Effects of Multiple Connectivity on a Finite-Element Barotropic Model

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ABSTRACT

A finite-element model recently designed for calculation of oceanic normal modes is amended here with a prescription for the proper treatment of multiple connectivity, the main effects of which are likely to be located in the Southern Ocean. For this purpose line bases, as well as point bases, are needed in the finite-element representations of volume-flux and energy-flux streamfunctions. The dynamical conditions that should be met by the circulation of velocity and energy on each boundary are found to arise as natural boundary conditions for the finite-element equations.

1. Background

Recently, I described a finite-element model designed for calculation of general time-dependent motion or normal modes in a barotropic ocean, and showed some modes obtained with this model in a domain consisting of the Atlantic and Indian Oceans (Platzman, 1978). Subsequently, I used the model to calculate normal modes in a connected world-ocean domain comprising the Arctic, Atlantic, Indian and Pacific Oceans. (A description is now being prepared for publication.) One of these modes is the subject of a brief report (Platzman, 1979) and will be taken as an example in the present note.

A diagnostic useful in analyzing results of normal-mode calculations is the distribution of energy flux as represented by an energy-flux streamfunction. This function is determined by the local energy-flux vector obtained from space-dependent aspects of the normal-mode elevation and velocity. In the course of examining energy-flux streamlines for the mode mentioned above, I became aware of a defect in my formulation of the boundary condition for the energy-flux streamfunction. The defect arose because I overlooked the consequences of going from the simply-connected domain used in a model of the Atlantic and Indian Oceans (Platzman, 1975) to the multiple connectivity of the world-ocean model. The effects of connectivity on formulation of dynamical models have been considered by several authors (some cited below), but I had the
mistaken belief that normal modes were immune to these effects. The purpose of the present note is to show how the finite-element design (Platzman, 1978) can accommodate them in a simple and natural way.

Connectivity of a numerical model is less than that of the actual ocean because the model must ignore islands and straits not resolvable by the grid. For the domain consisting only of the Atlantic and Indian Oceans I used a finite-difference grid of 6° Mercator squares which did not resolve even the largest island (Madagascar), so this model was simply connected. For the world-ocean domain I am using a finite-element grid of spherical triangles with average area equal to that of a 4.6° square. This grid does not resolve the Bering Strait but does see the Drake Passage (although at the limit of resolution), so the model domain is at least doubly connected since any circumboreal circuit in the Southern Ocean, for example, is irreducible. In fact, the model is quadruply connected because it also resolves two islands (Madagascar and New Zealand). Thus it has four disconnected land masses: the combined Eurasian and American continents, Antarctica and the two islands. In the following discussion the first of these land masses will be referred to as the "mainland" and the other three as "islands".

2. Energy-flux streamfunction

The local transport of wave energy in a linear barotropic model is $\rho g$ times the average of $(\text{Re} \xi)(\text{Re} \eta)$ over a wave period, where $(\xi, \eta) = (Z, U)e^{\text{i} \omega t}$ are the (complex) surface elevation and volume transport (depth $h$ times velocity) with space-dependent aspects $(Z, U)$. This gives the transport vector

$$J = \frac{1}{2} \rho g \text{Re}(Z^*U).$$

We seek to represent the calculated distribution of $J$ by means of Stokes/Helmholtz potentials $\theta$, $\chi$, i.e.,

$$J = \varepsilon - w \nabla \theta + k \times \nabla \chi,$$  \hspace{1cm} (2.1)

where $w$ is an assigned positive function. (The illustration given at the end of this note was calculated with $w = 1$.) The residual $\varepsilon$ arises when the number of degrees of freedom available in $\theta$ and $\chi$ is insufficient to represent $J$ exactly. For example, in the finite-element model that gave the results shown in Figs. 2–4, there are 2600 degrees of freedom in $J$ (corresponding to one value of $J$ in each of 1300 triangular grid elements) but only 1298 (747 and 551, respectively) in $\theta$ and $\chi$.

A classical method of determining $\theta$ and $\chi$ is that of least-squares approximation:

minimize $Q = \int w^{-1} \varepsilon^2 dS$,  \hspace{1cm} (2.2)

with $\chi$ uniform on each boundary.

