Beta-Plane Arrested Topographic Wave as a Linkage of Open Ocean Forcing and Mean Shelf Circulation

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ABSTRACT: Pressure anomaly set by the open ocean affects the dynamic topography and associated circulation over the continental shelf, which is explored here on a linearized β-plane arrested topographic wave framework that considers the variation in Coriolis parameter with latitude. It was found that on a meridional shelf, a nondimensional parameter Pe, termed the Péclet number, signifies the characteristics of open ocean–shelf interaction. The Pe = Dp/α is determined by the ratio of long-wave-limit planetary to topographic Rossby wave speeds, i.e., the β drift Dp, and the linear Ekman number α. On the western boundary shelf, due to the westward planetary Rossby wave, open ocean pressure propagates shoreward as Pe > 1, and shelf circulation peaks where Pe drops to 1. At this location, the planetary β effect is balanced by the bottom friction. The Pe = 1 must occur either on the shelf or on the coastal wall when Pe > 1 is observed at the shelf edge. On the eastern boundary shelf, however, Pe < 0, the pressure anomaly is removed from the shelf, and hence the inductive circulation decays rapidly from the shelf edge. This β effect is robust on gently sloping meridional shelves. For zonal shelves, the planetary β increases the effective bottom slope on the northern boundary shelf but decreases it on the southern one, in a sense of potential vorticity conservation. However, this effect could be less significant in reality, given the complex dynamics involved. The above mechanism can explain the dynamics driving the Taiwan Warm Current in the East China Sea and its bifurcation around 28°N.

KEYWORDS: Continental shelf/slope; Barotropic flows; Currents; Rossby waves; Topographic effects

1. Introduction

General circulation in the open ocean is constrained to the upper layer and thus hardly feels the abyssal topography, and the pressure anomaly away from the equator propagates westward due to Earth’s curvature, i.e., the planetary β effect. On reaching the shelf, the steep topography serves as a waveguide and deflects the oceanic information equatorward, i.e., the topographic Rossby wave, before it reaches the coastal region. In this case, the bathymetric slope, i.e., the topographic β effect, is dominant. Due to this dramatic contrast in dynamic regimes, the coastal ocean is to some degree insulated from the influence of the open ocean. Therefore, whether and how the oceanic pressure anomaly penetrates into the continental shelf, thus affecting the sea surface height (SSH), or more precisely, the dynamic topography and the associated circulation on the inner shelf, have become a central focus in coastal oceanographic studies.

Given the steep topography near the coast, the f-plane approximation seems to be adequate. Csanady (1978) suggested that bottom friction can be an agent for breaking the potential vorticity (PV) barrier set by the shelf slope. For an f plane, steady-state, linearized, and barotropic shelf with no along-shelf variations (see Fig. 1), and by neglecting cross-shelf bottom friction, he derived the arrested topography wave (ATW) equation:

\[ \partial_t \eta_y + \frac{f}{\beta} \partial_x \eta_x = 0. \]  

In (1) x is the offshore direction, y is the upshelf direction (i.e., opposite to the direction of the topographic wave), η is the SSH representing the pressure anomaly, f is the Coriolis parameter, s = dh/dx is the cross-shelf slope (positive value assumed), and r is the coefficient for linearized bottom friction, defined as τy/ρ = rV. Csanady suggested that ηb imposed at the shelf edge will “diffuse” shoreward along with the downshelf (i.e., the −y direction)-propagating topographic Rossby wave. He referred to this mechanism as a “heat conduction analogy,” in which the −y direction plays the role of time and κ = r/β plays the role of conductivity. Consequently, the entire shelf experiences the same along-shelf pressure gradient as on the shelf edge, i.e., −gηb, except for an initial insulation area. The inductive shelf circulation was then estimated as ν = −ghηb/r, which is proportional to water depth h. The f-plane ATW theory has been used to explain the circulation in the northern South China Sea (Hsueh and Zhong 2004), the West Florida Shelf (Csanady and Shaw 1983; Hetland et al. 1999), the Japan/East Sea (Ohshima 1994), and the Southwestern Atlantic Shelf (Matano et al. 2010), among many others.
However, the $f$-plane approximation could be problematic if the meridional extent of the shelf is large and/or the shelf slope is gentle, thus the variation of ambient PV, i.e., $\beta h$, is determined not only by $h$, but also by $f$. This was pointed out by Middleton and Thomson (1985), who found that the wind-driven circulation was narrow and strong along the western ocean boundary coast, but wide and weak along the eastern counterpart, due to the planetary $\beta$ effect. For the geostrophic and barotropic flow over topography, Tyler and Käse (2000) suggested that the pressure anomaly is “advected” along the $gh/f$ contour at a velocity

$$ C = \nabla(gh/f) \times \mathbf{k}; $$

$gh/f$ is essentially the inverse of PV, which they called the “string function.” Apparently, if the variation of $h$ dominates the gradient of the string line, Csanady’s $f$-plane ATW theory can apply, whereas if the shelf slope is gentle and the meridional extent is large, the string lines deviate from the isobaths, and the $\beta$ effect must be taken into account. As pointed out by Wise et al. (2018) and Hughes et al. (2019), the penetration of open ocean pressure anomaly to the coast can increase not only with large bottom friction $r$, but also with decreased shelf slope.

