Influence of the Ocean Mesoscale Eddy–Atmosphere Thermal Feedback on the Upper-Ocean Haline Stratification

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ABSTRACT

The ocean mesoscale eddy–atmosphere (OME-A) interaction through the eddy-induced sea surface temperature anomaly can feedback on ocean dynamics in various ways (referred to as the OME-A thermal feedback). In this study, the influence of the OME-A thermal feedback on the upper-ocean haline structure is analyzed based on high-resolution coupled simulations. In the Oyashio Extension where pronounced surface temperature and salinity fronts are collocated, the haline stratification in the upper 200 m is significantly enhanced by the OME-A thermal feedback. This enhancement is mainly attributed to the weakening of the upward eddy salinity transport in response to the OME-A thermal feedback. The OME-A thermal feedback influences the vertical eddy salinity transport through its differed impacts on the mesoscale buoyancy and temperature anomaly variances. As temperature and salinity in the Oyashio Extension are strongly compensated for their effects on buoyancy, the dissipation of the mesoscale buoyancy anomaly variance $b^2$ by the OME-A thermal feedback is considerably weaker than that estimated from the mesoscale temperature anomaly alone, i.e., $(g\alpha T')^2$, with $g$ the gravity acceleration and $\alpha$ the thermal expansion coefficient. Correspondingly, the vertical eddy buoyancy transport $(w' b')$ is weakened by the OME-A thermal feedback to a lesser extent than its thermal component $(gaw' T')$. The different responses of $w' b'$ and $gaw' T'$ to the OME-A thermal feedback are reconciled by the reduced vertical eddy salinity transport.

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1. Introduction

Ocean mesoscale eddies are ubiquitous in the global ocean. They are remarkably vigorous in regions such as the Kuroshio and Oyashio and their extensions, the Gulf Stream and its extension, the Agulhas Return Current, and the Antarctic Circumpolar Current (Chelton et al. 2011). Besides dominating oceanic kinetic energy (Ferrari and Wunsch 2009), mesoscale eddies are key contributors in transporting heat and materials, leaving profound impacts on climate and biogeochemical processes (e.g., Dong et al. 2014; Zhang et al. 2014; Dong et al. 2017). They are also intensely coupled with the overlying atmosphere and much progress has been made on this topic.

Unlike air–sea interactions at large scales, the ocean mesoscale eddy–atmosphere (OME-A) interaction is regarded as an ocean-driven process. The interaction can be divided into two categories: the effects arising from the sea surface current anomaly and from the sea surface temperature anomaly (SSTA) induced by mesoscale eddies (e.g., Small et al. 2008; Chelton and Xie 2010; Seo et al. 2016; Shan et al. 2020b). On the one hand, the surface velocity associated with mesoscale eddies leaves an imprint on surface wind stress, weakening or enhancing the stress when wind aligns with (opposes to) ocean current (e.g., Cornillon and Park 2001; Renault et al. 2016). Meanwhile, it has a countering effect on wind itself (Renault et al. 2016), accelerating (decelerating) the wind when wind and current are in the same (opposite) direction. On the other hand, the SSTA carried by mesoscale eddies has a profound influence on the atmospheric boundary layer, exerting a significant imprint on local turbulent heat flux, surface wind, and rainfall (e.g., Chelton et al. 2004; Chelton and Xie 2010; Frenger et al. 2013; Bishop et al. 2017; Liu et al. 2018).

As a two-way coupling, the OME-A interaction feeds back onto mesoscale eddies in a variety of ways. The imprint of the surface eddy current anomaly on wind stress acts to damp eddy kinetic energy (EKE) by deflecting energy to the atmosphere (e.g., Duhaut and Straub 2006; Zhai and Greatbatch 2007; Seo et al. 2016). Such an effect is partially compensated by the influence of the surface eddy current anomaly on wind itself (Renault et al. 2016). The SSTA associated with mesoscale eddies induces a dipolar wind stress curl anomaly, affecting eddies’ propagation through Ekman pumping (Gaube et al. 2015; Seo et al. 2016). In addition, the OME-A thermal feedback influences the mesoscale eddy potential energy (EPE) budget through the eddy-induced surface heat flux anomaly (Ma et al. 2016). Extensive observations and model simulations (e.g., Kirtman et al. 2012; Li et al. 2017; Yang et al. 2018; Small et al. 2019) have revealed that a positive (negative) mesoscale SSTA induces an upward (downward) heat flux anomaly. This leads to efficient destruction of the mesoscale temperature anomaly variance, thus eddy available potential energy (Ma et al. 2016).

Salinity of seawater is seldom discussed in the existing literatures of the OME-A interaction and its feedback on ocean dynamics. On the one hand, it is well understood that salinity does not affect air–sea exchanges and exerts no direct influence on the overlying atmosphere. On the other hand, the effect of the OME-A feedback on ocean salinity variability is unexplored. In particular, whether the OME-A feedback can shape the upper-ocean salinity structure by effecting eddy activity and associated transport is unknown. In this study, we attempt to address this question, focusing on the Kuroshio–Oyashio Extension (KOE) region. As a confluence of western boundary currents of subtropical and subarctic gyres, the KOE region is characterized by complex frontal structures where the Kuroshio Extension front, the Kuroshio bifurcation front, the subarctic front and the subarctic boundary exist (Yasuda 2003). Mesoscale eddies in this region are not only highly active but also strongly coupled with the overlying atmosphere through their associated SSTA (e.g., Bishop et al. 2017; Small et al. 2019). Furthermore, the strong salinity gradient associated with the subarctic front (Yasuda 2003) results in a pronounced salinity anomaly carried by mesoscale eddies (Itoh and Yasuada 2010; Dong et al. 2017) and a significant contribution of eddy salinity transport in the upper-ocean salinity budget (see section 3 for details), making the KOE region an ideal place for assessing the influence of the OME-A thermal feedback on salinity structure.

The remainder of this paper is organized as follows. Section 2 provides detailed description of the methodology. Based on high-resolution coupled simulations, the influence of the OME-A thermal feedback on the upper-ocean salinity structure and its reasons are presented in section 3. Discussion is included in section 4, followed by section 5 summarizing the main results in this study.