This is an area integral over the complete ocean domain. The purpose of the factor $w^{-1}$ and the boundary constraint on $\chi$ will be seen presently. The procedure for solving (2.2) is well known. I will summarize it briefly in order to call attention to the role of boundary conditions.

At a minimum of $Q$, since $\delta Q = 0$, it is apparent that any variation of the residual must be orthogonal to the residual itself, i.e.,

$$\int w^{-1} \varepsilon \cdot \delta \varepsilon dS = 0. \hspace{1cm} (2.3)$$

We first take the variation with respect to $\theta$ alone. Then $\delta \varepsilon = w \nabla \delta \theta$ so

$$0 = \int \varepsilon \cdot \nabla \delta \theta dS. \hspace{1cm} (2.4)$$

The implications of (2.4) can best be seen in the case when the individual terms of (2.1) are continuous and differentiable. Then (2.4) can be integrated partially to give

$$0 = \oint \varepsilon \delta \theta dl - \iint (\nabla \cdot \varepsilon) \delta \theta dS.$$  \hspace{1cm} (2.5)

Here $n$ denotes the outward normal and $l$ is on the left of $n$. There is no constraint on $\theta$ so the preceding equation can be satisfied only if $\nabla \cdot \varepsilon = 0$ at each interior point and $\varepsilon = 0$ at each boundary point. In view of (2.1) and the boundary constraint on $\chi$, these conditions are equivalent to

$$\nabla \cdot w \nabla \theta = -\nabla \cdot J \hspace{1cm} \text{and} \hspace{1cm} w \partial \theta / \partial n = -J_n \text{ on all boundaries}, \hspace{1cm} (2.5)$$

i.e., a complete boundary-value problem for $\theta$. The boundary condition in (2.5) ensures that $\theta$ will correctly represent the distribution of energy flux (if any) normal to the boundary. I assume the solution of (2.5) to be single-valued, thereby preventing $\theta$ from contributing to any circulations that might be present in $J$. In particular, $\iint (\partial \theta / \partial \theta) dl = 0$ on each boundary circuit.

Let $\chi_r$ be the uniform value of $\chi$ on the $r$th boundary circuit ($r = 0$ for the mainland circuit, $r = 1$ to $N$ for the others). Since (2.1) is unaffected by adding to $\chi$ any value uniform over the whole domain (the same is true of $\theta$), we can without loss of generality set $\chi_0 = 0$, thus leaving $\chi_r$ to be determined for each of the $N$ islands. We now take the variation of $Q$ with respect to $\chi$ alone with the result

$$0 = \int w^{-1} \varepsilon \times k \cdot \nabla \chi dS. \hspace{1cm} (2.6)$$

Again assuming the individual terms of (2.1) continuous and differentiable, we find by partial integration

$$0 = \sum_{r=1}^{N} \Gamma_r \delta \chi_r - \iint (\nabla \cdot w^{-1} \varepsilon \times k) \delta \chi dS,$$
where
\[ \Gamma_r = \int_{(r)} w^{-1} \epsilon dl. \]
This implies that \( \nabla \cdot w^{-1} \epsilon \times k = 0 \) at each interior point, since there is no interior constraint on \( \chi \). Further, there is no constraint on any of the uniform island values \( \chi_r \), so \( \Gamma_r = 0 \) for each island \( (r = 1 \text{ to } N) \). Therefore the boundary-value problem for \( \chi \) is
\[
\begin{align*}
\nabla \cdot w^{-1} \nabla \chi &= \nabla \cdot w^{-1} J \times k \\
\int_{(r)} w^{-1} \frac{\partial \chi}{\partial n} dl &= \int_{(r)} w^{-1} J_d dl \quad (r = 1 \text{ to } N)
\end{align*}
\] (2.7)

The boundary condition ensures that \( \chi \) will correctly represent the "circulation" of energy flux on each island boundary. It is also satisfied on the mainland boundary because integration of \( \nabla \cdot w^{-1} \epsilon \times k = 0 \) over the domain gives
\[ \Gamma_0 + \sum_{r=1}^{N} \Gamma_r = 0, \]
so the island conditions \( \Gamma_r = 0 \) jointly make \( \Gamma_0 = 0 \).

