On the Accuracy of Overturn-Based Estimates of Turbulent Dissipation at Rough Topography

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ABSTRACT
Evidence in support of overturn-based methods, often used to infer turbulent dissipation rate from density profiles, is typically from regions with weaker turbulence than that at rough-topography hotspots. The present work uses direct numerical simulations (DNS) of an idealized problem of sloping topography as well as high-resolution large-eddy simulation (LES) of turbulent flow at more realistic topography in order to investigate the accuracy of overturn-based methods in sites with internal wave breaking. Two methods are assessed: Thorpe sorting, where the overturn length $L_T$ is based on local distortion of measured density from the background, and inversion sorting, where the inversion length scale $L_I$ measures the statically unstable local region. The overturn boundaries are different between the two methods. Thorpe sorting leads to an order of magnitude overestimate of the turbulent dissipation in the DNS during large convective overturn events when inversion sorting is more accurate. The LES of steep, realistic topography leads to a similar conclusion of a substantial overestimate of dissipation by Thorpe sorting. Energy arguments explain the better performance of inversion sorting in convectively driven turbulence and the better performance of Thorpe sorting in shear-driven turbulence.

1. Introduction
Abyssal turbulence plays an important role in maintaining the observed oceanic stratification (Munk and Wunsch 1998; Wunsch and Ferrari 2004). Internal waves in the vicinity of rough topography in the deep ocean are thought to be a major contributor to abyssal turbulence based on observations of bottom-enhanced turbulence and mixing (Polzin et al. 1997; Ledwell et al. 2000; Laurent et al. 2001; Klymak et al. 2006) at these sites. At deep rough topography, there is conversion from the oscillating barotropic tide into internal wave energy. The fraction of energy that is dissipated locally by turbulence instead of radiating into the far field requires quantification. Similarly, the dissipation suffered by internal waves incident on rough topography also needs quantification. However, the complexity and cost of direct microstructure measurements of turbulent dissipation restricts such measurements to limited areas and for a limited period of time. To address the need for a broader spatial and temporal coverage of turbulent dissipation and mixing, easily available CTD data have been processed to infer dissipation from density profiles in several locations, including hotspots of internal tide generation such as the Hawaiian Ridge and the double-ridged system at Luzon Strait.

A simple objective method to infer the dissipation rate is based on the vertical overturning length scale computed from measured density profiles. Thorpe (1977) originally proposed a density sorting method to estimate the vertical length scale of density overturns in a stably stratified fluid. This scale has since been called the Thorpe scale $L_T$. It is theorized that there is a linear relationship (at least statistically) between $L_T$ and the Ozmidov length scale $L_O = \sqrt{\varepsilon/\nu}$. Knowledge of $\varepsilon$ through $L_O$ then leads to the eddy diffusivity through
\[ K_p = \Gamma \varepsilon / N^2, \] where \( \Gamma \) is the mixing efficiency. The Thorpe scale \( L_T \) is a kinematic measure of the length scale of an overturn that is computed as the root-mean-square (rms) of parcel displacements that would be required to achieve a statically stable density profile in that overturn. The Ozmidov scale \( L_O \) is the size of the largest eddy unaffected by buoyancy in stratified turbulence, a length scale that can be constructed from \( \varepsilon \) and \( N \) by
\[ L_O = \sqrt{\varepsilon / N^3}, \]
where \( \varepsilon \) is the rate of dissipation of turbulent kinetic energy (TKE) and \( N \) is the buoyancy frequency (Ozmidov 1965; Dougherty 1961); \( L_O \) is determined by dynamics, that is, it is the length scale of turbulent motion at which there is a balance between buoyancy force and inertial force.

There are observational studies (Ferron et al. 1998; Wesson and Gregg 1994; Moum 1996) suggesting that statistical measures of \( L_T \) and \( L_O \) are linearly related, and in some cases may even be approximately equal:
\[ L_T \approx L_O \] (Peters et al. 1988; Dillon 1982). There are several studies concerning the relationship between \( L_T \) and \( L_O \); for example, Dillon (1982) found that \( L_O = (0.8 \pm 0.4) L_T \). Crawford (1986) found \( L_O = (0.66 \pm 0.27)L_T \), and Ferron et al. (1998) proposed \( L_O = (0.95 \pm 0.6)L_T \).

Based on these results, the dissipation rate has been inferred from density measurements in areas where direct measurement with microstructure profiles is not possible. However, it is worth noting that the aforementioned observational studies pertain to shear-driven turbulence in the stratified upper ocean with relatively small \( O(1\text{-}5) \) m overturns, while topographic hotspots in regions like Luzon Strait often have \( O(10\text{-}100) \) m overturns with the largest turbulent features resulting from convective instability, not shear instability.

The basis for a linear relationship between \( L_T \), a kinematic measure of an overturn length scale based on sorting a density profile, and \( L_O \), defined through a dynamic measure, is not clear a priori, motivating investigations into when and why does the linear relationship hold. Smyth et al. (2001) considered the nonlinear evolution of a KH billow in a stratified layer that is an example of turbulence that is primarily driven by shear instability and, using direct numerical simulation (DNS), found that \( L_O / L_T \) in this problem continuously increased from a small value to an \( O(1) \) value. Thus, the Thorpe estimate of dissipation with an \( O(1) \) proportionality between \( L_T \) and \( L_O \) was inferred to be more applicable to “old” turbulence. Mater et al. (2013) analyzed DNS simulations of decaying homogeneous turbulence, finding agreement between \( L_T \) and \( L_O \) only for the special case when the buoyancy and turbulence time scales are equal. It was also found that \( L_T \) correlated well with length scales based on turbulent kinetic energy \( K \); the large-eddy length scale \( K^{1/2}/\varepsilon \), when buoyancy was weak; and the buoyancy length scale \( K^{1/2}/N \), when buoyancy was strong.