Here in this study, we focus on the structure of the shelf circulation induced by an open ocean pressure anomaly on a $\beta$ plane. This was motivated by an attempt to explain the dynamics driving the East China Sea (ECS; Fig. 2a) circulation. The ECS shelf is on the western boundary of the subtropical North Pacific Ocean, with the mighty western boundary current, Kuroshio, flowing poleward on its shelf break. On its inner shelf there is a year-round current with the same direction as Kuroshio, i.e., the Taiwan Warm Current (TWC) (Liang and Su 1994; Guan and Fang 2006; Zhu et al. 2004; sketched in Fig. 2a), regardless of the seasonal reversal of monsoonal winds (summerly in summer season and northerly otherwise). Although the water mass of the TWC originates from the Taiwan Strait or from the Kuroshio subsurface water intrusion in the south (Guan and Fang 2006; Yang et al. 2011), previous modeling studies indicated that the Kuroshio on the shelf break and the Tsushima Warm Current in the north are more important in sustaining the TWC (Zheng et al. 2009; Ma et al. 2010). “North” is the upshelf direction of the topographic Rossby wave in ECS, which implies that the ATW theory might be applicable. Fang and Zhao (1988) and Wang and Oey (2016) suggested that the TWC is dynamically driven by an along-shelf pressure gradient under a dominant northward descending sea surface slope. This postulate seems to be supported by the long-term mean absolute dynamic topography (MADT) derived from Archiving, Validation, and Interpretation of Satellite Oceanographic Data (AVISO, https://marine.copernicus.eu) (Fig. 2a), which shows that the MADT contours extend from the shelf edge to the coastal waters, with a limited insulation area.

However, if the $f$-plane ATW applies, the shelf currents should decay rapidly toward the coast. Indeed, in the South Atlantic Bight, the MADT in coastal water and in the Gulf Stream region are decoupled, and no strong inductive current in the inner shelf is developed (Fig. 2b). For the West Florida Shelf, which is located at the eastern boundary of the Gulf of Mexico, the Loop Current establishes an elevated SSH and therefore an associated southward current along the shelf break (HETLAND et al. 1999; WEISBERG and HE 2003). Although the West Florida Shelf is wide and gentle as the ECS shelf, its shelf circulation decays rapidly toward the coast, unlike that in the ECS.

The distinct dynamic regimes on these shelves raise the question of whether and how the planetary $\beta$ modulates the shelf response to the open ocean. To this end, a generalized $\beta$-plane ATW framework is developed in section 2 based on steady-state and linearized equations. In section 3, the $\beta$-plane ATW equations are solved directly on idealized shelves with meridional or zonal orientations, various slopes, and different frictions. In section 4, a primitive equation numerical model is used to simulate the dynamics on the idealized shelf, so as to verify the fidelity of the $\beta$-plane ATW that simplifies the dynamics. The ECS circulation is then used as a realistic example to examine the theory in section 5, and the dynamics of the TWC is explored. Finally, conclusions are drawn in section 6.

2. Theory

Although the original motivation was to explore the shelf sea response to the western boundary current, here we begin with an arbitrary shelf without loss of generality. Restricted by the focus of this study, the local wind was neglected, and the following steady-state, barotropic, and linearized equations were used:

$$ r \frac{u}{h} - fv = -g \eta_x, $$

$$ fu + \frac{r}{h} v = -g \eta_y, $$

$$ (uh)_x + (vh)_y = 0. $$

The temporal derivative terms are neglected in (3)–(5), which means that only the “mean” current with time scale much longer than the frictional spindown time [i.e., $O(h/r)$] was considered. Solving (3) and (4) yields

$$ u = -\frac{g}{f} (\alpha \eta_y + \eta_x), $$

$$ v = -\frac{g}{f} (-\eta_y + \alpha \eta_x). $$
where $\alpha = r/\bar{h}$ is the ratio between the friction and Coriolis forcing, termed the linear Ekman number. Terms of order of $\alpha^2$ or higher were dropped. For a typical $r = 0.0005$ m s$^{-1}$ and middle latitude of $27^\circ$N, $\alpha^2 = O(0.1)$ requires $h > 23$ m, which is not a strong restriction for a regular shelf, and therefore the associated terms can be safely dropped. Substituting (6) and (7) into (5) yields

$$J \left( \frac{gh}{f} \nabla \eta \right) + \nabla \cdot \left( \frac{gr}{f} \nabla \eta \right) = 0,$$

where $J$ is the Jacobian operator. Equation (8) suggests that the propagation of pressure anomaly follows an “advection–diffusion” equation, with the streamfunction of advection velocity being $ghf$ and the diffusivity being $grf^2$. The advection velocity here is different from the water mass transport velocity. Hence, to avoid the confusion, Tyler and Käse (2000) termed $ghf$ as the string function, and the associated streamline can be called the string line. The above equation is the same as that in Wise et al. (2018), except that the latter used the assumption that the linearized bottom stress was driven by the geostrophic current, whereas here an alternative assumption $\alpha^2 \ll 1$ was adopted.

Let the shelf be uniform in the along-shelf direction, as shown in Fig. 1, and let the angle between the $y$ axis and the due north direction be $\theta$ (positive rotating clockwise). Then (8) becomes

$$C^p \cdot \nabla \eta - \frac{gs}{f} \eta_y = \frac{gr}{f} \eta_{xx},$$

in which

$$C^p = -\beta \frac{gh}{f^2} (\cos \theta, \sin \theta)$$

is the speed of the planetary Rossby wave and $-gs/f$ is the speed of the topographic Rossby wave, respectively, both in the long-wave limit. The shelf slope $s$ is not necessarily a constant. The term $\nabla \cdot (grf^2) \nabla \eta$ was simplified to $(grf^2) \eta_{xx}$, i.e., the term $[(grf^2) \eta_y] = (grf^2) \eta_{yy} - (2gr\beta) \eta_{xy} - (2grf^2) \eta_x + (2grf^2) \eta_{xx}$ was dropped, a treatment much like those of Csanady (1978) and Wise et al. (2018). The approximation $(grf^2) \eta_{yy} = 0$ is exact for the asymptotic solution of (8), that after some initial adjustments, the inductive shelf current essentially follows the shelf (i.e., $u = 0$, as will be seen later in the numerical experiments. At this limit, according to (3)–(5), $\eta_{yy} = -r_0/gh = 0$, because $h$ depends on $x$ only. It is also easy to verify that the ratio between $-2gr\beta \eta_{xy}$ and $(grf^2) \eta_{xx}$ is $2\alpha \beta B/f$ when the current is trapped [note that $\eta_{yy} \approx O(B)$], which is $O(0.01)$ for $\alpha = O(0.1)$ and a reasonable middle-latitude shelf width (say, 400 km).

