Wind drift in a homogeneous equilibrium sea

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ABSTRACT: A simple dynamical model is proposed for the near-surface drift current in a homogeneous, equilibrium sea. The momentum balance is formulated for a mass-weighted mean in curvilinear surface-conforming coordinates. Stokes drifts computed analytically for small wave slopes by this approach for inviscid linear sinusoidal and Pollard-Gerstner waves agree with the corresponding Lagrangian means, consistent with a mean momentum balance that determines mean parcel motion. A wave-modified mixing length model is proposed, with a depth-dependent eddy viscosity that depends on an effective ocean surface roughness length $z_{0o}$, distinct from the atmospheric bulk-flux roughness length $z_{0r}$, and additional wave-correction factor $\phi_w$. Kinematic Stokes drift profiles are computed for two sets of quasi-equilibrium sea states and are interpreted as mean wind drift profiles to provide calibration references for the model. A third calibration reference, for surface drift only, is provided by the traditional 3%-of-wind rule. For 10-m neutral wind $U_{10N} \leq 20$ m s$^{-1}$, the empirical $z_{0o}$ ranges from $10^{-4}$ m to 10 m, while $\phi_w$ ranges from 1.0 to 0.1. The model profiles show a shallow log-layer structure at the smaller wind speeds and a nearly uniform near-surface shear at the larger wind speeds. Surface velocities are oriented 10°-20° from downwind for $U_{10N} \leq 10$ m s$^{-1}$ and 20°-35° from downwind for $10 \leq U_{10N} \leq 20$ m s$^{-1}$. A small correction to the drag coefficient is implied. The model predictions show a basic consistency with several sets of previously published near-surface current measurements but the comparison is not definitive.
1. Introduction

When the wind blows over the sea surface, the surface of the ocean is set in motion. This motion involves the interaction and coupling of two different fluids, the distortion of the interface by spontaneously generated waves with a wide range of scales, and momentum transport by irregular, nonlinear wave breaking and turbulence. The associated physical problem of understanding and quantifying these various dynamical processes and the resulting mean horizontal near-surface motion has been an object of focused study for over a century and remains an area of active research (see, e.g., Ekman 1905; Miles 1957; Phillips 1957, 1977; Rapp and Melville 1990; Xu and Bowen 1994; Anis and Moum 1995; Melville 1996; Banner and Peirson 1998; McWilliams and Restrepo 1999; Greenan et al. 2001; Sullivan and McWilliams 2010; Sutherland and Melville 2015b; Pizzo et al. 2016; Grare et al. 2018 and numerous related studies). The determination of the mean motion – the near-surface wind and wave drift – and its dependence on the wind and wave state is of interest both for intrinsic scientific reasons and because of its practical importance for understanding and predicting the motion of oil spills and other floating materials or objects (e.g., Ardhuin et al. 2009; Rascle and Ardhuin 2009; Zelenke et al. 2012; Breivik et al. 2016; Ardhuin et al. 2017; Morey et al. 2018; Rodriguez et al. 2018, 2019; Pizzo et al. 2019; Laxague and Zappa 2020). A basic model of the near-surface neutral atmospheric boundary layer has been available for decades (e.g., Taylor 1915; Ellison 1956; Blackadar and Tennekes 1968; Garratt 1992) but there is no analogous, broadly accepted description of the upper few meters of the ocean surface boundary layer.

The idealized case considered here is that of a homogeneous, equilibrium sea. The most general meaning of homogeneous is implied, so the ocean is taken to have uniform density and neutral stability. The equilibrium condition means, among other things, that the surface wave field is in statistical equilibrium with the wind, so that mean wave amplitudes are independent of time as well as space and there are no horizontal divergences of wave momentum fluxes. Further, there are no temporal or horizontal gradients of any fields – except, of course, the large-scale pressure gradients associated with the geostrophic wind and current – and the only mean divergence of momentum flux is that from the vertical gradient of the vertical flux of horizontal momentum. The form of the associated momentum balance, with Coriolis force balancing vertical stress divergence, is familiar, as this approach either, explicitly or implicitly, has been standard since Ekman (1905). Nonetheless,
it has not been clear whether or how wave motions are to be included in this mean momentum balance. This question is especially important for the mean velocity in the near-surface, wave-affected layer. Special attention is therefore paid to the specification of the appropriate averaging procedure.

Means are computed with respect to a mass-weighted spatial average in a curvilinear coordinate system that conforms to the instantaneous sea surface (Section 2.a). The resulting mean momentum balance still takes the familiar classical form, as this form follows from general arguments – essentially the translation invariance of the mean state – that do not require explicit consideration of the time-dependent curvilinear coordinate system (Section 2.b). However, the averaging procedure implies a specific interpretation of the mean horizontal velocity and stress divergence. Further, it is shown analytically that the mean inviscid wave-drift computed by this method agrees with the classical Stokes (1847) Lagrangian mean – that is, the mean captures the Stokes drift – for linear sinusoidal and weakly nonlinear Pollard-Gerstner waves (Sections 2.c,d).