Chalamalla and Sarkar (2015) considered turbulence driven primarily by convective instability of overturns in a stratified, oscillating boundary flow, an idealization of near-bottom flow driven by a topography-affected internal wave. A smaller-scale problem (\( L_T \) as large as 2 m) was examined using DNS, and a larger-scale problem (\( L_T \) as large as 20 m) was examined using high-resolution LES. The Ozmidov length scale and Thorpe length scale were found to behave differently during a tidal cycle: \( L_T \) decreased as the large convective overturn event (LCOE) evolved in time, while \( L_O \) increased, there was a significant phase lag between the maxima of \( L_T \) and \( L_O \), and, finally, \( L_T \) was not linearly related to \( L_O \). The ratio \( L_T / L_O \) could be larger or smaller by one to two orders of magnitude. Interestingly, the cycle-averaged values of \( L_T \) and \( L_O \) were found to have an \( O(1) \) ratio in both DNS and LES, a result that was explained by the available potential energy in the LCOE being the predominant driver of turbulence in that flow. It is worth noting that Chalamalla and Sarkar (2015) employed an inversion-based method to calculate \( L_T \) (section 3b) and not the conventional Thorpe sorting method (section 3a). Mater et al. (2015) compared \( L_T \) and \( L_O \) using data from three observational campaigns: Internal Waves in Straits Experiment (IWISE) conducted in Luzon Strait where there are \( O(100) \) m overturns associated with steep, supercritical topography; Brazil Basin Tracer Release Experiment (BBTRE) with \( O(3) \) m overturns in a site with bottom-enhanced turbulence; and the North Atlantic Tracer Release (NATRE) where turbulence is representative of less energetic, interior ocean turbulence. Instantaneous profiles of \( L_T \) were found to have bias with respect to those of \( L_O \); the resultant bias in turbulent dissipation was mitigated by ensemble averaging but not by either depth or time integration. The bias was especially large and positive for a series of profiles taken at Luzon Strait (large overturns with turbulence primarily driven by convective instability), resulting in an \( O(10) \) overestimate in the depth- and time-integrated dissipation. Scotti (2015) used DNS to evaluate the accuracy of overturn-based turbulent dissipation during the nonlinear evolution of an initially unstable patch in an otherwise stably stratified fluid as an idealized example of convectively driven mixing and found that \( L_T \) is much larger than \( L_O \). On the other hand, \( L_T \) was able to diagnose turbulent dissipation in stratified Couette flow between two walls in relative motion, an example of shear-driven turbulence.

Bias in overturn estimates of turbulent dissipation at rough topography has been investigated using...
observational data by Mater et al. (2015), but the simulation-based investigations have so far been restricted to canonical flows as described above that do not have topographically generated internal waves or rough boundaries. This motivates the present study of a northern section of the western ridge of Luzon Strait with multiscale topography. Focusing on the western ridge allows high resolution of turbulent features but has the disadvantage of excluding the physical effect of wave resonance between the two ridges (Alford et al. 2011; Buijsman et al. 2012) at Luzon Strait. The computational method, LES technique, and model setup are described in section 2. Section 3 describes two alternate methods from prior studies that are adopted here for obtaining overturn-based dissipation estimates from the simulation data. Large-eddy simulations to capture flow near such a complex bathmetry are the first of their kind, with various turbulence mechanisms contributing to the energy cascade to smaller scales. Therefore, before venturing into the complex problem of Luzon Strait, the DNS simulation data of Chalamalla and Sarkar (2015) will be reexamined in section 5 of this paper, where the dissipation rates calculated from the simulation data will be compared against dissipation rates inferred from both conventional Thorpe and inversion sorting methods. The realism of the present model in capturing the baroclinic response of the system over the local topography is demonstrated in section 4 by performing a simulation (case M) with barotropic forcing that was measured at station N2 during a period of M2-dominant tide in August 2010 and reported as dataset N2a by Alford et al. (2011). We then turn to the main objective of the paper: evaluation of the accuracy of overturn-based estimates of dissipation at convectively unstable regions above rough topography. To assess the broad range of flow conditions extant at rough topography, the comparison of overturn-based estimates with the turbulent dissipation rate is performed at several spatial stations and with two different values of the forcing amplitude. Results of the comparison are given in section 6. The accuracy of overturn-based dissipation rates has been found to depend on the mechanism of transition to turbulence at different locations, as discussed in section 7. Finally, a summary is given and conclusions are drawn in section 8.

2. Formulation and setup of the LES problem

Luzon Strait is a strong internal tide generation site with broad, rough topography, as illustrated by the transect at 20.6°N (Fig. 1a). It is a double-ridge system with each primary ridge having smaller subridges and topographic bumps. A semidiurnal oscillating tide over a section of the Luzon Strait is studied using fully nonlinear, three-dimensional LES. The model is a scaled-down version of the realistic geometry of an approximately 40-km-long region of the west ridge (shown with a box in Fig. 1a) at 20.6°N that contains station N2, a site of the observational study in 2010 whose data will be compared with the present simulations. The topography shown in Fig. 1c corresponds to high-resolution bathmetry used by Buijsman et al. (2012) and was extracted from a global topography/bathymetry grid (SRTM30_PLUS) with a resolution of 30 arc seconds as well as the Smith and Sandwell database. Our focus in this study is on the west ridge and in particular the two subridges boxed in Fig. 1.

a. Governing equations and numerical method

The Navier–Stokes equations that are numerically solved under the Boussinesq approximation in a non-rotating environment are as follows:

$$\nabla \cdot \mathbf{u} = 0,$$

$$\frac{D\mathbf{u}}{Dt} = -\nabla p^* + F_0(t)\mathbf{i} + \frac{1}{Re}\nabla^2\mathbf{u} - B\rho^*\mathbf{k} - \nabla \cdot \mathbf{\tau},$$

and

$$\frac{DP^*}{Dt} = \frac{1}{RePr} \nabla^2 \rho^* + w \frac{dp^b}{dz} - \nabla \cdot \mathbf{\lambda}.$$

Here, $u$, $v$, and $w$ denote velocity in streamwise (x), spanwise (y), and vertical (z) directions, respectively. Bold letters stand for vector/tensor variables. The non-dimensional variables in the governing equations are

$$\begin{align*}
    t &= t_0 \frac{U_0}{\Omega},
    \mathbf{x} &= (x,y,z) = \left(\frac{x_d,y_d,z_d}{U_0/\Omega}\right),
    p^* &= \frac{p_d^*}{\rho_0 U_0^2},
    \mathbf{u} &= \left(\frac{u_d,v_d,w_d}{U_0}\right),
    \rho^* &= \frac{\rho_d^*}{U_0 \frac{dp_d^b}{dz_d}|_{z_d}}.
\end{align*}$$

(2)

The non-dimensional parameters Reynolds number $Re$, buoyancy parameter $B$, and Prandtl number $Pr$ are defined as

$$\begin{align*}
    Re &= \frac{l_{xx} U_0}{\nu} = \frac{U_0^2}{\nu},
    B &= -g \frac{dp_d^b}{dz_d}|_{z=0} \frac{1}{\rho_0 \Omega^2} = \frac{N^2}{\Omega^2},
    Pr &= \frac{\nu}{k}.
\end{align*}$$

(3)

1 Kindly shared by M. Buijsman (2015, personal communication).
Here, \( p^* \) stands for the deviation from the background hydrostatic pressure, and \( \rho^* \) denotes deviation from the background density state \( \rho_b(z) \). The variable \( n \) is the molecular viscosity, \( \kappa \) is the thermal diffusivity, and \( \rho \) is the density; \( N' \) is a characteristic value of the background buoyancy frequency taken here to be the value at the ridge crest; \( V \) is the tidal frequency; and \( F_b \) is a barotropic forcing term. It is also common in tidal flows to alternatively use the Reynolds number based on the Stokes boundary layer thickness \( d_s = \sqrt{2 \delta \Delta} \).

