIG. B1. As in Fig. 8, but for the relation between the ice-band and the internal wave flow fields generated by the Ekman transport difference caused by ∆θ. (a) Phase relationship when the ice band damps the internal wave. (b) Neutral mode when the vertical velocity due to the ice band and vertical velocity associated with the internal wave are in phase.