### a. Western boundary shelf

A “western boundary shelf” here means the continental shelf on the western side of a large ocean body (e.g., the Pacific Ocean or the Gulf of Mexico). In this case $\theta = 0$, and (9) becomes

$$-\eta_y - \frac{\beta h}{fs} \eta_y = \frac{r}{fs} \eta_{xx}.$$

Csanady (1978) introduced a heat-conduction analogy for the $f$-plane ATW, which was also used here. The dimensional parameter $\kappa = r/fs$ plays the role of “heat conductivity,” which transmits the shelf edge pressure anomaly shoreward under the bottom friction. The nondimensional parameter $D_{\beta} = \beta h/fs$, termed the $\beta$ drift, plays the role of shoreward advection speed due to the propagation of the planetary Rossby wave. The $\beta$ drift $D_{\beta}$ is the ratio between planetary and topographic Rossby wave speeds. It suggests that the planetary $\beta$ can be important on shelves with small $s$. Equation (11) is subjected to following “initial” and boundary conditions:

$$\eta(x,0) = \eta_i(x),$$

$$\eta(B,y) = \eta^p(y),$$

$$\alpha \eta(0,y) + \eta_x(0,y) = 0.$$

\[ \text{Fig. 2. Topography and AVISO mean absolute dynamic topography (MADT) of (a) the East China Sea and (b) the South Atlantic Bight. The MADT contour interval is 0.03 m. The bathymetry along the thick black line is shown in the insets. The main circulation is also sketched, in which TWC represents the Taiwan Warm Current and TSWC the Tsushima Warm Current.} \]
The quotation mark means that it is an analogous “initial condition” that takes the $-\gamma$ direction as time. It in fact refers to the inflow condition from the upshelf. Equation (14) is the no-normal flux condition at the coastal boundary given that the along-shelf wind stress is zero. Middleton and Thomson (1985) provided a highly skillful analytical solution for the coastal trapped solution of (11) with uniform slope $s$. However, for nonuniform $s$ and the open ocean boundary condition the analytical solution is difficult to find, and therefore (11)–(14) were solved numerically.

Before doing the numerical calculation, let us look at the characteristics of the asymptotic solution. When the current driven by an imposed pressure anomaly is trapped and running along the shelf (see Fig. 4 for a reference), $u = 0, v = g\eta_x/f$, and $\eta_x = -\alpha \eta_y$ according to (3) and (4), (11) becomes

$$\left( \frac{r}{fh} - \frac{\beta h}{fs} \right) \eta_x = \frac{r}{fs} \eta_{xx}. \quad (15)$$

The solution for $\eta_x$, and hence for the along-shelf velocity $v$, is

$$v = V \frac{h}{H} \exp \left( \frac{\beta h}{r} \int_s^0 h \, dx \right). \quad (16)$$

in which $V$ and $H$ are the along-shelf current and the water depth at the shelf edge, respectively. If $\beta = 0$, $v$ decreases shoreward with $h$, and the $f$-plane ATW is recovered. However, as $\beta > 0$, the term $\exp \left( (\beta/r) \int_s^0 h \, dx \right)$ increases exponentially with the cross-shelf area from $x$ to the shelf break, which compensates for the decrease in $h$.

The location where the along-shelf velocity $v$ peaks, which is the central focus of this study, requires $\eta_{xx} = 0$. From (15) it immediately follows that $\beta h^2/rs = 1$. The nondimensional number

$$Pe_{\beta} = \frac{\beta h^2}{rs} \quad (17)$$

is hence a measure for indexing the open ocean–shelf interaction. Apparently, $Pe_{\beta} = D_{\beta}/\alpha$, i.e., the ratio between the $\beta$ drift $D_{\beta}$ and the linear Ekman number $\alpha$. The $\beta$ drift is the ratio between planetary and topographic Rossby wave speeds, or equivalently the (tangent of the) angle between the string line $gh/f$ and the isobath. On the other hand, when the shelf current is trapped, $\alpha = -\eta_x/\eta_y$, i.e., the (tangent of the) angle between the SSH contour and the isobath. Therefore, $Pe_{\beta}$ represents the ratio between westward “advection” due to the planetary Rossby wave and “diffusion” due to the bottom friction, if riding on a coordinate moving with the topographic Rossby wave. It is analogous to the conventional Péclet number for the scalar transport in a continuum, and hence is termed the $\beta$ Péclet number. However, one should bear in mind that for this $\beta$ Péclet number, the scalar quantity transported is the pressure anomaly, not the material; the advection is due to the Rossby wave, not the flow; and the diffusion is caused by bottom friction, not the fluid diffusivity.

For a shelf with constant $s$, $Pe_{\beta}$ decreases monotonically from deep to shallow water. Let $\eta_x > 0$ without loss of generality, it is apparent that in regions with $Pe_{\beta} > 1$, $\eta_{xx} < 0$, and $\eta_x$ (i.e., $v$) increases shoreward; whereas if $Pe_{\beta} < 1$, $\eta_{xx} > 0$, and $\eta_x$ decreases shoreward. Therefore, peak velocity occurs where $Pe_{\beta} = 1$ and $dPe_{\beta}/dx > 0$. On approaching the vertical coastal wall, $s \to \infty$, and hence $Pe_{\beta} \to 0$. Hence, even though $Pe_{\beta}$ is greater than 1 on the entire shelf, it still drops to this critical value on the coast. For nonuniform $s$ on the shelf, because $h$ and $s$ both vary, $Pe_{\beta}$ may lose monotonicity, and multiple places with $Pe_{\beta} = 1$ could be found.