2. Methodology

a. The coupled regional climate model

A coupled regional climate model (CRCM) developed at Texas A&M University and implemented at Pilot National Laboratory for Marine Science and Technology (Qingdao) is used to assess impacts of the OME-A thermal feedback. The model includes the
Regional Oceanic Modeling System (ROMS) (Shchepetkin and McWilliams 2005) as the ocean component, the Weather Research and Forecasting (WRF) Model (Skamarock et al. 2008) as the atmosphere component, and a coupler that allows the exchanges of mass, momentum, and energy between ocean and atmosphere components (Ma et al. 2016).

The model covers the entire North Pacific from 3.6° to 66°N and from 99° to 270°E, with a horizontal resolution of 9 km for WRF and ROMS. WRF is divided into 30 vertical levels with Lin et al.’s (1983) scheme for microphysics, RRTMG and Goddard scheme for longwave and shortwave radiation (Mlawer et al. 1997; Chou and Suarez 1994), YSU scheme for planetary boundary layer (Hong and Pan 1996), Smagorinsky scheme for calculating eddy coefficient (Smagorinsky 1963), Kain–Fritsch scheme for cumulus parameterization (Kain 2004), and Noah scheme for land surface. ROMS has 50 levels in a vertical terrain-following coordinate, including a K-profile parameterization turbulent mixing closure scheme for vertical mixing (Large et al. 1994) and a biharmonic horizontal Smagorinsky-like mixing for momentum (Griffies and Hallberg 2000). In our configuration, WRF and ROMS are coupled hourly. WRF gives heat, freshwater flux, and wind stress to ROMS, while ROMS provides SST and surface current velocity for WRF.

b. Experiment design

To assess influences of the OME-A thermal feedback, a set of twin experiments are performed. Both experiments consist of an ensemble of five winter half-year integrations. The five cases for each experiment are initialized on 1 October 2003, 2004, 2005, 2006, and 2007, respectively. The detailed information for initial and boundary conditions can be found in Ma et al. (2016). The control experiment (CTRL) is a fully coupled simulation, while in the filter experiment (FILT), a low-pass Loess filter with a 15° (longitude) × 5° (latitude) half-width is applied to the ROMS simulated SST before being given to WRF at each coupling step. In FILT, the atmosphere cannot “feel” the SSTA associated with mesoscale eddies so that the OME-A thermal feedback is largely suppressed. Therefore, the difference between CTRL and FILT enables us to evaluate the influence of the OME-A thermal feedback on haline structure.

c. Salinity budget

To reveal processes shaping haline structure in the upper ocean and their potential regulation by the OME-A thermal feedback, a salinity budget analysis is performed:

\[
\frac{\partial S}{\partial t} = -\left( \mathbf{u} \cdot \nabla S \right) - \left( \nabla_h \cdot (\mathbf{u}_h S') \right) - \frac{\partial w' S'}{\partial z} + \left( \frac{\partial F_S}{\partial z} \right) - (\mathbf{u} \cdot \nabla S + \mathbf{u} \cdot \nabla S'), \tag{1}
\]

where \( S \) is salinity, \( \mathbf{u} = (u, v, w) \) is the three-dimensional velocity with \( \mathbf{u}_h = (u, v) \) denoting its horizontal component, \( \nabla = (\partial \partial x, \partial \partial y, \partial \partial z) \), \( \nabla_h = (\partial \partial x, \partial \partial y) \), \( F_S \) is the vertical turbulent salinity transport (defined positive downward), the overbar denotes large-scale background fields, the prime denotes mesoscale anomalies, and the angle brackets represent the average over an interested region within a certain time period. The mesoscale and background motions are separated using a 2D Gaussian filter in the following analysis. To keep consistency, the standard deviation parameter of the Gaussian filter is tuned to mimic the performance of the 15° (longitude) × 5° (latitude) Loess filter used in CRCM FILT (Shan et al. 2020a). We use the Gaussian filter instead of the Loess filter because the latter carries a formidable computational burden.

The term on the left-hand side of Eq. (1) is the salinity tendency, which is balanced by terms on the right-hand side of Eq. (1). The terms on the right-hand side in sequence are the advection by large-scale background flows, the horizontal eddy salinity transport convergence, the vertical eddy salinity transport convergence (referred to as the vertical mixing), and the nonconventional term. The nonconventional term would vanish if large-scale motions were defined using a time or zonal mean as in the classical eddy-mean flow interaction theories (e.g., Jeffreys 1926; Hoskins et al. 1983) but is not so for the Gaussian filter used in this study. The horizontal mixing is dropped in Eq. (1) as the horizontal diffusivity is set as zero in the simulations.

d. EPE budget

The OME-A thermal feedback acts to damp EPE (Ma et al. 2016). To examine this effect, a budget of the mesoscale buoyancy anomaly variance (the detailed derivation can be found in appendix A) is performed:

\[
\frac{\partial \langle b'^2 \rangle}{\partial t} = -\left\{ \begin{array}{l}
\mathbf{ADVP}_e \cdot \left( \mathbf{u}_h b' \cdot \nabla b' \right) \\
\mathbf{p}_m \mathbf{pe} \\
-\langle w' b' \rangle \right\} N.C., \tag{2}
\]

where \( b = -(g/\rho_0)(\rho - \rho_0) \) is buoyancy with density denoted by \( \rho, g = 9.8 \text{ m s}^{-2} \) is the gravitational acceleration,
\( \rho_0 = 1025 \text{ kg m}^{-3} \) is the reference density, \( \overline{N^2} = g a \overline{\alpha T'}/\partial z - g \beta \overline{\alpha S'}/\partial z \) is the squared buoyancy frequency of background fields (McDougall 1987), with \( T \) the temperature, \( \alpha \) the thermal expansion coefficient, and \( \beta \) the haline contraction coefficient, \( F_b \) is the vertical turbulent buoyancy transport (defined positive downward) that can be related to the vertical turbulent temperature transport in Eq. (2) \( F_r \) and \( F_s \) as \( F_b = gaF_r - g \beta F_s \). The abbreviation \( \text{N.C.} \) represents the nonconventional terms that have no counterparts in the classical eddy–mean flow interaction theories (see its expression in appendix A). They are found to have minor contribution to Eq. (2). The budget of EPE can be derived by dividing each term in Eq. (2) by \( \overline{N^2} \) (for more detailed discussion, see Aiki et al. 2016). As \( \overline{N^2} \) occasionally becomes close to zero in the mixed layer, resulting in singularities for EPE, we will use Eq. (2) as a proxy for the EPE budget.