The boundary conditions in (2.5) and (2.7) are natural conditions for the variational problem (2.2): they must be satisfied in order to allow \( Q \) to attain its minimum value, and thus are \textit{a posteriori} rather than \textit{a priori} conditions. The minimum \( Q \) can be calculated from
\[ Q = \int w^{-1} J_d ds - \int w(\nabla \theta)^2 ds - \int w^{-1}(\nabla \chi)^2 ds, \quad (2.8) \]

after \( \theta \) and \( \chi \) have been found from (2.5) and (2.7). In the continuum (rather than for discrete approximation), the Stokes/Helmholtz potentials are capable of representing an arbitrary vector field so we should expect \( \epsilon = 0 \) and hence \( Q = 0 \) in (2.8).

It should also be remarked that problems (2.5) and (2.7) are completely decoupled. This is a consequence of two features of the variational problem (2.2): the weight factor \( w^{-1} \) and the boundary constraint on \( \chi \), which jointly cause the \( \theta \) term in (2.1) to be orthogonal to the \( \chi \)-term, i.e.,
\[ \int w^{-1}(w \nabla \theta) \cdot (k \times \nabla \chi) dS = 0. \] (2.9)

Moreover, since the \( \theta \) term and \( \chi \) term are individually orthogonal to \( \epsilon \) [according to (2.4) and (2.6)], we see that the three parts of \( J \) in (2.1) are mutually orthogonal. For the purpose of determining \( \chi \) we can therefore regard the \( \theta \) term as part of an enlarged residual. Likewise, for determining \( \theta \) we can include the \( \chi \) term with the residual.

The boundary condition gives (2.7) the appearance of an integro-differential problem of Neumann type but, in fact, Eq. (2.7) is equivalent to \( N + 1 \) independent differential problems because there are just \( N \) degrees of freedom in the boundary values of \( \chi \). This was pointed out by Kamenovich (1961) in connection with determination of the volume-transport streamfunction for steady wind currents in a homogeneous ocean, which leads to a problem formally identical to (2.7). Each of the differential problems is a Dirichlet problem and the solution of (2.7) is expressed as a linear combination of these. The boundary values \( \chi_r (r = 1 \text{ to } N) \) are then obtained by solving \( N \) linear simultaneous algebraic equations. The same method was used for the same purpose by Gedney and Lick (1972).

Coming to the finite-element model we find that only one part of the foregoing analysis must be changed, namely, the transitions from (2.4) to (2.5) and (2.6) to (2.7), which require continuity and differentiability of the individual terms in (2.1). In the finite-element model \( \theta \) and \( \chi \) are discretized on nodes consisting of the vertices of a grid of triangles and are assumed to vary linearly within each triangle. The calculated energy flux \( J \) and the individual terms on the right in (2.1) are uniform within each triangular grid element, and therefore are generally discontinuous at the junctions between elements.

In these circumstances we must return to (2.4) and (2.6), which are still valid statements of the consequences of (2.2). Taking (2.4) first, we give to \( \theta \) the finite-element representation
\[ \theta(t, P) = \sum_j \theta_j(t) \beta_j(P). \] (2.10)

Here \( P \) is any point in the domain, \( \theta_j \) is the value of \( \theta \) at node \( j \), and \( \beta_j \) are prescribed finite-element basis functions. The sum is over all nodes of the grid except one. [Omission of one node corresponds to the fact that (2.1) is not affected by adding to \( \theta \) any value uniform over the whole grid, so without loss of generality we can set \( \theta = 0 \) at any one point.] The variation \( \delta \theta \) is now restricted to the function obtained by replacing \( \theta_j \) in (2.10) with \( \delta \theta_j \). Since all of the \( \delta \theta_j \) are arbitrary, we find that an equation of the form (2.4) must be satisfied for each node (except the omitted one), but with \( \delta \theta \) replaced by \( \delta \beta \). With the help of (2.1) this leads to
\[ \int w \nabla \theta \cdot \nabla \beta ds = - \int J \cdot \nabla \beta ds. \] (2.11)

On substitution of (2.10) for \( \theta \) in (2.11), we obtain a set of inhomogeneous linear simultaneous equations for the nodal values \( \theta_j \). The coefficient matrix of this system is symmetric and positive definite.

Eq. (2.11) is the finite-element analogue of (2.5). Indeed, it can be shown, in the limit of zero discretization interval, that (2.11) coalesces with the differential equation in (2.5) when \( j \) is an interior
node, and with the boundary condition when \( j \) is a boundary node, provided \( \nabla \theta \) and \( J \) become continuous and differentiable in the limit.