In Wise et al. (2018) a similar parameter $Pa = \beta HL/\gamma r$ was defined for a uniform and zero-coastal-depth shelf through scaling analysis, in which $L$ is the scale of the shelf width. The parameter $Pa$ is equivalent to $Pe_{\beta}$ at the shelf break (where $h = H$) for this particular shelf configuration because $s = H/L$, and therefore $Pe_{\beta} = \beta H^2/\gamma r (H/L) = Pa$. As $Pe_{\beta}$ approaches 0 on the coast, if $Pe_{\beta} > 1$ on the shelf edge, there must be a place on the shelf where $Pe_{\beta}$ drops to 1. Although $Pa$ is a good measure of the bulk characteristics of the shelf–ocean interaction, $Pe_{\beta}$ provides more details.

b. Arbitrary direction of the shelf

For arbitrary $\theta$, (9) can be rewritten as

$$-\eta_x - \frac{\beta h \cos \theta}{fs + \beta h \sin \theta} \eta_x = \frac{r}{fs + \beta h \sin \theta} \eta_{xx}. \quad (18)$$

In this case, the $\beta$ drift and “conductivity” become

$$D_{\beta}^\theta = \frac{\beta h \cos \theta}{fs + \beta h \sin \theta}, \quad (19)$$

$$\kappa^\theta = \frac{r}{fs + \beta h \sin \theta}, \quad (20)$$

where $fs + \beta h \sin \theta$ is the sum (with factor $-g/r^3$) of the topographic Rossby wave and the projected planetary Rossby wave on the isobath. Using the same procedure as in section 2a, the along-shelf velocity $v$ being trapped is

$$v = V \frac{h}{H} \exp \left[ \frac{\beta \cos \theta}{r} \int_s^0 h \, dx + \frac{\beta \sin \theta}{f} (x - B) \right], \quad (21)$$

and $v$ peaks on the shelf if

$$Pe_{\beta}^\theta = \frac{\beta h^2}{rs} \frac{\cos \theta}{(1 + \beta h \sin \theta/fs)} \quad (22)$$

is 1. Note that (21) and previous (16) are valid for an arbitrary cross-shelf $h$ profile.

However, $Pe_{\beta}^\theta = 1$ cannot be reached for certain $\theta$. For an eastern boundary shelf, $\theta = \pi$, and hence $Pe_{\beta}^\theta = -\beta h^2/rs$, always negative, and $v = V(h/H) \exp \left[ -(\beta/r) \int_s^0 h \, dx \right]$ that decreases rapidly toward the coast. Physically, this occurs because the $\beta$ drift is in the offshore direction, which removes the oceanic influence from the shelf. At $Pe_{\beta}^\theta = -1$, the removal of the pressure anomaly due to the planetary Rossby wave is balanced by the friction effect. Hence, shoreward of this critical location, the shelf is frictional and hardly feels the $\beta$ effect. On zonal (i.e., southern or northern boundary) shelves, $D_{\beta}^\theta$ is zero, Eq. (18) resembles the $f$-plane ATW and the frictional effect is dominant.
According to (22), for a reasonable shelf with \( \beta h \sin \theta / \sqrt{s} < 1 \), i.e., \( s > h/R \) in which \( R \) is Earth’s radius, the sign of \( \text{Pe}_\beta \) is determined by \( \cos \theta \). Hence, \( \text{Pe}_\beta = 1 \) can be reached only for \( \cos \theta > 0 \), namely, \( \theta \in (-\pi/2, \pi/2) \). In another word, the shelf should have a sense of “western boundary.” Hence, the western boundary shelf is the only place that the planetary \( \beta \) produces an intensified current on the shelf.

3. Direct solution of \( \beta \)-plane ATW on the idealized shelf

Equations (12)–(14) and Eq. (18) were numerically solved with different shelf bathymetries as shown in Fig. 3. The shelf width was set to 400 km and the depth at shelf edge was set to 100 m, which is much like the ECS shelf (see Fig. 2a). In Figs. 3a and 3b, the cross-shelf slope is constant, with a water depth at the coastal wall of 80 and 40 m, respectively, corresponding to slopes of \( 0.5 \times 10^{-4} \) and \( 1.5 \times 10^{-4} \), respectively. In Fig. 3c the slope becomes steep shoreward of 125 km, mimicking the bathymetry of the ECS inner shelf. The reference latitude was 27°N, which is representative of both the Kuroshio and the Gulf Stream (Fig. 2), resulting in a planetary \( \beta \) of \( 2.03 \times 10^{-11} \text{s}^{-1} \). The friction coefficient \( r \) was 0.0005 m s\(^{-1}\). The boundary condition at the shelf edge was set as follows:

\[
\eta^B(y) = \begin{cases} 
-10^{-6}y, & y > -200 \text{ km} \\
0.2 \text{ m}, & y \leq -200 \text{ km} 
\end{cases}
\]  

(23)
i.e., a source was added on the upper-right corner of the shelf (see Fig. 3). The inflow condition from upshelf was zero. The slope and friction were further adjusted in the sensitivity experiments.

a. Effect of shelf orientations

3. Direct solution of \( \beta \)-plane ATW on the idealized shelf

Figure 4 shows the SSH and associated barotropic currents with the shelf bathymetry shown in Fig. 3a for the western boundary, northern boundary, eastern boundary, and southern boundary shelves, respectively. The associated \( f \)-plane solution is shown in Fig. 5b. At first glance, these results were similar. The pressure anomaly propagated downshelf and was then transmitted shoreward, forming a current toward the source. In the first several hundred kilometers, the coastal area was free from the oceanic influence, serving as an insulation area. However, further downshelf, the coastal area felt the oceanic...
influence and the overall circulation was in the along-shelf direction.