The term on the left-hand side of Eq. (2) corresponds to the tendency of the mesoscale buoyancy anomaly variance (denoted as \( TP_e \)), which is controlled by the production and destruction processes shown on the right-hand side of the equation. The production comes from two processes, i.e., the advection of the mesoscale buoyancy anomaly variance (denoted as \( \text{ADVP}_e \)) and the horizontal eddy buoyancy transport acting on the horizontal buoyancy gradient of mean flows which is associated with a conversion of the potential energy of mean flows to EPE (denoted as \( P_mP_e \)). As these two terms tend to cancel each other locally (Marshall and Shutts 1981), we will combine them together \( (\text{ADVP}_e + P_mP_e) \) in the following analysis. The destruction of the mesoscale buoyancy anomaly variance is attributed to the third and fourth terms on the right-hand side: the vertical eddy buoyancy transport acting on the background vertical buoyancy gradient associated with the conversion of EPE to EKE (denoted as \( P_eK_e \)) and the dissipation of the mesoscale buoyancy anomaly variance through vertical mixing (denoted as \( DP_e \)). With the horizontal diffusivity set as zero in the simulations, there is no dissipation through horizontal mixing.

The OME-A interaction contributes to the EPE budget through \( DP_e \). This can be shown by integrating \( DP_e \) from an arbitrary depth \( z_b \) to the sea surface:

\[
\left\langle \int_{z_b}^{\infty} b^r \frac{\partial F_b^r}{\partial z} \ dz \right\rangle = \left\langle F_b^r b^r \Big|_{z_b=0}^{z=0} \right\rangle - \left\langle \int_{z_b}^{\infty} \frac{\partial b^r}{\partial z} F_b^r \ dz \right\rangle. \tag{3}
\]

As shown in section 3a, due to the negative correlation between \( F_b^r \) and \( b^r \) at the sea surface, the OME-A thermal feedback leads to a destruction of the mesoscale buoyancy anomaly variance through \( F_b^r b^r \Big|_{z_b=0}^{z=0} \). Such an effect is largely suppressed in FILT as the atmosphere cannot feel mesoscale SSTA so that there is nearly no correlation between \( F_b^r \) and \( b^r \) at the sea surface.

As the dissipation of EPE by the OME-A thermal feedback is essentially attributed to the negative \( F_b^r T \Big|_{z_b=0} \) (Ma et al. 2016; Shan et al. 2020a), the OME-A thermal feedback is expected to affect the mesoscale temperature anomaly variance and the related eddy temperature transport as well. Similar to the derivation of Eq. (2), the mesoscale temperature anomaly variance budget can be obtained as follows (the detailed derivation can be found in appendix A):

\[
\left\langle \frac{g^2 \alpha^2 \partial T^2}{\partial t} \right\rangle = \left\langle \frac{g^2 \alpha^2 u \cdot \nabla T'^2}{2} \right\rangle - \left\langle \frac{g^2 \alpha^2 u_b T' \cdot \nabla u_b T'}{2} \right\rangle - \left\langle \frac{g^2 \alpha^2 w T' \partial T}{\partial z} \right\rangle - \left\langle \frac{g^2 \alpha^2 T' \partial F_T}{\partial z} \right\rangle + \text{N.C.} \cdot T, \tag{4}
\]

where \( gaT' \) is the contribution of the mesoscale temperature anomaly to the mesoscale buoyancy anomaly. The budget for \( \langle (gaT')^2 \rangle \) instead of \( \langle T'^2 \rangle \) is constructed to facilitate its comparison with Eq. (2). In fact, Eq. (2) is reduced to Eq. (4) when the influence of salinity on buoyancy can be neglected. The terms in Eq. (4) are the counterparts of \( TP_e, \text{ADVP}_e, P_mP_e, P_eK_e, DP_e, \) and \( \text{N.C.} \) in Eq. (2) and will thus be denoted as \( TP_eT, \text{ADVP}_eT, P_mP_eT, P_eK_eT, DP_eT, \) and \( \text{N.C.} \cdot T, \) respectively.

**3. Result**

**a. Overview of CRCM simulations**

CRCM CTRL reproduces the frontal structures in the KOE region reasonably well despite a slight northward overshoot (Figs. 1a–c). The simulated Kuroshio Extension (35°–40°N, 146°–158°E, shown by the green box in Fig. 1a), centered around 36.5°N, is characterized by strong sea surface height (SSH) front. This front is largely attributed...
to the deeply extended temperature front peaking in the subsurface region (Fig. 2b). In contrast, there are pronounced SST and sea surface salinity (SSS) fronts in the Oyashio Extension (40°N–44°N, 146°E–158°E, shown by the pink box in Fig. 1a) around 41°N. The lack of SSH front in the Oyashio Extension region is due to the shallower vertical structure of temperature/salinity fronts and the strong cancellation between contributions of temperature and salinity to buoyancy (Figs. 2b,c). Oceanic mesoscale variability is intensified along fronts as a result of baroclinic instability (Figs. 1d–f and 2d–f). There is a local enhancement of the mesoscale SSTA variance in both the Kuroshio and Oyashio Extensions, with the latter more evident as a result of the stronger SST front there. The significant mesoscale SSSA variance is only confined to the Oyashio Extension due to the lack of evident SSS front in the Kuroshio Extension. All these features are qualitatively consistent with observations and reanalysis products (Jing et al. 2019), lending supports to the validity of CRCM simulations.