Turning to (2.6) we can follow a similar procedure, starting from the finite-element representation

\[
\chi(t, P) = \sum_j \chi_j(t) \gamma_j(P).
\]

The sum is over all nodes of the grid except those on the mainland boundary (since we have assigned \( \chi_j = 0 \) there). If \( j \) is any node on island boundary circuit \( r \), we have \( \chi_j = \hat{\chi}_r \) so the preceding representation can be written

\[
\chi(t, P) = \sum_j' \chi_j(t) \gamma_j(P) + \sum_{r=1}^N \hat{\chi}_r(t) \tilde{\gamma}_r(P),
\]

where \( \sum' \) ranges over interior nodes only and \( \sum^r \) over the nodes of island boundary circuit \( r (= 1 \text{ to } N) \). The basis \( \gamma_j(P) \) differs from zero only in the grid elements that contain node \( j \) and thus has "local support" at a single point. The basis \( \tilde{\gamma}_r(P) \) differs from zero in a boundary strip consisting of all elements with at least one node on circuit \( r \) and thus has local support along a line (Fig. 1).

The variation \( \delta \chi \) is now restricted to the expression obtained by replacement of \( \chi_j \) and \( \hat{\chi}_r \) in (2.12) with \( \delta \chi_j \) and \( \delta \hat{\chi}_r \). On substitution of this expression into (2.6), and in view of the fact that all the \( \delta \chi_j \) and \( \delta \hat{\chi}_r \) are arbitrary, we find that an equation of the form (2.6) must be satisfied for each interior node and each island boundary circuit, but with \( \delta \chi \) replaced by \( \gamma \) for the former and by \( \tilde{\gamma} \) for the latter. With the help of (2.1) this leads to

\[
\begin{align*}
\int w^{-1} \nabla \chi \cdot \nabla \gamma' dS = & \int w^{-1} J \times k \cdot \nabla \gamma dS \\
\int w^{-1} \nabla \hat{\chi} \cdot \tilde{\gamma}' dS = & \int w^{-1} J \times k \cdot \nabla \hat{\chi} \tilde{\gamma} dS
\end{align*}
\]

where \( j \) ranges over the interior nodes and \( r \) over the island boundaries. These two equations are respectively the finite-element analogues of the two in (2.7). They approach (2.7) in the limit of zero discretization interval, provided \( \nabla \chi \) and \( J \) become continuous and differentiable in the limit.

On substitution of expansion (2.12) for \( \chi \) in (2.13), we obtain a set of inhomogeneous linear simultaneous equations for joint determination of all the interior \( \chi_j \) and boundary \( \hat{\chi}_r \).

3. Volume-flux streamfunction

My colleague Joseph Pedlosky, on hearing me comment about the problem of boundary conditions for the energy-flux streamfunction, called my attention to the work of McWilliams (1977) and of Flierl (1977) on formulation of boundary conditions for dynamical equations governing quasi-geostrophic flow in multiply-connected domains. This conversation brought to light another consequence of my having given no consideration to connectivity in formulating the finite-element model, namely, the effect of connectivity on the volume-flux streamfunction. This question had been put to me earlier by another colleague, Mitchell Luskin, but I responded to it with the mistaken belief that, in contrast with wind-driven circulations (for example), the normal-mode problem was not affected by connectivity because it concerns only linear waves in which there is no net volume transport. The work of McWilliams and of Flierl cited above makes it clear that even though there is no net volume transport, boundary values of volume-flux streamfunction are nevertheless affected by connectivity.

Proper treatment of the volume-flux streamfunction \( \psi \) is a more serious matter than that of the energy-flux streamfunction because the former is one of the governing dependent variables while the latter is a diagnostic quantity. As previously mentioned, the earlier model of a domain consisting of the Atlantic and Indian Oceans was simply connected. The boundary condition \( \psi = 0 \) imposed in that model was correct for that reason. However, in preliminary calculations for the world-ocean domain described above, with quadruple connectivity, I used the same condition \( \psi = 0 \) on all

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**Fig. 1.** South-polar stereographic projection of a portion of the finite-element grid used for calculation of normal modes of the world ocean. Boundary line bases are shown for Antarctica and New Zealand.
boundaries. While this leads to a problem that is well-posed mathematically, it gives a solution in general inconsistent with dynamical conditions on the circulation in individual boundary circuits.