Notable differences can be found for cross-shelf gradients of SSH and alongshore velocity. On the western boundary shelf, the SSH contours were squeezed toward the coast, which decreased the area of insulation, and the along-shelf velocity peaked near the coast. For the chosen bathymetry adopted (Fig. 3a), $P_e \beta > 5.2$ over the shelf, and reached the critical value 1 only when approaching the coastal wall. This explained the occurrence of peak velocity along the coast. On the other hand, on the eastern boundary shelf, the SSH contours were squeezed toward the shelf edge, which increased the area of insulation, and the along-shelf velocity peaked on the shelf edge. The opposite drift directions (relative to the shelf) on the western and eastern boundary shelves caused this dramatic difference. However, the “conductivity” $\kappa^\theta$ was not altered by planetary $\beta$ on the meridional shelves because the projection of the planetary Rossby wave on the isobath was zero.

Along-shelf velocity in general also decayed from the shelf edge on the southern and northern boundary shelves. In these two cases, the planetary $\beta$ modulated the conductivity $\kappa^\theta$ rather than the cross-shelf $\beta$ drift and hence the resulting ATW behaved more like that on an $f$ plane. For the northern boundary shelf, i.e., $\theta = \pi/2$, the directions of the planetary and topographic Rossby waves were the same, which increased the denominator in (20), thus reducing the conductivity $\kappa^\theta$. For the southern boundary shelf, $\theta = 3\pi/2$, the directions of the planetary and topographic Rossby waves were opposite, which increased the $\kappa^\theta$. Because the southern boundary shelf had higher conductivity, the pressure anomaly reached the coast slightly faster, and the cross-shelf gradient of the along-shelf velocity was smaller (Fig. 4). Because $\kappa^\theta = r/|fs \pm \beta h| = r/|fs[1 \pm (\beta h/fs)] = r/f s^\theta$ for northern and southern boundary shelves, respectively, the planetary $\beta$ acted to increase or decrease the effective bottom slope $s^\theta$. Note that on the northern boundary shelf, the gradient of $f$ and the gradient $1/h$ are in the same direction and together amplify the PV($=f/h$) gradient; hence, the presence of a planetary $\beta$ is equivalent to a steepened bathymetric slope in the sense of PV conservation. On the southern boundary shelf their directions are opposite.

FIG. 4. Numerical solution of the $\beta$-plane arrested topography wave for the bathymetry shown in Fig. 3a on (a) western, (b) northern, (c) eastern, and (d) southern boundary shelves.
Fig. 5. Numerical solution of the arrested topography wave on the western boundary shelf, with bathymetries shown in (a)–(c) Fig. 3a, (d)–(f) Fig. 3b, and (g)–(i) Fig. 3c. In the left column, friction is not considered; in the middle column, the $\beta$ effect is not considered; and in the right column, both friction and $\beta$ effects are considered. The location $Pe_\beta = 1$ is indicated by a blue line.
Hence, on meridional shelves the planetary \( \beta \) affects the open ocean–shelf interaction through \( \beta \) drift, which advected the pressure anomaly across the shelf; but on zonal shelves the planetary \( \beta \) modulated the effective bottom slope and thereby the conductivity, that is, the diffusion of the pressure anomaly. In relative terms, the effect of planetary \( \beta \) on zonal shelves was less significant, as expected.

**b. Effect of bottom friction and shelf slope on the western boundary shelf**

Without bottom friction, the pressure anomaly propagated exactly along the string line (Figs. 5a,d,g), i.e., along the \( gh/f \) contours labeled in Fig. 3. The SSH signal reached the coast faster in the case of a gentle slope, i.e., the large \( \beta \) drift, as expected. For the case of a steep slope in the inner shelf, the string lines converged dramatically (Fig. 3c), resulting in a strong SSH gradient across the inner shelf slope. When friction was included, the SSH contours diffused significantly, and the coastal area felt the open ocean signal much more than in the frictionless case.

Peak velocity of the \( f \)-plane ATW occurred at the shelf edge, whereas that of the \( \beta \)-plane ATW occurred on the shelf. For a gentle shelf, i.e., with a slope of \( 0.5 \times 10^{-4} \) and a shelf width of 400 km, peak velocity was observed along the coast (Fig. 5c) because \( \text{Pe}_{B} \to 1 \) on the coast, as explained in section 3a. On the other hand, for a relatively steep shelf, say with a slope of \( 1.5 \times 10^{-4} \) (Fig. 3b), \( \text{Pe}_{B} = 1 \) on the middle shelf, and the associated depth \( h = \sqrt{rs/\beta} = 60.7 \) m. This location is labeled in Fig. 5f. Evidently, the along-shelf velocity reached a maximum exactly at this location, which can be seen more clearly in the cross-sectional velocity profile (Fig. 6a). If a steep inner shelf slope was set, the shelf current peaked at the foot of this slope, where again \( \text{Pe}_{B} = 1 \) (Fig. 5i). This criterion was further justified with model results for other slopes. For example, with a steeper slope \( 2 \times 10^{-4} \), the location with \( \text{Pe}_{B} = 1 \) occurred near the shelf edge (Fig. 6a). In this case, the resulting shelf current could be indistinguishable from the strong boundary current on the shelf break. For an even steeper shelf, i.e., \( \text{Pe}_{B} < 1 \) for the entire shelf, the response of the shelf was much like that of the \( f \)-plane ATW.

The bottom friction coefficient \( r \) is another controlling parameter. Model results with different choice of \( r \) for the gentle shelf were obtained (Fig. 6b). It was apparent that with larger friction, the coastal sea level was higher because of the higher conductivity \( k \), which increased the penetration, and the along-shelf velocity was more uniformly distributed over the shelf. On the other hand, with little friction, coastal sea level was less affected, the cross-shelf SSH gradient was large, and the along-shelf velocity was higher and more concentrated.