The coincidence of the pronounced mesoscale SSTA and atmospheric synoptic variability in the KOE region leads to the strong OME-A thermal interaction (e.g., Bishop et al. 2017; O’Neill et al. 2012; Small et al. 2019). There is a large negative covariance between the mesoscale SSTA and the surface heat flux anomaly, i.e., $Q^T_T(z=0)$, in the region (Fig. 3a). The Ekman pumping on the mesoscale, i.e., $w^*_e = k \cdot \nabla \times (\tau/\rho_0 f)$ with $\tau$ the

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**Fig. 1.** The winter-mean (1 Oct to 31 Mar) (a) SSH [contour interval (CI) = 0.1 m], (b) SST (CI = 1°C), (c) SSS (CI = 0.1 psu), (d) the mesoscale SSHA variance, (e) the mesoscale SSTA variance, and (f) the mesoscale SSSA variance. In (a) the pink box shows the Oyashio Extension (40°–44°N, 146°–158°E) and the green box denotes the Kuroshio Extension (35°–40°N, 146°–158°E). All values shown here are averaged over the five members of CRCM CTRL.

**Fig. 2.** The zonal (146°–158°E, denoted by the white dashed lines in Fig. 1a) and winter-mean (a) buoyancy (CI = 0.02 m s$^{-2}$), (b) temperature (CI = 1°C), (c) salinity (CI = 0.1 psu), (d) mesoscale buoyancy anomaly variance, (e) mesoscale temperature anomaly variance, and (f) mesoscale salinity anomaly variance. All values shown here are averaged over the five members of CRCM CTRL.
wind stress and the local Coriolis parameter, exhibits enhancement in regions with strong mesoscale SSTA (Fig. 4a) as a result of the mesoscale SSTA’s imprint on surface wind. Moreover, the OME-A thermal interaction results in negative $F^\theta_b|_{z=0}$, acting to dissipate EPE (Fig. 3c). However, unlike $Q^\theta_T|_{z=0}$ that exhibits comparable values in the Kuroshio and Oyashio Extensions, the magnitude of $F^\theta_b|_{z=0}$ is much more larger in the former region. To understand the discrepancy between the spatial pattern of $Q^\theta_T|_{z=0}$ and $F^\theta_b|_{z=0}$, we decompose $F^\theta_b|_{z=0}$ into:

$$F^\theta_b|_{z=0} = g^2\alpha^2F^\theta_T|_{z=0} - g^2\alpha\beta F^\theta_S|_{z=0} - g^2\beta F^\theta_S^3|_{z=0}$$

where $F^\theta_T$ can be related to $Q^\theta_T$ according to $Q^\theta_T = \rho_0 C_p F^\theta_T$, with $C_p$ the ocean heat capacity. Consistent with $Q^\theta_T|_{z=0}$, the magnitudes of $g^2\alpha^2F^\theta_T|_{z=0}$ are similar in the Kuroshio and Oyashio Extensions (Fig. 5a). The value of $g^2\alpha^2F^\theta_T|_{z=0}$ agrees roughly with that of $F^\theta_b|_{z=0}$ in the Kuroshio Extension with an area-mean difference $(g^2\alpha^2F^\theta_T|_{z=0} - F^\theta_b|_{z=0})/F^\theta_b|_{z=0}$ about 36%. In contrast, the area-mean $g^2\alpha^2F^\theta_T|_{z=0}$ is about 4 times as large as the area-mean $F^\theta_b|_{z=0}$ in the Oyashio Extension. The significant discrepancy between $g^2\alpha^2F^\theta_T|_{z=0}$ and $F^\theta_b|_{z=0}$ in the Oyashio Extension is mainly attributed to the large cancellation between $g^2\alpha^2F^\theta_T|_{z=0}$ and $-g^2\alpha\beta F^\theta_S|_{z=0}$ (Figs. 5a,b). Such cancellation in the Oyashio Extension is a reflection of the strong compensation effect of SSTA and SSSA on the sea surface density or equivalently buoyancy anomaly (i.e., a positive SSTA–SSSA relationship) due to the collocated SST and SSS fronts there.

The OME-A thermal interaction is largely suppressed in FILT (Fig. 3b). There is no notable negative covariance between the mesoscale SSTA and $Q^\theta_T|_{z=0}$ in the KOE region. So is the case for $F^\theta_b|_{z=0}$ (Fig. 3d). The Ekman pumping at mesoscale becomes significantly weaker in FILT (Fig. 4b) due to the removal of the mesoscale surface wind anomaly originated from the mesoscale SSTA. The area-mean mesoscale Ekman pumping velocity anomaly variance, i.e., $\langle w^2_\tau \rangle$, in the KOE region (35°–44°N, 146°–158°E) in FILT is only 63% of that in CTRL. Consistent with Ma et al. (2016), the mean flow in the Oyashio Extension is stronger in CTRL than FILT (Fig. 6). But the enhancement of mean flow by the
OME-A thermal feedback is less pronounced in the Oyashio Extension. This is probably because the destruction of EPE by the OME-A thermal feedback is less efficient in the Oyashio Extension due to the stronger compensation between temperature and salinity variations.

b. Influence of the OME-A thermal feedback on the upper-ocean haline structure

Figure 7a shows the difference (CTRL – FILT) of zonal-mean salinity in the KOE region averaged over the last-90-days simulations. In presence of the OME-A thermal feedback, the upper-50-m water column in the Oyashio Extension becomes fresher, whereas the deeper ocean becomes saltier. Such salinity changes result in an enhanced haline stratification defined as $\frac{\partial S}{\partial z}$ (Fig. 7d). In FILT, the value of $\langle -g\beta S/\partial z \rangle$ averaged in the Oyashio Extension over the last-90-days simulations peaks around 100 m with a value of $1.6 \times 10^{-5}$ s$^{-2}$. But the peak value increases to $2.2 \times 10^{-5}$ s$^{-2}$ in CTRL, a 38% intensification. We remark that such responses of the upper-ocean haline structure to the OME-A thermal feedback are robust, insensitive to the simulation periods. Repeating the above analysis for the simulations in the last 120 days or the entire winter half-year yields qualitatively similar results (Figs. 7b,c,e,f).