Reference to the minimization problem (2.2) will help to illustrate this point. If in (2.2) the constraint on \( \chi \) were replaced with the stronger one that \( \chi = 0 \) on all boundaries, the problem would still be well-posed but the minimum value of \( Q \) would not be as small as that attained with the weaker constraint of (2.2). The problem (2.5) for \( \theta \) would be the same in the two cases but in (2.7) for \( \chi \), the boundary condition would be simply \( \chi = 0 \), and this would produce a different interior \( \chi \)—one that would not in general satisfy the energy-circulation boundary condition of (2.7).

Formally, the boundary problem for \( \psi \) is identical to that for \( \chi \), so the same remedy can be used in both cases. The analogy can be seen by writing the depth-integrated-momentum equation

\[
M = -\mu + \partial w/\partial t. \tag{3.1}
\]

Here \( M \) is \( h \) times the sum of all forces (per unit mass) and \( -\mu \) the residual for a particular determination of \( \partial w/\partial t \). The volume transport \( u \) is represented by Stokes/Helmholtz potentials

\[
u = -h\nabla \varphi + k \times \nabla \psi, \tag{3.2}
\]

where, in order to orthogonalize the \( \varphi \) and \( \psi \) terms, \( \psi \) is constrained to be uniform on each boundary, and in particular, \( \psi = 0 \) on the mainland boundary.

The residual appears in (3.1) rather than (3.2) because the dynamical problem consists in seeking a distribution of \( \partial w/\partial t \) that approximates an assigned distribution of \( M \). The representation (3.2) is regarded as a definition of \( u \); thus, in seeking \( \partial w/\partial t \) we are in effect seeking \( \partial \psi/\partial t \) and \( \partial \psi/\partial t \). By combining (3.1) and (3.2) we have

\[
M = -\mu - h\nabla \frac{\partial \varphi}{\partial t} + k \times \nabla \frac{\partial \psi}{\partial t} \tag{3.3}
\]

and this is strictly analogous to (2.1).

In the design recently published (Platzman, 1978) the constraints on \( \mu \) were formulated from energy considerations rather than on the basis of least squares. They are given by Eqs. (4.8b) and (4.8c) of that publication, which are equivalent to

\[
0 = \int_{-\infty}^{\infty} h^{-1} \mu \cdot \delta \mu dS + \int_{\Omega} h^{-1} \kappa \cdot \delta \mu dl. \tag{3.4}
\]

The boundary integral here is taken over those portions of the boundary (if any) where the surface elevation (rather than the volume transport) is prescribed. The boundary residual \( \kappa = h(\zeta' - \zeta)n \) causes the actual elevation \( \zeta' \) to approximate the prescribed value \( \zeta \) as a natural boundary condition. If there are no such "ports" in the boundary, (3.4) does not have the boundary integral and is identical to the variational condition for least squares.

Assuming there are no ports, the treatment of \( \partial \psi/\partial t \) is exactly analogous to that of \( \theta \) in the preceding section. In particular, the continuum equations are exactly of the form (2.5) and the finite-element equations exactly of the form (2.11), but in both cases with \( \theta \) replaced by \( \partial \psi/\partial t \) and \( J \) by \( M \) (also \( w \) by \( h \)). Similarly, the treatment of \( \partial \psi/\partial t \) is analogous to that of \( \chi \) in the preceding section.

The continuum equations are exactly of the form (2.7) and the finite-element equations exactly of the form (2.13), but in both cases with \( \chi \) replaced by \( \partial \psi/\partial t \) and \( J \) by \( M \) (also \( w^{-1} \) by \( h^{-1} \)). In particular, the boundary condition for \( \partial \psi/\partial t \) that corresponds to the one in (2.7) is

\[
\frac{\partial}{\partial t} \int_{\Omega} h^{-1} \frac{\partial \psi}{\partial n} dl = \int_{\Omega} h^{-1} M dl.
\]

This is the desired dynamical condition on the circulation (of velocity) in each island boundary circuit. It is a natural condition for the variational statement (3.4) (subject to the constraint that \( \psi \) is uniform on each boundary). It is satisfied automatically on the mainland boundary for the same reasons as given below (2.7).