In Eq. (1), \( \tau \) is the subgrid-scale (SGS) stress tensor, and \( \lambda \) is the SGS heat flux. The popular dynamic Smagorinsky model (Smagorinsky 1963; Germano et al. 1991; Zang et al. 1993) is used to calculate the SGS stress tensor \( \tau \) as

\[
\tau_{ij} = -2n_i \delta_{ij} \nu_t = (C_s \Delta)^2 |\hat{\mathbf{S}}|.
\]  

(5)

Here, the variable \( \hat{\mathbf{S}} \) is the rate-of-strain tensor and \( C_s \) is the Smagorinsky coefficient that is dynamically evaluated using a test filter \( \Delta_t \Delta = \alpha \), equal to \( \sqrt{6} \) (Lund 1997). The model coefficient \( C_s \) is given by

\[
(C_s^2) = \frac{\langle L_{ij} M_{ij} \rangle}{\langle M_{ij} M_{ij} \rangle},
\]  

(6)

where

\[
L_{ij} = \bar{u}_i \bar{u}_j - \bar{u}_i \bar{u}_j, \quad M_{ij} = 2\Delta^2(\bar{\mathbf{S}}_{ij} - \alpha^2 \bar{\mathbf{S}} \cdot \bar{\mathbf{S}}_{ij}).
\]  

(7)

The test and grid filters are denoted by \( \bar{\cdot} \) and \( \langle \cdot \rangle \), respectively, where \( \langle \cdot \rangle \) denotes the spanwise average. The magnitude of the resolved strain rate tensor is given by

\[
|\hat{\mathbf{S}}| = \sqrt{2 \bar{S}_p S_{ij}}.
\]  

Similarly, the SGS heat flux \( \lambda \) is obtained using a dynamic eddy viscosity model (Armenio and Sarkar 2002) as given below:

\[
\lambda_j = -\kappa_i \frac{\partial \rho}{\partial x_j}, \quad \kappa_i = (C_\rho \Delta)^2 |\hat{\mathbf{S}}|.
\]  

(8)

Here, \( \rho = \rho^* + \rho^b \) is the density, and the model coefficient \( C_\rho \) is given by

\[
\text{FIG. 1. (a) Cross section of the double-ridge Luzon Strait. (b) The body-fitted computational domain; the grid has been shown partially with one of every 10 horizontal and one of every two vertical grid points. (c) The vertical profile of density and buoyancy frequency profile inferred from observed temperature profiles (Alford et al. 2011). The body-fitted computational domain in (b) is a 1:100 scaled-down model of the boxed region in red shown in (a) while the buoyancy frequency and density are the scaled-up values. Such scaling keeps the key nondimensional parameters identical between the ocean and the computational model as discussed in Table 1. The viscosity in the simulations is scaled up by a factor of 20 to } 2 \times 10^{-5} \text{ m}^2 \text{s}^{-1}.\]
TABLE 1. Key dimensional and nondimensional parameters are compared between the computational model and the ocean at the Luzon western ridge. The key nondimensional parameters, except for the Reynolds number, are the same between the model and the ocean. The computation domain has a streamwise length of \( L_x = 800 \text{ m} \) (80-km ocean scale), height of \( L_z = H = 36 \text{ m} \) (3.6-km ocean scale), and spanwise length of \( L_y = 24 \text{ m} \) (2.4-km ocean scale). The variable \( l \) represents the half-width of the hill; Froude number \( Fr \) is defined as \( Fr = U_0/Nh \); and criticality \( \epsilon \) is the ratio of the topographic slope to the slope of the internal wave characteristic.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit</th>
<th>Model</th>
<th>Ocean</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U_0 )</td>
<td>( \text{m s}^{-1} )</td>
<td>1.132</td>
<td>1.132</td>
</tr>
<tr>
<td>( \Omega )</td>
<td>s(^{-1} )</td>
<td>( 1.405 \times 10^{-2} )</td>
<td>( 1.405 \times 10^{-4} )</td>
</tr>
<tr>
<td>( N_e )</td>
<td>s(^{-1} )</td>
<td>11</td>
<td>0.0011</td>
</tr>
<tr>
<td>( h )</td>
<td>m</td>
<td>21</td>
<td>2100</td>
</tr>
<tr>
<td>( H )</td>
<td>m</td>
<td>36.94</td>
<td>3694</td>
</tr>
<tr>
<td>( l )</td>
<td>m</td>
<td>400</td>
<td>40000</td>
</tr>
<tr>
<td>( \nu )</td>
<td>( \text{m}^2 \text{s}^{-1} )</td>
<td>( 2 \times 10^{-5} )</td>
<td>( 10^{-6} )</td>
</tr>
</tbody>
</table>

\[
C_r^2 = \frac{\langle L_p^6 M_p^6 \rangle}{\langle M_p^6 \rangle}, \quad (9)
\]

where

\[
L_p^6 = \overline{\rho \mathbf{t} \cdot \mathbf{u}} - \overline{\rho \mathbf{u}}, \quad M_p^6 = 2\Delta^2 \left( \overline{\mathbf{S} \frac{\partial \rho}{\partial x_j}} - \alpha^2 \overline{\mathbf{S} \frac{\partial \rho}{\partial x_i}} \right). \quad (10)
\]

The governing equations, written in generalized curvilinear coordinates, are solved on a body-conforming grid. Derivatives in the spanwise horizontal \( y \) direction are computed using a spectral method, and derivatives in the streamwise and vertical directions are computed with a second-order, central, finite-difference scheme. A third-order Runge–Kutta–Wray time advancement is used within the fractional step method, and a multigrid solver is employed to solve the Poisson equation for pressure. A sponge layer or Rayleigh damping is added at the left and right boundaries to minimize spurious reflections. Boundary conditions, sponge layer, and numerical methods are designed and implemented along the lines of Rapaka et al. (2013).

b. Simulation setup

The LES model is applied to simulate flow at topography that is two-dimensional. A three-dimensional computational domain that enables the simulation of three-dimensional turbulent motions is discretized with 1281 grid points in the streamwise \( x \) direction, 193 grid points in the vertical \( z \) direction, and 64 equispaced points in the spanwise \( y \) direction. The stretched grid has higher resolution near the bottom and also near the ridge top, as illustrated in Fig. 1b. Three different cases have been simulated with different barotropic forcing. The first simulation, case M, is constructed with the measured barotropic forcing to run for 2 days during a period of semidiurnal dominant tides (yeardays 238.6–240.6). This period covers measurements of Alford et al. (2011) at station N2a in late August 2010. Two additional simulations (A and B) with semidiurnal frequency and different amplitudes of tidal forcing are also performed.

Table 1 gives dimensional values of the key variables in case M at both ocean scale (column 4) and model scale (column 3). The term \( U_0 \) is the barotropic velocity and the tidal frequency is \( \Omega \). The depth at the deepest point is \( H \), the ridge crest (at model coordinates of \( x = 0, z = -16 \)) is at a height \( h \) with respect to the deepest point, \( N_e \) is the buoyancy frequency at the ridge crest, and \( l \) is the streamwise length of the topography. The barotropic forcing for case M is kept the same between the computational model and observations at station N2. To achieve high resolution of turbulent motions, the model length scales are decreased (1:100 in both horizontal and vertical) without changing the aspect ratio. The characteristic frequencies (\( N \) and \( \Omega \)) are increased by a factor of 100 to keep the nondimensional parameters excluding the Reynolds number the same between model and ocean. The model features a background barotropic forcing obtained from curve fitting of the measured barotropic velocity at location N2.