To find out the underlying process strengthening the haline stratification in CTRL, a salinity budget over the Oyashio Extension is performed for the last-90-days simulations (Fig. 8). In both CTRL and FILT, the upper-ocean haline stratification is, to a large extent, shaped by the jointed effects of the vertical mixing, the vertical eddy salinity transport convergence and the advection by mean flows. The horizontal convergence of eddy salinity transport plays a secondary role with the tendency and nonconventional terms negligible. Both the vertical eddy salinity transport convergence and the vertical mixing change significantly between CTRL and FILT, whereas the advection by mean flows remains nearly unchanged. As a response to the OME-A thermal
feedback, the vertical eddy salinity transport convergence leads to more salinity decrease (increase) in the upper (lower) layer, accounting for the enhanced haline stratification in CTRL. Such an effect is counteracted by the change of the vertical mixing so that a new equilibrium state can be established. The change of tendency term between CTRL and FILT is more than an order of magnitude smaller than those of the vertical eddy salinity transport and the vertical mixing, suggesting that the CRCM simulation provides a robust representation on the influence of the OME-A thermal feedback on the upper-ocean salinity budget and haline stratification.

The differed vertical eddy salinity transport convergence between CTRL and FILT results from the influence of the OME-A thermal feedback on the vertical eddy salinity transport (Fig. 9). In both CTRL and FILT, there is an upward eddy salinity transport acting to destroy the haline stratification. The intensity of the vertical eddy salinity transport is significantly weakened in response to the OME-A thermal feedback. Its peak value reduces from $4.2 \times 10^{-6}$ psu m s$^{-1}$ in FILT to $2.4 \times 10^{-6}$ psu m s$^{-1}$ in CTRL.
in CTRL. In the following subsection, we explore the underlying reasons accounting for this reduction.

c. Reasons for the weakened vertical eddy salinity transport by the OME-A thermal feedback

As reviewed in section 1, the OME-A thermal interaction can feedback on ocean eddy dynamics in two major ways. The Ekman pumping induced by the OME-A thermal feedback may potentially affect the vertical eddy salinity transport. To test this hypothesis, we compute the difference of $\langle w'S' \rangle$ in CTRL and FILT in the Oyashio Extension and compare it to the difference of $\langle w'S' \rangle$ at the bottom of the Ekman layer. It should be noted that the exact value of the Ekman layer thickness $\delta_e$ is not available. Existing literature suggests $\delta_e = 0.4u*/f$, where $u* = \sqrt{\tau/ρ_0}$ is the frictional velocity (Wimbush and Munk 1971). Given the values of $\tau$ and $f$ in the Oyashio Extension over the last-90-days simulations, $\langle \delta_e \rangle$ is estimated to be $65 \pm 2$ m (64 $\pm$ 3 m) in CTRL (FILT). Within this depth range, the value of $\langle w'S' \rangle$ is an order of magnitude smaller than that of $\langle w'S \rangle$. So are their differences between CTRL and FILT ($-2 \times 10^{-7}$ psu m s$^{-1}$, Fig. 9). Moreover, the magnitude of $\langle w'S' \rangle$ becomes actually larger in CTRL than FILT. We thus conclude that the reduced vertical eddy salinity transport in CTRL is unlikely to be due to the Ekman pumping induced by the mesoscale SSTA’s imprint on surface wind stress.

Besides affecting Ekman pumping, the OME-A thermal feedback acts as an important dissipater for $\langle b'^2 \rangle$ and $\langle (gαT')^2 \rangle$. Accordingly, other processes influencing $\langle b'^2 \rangle$ and $\langle (gαT')^2 \rangle$ are expected to respond significantly to the OME-A thermal feedback. Given that $\langle gβw'S' \rangle = \langle gaw'T \rangle - \langle wb' \rangle$ (Fig. A1), we suspect that the weakened $\langle w'S' \rangle$ in CTRL may result from the different impacts of the OME-A thermal feedback on $\langle b'^2 \rangle$ and $\langle (gαT')^2 \rangle$ budgets. Table 1 compares the vertically integrated $\langle b'^2 \rangle$ budgets in the Oyashio Extension between CTRL and FILT. The lower bound for the vertical integration is chosen as 200 m below which the difference between CTRL and FILT is negligible. In presence of the OME-A thermal interaction, $DP_e$ exhibits significant enhancement with its value about 2.7 $×$ $10^{-11}$ m$^3$ s$^{-5}$ larger in CTRL than in FILT. Most of its enhancement is attributed to $\langle F_{b'|z=0} \rangle$, whereas the interior dissipation changes a little. The intensified $\langle b'^2 \rangle$ dissipation through $DP_e$ is largely balanced by the increased $\langle b'^2 \rangle$ production through $\text{ADVP}_{e} + P_mP_e$ and the reduced $\langle b'^2 \rangle$ destruction through $P_{Ke}$. The former and latter compensate for 63% and 15% of the increased $DP_e$ in presence of the OME-A thermal feedback, respectively. The reduced $\langle b'^2 \rangle$ destruction through $P_{Ke}$ corresponds to the weakened $\langle wb' \rangle$ in CTRL.

Similar to $DP_e$, $DP_eT$ is also significantly enhanced in CTRL due to the contribution of $\langle g^2α^2F_T'|z=0 \rangle$. The enhanced dissipation of $\langle (gαT')^2 \rangle$ is largely balanced by the increased $\langle (gαT')^2 \rangle$ production through $\text{ADVP}_{eT} + P_mP_eT$ and the reduced $\langle (gαT')^2 \rangle$ destruction through $P_{Ke}T$ (Table 2). The magnitude of $\langle g^2α^2F_T'|z=0 \rangle$ in CTRL is much larger than that of $\langle F_{b'|z=0} \rangle$ due to the strong compensation effect of temperature and salinity on buoyancy in the Oyashio Extension region. Correspondingly, the responses of $\text{ADVP}_{eT} + P_mP_eT$ and $P_{Ke}T$ to the OME-A thermal feedback are significantly larger than their counterparts in the $\langle b'^2 \rangle$ budget. In particular, the difference of $P_{Ke}T$ between

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Table 1. The upper-200-m-integrated mesoscale buoyancy anomaly variance budget ($\langle b'^2 \rangle$; unit: $×10^{-11}$ m$^3$ s$^{-5}$) in the Oyashio Extension. The values in parentheses denote the contribution of $\langle F_{b'|z=0} \rangle$ to $DP_e$. All values shown here are averaged over the last-90-days simulations of the five ensemble members.