These computed wave-drifts are consistent with the assertion that this average yields a mean momentum balance that incorporates the total mean parcel motion, including wave effects. For example, if linear sinusoidal waves are present in the state over which the mean is taken, the Coriolis force induced by the associated Stokes drift (the so-called “Stokes-Coriolis” terms; e.g., Hasselmann 1970) will be captured in the mean momentum balance. In this context, it is worth recalling that the fixed-depth, Eulerian mean does capture the mean momentum of the linear sinusoidal wave but places all the momentum above trough level. In contrast, the mass-weighted, surface-conforming spatial mean distributes the mean momentum vertically in the same way as the classical Lagrangian mean.

The proposed model for the unknown vertical flux of horizontal momentum in the homogeneous-equilibrium setting is described in Section 3. The model resembles a classical mixing-length eddy viscosity but depends on both an ocean surface roughness length \( z_{0o} \) – which is distinct from the atmospheric roughness length appearing in bulk-stress algorithms such as that of Edson et al. (2013) – and a novel wave-correction factor \( \phi_w \). The wave-correction factor distinguishes it from previous, related models such as those of Madsen (1977), Weber (1983), Bye (1988), Jenkins (1989) and Bourassa (2000). The model can be viewed as a simplified version of the mixing-length eddy-viscosity models explored by Craig and Banner (1994) and Rasche et al. (2006), which
were formulated instead for general wind-wave conditions and included an independent evolution equation for a turbulent kinetic energy quantity. The latter is effectively replaced here by the factor $\phi_w$, which along with $z_{0w}$ must be determined empirically as a function of wind speed.

An initial attempt is made here to calibrate the wind-speed dependence of these empirical constants by fitting the resulting model velocity profiles to theoretical Stokes-drift profiles for wind-dependent equilibrium wave states, supplemented by the 3%-of-wind rule (e.g., Keulegan 1951; Van Dorn 1953) for the surface current. The rationale and background for this approach is explained in Section 4. The calibration gives a nominal model of near-surface wind-drift that can be paired with a bulk-stress algorithm such as that of Edson et al. (2013) to provide a nominal wind-dependent description of the near-surface mean flow. It is emphasized that this calibration is necessarily to be regarded as preliminary and tentative, given the indirect method for estimation of the “observed” relation between wind and near-surface currents. A preliminary comparison, in Section 5, of the predicted mean flow from the nominal calibrated model with mean near-surface currents from several existing observational datasets nonetheless shows a basic consistency of the model with the observations.

Implicit in the approach is the assumption that the homogeneous, equilibrium state is a useful approximation to typical open ocean conditions. This assumption is supported by observations of frequent approximate wind-wave equilibrium (Thomson et al. 2013; Vincent et al. 2019) but, in some cases, is clearly violated. Essentially the same assumption is made in standard bulk flux parameterizations of air-sea momentum flux (e.g., Garratt 1992; Edson et al. 2013). From this point of view, the present study could be seen as having the goal of extending the near-surface atmospheric wind and stress profiles that are implicit in bulk-stress algorithms through the uppermost few meters of the water column, to obtain matching mean near-surface ocean current and stress profiles that depend in an analogous way upon the 10-m wind.

2. Formulation: mean momentum and wave-drift

a. Mass-weighted curvilinear-coordinate mean

Consider the problem of averaging beneath a moving interface in a homogeneous, equilibrium sea. Far from the interface, horizontal averaging could be done in standard Cartesian coordinates, after which all horizontal gradients in the resulting averaged momentum equations would vanish.
except the uniform, constant, large-scale pressure gradient associated with the geostrophic current. For the near-surface problem, however, this averaging would result in a smearing of the interface across a vertical transition zone defined by the amplitude of the free-surface displacements.