To investigate the accuracy of overturn-based estimates, two other cases have been simulated with high and low barotropic forcing amplitudes. The forcing is at a scaled semidiurnal frequency of \( \Omega = 0.01406 \text{ s}^{-1} \):

\[
U_{bt}(t_d) = U_0 \sin(\Omega t_d), \quad (11)
\]

where the amplitude \( U_0 \) takes the values 0.05 and 0.1 ms\(^{-1} \) in cases A and B, respectively. The subscript “bt” denotes barotropic. The forcing is accomplished by an imposed horizontal pressure gradient that oscillates in time \( t_d \):

\[
F_0(t_d) = \rho_0 U_0 \Omega \sin(\Omega t_d). \quad (12)
\]

All other settings and parameters in cases A and B are as for case M.

3. The computation of turbulent dissipation rate

The turbulent dissipation rate \( \epsilon \) can be inferred from the overturn thickness and buoyancy frequency \( N \) by assuming...
that an overturn length scale defined by the Thorpe scale $L_T$ is proportional to the Ozmidov length scale (Ozmidov 1965). The procedure for calculating $L_T$ involves resorting the instantaneous potential density profile at a particular observational station and time into a stable monotonic profile as well as identifying overturns. We continue below with a description of the conventional Thorpe sorting method and the proposed inversion sorting method.

a. Thorpe sorting method

The Thorpe sorting method appears in several previous studies, for example, Galbraith and Kelly (1996), Klymak et al. (2008), and Gargett and Garner (2008). The following procedure is based on the discussion in Klymak et al. (2008), and the associated algorithm is given in the appendix (algorithm 1). The density profile containing inversions is resorted into a stable monotonic density profile. Note that if the vertical grid is not constantly spaced, an interpolation of the density profile to an evenly spaced grid is performed to conserve the average density of the profile in the resorting process. The Thorpe-scale displacement is given by the difference in depth of each fluid parcel between the unsorted and sorted profiles:

$$d_p = Z(\rho) - Z(\rho_i).$$

An auxiliary variable $S_p$ is introduced. At each vertical point, $S_p$ is the depth sum of individual Thorpe displacements $d_p$ from the upper bound of the density profile up to that point. The bounds of an overturn are determined by the locations where $S_p$ drops back to zero (in practice, a threshold close to zero is adopted). The Thorpe length scale $L_T$ associated with each so identified overturn is calculated as the root-mean-square of displacements for the points within that overturn:

$$L_T = \sqrt{\langle d_p^2 \rangle},$$

where $N_{ov}^2$ is computed as the averaged density gradient within the overturn, exactly as in the original Thorpe method. We filter out small overturns from inversions that span less than or equal to three grid points since they are marginally resolved. In application of this method to an oceanic dataset, a similar thresholding based on noise in the data will need to be adopted.

c. An example computation of the Thorpe scale

Figure 2 shows the overturn bounds that were employed for the dissipation calculation at a particular time $t = 239.93$ (yearday) at N2 station using measurements (Alford et al. 2011) at Luzon Strait that were kindly shared by the authors. Figure 2a shows the unsorted and sorted profiles in the Thorpe sorting method, and Fig. 2b shows the Thorpe displacement and the summation of displacements $S_p$. The region between successive zero values of $S_p$ is identified as the vertical extent of an overturn (as indicated by the double arrow line for each overturn) in the Thorpe sorting method. Figures 2c and 2d show overturn bounds using the inversion and Thorpe sorting approaches, respectively, in a 250-m vertical section. The Thorpe sorting approach in Fig. 2d detects a big overturn between depths of approximately 1500 and 1740 m [in agreement with Alford et al. (2011)], while the inversion sorting approach in...
FIG. 2. Example of Thorpe-scale calculation by the different overturn sorting methods using the potential density profile measured at station N2 of the IWISE experiment (Alford et al. 2011) at \( t = 239.93 \) (year-day). (a) Unsorted (black solid) and sorted (red dashed) potential density profile used by the Thorpe sorting method. (b) Local vertical displacement \( d \) in dashed black and summation of displacements \( S \) in red. Overturn boundaries identified by the Thorpe sorting method are shown with blue dashed–dotted lines, and the vertical extent of each identified overturn is indicated by a blue double-arrowed line. (c) Zoom of unsorted density profile (black) showing overturn boundaries identified by the new inversion sorting method that is based on local density inversions. Several overturns are identified. (d) Same zoom of unsorted density (black) as in (c). The red dashed line is a zoom of the sorted density profile that was shown in part (a). The Thorpe sorting method identifies a large 250-m overturn instead of the smaller overturns identified by the inversion sorting method.
Fig. 2c detects multiple smaller overturns within the same region.

d. Model dissipation from LES

The dynamic Smagorinsky model, used in this simulation, is a well-established subgrid model for LES that has been applied successfully in several previous studies, for example, in a rotating channel flow (Piomelli 1993), mixing layers (Vreman et al. 1997), scalar mixing in axisymmetric jets (Akselvoll and Moin 1996), turbulent boundary layer over a bump (Wu and Squires 1998), and flow over bluff bodies (Rodi et al. 1997). The accuracy of turbulent dissipation rate obtained from the subgrid model has been validated in several studies, for example, by Martin et al. (2000), who found close agreement between the dynamic Smagorinsky model and DNS in the case of isotropic turbulence, and by Foysi and Sarkar (2010) in a compressible mixing layer. Recently, Pham and Sarkar (2014) compared DNS and LES results in a stratified shear layer and found good agreement between them with regards to the buoyancy-induced reduction in shear layer growth rate and the TKE balance.

Resolved dissipation is defined as

\[ e_{\text{resolved}} = \mu \left( \frac{\partial u_i'}{\partial x_j} \right)_y, \]  

(17)

where \( u_i' \) stands for velocity fluctuations with respect to the spanwise-averaged mean velocity and the subgrid-scale dissipation \( e_{\text{sgs}} \) is given as

\[ e_{\text{sgs}} = -\langle \tau_{ij} S_{ij} \rangle_y, \]  

(18)

where \( \tau_{ij} \) is the subgrid-scale stress tensor, and \( S_{ij} \) is the strain rate of the LES field.

The setup has a stretched grid in the vertical direction in addition to conforming with the bottom boundary. This allows enhanced resolution of the boundary layer and processes near the ridge top. In the vicinity of the ridge top, the grid spacing is \( \Delta x = 0.089 \text{ m} \) and \( \Delta z = 0.0475 \text{ m} \). The spanwise direction has a uniform grid size of \( \Delta \gamma = 0.375 \text{ m} \), and spanwise derivates are computed using a spectral method. The resolution in the boundary layer is adequate, for example, at the no-slip boundary; case B has \( \Delta x^+ = 3.83 \), \( \Delta y^+ = 15.97 \), and \( \Delta z^+ = 2.02 \) in units of the viscous scale \( \nu/u_a \), with \( u_a \) denoting the average friction velocity calculated using simulation results.