<table>
<thead>
<tr>
<th>Expt</th>
<th>$T_P$</th>
<th>$\text{ADVP}_p + P_mP_e$</th>
<th>$P_{Ke}$</th>
<th>$DP_e$</th>
<th>N.C.</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTRL</td>
<td>-1.3</td>
<td>7.3</td>
<td>-4.3</td>
<td>-5.1</td>
<td>(-2.6)</td>
</tr>
<tr>
<td>FILT</td>
<td>-1.0</td>
<td>5.6</td>
<td>-4.7</td>
<td>-2.4</td>
<td>(-0.6)</td>
</tr>
<tr>
<td>CTRL – FILT</td>
<td>-0.3</td>
<td>1.7</td>
<td>0.4</td>
<td>-2.7</td>
<td>(-2.0)</td>
</tr>
</tbody>
</table>

---

FIG. 9. The vertical eddy salinity transport averaged in the Oyashio Extension over the last-90-days simulations. The blue (red) line shows $\langle w'S' \rangle$ in CTRL (FILT). The solid black line indicates their difference (CTRL – FILT), $\Delta \langle w'S' \rangle$. The short dashed line denotes the difference of the Ekman-induced vertical eddy salinity transport $\Delta \langle w'S \rangle$. All values shown here are averaged over the five ensemble members. The shading is the error bar defined as $[X - 3σ_e, X + 3σ_e]$. 

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CTRL and FILT, $\Delta P_s K_r$, is 8 times as large as $\Delta P_s K_r$. This corresponds to a larger $\Delta \langle gw' T' \rangle$ than $\Delta \langle w' b' \rangle$ (Fig. 10), explaining the weakened $\langle g\beta w' S' \rangle$ (thus $\langle w' S' \rangle$) in CTRL.

4. Discussion

The above analysis suggests that a weakened $\langle g\beta w' S' \rangle$ is required to reconcile the different responses of $\langle w' b' \rangle$ and $\langle gw' T' \rangle$ to the OME-A thermal feedback. Such different responses result from the positive $T'$–$S'$ relationship of mesoscale eddies in the Oyashio Extension. In this section, we explore more for the underlying dy-

nations. Specifically, should the OME-A thermal feedback affect $\langle g\beta w' S' \rangle$ by changing $\langle w'^2 \rangle$, $\langle (g\beta S')^2 \rangle$, or their correlation? Figs. 11a,d,e display these three quantities in CTRL and FILT. The values of $\langle w'^2 \rangle$ are almost the same in CTRL and FILT. The value of $\langle (g\beta S')^2 \rangle$ is even slightly larger in CTRL than FILT possibly because the eddy-induced freshwater flux acts to enhance $S'$ (note that warm/cold eddies evaporate more/less, leading to positive $\langle F'_i S' \rangle$ due to the positive $T'$–$S'$ relationship). The weakened $\langle g\beta w' S' \rangle$ in CTRL than FILT is mainly due to the pronounced reduction of the correlation between $w'$ and $g\beta S'$, suggesting that the reduced $\langle g\beta w' S' \rangle$ in CTRL cannot be simply ascribed to the weakened mesoscale eddy activities by the OME-A thermal feedback. To throw light on the cause of this reduction, $\langle g\beta w' S' \rangle$ is decomposed into the following two components:

$$\langle g\beta w' S' \rangle = \langle g\beta w'_i S' \rangle + \langle g\beta w'_j S' \rangle,$$

where $w'_i$ and $w'_j$ are the vertical eddy velocity component relevant and irrelevant to the vertical buoyancy transport, respectively. Here $w'_i$ is determined by regressing the time series of $w'$ onto that of $b'$ at individual points and $w'_j$ is computed as the residue of the regression. By definition, $w'_j$ does not result in any vertical buoyancy transport. In both CTRL and FILT, it is found that $\langle g\beta w'_i S' \rangle$ makes dominant contribution to $\langle g\beta w' S' \rangle$ especially in the upper 100 m where $\langle g\beta w'_i S' \rangle$ is close to zero (Fig. 12), suggesting that the vertical eddy salinity transport is tightly related to the release of available potential energy.

Both $\langle w'_i w'_i \rangle$ and $\langle w'_j w'_j \rangle$ remain almost unchanged between CTRL and FILT (Figs. 11b,c). The reduced correlation between $w'$ and $g\beta S'$ is thus mainly due to the reduced correlation between $w'_i$ and $g\beta S'$ (Fig. 11f). The latter can be expressed as (see appendix B for details)

$$\text{corr}(w'_i, g\beta S') = \frac{\text{cov}(w'_i, g\beta S')}{{\sqrt{\text{var}(w'_i)} \text{var}(g\beta S')}}} = \frac{\text{sgn}(p_{bw})(p_{st} - 1) \text{var}(g\beta S')}{\text{var}(b')}, \quad (7)$$

where sgn represents the sign of its argument, var and cov represent variance and covariance, $p_{bw}$ is the regression coefficient of $w'$ onto $b'$, and $p_{st}$ is the regression coefficient of $g\alpha T'$ onto $g\beta S'$ measuring the $T'$–$S'$ relationship.
of mesoscale eddies. The emergence of $p_{st} - 1$ in Eq. (7) has a clear dynamical interpretation. When $p_{st} - 1$ is close to zero, the contributions of $gT' \beta$ and $gS' \beta$ to $b'$ largely cancel each other out. In this case, only a small portion of $gS' \beta$ is projected onto $b'$ (see appendix B) and transported by $w_{jj}$, leading to a weak vertical salinity transport.

Figure 13 displays the values of $\text{sgn}(p_{bw})$, $p_{st} - 1$ and $\sqrt{\text{var}(gS') \text{var}(b')}$ in CTRL and FILT, respectively. The term $\text{sgn}(p_{bw})$ remains the same in the two experiments. The term $\sqrt{\text{var}(gS') \text{var}(b')}$ becomes slightly larger in CTRL than FILT due to the weakened $(b'^2)$ and enhanced $(S'^2)$ by the OME-A thermal feedback. It is the term $p_{st} - 1$ exhibiting pronounced reduction in response to the OME-A thermal feedback that accounts for the reduced correlation between $w_{jj}$ and $gS' \beta$ and thus the weakened $\langle gS' w' S' \rangle$.