In summary it will be useful to point out what changes are needed in the published finite-element design (Platzman, 1978) in order to provide for multiple connectivity. The matter can be stated quite simply: no formal change is needed in any part of the design. It suffices to give the volume-flux streamfunction a finite-element representation that includes an unassigned but uniform value on each island boundary, and a corresponding line basis to represent each such value.

4. Illustration

As an illustration of the effect of multiple connectivity on a computed normal mode I refer to the wave described recently (Platzman, 1979) as a mode that might make a significant contribution to the \( \mathbf{M}_o \) tide in the eastern North Pacific.

The upper map in Fig. 2 shows energy-flux streamlines calculated improperly by setting the volume-flux and energy-flux streamfunctions to zero on all boundaries, and thus without allowance for multiple connectivity. The lower map is the result obtained using proper boundary conditions on the coasts of Antarctica, Madagascar and New Zealand, by means of the procedure described in this note, and thus with allowance for multiple connectivity. The periods in the two cases are respectively 15.59 and 15.52 hours in the eastern Pacific there is a slight difference between the two maps near the center of the energy gyre, but the intensity of the gyre is
virtually the same in the two cases (4.2 vs 4.1 \times 10^{11}\text{W}).

The main difference between the improper and proper solutions is in the Southern Ocean, where the improper solution does not allow the westward flow of energy along the Antarctic coast that must accompany the Kelvin-like wave present there (Fig. 3). This wave has a hemispheric wavenumber of 2. Paradoxically, the elevation is very nearly the same in the improper and proper solutions, so presumably the governing variables in the two solutions differ mainly in the volume-flux streamfunction.

Fig. 4 shows that when proper boundary conditions are used (lower map), the energy transported by the Kelvin wave in the Southern Ocean is revealed. The total flux is about $2.2 \times 10^{11}\text{W}$ (the magnitude of the energy-flux streamfunction on the Antarctic coast). In an ocean of depth 4000 m at latitude 60° this is consistent with the energy flux $\frac{1}{4} \rho g h f^{-1} \zeta^2$ of an ideal Kelvin wave of amplitude 54 cm, in rough agreement with the lower map of Fig. 3. It is noteworthy that the wave on the Antarctic coast has about the same intensity as the one on the west coast of North America.

When improper boundary conditions are used
that the energy of the rotational part of the volume flux [the $\psi$ term of (3.2)] is only \( \sim 7\% \) of the total kinetic energy, in both cases. It is likely, therefore, that improper boundary conditions for the volume-flux streamfunction do not prevent the elevation and volume flux from providing for approximately the correct energy flux. Thus, the difference between the upper and lower maps in Fig. 4 can be ascribed primarily to improper boundary conditions for the energy-flux streamfunction—a purely diagnostic problem. This interpretation is plausible for

(upper map, Fig. 4) there is virtually no flow of energy in the Southern Ocean, notwithstanding the propagation of phase (upper map, Fig. 3). That anomaly is what made me aware of the problem discussed in this note.

An interpretation of the foregoing comparisons is suggested by the fact that the elevation is about the same in the two solutions (Fig. 3), and the fact

**Fig. 3.** South polar stereographic projection of amplitude and phase of the same mode as shown in Fig. 2. Amplitude contours (dashed lines) are drawn at intervals of 10 cm, phase contours (solid lines) at intervals of 30°. An arrowhead is attached to the line of zero phase and points in the direction of propagation.

**Fig. 4.** South polar stereographic projection of energy-flux streamfunction (unit and interval \( 10^{11} \text{W} \)) for the same mode as shown in Fig. 2.
a gravity wave but should not be expected to apply to a vorticity wave.

Finally, in Table 1 are listed values of volume-flux and energy-flux streamfunctions on the three “islands” of the finite-element model, for the 15.52 h mode shown in the lower parts of Figs. 2–4. The amplitude of the mode is scaled to 50 cm at La Jolla.

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<table>
<thead>
<tr>
<th></th>
<th>Magnitude of volume-flux streamfunction (10^6 m^3 s^-1)</th>
<th>Magnitude of energy-flux streamfunction (10^6 W)</th>
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<tr>
<td>Madagascar</td>
<td>2.5</td>
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<tr>
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<td>Maximum in domain</td>
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