Figures 3a and 3b show the spanwise spectra of the TKE at two locations S2 and S3. The spectra are calculated at the phase of flow reversal from down to upslope, where the turbulence due to wave breaking is at its peak. The three orders of magnitude drop in the wavenumber spectra shows that the model is able to capture almost all the turbulent kinetic energy that is indicative of LES with sufficient resolution (Pope 2000). However, to make sure the stretched grid does not lead to an underestimation of turbulent dissipation, we have added a positive residual component when computing the model dissipation:

\[ e = e_{\text{resolved}} + e_{\text{sgs}} + e_{\text{residual}}. \]  

(19)

The residual term is based on the numerical balance of the TKE equation

\[ \frac{\partial K}{\partial t} = P - e + B - \nabla \cdot \mathbf{T}. \]  

(20)

Here, \( K \) is TKE defined by \( (1/2) \langle u_i' u_i' \rangle; P \) is the production term defined as

\[ P = -\langle u_i' u_i' \rangle \langle S_{ij} \rangle - \langle \tau_{ij} \rangle \langle S_{ij} \rangle, \]

where the last term is the SGS production. The turbulent dissipation rate \( e \) is defined as the sum of the resolved and SGS components, and \( B \) is the buoyancy flux defined as

\[ B = -\frac{g}{\rho_0} \langle p' w' \rangle. \]

The term \( \nabla \cdot \mathbf{T} \) denotes the transport of the turbulent kinetic energy and contains pressure transport, turbulent transport, viscous transport, and SGS transport.

The Ozmidov length scale at the same locations and phase described above are also shown in Figs. 3c and 3d. At location S2, shown in Fig. 3c, the average of \( L_O \) spanning between the bottom surface and \( z = -16 \text{ m} \) is about 0.326 m, which is equivalent to approximately 8\( \Delta \gamma \), suggesting that the Ozmidov scale is well resolved. Figure 3c also has a subpanel zoomed in near an energetic turbulence patch at \( z = -17 \text{ m} \). The boundaries of the patch are marked by dashed lines, and the patch has a vertical extent equivalent to 13 grid points. At location S3 shown in Fig. 3d, the average \( L_O \) spanning the region between the bottom boundary and \( z = -15 \text{ m} \) is 0.286 m, which is approximately 8\( \Delta \gamma \) based on the grid spacing in the local region. The good resolution of the Ozmidov scale by the grid is another indicator of adequate LES resolution. The CFL number of 0.57 constrains the time step \( \Delta t \)
FIG. 3. (top) Two examples of spanwise wavenumber $k_y$ spectra of the turbulent kinetic energy. Spectra at three different heights are shown for each example. (bottom) Turbulent dissipation rate (blue), background buoyancy frequency (black), and Ozmidov length scale (red) plotted in SI units. (a),(c) S2 location at $t = 2.47T$; (b),(d) S3 location at $t = 2.53T$. The location of stations S2 and S3 on the ridge are shown in Fig. 7h.
to a small value of approximately 0.04 s, resulting in approximately 11,000 time steps in a tidal cycle. The model LES dissipation rate is compared with the overturn-based dissipation rates at the modeled Luzon Strait in section 6.

4. Model results for case M

Results for case M are shown to illustrate similarity between the velocity and density between numerical simulation and field observation. The spatial structure of the internal wave (IW) field at the west ridge shows small-scale, nonlinear features and tall overturns consistent with observations (Alford et al. 2011) and simulations (Buijsman et al. 2012) except for a short phase when the resonant interaction with the wave field from the east ridge (absent in the present model) is important locally to the flow. Figure 4 shows the internal wave response at four different phases of barotropic velocity (shown in the header). Coherent internal wave beams with velocity that is markedly larger than the background barotropic velocity radiate from the topography and interact with the surface. Figures 4a and 4b show the velocity field at two instances of time when the westward (negative) and eastward (positive) velocity reach maximum magnitude, respectively. The internal wave beams are especially strong at these times. Rays corresponding to harmonics and interharmonics can also be seen, and, in addition, features corresponding to asymmetric response at the ridge top such as downslope jets and lee waves are also evident. During the phases of zero barotropic flow (Figs. 4c,d), the valley between the two subridges exhibits a strong baroclinic flow with an interesting pattern and significant distortion of the isopycnals.

The time evolution of the velocity profile and isopycnal displacement is presented (Fig. 5) at station N2 (location marked in Fig. 4) for both simulation and field data (Alford et al. 2011) taken during yeardays 239.04–240.37. Simulation and observation are in general agreement with respect to the spatial structure and

2 The measured field data at N2 was kindly shared by the authors.
phasing of the velocity and the associated isopycnal displacements. In both cases, strong eastward flow during the peak of the barotropic tide is found at N2 as a result of velocity intensification in the IW beam shown as A2 and B2 for the simulation in Fig. 5b and A'2 and B'2 for the observation in Fig. 5a. This strong flow depresses the isotherm at N2 during the downslope phase, but the depressed isopycnals rebound during the upslope phase.

Although, the features denoted by B2 and A2 compare well with observations at station N2, there is an important difference as well. The east ridge is absent in the present simulations and so is the constructive resonance between the internal wave generated at the east ridge and the locally generated internal wave at N2. The near-bottom velocity during eastward flow at N2 is therefore weaker, leading to a less powerful downslope jet and therefore smaller velocity and isopycnal displacement during feature B3 compared to observations. The scattering effect of the three-dimensionality in the ocean is also worth noting. As a result, the vertical beam structure in the model is cleaner and more coherent relative to observations. The higher Reynolds number in the ocean, larger by three orders of magnitude, also has an impact: the measured isopycnals are sharper and exhibit more small-scale fluctuations than the modeled ones.

5. Idealized simulation of internal tide breaking

In this section, results from a previous numerical study by Chalamalla and Sarkar (2015) will be revisited to compare dissipation rates inferred from overturn-based methods against the dissipation rates calculated from the simulation data. They performed DNS of an oscillatory bottom layer on a slope underneath an internal tide to quantify turbulence and mixing brought about by density overturns during the flow reversal from down to upslope. Convective instability is found to be the primary mechanism of transition to turbulence, where the large density overturn formed during the transition from down to upslope flow breaks down into small-scale turbulence at a later time. The dominant fraction of the turbulent dissipation in this flow is associated with convectively driven turbulence ensuing from the LCOE, and a smaller fraction is due to boundary layer dissipation during upslope flow. Thus, it is an idealized scenario to study the accuracy of various overturn-based dissipation rates during turbulence driven by wave breaking.
and density overturns. They, without explicitly stating it, adopted the inversion sorting method instead of the conventional Thorpe sorting method in their Thorpe-scale analysis [more details about this study can be found in Chalamalla and Sarkar (2015)].