**4. Simulated \( \beta \)-plane ATW on the idealized shelf with primitive equation model**

One apparent risk of the above analyses is that Eqs. (3)–(5) neglects several realistic dynamic processes, such as the inertia and the unsteadiness, which could be important for the initial adjustments; the linearization of bottom friction could be another oversimplification. It is therefore necessary to verify the fidelity of the above \( \beta \)-plane ATW by the direct simulation using a primitive equation numerical model.
The primitive equation model ECOM-si (Blumberg 1994) was configured to simulate the open ocean–shelf interaction for the gently sloping shelf, as shown in Fig. 3a. Model resolution was 5 and 10 km in the cross-shelf and along-shelf directions, respectively, and three uniform sigma layers were set in the vertical direction. The downshelf end was artificially extended for 8000 km to avoid the difficulty of treating the open boundary condition. A “flow-in-and-out” condition was set at the open boundary to mimic the open ocean current influence; a negative $\eta_{by}$ corresponding to a 1.5 Sv (1 Sv = $10^6$ m$^3$ s$^{-1}$) outflow was set from $-200$ to $0$ km and a positive $\eta_{by}$ corresponding to a 1.5 Sv inflow was set from $-800$ to $-1000$ km on the open boundary, respectively. The same representative latitude ($27^\circ$N) and the associated $\beta$ were set. The quadratic bottom friction parameterization was used with a drag coefficient $C_D$ of 0.0025. The model reached steady state rapidly, and the results of the fifth day were used for analysis.

For the $f$ plane, the model produced a circulation throughout the shelf (Fig. 7), and the currents decayed gradually from the shelf edge to the coast, which is consistent with previous theoretical results. When the planetary $\beta$ was considered, the shelf currents changed dramatically on the meridional shelves. The primitive equation model simulated a consistent pattern with the direct calculation of $\beta$-plane ATW (Fig. 8). The currents were intensified strongly along the coast for the western boundary shelf and along the shelf edge for the eastern boundary shelf, respectively. On the zonal shelves, however, the planetary $\beta$ caused little change in the model results. The way that planetary $\beta$ affects the ATW on a zonal shelf is by adjusting the conductivity. Indeed, on close inspection, one can find slight differences between the results on the northern and southern boundary shelves. Nevertheless, with the nonlinear bottom friction and more dynamics involved, the planetary $\beta$ was less important on a zonal shelf.

5. Mean circulation on the ECS shelf

The $\beta$-plane ATW was then used to explore the dynamics of ECS shelf circulation. The ECS has one of the broadest continental shelves in the world, with a width of 400–500 km in the northern and middle parts and then narrowing southward to $\sim 200$ km near the Taiwan Island (Fig. 2a). The ECS shelf features a double-slope structure in the cross-shelf direction (see the inset in Fig. 2a). Besides the steep shelf break offshore of the $\sim 120$-m isobath, there is also a nearshore slope between the 20- and 60-m isobaths in $\sim 60$-km distance. The inner shelf slope was formed by the past sediments deposition from the Changjiang River (Liu et al. 2018). Given this origin, it is conventionally called the Mud Belt.

The AVISO data were again used to look at the MADT in the ECS, but for the February (winter) and July (summer) separately (Figs. 9a,b). The overlays show the monthly mean (in 2003–16) sea surface temperature (SST) derived from the Moderate Resolution Imaging Spectroradiometer (MODIS) Level-3 data, and the monthly winds from the European Center for Medium-Range Weather Forecasts (ECMWF). A 3D primitive equation numerical model was configured by Wu et al. (2011) to simulate the circulation in the ECS. The model domain covered the entire ECS and adjacent waters. The model has been extensively validated against in situ salinity, temperature, currents, and tide data and has shown reasonable capabilities (see Wu et al. 2011, 2014). In this study the model was run with climatological monthly forcings. The first two years served for spinup, and the output of the third year was analyzed.

a. Circulation pattern and sensitivities to local forcings

The satellite and model results showed consistent dynamics patterns in both winter and summer seasons (Fig. 9). The depth-mean shelf currents and the SST over the ECS shelf (Figs. 9c,d) showed a clear TWC in the inner shelf in both winter and summer. Part of the TWC veered offshore around 28$^\circ$N and merged into the western flank of the Kuroshio, although wind directions were opposite in these two seasons. In summer, the TWC was associated with the intense SSH
contour extending along the Mud Belt from the Taiwan Strait to the south of the Changjiang River Estuary, then veering offshore and exiting through the Tsushima/Korea (T/K) Strait. The shelf circulation seemed to follow the geostrophic balance (Fig. 9d). The wintertime SSH pattern was quite different, with contours in the inner shelf in a cross-shelf direction (Fig. 9c). This seemed to support the previous argument that the along-shelf pressure gradient drives the TWC (Guan and Fang 2006; Wang and Oey 2016).

The upwelling-favorable southerly wind in summer has been considered to cause the strong cross-shelf SSH slope in some studies. A simple estimation can be made to assess its contribution. Constrained by the coast, the depth-mean cross-shelf velocity vanishes, resulting in a simple steady-state momentum balance \( \frac{\partial \eta}{\partial x} = f r^* \rho g r \), in which \( r^* \) is the along-shelf wind stress. The term \( r \) can be determined as \( r = \delta C_D U_T \), where \( C_D \) is the drag coefficient (0.0025 used in this study), \( U_T \) is the amplitude of the background tidal current, and \( \delta \) is a parameter of order 1 (Parker 1991). Typical \( U_T \) over the ECS shelf is \( \sim 0.5 \text{ m s}^{-1} \). Parameter \( \delta \) was determined by the regression between the modeled bottom stress and the linearized bottom stress on each grid and was \( \sim 0.4 \). This resulted in \( r \) of \( \sim 0.0005 \text{ m s}^{-1} \). Given a mean summertime wind stress of \( \sim 0.02 \text{ N m}^{-2} \) in this area, the associated cross-shelf SSH gradient around 27°N should be \( \sim 2.6 \times 10^{-7} \). However, estimated from AVISO or model results, the SSH gradient in this location was \( \sim 2 \times 10^{-6} \), one order of magnitude steeper. Hence, although the wind forcing is favorable in summer, it is not the major contributor driving the TWC.