The reduced $p_{st}$ in response to the OME-A thermal feedback is consistent with its different impacts on $(b'^2)$ and $\langle (gT')^2 \rangle$. Both are eventually attributed to the positive $T' - S'$ relationship of mesoscale eddies in the Oyashio Extension region. On the one hand, this positive $T' - S'$ relationship makes the OME-A thermal feedback damps $T'$ but enhances $S'$, leading to the reduced $p_{st}$. On the other hand, it results in positive $-g^2 a \beta F_0 S' |_{z=0}$ so that $\langle b'^2 \rangle$ is dissipated less pronouncedly than $\langle (gT')^2 \rangle$ by the OME-A thermal feedback as discussed in section 3.

Finally, as suggested by the factor $p_{st} - 1$ in Eq. (7), the influence of the OME-A thermal feedback on $\langle gS' w' S' \rangle$ will be pronounced in regions like the Oyashio Extension where temperature and salinity are strongly compensated for their effects on buoyancy. In such regions, the value of $p_{st} - 1$ is already so close to zero that any further change of $p_{st}$ by the OME-A thermal feedback will lead to considerable relative change of $p_{st} - 1$. Indeed, we find that the influence of the OME-A thermal feedback on $\langle gS' w' S' \rangle$ is much weaker in the Kuroshio Extension (not shown) where $\langle p_{st} \rangle$ is 3–5 in the upper 200 m. This may explain why the difference of

![Figure 11](image-url)
haline stratification between CTRL and FILT is insignificant there (Fig. 7).

5. Conclusions

The SSTA carried by ocean mesoscale eddies exerts significant imprints on the overlying atmosphere that can further feedback on ocean dynamics. In this study, we assess how this OME-A thermal feedback affects the upper-ocean haline stratification in the KOE region based on high-resolution regional coupled simulations. Two experiments (CTRL and FILT) are performed. The CTRL is a standard coupled simulation, whereas in the FILT the atmosphere cannot feel the mesoscale SSTA so that the OME-A thermal feedback is largely suppressed. The major findings obtained from the comparison of the two experiments are as follows:

1) The haline stratification, defined as $-\gamma \beta S / \partial z$, becomes more stable in the upper-200 m water column of the Oyashio Extension as a response to the OME-A thermal feedback. The enhanced haline stratification in CTRL than FILT results from the weakened upward eddy salinity transport.

![Diagram](image_url)
2) The OME-A thermal feedback influences the vertical eddy salinity transport through its different impacts on the budget of \((b'^2)\) and \((<gaT'>^2)\). Due to the strong compensation effect of temperature and salinity on buoyancy in the Oyashio Extension, the dissipation of \((b'^2)\) by the OME-A thermal feedback is much weaker than that inferred from temperature alone, i.e., \((<gaT'>^2)\). Correspondingly, the vertical eddy buoyancy transport \((w'b')\) is less weakened by the OME-A thermal feedback compared to its thermal component \((<gw'T'>)\). This difference is reconciled by the reduced \((w'S')\) in CTRL than FILT.

Salinity has rarely been discussed in the studies of air–sea interactions (e.g., Small et al. 2008; Chelton and Xie 2010) probably because it has no effect on the momentum, heat and freshwater exchanges between the ocean and atmosphere. Nevertheless, this study demonstrates that haline structure in the upper ocean can be largely influenced by the OME-A thermal feedback. Somewhat counterintuitively, such an influence is not achieved through the eddy-induced freshwater flux anomaly at the air–sea interface. Instead, it results from the strong compensation effect of temperature and salinity on buoyancy. This study provides a new perspective on the dynamics controlling haline structure in the upper ocean and highlights the necessity of using high-resolution coupled models for reliable simulations of haline stratification.

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Data availability statement: The data supporting our study are publicly available from Figshare repository (https://doi.org/10.6084/m9.figshare.12587720.v1).

APPENDIX A

Derivations of Eqs. (2) and (4)

The ROMS model uses the equation of state derived by Jackett and McDougall (1995), expressing density as a nonlinear function of potential temperature \((\theta)\), salinity, and pressure \((p)\). Its differential form is

\[
dp = -\rho \alpha d\theta + \rho \beta dS + \frac{1}{c_s^2} dp, \tag{A1}
\]

where \(c_s\) is the speed of sound. Equation (A1) can be simplified by noting that under the Boussinesq approximation adopted by the ROMS model, \(p \approx p_0(z)\) with \(dp_0(z)/dz = -\rho g\). Substituting \(p \approx p_0(z)\) and \(b = -g(p - p_0)/\rho_0\) into Eq. (A1) yields

\[
\frac{db}{dt} \frac{g^2}{c_s^4} w = g\alpha \frac{d\theta}{dt} - g\beta \frac{dS}{dt}, \tag{A2}
\]

with

\[
\frac{d\theta}{dt} = \frac{\partial \theta}{\partial t} + \mathbf{u}_h \cdot \nabla \theta + w \frac{\partial \theta}{\partial z} = \frac{\partial F_{\theta}}{\partial z}, \tag{A3}
\]

\[
\frac{dS}{dt} = \frac{\partial S}{\partial t} + \mathbf{u}_h \cdot \nabla S + w \frac{\partial S}{\partial z} = \frac{\partial F_{S}}{\partial z}. \tag{A4}
\]

Decomposing \(b\) into \(b = \bar{b} + b'\) where \(\bar{b} = b(\bar{\theta}, \bar{S}, z)\) is the large-scale background field and \(b'\) is the anomaly generated by mesoscale eddies. The nonlinearity of the equation of state leads to nonlinear dependence of \(\beta\) on \(\theta'\) and \(S'\), making it impossible in practice to construct a budget for \((b'^2)\) based on Eqs. (A2)–(A4). For this sake, we make the following two assumptions. First, it is assumed that the magnitudes of \(\theta'\) and \(S'\) are sufficiently small so that \(a(\theta, S, z)\), \(b(\theta, S, z)\), and \(c_i(\theta, S, z)\) can be approximated as \(a(\bar{\theta}, \bar{S}, z)\), \(b(\bar{\theta}, \bar{S}, z)\), and \(c_i(\bar{\theta}, \bar{S}, z)\), respectively. In this case, \(b'\) depends linearly on \(\theta'\) and \(S'\), satisfying \(db' \approx ga\theta' - g\beta dS'\). We further assume \(a(\bar{\theta}, \bar{S}, z)\) and \(b(\bar{\theta}, \bar{S}, z)\) to evolve on spatial and time scales much longer than those of \(\theta'\) and \(S'\) so that \(b' \approx ga\theta' - g\beta S'\).