Figure 6a shows the temporal evolution of Richardson number profiles at the midslope of the DNS simulation for one cycle. Richardson number \( R_i = N^2/S^2 \) can be used as an indicator of possible instability. Here, \( S \) is the mean (span averaged) shear, and \( N \) is the mean (span averaged) buoyancy frequency. We plot values of negative \( R_i \) that correspond to statically unstable regions. In Fig. 6a, the interval between \( \phi = -0.3\pi \) and \( 0.2\pi \), which has spatially coherent and highly negative \( R_i \), corresponds to a LCOE.

Figure 6b shows the time evolution of spatially averaged dissipation calculated from both Thorpe-scale and inversion-based methods. The ensemble average at any given time is computed by averaging the results of overturn-based analysis at that time, conducted for all vertical lines in the computational domain. We focus on the LCOE. Initially, when the overturn is still young and growing, both Thorpe-scale dissipation \( \varepsilon_T \) and inversion-scale dissipation \( \varepsilon_I \) increase. The turbulent dissipation \( \varepsilon \) from the simulation also exhibits an increase that commences at a phase of \( -0.1\pi \), and it peaks at the phase of \( 0.15\pi \) during the LCOE. However, the increase of \( \varepsilon_T \) is excessive during the overturn growth. The dissipation rate estimated from the Thorpe scale becomes higher by between one and two orders of magnitude when compared to either the model dissipation rate calculated from the simulation data or that obtained using the inversion method. Although there is a clear phase lag of the dissipation calculated from the simulation data with respect to the dissipation based on the inversion scale, the peak value of these dissipation rates are in good agreement.

The time-averaged dissipation rate over different periods of the cycle is of equal interest. During the life cycle of the LCOE, the dissipation rate from the Thorpe sorting method leads to a substantial overprediction, \( \Sigma \varepsilon_T/\Sigma \varepsilon = 14.6 \), while the time-averaged dissipation based on the inversion sorting method leads to a better estimate of \( \Sigma \varepsilon_I/\Sigma \varepsilon = 0.6 \). On the other hand, outside of the LCOE,
the Thorpe sorting method is substantially more accurate. The time average over the non-LCOE part of the cycle is \( \epsilon / \epsilon = 0.42 \), while in the same period \( \epsilon / \epsilon = 0.014 \). It highlights the fact that the inversion sorting method underestimates the dissipation when the source of turbulence is not convective instability. Looking at the time-averaged dissipation rate during the entire cycle, the ratios of \( \epsilon / \epsilon = 5.39 \) for the Thorpe sorting method and \( \epsilon / \epsilon = 0.21 \) for inversion sorting indicate deficiencies of each method within and outside of LCOE, respectively. Ideally, inversion sorting should be used for LCOE and, otherwise, Thorpe sorting.

Sensitivity of the two methods to the resolution of the grid, which samples the density, is also investigated using the DNS data. Four different sampling grids are used to calculate overturn-based dissipation estimates from the single dataset coming from the DNS of Chalamalla and Sarkar (2015). The same algorithm and threshold are used for all the sampling grids. The vertical grid resolution of the sampling grids is 1.5, 2, 2.5, and 4 times larger than that of the original DNS grid. The results that are presented in Table 2 show that the Thorpe sorting method is less sensitive to the size of the sampling grid, both inside and outside of the LCOE. The variation of the Thorpe sorting method as a function of sampling grid resolution is found to be between 10% and 20%. Meanwhile, the inversion sorting estimate is more influenced by the resolution of the sampling grid. The dissipation tends to increase with the grid spacing in the inversion method because the overturn size increases. The inversion sorting method is more dependent on the local behavior of the density, while the Thorpe sorting method is dependent on the entire density profile since overturn boundaries are marked using the summation of displacement for the entire column.

6. A broader assessment of turbulent dissipation rate

In the present section, we broaden the scope of our assessment of overturn-based estimates of turbulent dissipation rate to investigate regions with dominant convective instabilities in a more realistic setup. Both Thorpe sorting as well as inversion sorting estimates are compared with the model dissipation rate obtained from simulation results. The comparison is performed at several locations in the vicinity of the ridge top for case A with \( U_0 = 0.05 \text{ m s}^{-1} \) and case B with \( U_0 = 0.1 \text{ m s}^{-1} \). The time evolution of the vertically integrated value of turbulent dissipation is compared among the two overturn-based methods and the LES model. Four different spatial locations with large wave-breaking features are chosen for the comparison in this case study, as shown in Fig. 7h: S1 is at the right flank approximately corresponding to the measurement station of N2, S3 is at the top of the ridge, S2 is midway between S1 and top of the ridge, and S4 is on the left flank.

The time evolution of depth-integrated dissipation is presented in Fig. 7. Model dissipation \( \epsilon \) is compared with both the Thorpe sorting method \( \epsilon_T \) applied to the simulation results and the inversion sorting method \( \epsilon_I \) also applied to the simulation results. The evolution of turbulent dissipation is shown over three tidal cycles with the first tidal cycle corresponding to a ramp up from the state of no turbulence. As elaborated below, at these four locations, the original Thorpe method significantly overestimates the turbulent dissipation in terms of magnitude and temporal duration. The inversion sorting method shows better agreement with the turbulent dissipation rate directly computed from the simulation. For example, the inversion sorting method (blue curve in Fig. 7a) quantitatively matches the turbulent dissipation rate computed from the model (black curve) at location S2, while the Thorpe sorting method (red curve) generally overestimates it, often by an order of magnitude.

The depth-integrated turbulent dissipation near the summit (station S3) is shown in Fig. 7b. It is noteworthy that, similar to DNS results of Jalali et al. (2014), there are two dissipation maxima per cycle corresponding to the passage of turbulence from each of the two internal wave beams generated at the ridge flanks, while there is only one maximum at the left or right midslope locations. Similar to the situation at the S1 virtual mooring,
the conventional Thorpe sorting method overestimates the dissipation at S2, S3, and S4.

Thus, our results show that the Thorpe sorting estimate of dissipation at S1–S4, which are locations with breaking internal waves, lead to significantly larger turbulent dissipation rate relative to the value in the simulation. Comparison between overturn-based estimates and direct measurements of dissipation at sites with direct wave breaking are scant. However, Nash and Moum (2013) present a comparison of dissipation rate from microstructure measurement using Chipods with the Thorpe sorting estimate at the N2 mooring in the IWISE-2011 experiment. Their results suggest that the Thorpe sorting estimate is an overprediction relative to
the direct measurement, consistent with the finding in our simulations.

An overall view of the accuracy of Thorpe sorting estimates in locations with large convective overturns is provided by comparing their cycle average with the cycle-averaged dissipation $\Sigma e$ in the model. Table 3 shows that the Thorpe sorting method leads to a substantial overestimate at those locations. The overestimate can be as much as a factor of 10. This shows that the inversion sorting method is a better predictor near the ridge top, where internal wave breaking leads to turbulence, albeit with occasional underestimation during some portions of the tidal cycle.