On the other hand, in winter, although the inner shelf current was nearly in the down-SSH slope direction, which would seem to support the previous “along-shelf pressure gradient” hypothesis, it should be noted that in this season the coastal SSH was strongly influenced by the northerly wind and the extension of the Changjiang River plume. Specifically, the Changjiang River plume that extends southward along the coast in winter (Wu et al. 2013) can elevate the sea level near the coast due to its buoyancy, thus distorts the SSH.

Fig. 8. Numerical simulation of the \( \beta \)-plane arrested topography wave with the primitive equation numerical model on (a) western, (b) northern, (c) eastern, and (d) southern boundary shelves.
gradients over the ECS shelf. If the discharge from the Changjiang River in winter was removed, the northward intrusion of the TWC decreased somewhat, but the major patterns remained the same (Fig. 10a). If in addition the wind forcing was removed, the TWC in winter was stronger and resembled the pattern in summer, with a strong cross-shelf SSH gradient (Fig. 10b). South of 28°N, however, the modeled circulations were similar, regardless of wind or river influence. These results suggested that the local forcings on the ECS shelf only modulate the TWC, but are not intrinsic drivers. It is the inherent and robust pressure field as shown in Fig. 10b that substantially drives the TWC. This is consistent with the observational fact that TWC exists under various weather conditions, during various seasons (Huang et al. 2016; Guan and Fang 2006). The question has now been reduced to what determines the pressure field as shown in Fig. 10b.

b. β-plane ATW on the ECS shelf

Viewed from the upshelf to the downshelf (Fig. 10b), the SSH contours in the ECS originated from two sources: the T/K Strait in the north and the Kuroshio on the shelf break. The former was caused by the Tsushima Warm Current (TSWC), which in fact is also driven by the Kuroshio under the Island Rule that applies to the Japan Islands (Yang et al. 2013). This pattern, as well as the fact that the TWC existed regardless of local forcings, implied that the intrinsic pressure field in the ECS was the ATW induced by open ocean forcing. A sensitivity experiment has confirmed that, by turning off the Kuroshio and Tsushima Warm Currents, the TWC

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**Fig. 9.** (top) Long-term (2003–16) mean sea surface temperature derived from MODIS (colored shading), AVISO MADT (thin black contours at 3-cm intervals), and monthly wind derived from ECMWF (arrows) for (a) February (winter) and (b) July (summer). (bottom) Modeled sea surface temperature, sea surface height (thin black contours at 3-cm intervals), sea surface salinity (white contours at 3-psu intervals), and depth-mean velocity in (c) winter and (d) summer. In (c) and (d), only the velocities on the part of the shelf shallower than 200 m are shown.
disappeared as expected (not shown), as reported in previous studies (Zheng et al. 2009; Ma et al. 2010).

Due to the flat topography of the central ECS shelf, the string lines $ghf$ deviate strongly from the isobaths, or in other words, a large $\beta$ drift exists throughout the shelf (Fig. 11a). In the ECS, the regressed bottom friction $r$ from the numerical model was $\sim 0.0005$ m s$^{-1}$, and the calculated $Pe_\beta$ was as shown in Fig. 11a. From the Pacific Ocean to the coastal ECS, $Pe_\beta \to \infty$ before reaching the Okinawa Trough, then dropped sharply to 1 or an even lower value on the shelf break. Thereafter, $Pe_\beta$ rose again and was larger than 1 over a vast area of the central ECS shelf due to the flat topography. Finally, on approaching the Mud Belt, due to the dramatic increase in bottom slope, $Pe_\beta \to 1$ first and then dropped below 1 until reaching the coast. A comparison of Fig. 11a and the ECS circulation (e.g., Fig. 10b) shows that the two places with $Pe_\beta = 1$ correspond exactly to the Kuroshio and the inner shelf TWC. This is consistent with the characteristics of the $\beta$-plane ATW. The second place with $Pe_\beta = 1$, i.e., the foot of the Mud Belt, relies essentially on the flat topography of the central ECS shelf. For a comparison, in the South Atlantic Bight, another western boundary shelf that is narrow and much steeper, the string lines largely follow the isobaths, and $Pe_\beta = 1$ only at the shelf break (Fig. 11b), where Gulf Stream is located.

1) Effect of Planetary $\beta$

If the planetary $\beta$ was shut down, that is, if the Coriolis parameter was set to a uniform value at latitude $27^\circ$N, the shelf
circulation would still be formed if without the wind forcing (Fig. 12a). In this case, the shelf pressure anomaly originated only from the T/K Strait, which propagated downshelf as an $f$-plane ATW. The resulting currents in the entire shelf were much weaker than those considering the variation of $f$ with latitude, especially south of 28°N. Under the northerly wind forcing, the $f$-plane ECS shelf circulation almost disappeared (Fig. 12b). According to Csanady (1978), the along-shelf wind forcing acts as a setup (a downwelling-favorable wind) or set-down (an upwelling-favorable wind) coastal boundary condition for the ATW solution. Hence, the SSH contours in the central ECS became zonal and no longer followed the isobath, weakening the resulting current. Note that the model was forced by inflow/outflow condition at the open boundary and that hence the simulated Kuroshio still existed on an $f$-plane although was weakened. The $\beta$ effect is in fact necessary to form a western boundary current (Stommel 1948).

On the $f$ plane, the Kuroshio pressure anomaly could barely cross the steep shelf break (Figs. 12a,b), much like the situation addressed by Wang (1982) in the Middle Atlantic Bight. In that case, the shelf break was so steep that the ATW conductivity $\kappa$ was too low for the friction to break the PV barrier. However, with the $\beta$ effect, the Kuroshio was stronger, and its interaction with the shelf was intensified, which was favorable for the pressure anomaly to cross the shelf break. Thereafter, the increased $\beta$ drift on the flat central ECS shelf overwhelmed the friction effect (Fig. 11a), i.e., $Pe_\beta > 1$, which favored advection of the pressure anomaly toward the coast. In some areas, the local bathymetry even slightly deepened westward, resulting in a negative $\beta$ drift (thus $Pe_\beta$), which intensified the westward propagation of the pressure anomaly.