Spatially low-pass filtering Eq. (A3) yields the governing equation for \(\bar{\theta}\):

\[
\frac{\partial \bar{\theta}}{\partial t} + \mathbf{u}_h \cdot \nabla \bar{\theta} + w \frac{\partial \bar{\theta}}{\partial z} = \frac{\partial F_{\theta}}{\partial z}. \tag{A5}
\]

Subtracting Eq. (A5) from Eq. (A3) yields the governing equation for \(\theta'\):

\[
\frac{\partial \theta'}{\partial t} = -\mathbf{u}_h' \cdot \nabla \theta' - \mathbf{u}_h' \cdot \nabla \bar{\theta} - \mathbf{u}_h' \cdot \nabla \bar{\theta} - \mathbf{u}_h' \cdot \nabla \bar{\theta} + \frac{\partial F_{\theta}'}{\partial z} - w \frac{\partial \theta'}{\partial z} - w \frac{\partial \theta}{\partial z} - w \frac{\partial \bar{\theta}}{\partial z} + \frac{\partial F_{\theta}}{\partial z}. \tag{A6}
\]
where the abbreviation N.C. represents the nonconventional members.

\[ \frac{\partial b'}{\partial t} = -u \cdot \nabla b' - u_h \cdot \nabla \bar{b} - \bar{u}_h \cdot \nabla \bar{b} + u_h \cdot \nabla \bar{b} \]
\[ -w' \frac{\partial \bar{\theta}}{\partial z} - w \frac{\partial \bar{\theta}}{\partial z} + w \frac{\partial \theta}{\partial z} + \frac{\partial F_b'}{\partial z} \]  
(A7)

Similarly, we can get the governing equation for \( S' \):

\[ \frac{\partial S'}{\partial t} = -u \cdot \nabla S' - u_h \cdot \nabla \bar{S} - \bar{u}_h \cdot \nabla \bar{S} + u_h \cdot \nabla \bar{S} \]
\[ -w' \frac{\partial \bar{S}}{\partial z} - w \frac{\partial \bar{S}}{\partial z} + w \frac{\partial S}{\partial z} + \frac{\partial F'_b}{\partial z}. \]
(A8)

Multiplying Eq. (A7) by \( g \alpha \) and Eq. (A8) by \( -g \beta \) and adding them together yield the governing equation for \( b' \):

\[ \frac{\partial b'}{\partial t} = -u \cdot \nabla b' - u'_h \cdot \nabla \bar{b} - \bar{u}_h \cdot \nabla \bar{b} + g \alpha u_h \cdot \nabla \bar{\theta} \]
\[ -g \beta w_0 \frac{\partial S}{\partial z} + \frac{\partial F'_b}{\partial z}. \]  
(A9)

where \( N^2 = g \alpha (\bar{\theta}/\partial z) - g \beta (\bar{S}/\partial z) \). Multiplying Eq. (A9) by \( b' \) yields

\[ \frac{\partial b'^2}{\partial t} = -u \cdot \nabla b'^2 - u'_h b' \cdot \nabla \bar{b} - \bar{u}_h b' \cdot \nabla \bar{b} - w' b' N^2 + b' \frac{\partial F_b'}{\partial z} + \text{N.C.}, \]
(A10)

where the abbreviation N.C. represents the nonconventional terms defined as

\[ \text{N.C.} = -\bar{u}_h b' \cdot \nabla \bar{b} - w b' N^2 + b'(g \alpha u \cdot \nabla \bar{u} - g \beta u \cdot \nabla \bar{S}). \]
(A11)

Taking the area and time average of Eq. (A10) yields the mesoscale buoyancy anomaly variance budget in our paper. Dropping all the salinity-related terms yields Eq. (4).

The validity of the above two assumptions can be evaluated by comparing \( \langle w' b' \rangle \) with \( b' \) computed as \( b - \bar{b} \) and \( g \alpha \theta' - g \beta S' \), respectively. As shown in Fig. A1, the vertical profiles of \( \langle w' b' \rangle \) computed from these two different ways agree well with each other, lending supports to the robustness of the major conclusions in this paper.

Finally, it should be noted that the adiabatic lapse rate is quite small (on the order of 0.1°C km\(^{-1}\)) so that temperature and potential temperature are almost the same in the upper ocean (Vallis 2006). Therefore, temperature is used as a substitute for potential temperature in this paper to make the analysis concise.

**APPENDIX B**

Derivations of Eq. (7)

Construct the following regression model for \( g \alpha T' \) and \( g \beta S' \):

\[ g \alpha T' = p_s g \beta S' + g \alpha T'_r, \]
(B1)

where \( g \alpha T'_r \) is the residue of the regression. By definition, \( \langle g \alpha T'_r g \beta S' \rangle = 0 \).

Based on Eq. (B1), \( b' \) can be re-expressed as

\[ b' = (p_s - 1)g \beta S' + g \alpha T'_r. \]
(B2)

According to Eq. (B2), \( \text{cov}(b', g \beta S') = (p_s - 1) \text{var}(g \beta S') \), suggesting that when \( (p_s - 1) \) is close to zero, only a small portion of \( g \beta S' \) is projected onto \( b' \).
Moreover, we have
\[
\text{cov}(w'_j, gS') = \text{cov}(p_{bw} b', gS') = p_{bw}^2 \text{cov}(b', gS') = p_{bw}^2 (p_{st} - 1) \text{var}(gS'), \tag{B3}
\]
\[
\text{var}(w'_j) = \text{var}(p_{bw} b') = p_{bw}^2 \text{var}(b'). \tag{B4}
\]
Substituting Eqs. (B3) and (B4) into \( \text{corr}(w'_j, gS') = \text{cov}(w'_j, gS')/\sqrt{\text{var}(w'_j) \text{var}(gS')} \) yields Eq. (7) in our paper.

REFERENCES