### Table 3. Ratio of cycle-averaged Thorpe estimate to cycle-averaged turbulent dissipation ($\Sigma e_T/\Sigma e$) and a similar ratio of cycle-averaged values for the inversion sorting method ($\Sigma e_I/\Sigma e$). The cycle average is computed over the span of the second and third cycles.

<table>
<thead>
<tr>
<th>Location</th>
<th>Case A (Thorpe)</th>
<th>Case B (Thorpe)</th>
<th>Case A (inversion)</th>
<th>Case B (inversion)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S2</td>
<td>8.76</td>
<td>15.28</td>
<td>0.85</td>
<td>0.63</td>
</tr>
<tr>
<td>S3</td>
<td>11.14</td>
<td>12.38</td>
<td>1.55</td>
<td>0.66</td>
</tr>
<tr>
<td>S4</td>
<td>16.69</td>
<td>16.64</td>
<td>1.76</td>
<td>0.89</td>
</tr>
<tr>
<td>S5</td>
<td>8.45</td>
<td>6.16</td>
<td>1.05</td>
<td>0.57</td>
</tr>
</tbody>
</table>

### 7. Discussion

Based on the results discussed in the previous sections, it can be inferred that the inversion sorting method estimates the dissipation rate more accurately at locations where direct wave breaking is a major contributor to turbulence. In the present section, we seek a more complete understanding of the inversion sorting method and the flow conditions under which it is more accurate compared to the Thorpe sorting approach.

The fundamental difference between shear-driven turbulence and convectively driven turbulence is that in shear-driven turbulence there is a direct transfer from mean kinetic energy to turbulent kinetic energy through shear production. However, in this example of convectively driven turbulence (wave-breaking event in this study), mean kinetic energy drives the flow to a statically unstable density configuration, leading to an increase in the available potential energy, which is subsequently released to turbulence when the density overturn breaks down into small-scale turbulence. The conventional Thorpe-scale method estimates the total available potential energy in a density profile. However, the recent study by Scotti (2015) argues that only a fraction of the total APE is available for turbulent dissipation during convectively driven mixing.

Thus, a more accurate method of estimating turbulent dissipation rate during convectively driven turbulence is necessary. The fundamental criterion on which the inversion sorting method has been developed is that it is only the available potential energy within a density inversion that can be immediately released to cascade down to turbulence. The density inversion is the region where the density is statically unstable, that is, $dp/dz > 0$ or $N^2 < 0$, and therefore the APE of that overturn is instantaneously available to energize turbulence through flow instabilities. In the inversion method, the density profile is sorted only within an inversion, and the inversion length scale $L_N$ is defined as the rms of displacements of all fluid parcels within the inversion. The term $L_N^2 N^2$ is thus a measure of the instantaneously available potential energy (IAPE) associated with the inversion. Assuming that a substantial fraction of this energy goes into turbulent dissipation over a time scale of the order $1/N$, the turbulent dissipation rate can be estimated as $\epsilon_T \propto L_N^4 N^3$, which is expected to be more accurate in situations where density inversion is a dominant source of turbulence. This is consistent with the results discussed in the previous section, where the inversion sorting method is able to better estimate the dissipation rate of turbulence that originates from density inversions. On the other hand, the conventional Thorpe sorting approach considers zero sum of Thorpe displacements of all fluid parcels within the inversion. The Thorpe length scale is found to be almost 3–8 times larger compared to the inversion length scale at locations of large density inversions, and thus the dissipation rate from the Thorpe sorting method, which is proportional to the square of the length scale ($\epsilon_T \propto L_T^2 N^2$), is expected to be an order of magnitude larger when compared to $\epsilon_T$.

To better explain the differences between the two overturn-based estimates, Fig. 8 contrasts convectively driven turbulence with shear-driven turbulence. Figure 8a shows the density profile that is typically found during turbulence driven by convective overturns, taken from the idealized problem examined by Chalamalla and Sarkar (2015). The black line shows the unsorted
density profile, while the red dashed line shows the sorted density profile obtained using the Thorpe sorting method. The overturn boundaries from the Thorpe sorting method are represented by red dashed–dotted lines, whereas blue dashed–dotted lines represent overturn boundaries using the inversion sorting method. It is evident that the overestimation of dissipation using the Thorpe sorting method is due to the larger overturn height, which results in large Thorpe scale. Figure 8b of this figure will be discussed shortly.

There is an important caveat to the inversion sorting estimate. Since this method measures APE within regions where the density profile is actually unstable, it may not be accurate when turbulence arises because of shear that has a sufficiently small gradient Richardson number for instability or because of advection/transport from elsewhere. In other words, there may be turbulence in locations where the density profile does not necessarily contain an overturned region that spans the entire patch of turbulence.

Returning to the schematic contrasting convective- and shear-driven turbulence, Fig. 8b shows the density profile from a simulation, which has similar parameters to Armenio and Sarkar (2002), where stratified channel flow was studied. In that case, turbulence was mainly driven by shear production near the wall that was then distributed across the channel by turbulent transport. The region with density inversion can be seen to be much smaller than the region with mean shear and turbulence. Therefore, the inversion sorting method, which measures the available potential energy owing to the small inversion (between blue dashed lines), is unable to estimate turbulent dissipation correctly. The two scenarios that we have described here are idealized limits of realistic ocean mixing since ocean turbulence in general involves both shear and convectively driven instabilities. Nevertheless, the idealized scenarios in Fig. 8 allow us to understand the limitations of the alternative overturn-based methods.

The Thorpe sorting method has better agreement in situations where shear instability is responsible for turbulence (Dillon 1982; Crawford 1986; Ferron et al. 1998). In such cases, the dissipation scales as \( u^3/L \propto (SL)^3/L = S^3L^2 \), where \( S \) is the mean velocity shear of the overturn. Furthermore, since the Thorpe sorting overturn measured from the density profile is essentially caused by shear, \( L \) can be taken to be proportional to \( L_T \). Meanwhile, \( S \) is proportional to \( N \) in the overturn since the Richardson number \( N^2/S^2 \) required to get shear instability is \( O(1) \). Therefore, it is reasonable to assume \( \varepsilon \propto L_T^3N^3 \) for shear-driven turbulence. The coefficient has been calibrated in various studies (Dillon 1982; Crawford 1986; Ferron et al. 1998). Shear-driven turbulence does not necessarily need to have a large inversion region for large TKE (an extreme example is shear-driven turbulence in a nonstratified system).

To quantify the relative behavior of Thorpe length scale and Ozmidov length scale in regions with internal wave breaking, the time evolution of average Thorpe scale and Ozmidov scale is shown in Fig. 9 for case B at S1. Figure 9a depicts that at phases with large overturns (e.g., \( t/T = 1.5 \) for S1), \( L_T \) is significantly larger than \( L_O \). Figure 9b, displaying the ratio of \( L_T/L_O \), shows that the widely used assumption of \( L_T/L_O = 1/0.8 = 1.25 \) is
incorrect; $L_T/L_O$ has a large variation from 1 to a peak of 8 during the cycle.