One puzzling observation was that the shelf current was still intensified in the inner shelf on the $f$ plane in the absence of wind (Fig. 12a). This seemed to violate the above arguments...
about $f$-plane ATW. As a matter of fact, this phenomenon was caused by the narrowing of the ECS shelf in the downshelf direction (see Fig. 11a). On an $f$-plane, if friction is neglected, the string function equation (8) becomes $J(h, \eta) = 0$, meaning that the contours of $\eta$ simply follow the isobath. Hence, the sparse SSH contours from the upshelf converged upon reaching the narrow shelf area, producing a relatively stronger current. However, the friction-dominant $f$-plane ATW was fragile and could be easily destroyed by wind (Fig. 12b).

2) Effect of Topography

The TWC flows offshore of the Mud Belt. The idealized model results shown in Figs. 3c and 5i show that the inner shelf slope forces the streamlines to converge and prevents the further shoreward propagation of the planetary Rossby wave. To verify this, additional sensitivity experiments were conducted by removing the bathymetry of the Mud Belt, as shown in Figs. 12c and 12d. In this case, $P_{\beta} \rightarrow 1$ near the coast, the planetary Rossby wave reached the coast, and a strong current occurred along the coast. This again supported the hypothesis that the TWC was driven by the $\beta$-plane ATW.

These analyses indicated that the pressure field in the ECS was codetermined by the T/K Strait in an $f$-plane ATW manner and by the Kuroshio in a $\beta$-plane ATW manner. The latter, however, was limited in the south of 28$^\circ$N. These two different origins of the pressure anomaly, as well as the local bathymetry, determined the intrinsic dynamic regime of the ECS shelf circulation. South of 28$^\circ$N, the circulation was stable regardless of wind stress, and the TWC gradually veered offshore; north of this point, the TWC flowed mainly northward along the Mud Belt, but was suppressed by a strong local forcings such as wind. This may explain the bifurcated nature of the TWC around 28$^\circ$N, which has been reported in previous studies (e.g., Guan and Fang 2006).

6. Conclusions

The influence of open ocean forcing on shelf circulation was investigated within a $\beta$-plane arrested topographic wave (ATW) framework. It was found that the ratio between long-wave-limit planetary and topographic Rossby wave speeds, i.e., the $\beta$ drift $D_\beta$, and the ratio between friction and Coriolis forcing, i.e., the linear Ekman number $\alpha$, jointly determined the characteristics of shelf response to open ocean forcing. Their ratio, i.e., the $\beta$ Péclet number $P_{\beta} = D_\beta/\alpha = \beta \eta^2/\alpha$, can serve as an intrinsic measure to index inductive shelf circulation under open ocean influences.

In general, for a gently sloping shelf on a western ocean boundary, the planetary Rossby wave advected the pressure anomaly from the open ocean toward the coast, resulting in an intensified shelf current at the location where $P_{\beta} = 1$. At this location, the effects of the Rossby wave and bottom friction balanced. On a steep shelf, $P_{\beta} < 1$ throughout the shelf, and the shelf currents decayed rapidly toward the coast, resembling the situation predicted by conventional $f$-plane ATW theory. The South Atlantic Bight can serve as an example. If $P_{\beta} > 1$ on the edge or at some place on the shelf, there must be a place with $P_{\beta} = 1$ either on the shelf or on the coastal wall, where a strong inductive shelf current occurred along with the western boundary current. The Taiwan Warm Current in the East China Sea can serve as a typical example. For an eastern boundary shelf, the $\beta$ drift, and therefore $P_{\beta}$, was always negative, and the planetary Rossby wave removed pressure from the shelf, meaning that open ocean forcing drove a strong current only on the shelf edge. The West Florida Shelf can serve as an example. This $\beta$ effect on a meridional shelf is robust, and the resulting shelf circulation pattern remained predominant even under local forcing.

For zonal shelves, the $\beta$ drift was zero, and the planetary $\beta$ regulated the effective bottom slope in the sense of PV conservation. Hence, the $f$-plane ATW theory applied, and the inductive currents decayed from the shelf edge. On a northern boundary shelf, because the planetary and topographic waves were in the same direction, the effective bottom slope increased, and therefore the open ocean signal was transmitted shoreward slightly more slowly. In contrast, on a southern boundary shelf, because the planetary and topographic waves were in opposite directions, the effective bottom slope decreased, and therefore the open ocean signal reached the coast slightly faster. However, the $\beta$ effect on zonal shelves was less robust than on meridional shelves.

This $\beta$-plane ATW theory explained the dynamics of the observed Taiwan Warm Current in the inner East China Sea shelf. The slope of the inner-shelf East China Sea, i.e., the Mud Belt, was the place where $P_{\beta}$ dropped to 1, and hence where an intensified current formed. Another key point was that the energy driving this shelf current did not come from the south where the water mass originated, but from the Tsushima/Korea Strait and the Kuroshio in the manner of the downshelf-propagating ATW. Given these two different origins, the Taiwan Warm Current bifurcated around 28$^\circ$N.

The limitation of the above theory was apparent. The neglect of time dependency terms in Eqs. (3)–(5) restricted the validity of the theory only to the mean circulation averaged over a time scale longer than the frictional spindown time. The $\beta$-plane ATW only predicted where there should be a shelf current; it did not tell where the water mass of this current originated. To obtain this information, more detailed studies should be conducted. The theory assumed the shelf to be barotropic, which was not true for many cases. Given the heat flux from the atmosphere or the buoyancy input from rivers, the sea on the shelf was stratified, and therefore the shelf current and its exchange with the open ocean were influenced by baroclinicity. This can be seen from sensitivity experiments on the East China Sea shelf. According to Welander (1957), the problem of predicting the response of a shelf sea to external forcing should be separated into “local” and “global” problems. The $\beta$-plane ATW provided information about the global distribution of the mean pressure field that determined the overall features of shelf currents, but to obtain a complete picture, the local and synoptic dynamics must be considered.

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