8. Summary and conclusions

Three-dimensional, high-resolution large-eddy simulations are performed for tidal flow over a multiscale topography patterned after a cross section of Luzon Strait, a double-ridge generation site, which was the subject of the recent IWISE experiment. A 1:100 scaling of topography was employed, the molecular viscosity was increased by a factor of 20, and environmental parameters were chosen to match field values of important nondimensional parameters: the slope criticality, excursion number, and Froude number. Using the observational data for the local barotropic velocity at station N2 for model forcing in case M, the simulated velocity (dominated by the baroclinic response) and turbulent dissipation are demonstrated to exhibit good agreement with the corresponding field observations. An exception is a short duration of the tidal cycle when the resonant interaction with the east ridge (a topographic feature that is not included in the present computational model so as to optimally resolve turbulence at N2 with available computational resources) is important for the baroclinic response at N2.

The focus of the paper is an assessment of overturn-based estimates of turbulent dissipation at rough topography where the oscillating barotropic tide generates internal gravity waves. The conventionally used Thorpe sorting method lead to overprediction of turbulent dissipation relative to the model value at station N2. To broaden the assessment, two different cases A and B with low and moderate tidal velocities, respectively, are simulated, and several locations on the model ridge transect are considered. A new inversion sorting method is proposed, and its performance is compared with the widely used Thorpe sorting method. The Thorpe sorting method can qualitatively estimate dissipation in terms of spatial distribution and phasing but is found to be quantitatively inaccurate. In particular, the Thorpe sorting method overestimates dissipation by more than an order of magnitude at locations with large density inversions where turbulence primarily originates from the convective instability of these large overturns. However, the same method performs better in the case of turbulence driven by strong shear. The overprediction of the Thorpe sorting method in the case of convectively driven turbulent patches in a stratified background is consistent with prior numerical studies of idealized problems, for example, DNS by Scotti (2015) and DNS and LES by Chalamalla and Sarkar (2015). The overprediction is also consistent with the observational study by Mater et al. (2015) that reports a positive bias in overturn-based estimate of dissipation for large overturns in a strongly stratified background.

In the inversion sorting method, the overturn boundaries are determined based on local inversions in the measured density profile rather than by the sum of Thorpe-scale displacements. This inversion sorting method is found to estimate dissipation rate more accurately at locations on the model topography that have large overturns associated with internal waves. The reason is that an inversion that has already formed is statically unstable, and the potential energy inside the inversion is instantaneously available for turbulent mixing after its collapse. The Thorpe sorting method, on the other hand, computes the Thorpe scale and thus the APE in an overturn (defined as a flow-induced distortion of the density from its background profile) that may or may not break down into turbulence later in time in a problem where the shear and density profiles oscillate over a wave period. Release of the APE into turbulence depends on the details of the flow, for example, whether the mean state at later times is statically or shear unstable. The inversion sorting method has a disadvantage too. It underestimates turbulent dissipation when the source of turbulence is the kinetic energy present in the local (unstable because $S^2$ is much larger than $N^2$) sheared velocity profile. Under such flow conditions, the patch of turbulence is larger than what is suggested by the inversion scale in the measured profile since instantaneously available potential energy (IAPE) of the inversion is not the primary driver of turbulence. On the other hand, the standard Thorpe method works well in these shear-unstable situations with $Ri_b = O(1)$, since $L_T^2 N^3$ becomes
proportional to $S^3L^2$, which is the inertial estimate ($u^3/L$ with the turbulent velocity $u$ taken as $SL$) of turbulent dissipation. Neither overturn-based method is accurate when the turbulence is locally due to advection or turbulent transport from elsewhere.

To conclude, both overturn-based methods give overall qualitative trends regarding the turbulent dissipation in the case of internal waves at rough topography. However, quantitative accuracy is highly dependent on the details of the flow. The inversion sorting method is better at estimating turbulent dissipation when the turbulence results directly from available potential energy (breaking internal waves in the present problem), while the widely used Thorpe sorting method is better when the pathway to turbulence is directly from kinetic energy (near-bottom unstable regions of stratified shear). At hotspots of turbulence such as steep, rough topography similar to the flow simulated here, the dominant contributor to turbulence in the near field is likely to be breaking internal waves, and, if so, the inversion sorting method, after calibration for the proportionality coefficient, could provide a more accurate estimate of cycle-averaged turbulent dissipation than the Thorpe sorting method.

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APPENDIX

Algorithms to Estimate Dissipation from Overturns

a. Thorpe sorting method

Algorithm 1: Dissipation estimation using the Thorpe sorting method with potential density (temperature) profile. The value of threshold (used to identify overturn boundaries) is $10^{-6}$, but the results do not change in the present cases as long as it is less than $O(1)$. The algorithm is based on the implementation of Alford et al. (2011).

Input: Potential density (temperature)
Output: Dissipation
1) Find boundaries of input potential density data for a particular vertical profile;
2) Interpolate data to a equispaced grid and store as $\sigma_{th}$;
3) Sort density profile of the entire column into a stable monotonic profile $\sigma_{th,s}$;
4) Find vertical displacement for each point between sorted and unsorted profile $d_{\sigma}$;
5) Calculate sum of displacements from surface $s_{\sigma}$;
6) Set threshold. Mark the boundaries of overturn when $s_{\sigma} < \text{threshold}$;
7) Calculate Thorpe scale $L_T = \sqrt{\text{mean}(d_{\sigma})}$;
8) Calculate overturn $N$ using difference of max and min of density within the unsorted data $N_{ov} = \sqrt{-(g/\rho_0)[\max(\sigma_{th,u}) - \min(\sigma_{th,u})]/h}$; and
9) Calculate the dissipation of the overturn as $\varepsilon = 0.64L_T^3N_{ov}^3$;
end
10) Repeat the steps 1–9 for potential temperature if available and take the minimum of both calculated dissipations as the final estimate;
11) Make plots and report profile of dissipations.

b. Inversion sorting method

Algorithm 2: Dissipation estimation using the inversion sorting method with potential density profile. The value of threshold used here is three grid points and tied to the resolution of the numerical database.

Input: Potential density
Output: Dissipation
1) Find boundaries of input potential density data for a particular vertical profile;
2) Interpolate data to a equispaced grid and store as $\sigma_{th}$;
3) Set threshold. Mark the boundaries of overturn wherever there is an inversion in the profile and the length of inversion $H_i$ is bigger than preset threshold $H_i < \text{threshold}$;
for each marked overturn do
4) Sort the portion of potential density profile within the overturn into a stable monotonic profile $\sigma_{th,s}$;
5) Find vertical displacement for each point between sorted and unsorted profile within the overturn $d_{\sigma}$;
6) Calculate inversion scale $L_I = \sqrt{\text{mean}(d_{\sigma})}$;
7) Calculate overturn $N$ using difference of max and min of density within the unsorted data $N_{ov} = \sqrt{-(g/\rho_0)[\max(\sigma_{th,u}) - \min(\sigma_{th,u})]/h}$; and
8) Calculate the dissipation of the overturn as $\varepsilon = L_I^3N_{ov}^3$;
end
9) Make plots and report profile of dissipations.
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