To preserve the interface in the averaged equations, the averaging may be done instead on surfaces of constant \( \eta \) in time-dependent curvilinear coordinates \( (\xi_1, \xi_2, \eta) \), where \( \eta(x, y, t) \) is a pseudo-vertical coordinate with \( \eta = 0 \) at the interface (Phillips 1977). The corresponding spatial average \( \langle q \rangle \) of a quantity \( q(x, y, z, t) \) on the surface \( \eta = \eta_0 \) may be defined as

\[
\langle q \rangle(\eta_0, t) = \frac{1}{A_{\eta_0}} \int_{L_z} \int_{L_y} \int_{L_x} q(x, y, z, t) \delta(\eta - \eta_0) \, dx \, dy \, dz
\]

\[
= \frac{1}{A_{\eta_0}} \int_{L_{\xi_2}} \int_{L_{\xi_1}} \hat{q}(\xi_1, \xi_2, \eta_0, t) J(X, Y, Z)|_{\eta=\eta_0} \, d\xi_1 \, d\xi_2
\]  

(1)

where \( \delta(\eta) \) is the Dirac delta distribution, \( J(X, Y, Z) \) is the Jacobian of the transformation \( (x, y, z) = [X(\xi_1, \xi_2, \eta, t), Y(\xi_1, \xi_2, \eta, t), Z(\xi_1, \xi_2, \eta, t)] \), and the spatial scales \( L_x \) and \( L_y \) either correspond to periodicities of the field or are taken suitably large, such that the average converges. In (1),

\[
\hat{q}(\xi_1, \xi_2, \eta, t) = q[X(\xi_1, \xi_2, \eta, t), Y(\xi_1, \xi_2, \eta, t), Z(\xi_1, \xi_2, \eta, t), t]
\]

(2)

is the quantity \( q \) written as a function of the curvilinear coordinates, and

\[
A_{\eta_0} = \int_{L_z} \int_{L_y} \int_{L_x} 1 \cdot \delta(\eta - \eta_0) \, dx \, dy \, dz = \int_{L_{\xi_2}} \int_{L_{\xi_1}} J(X, Y, Z)|_{\eta=\eta_0} \, d\xi_1 \, d\xi_2
\]

(3)

is defined so that \( \langle 1 \rangle = 1 \). The transformation to curvilinear coordinates is necessary in order that the Dirac delta – defined by \( \int F(\eta) \delta(\eta) \, d\eta = F(0) \) for any function \( F \) – can be properly interpreted.

Under the equilibrium assumption, the spatial mean \( \langle q \rangle(\eta, t) \) from (1) may be further averaged over time, to obtain the spatio-temporal average \( Q(\eta) \),

\[
Q(\eta) = \frac{1}{T} \int_T \langle q \rangle(\eta, t) \, dt,
\]

(4)

where the timescale \( T \) either corresponds to a temporal periodicity of the field or is taken suitably large, so that the average converges. The spatio-temporal average \( Q \) computed by (1)-(4) is a function only of the pseudo-vertical curvilinear coordinate variable \( \eta \).
For the homogeneous fluid, (1) is a mass-weighted average in \((\xi_1, \xi_2, \eta)\)-space, with \(A_{\eta_0}\) being the mass-weighted area (3). The integral of (1) over \(\eta_0\) recovers the volume integral,

\[
\int_{L_q} A_\eta \langle q \rangle (\eta, t) \, d\eta = \int_{L_\xi} \int_{L_\xi_2} \int_{L_{\xi_1}} \hat{q} \, d\xi_1 \, d\xi_2 \, d\eta = \int_{L_x} \int_{L_y} \int_{L_z} q \, dx \, dy \, dz,
\]

so the mean captures the instantaneous total horizontal momentum if \(q\) is the horizontal velocity. Technically, the use of \(\eta\)-surfaces requires that the interface be at least smooth and single-valued but it is assumed here that the resulting averaged description relative to a well-defined interface is still valid under general conditions, including, for example, the presence of breaking waves. Even in relatively high sea states, breaking waves cover only a small fraction of the sea surface, so this assumption regarding the geometry of the interface is plausibly defensible.

Because the average (1) is taken over space at a fixed time, there is no distinction between Eulerian and Lagrangian means: the quantity \(q\) in (1) may be regarded as either an Eulerian or a Lagrangian property. The geometry of the surfaces \(\eta = \eta_0\) is fixed by the instantaneous free surface, which serves as an effective domain boundary, but (1) refers only to Eulerian coordinates. In Sections 2.c and 2.d, two explicit examples of this averaging are given, for the simpler but related case of surface gravity waves. In particular, it is shown for these examples that the mass-weighted curvilinear-coordinate mean (1) recovers the classical Stokes drift motion, illustrating that this spatial mean can capture the total mean horizontal momentum associated with the total mean parcel motion even in the presence of waves.

In contrast to more restrictive approaches such as that of Andrews and McIntyre (1978), which require that the fluctuating motion be assumed wave-like, the mean (1) allows any combination of wave and turbulent fluctuations and involves no separation of Eulerian and Lagrangian motions. For small surface slopes,

\[
J(X,Y,Z) \approx \frac{\partial Z}{\partial \eta} \frac{\partial X}{\partial \xi_1} \frac{\partial Y}{\partial \xi_2},
\]

and (1) can be written

\[
\langle q \rangle (\eta_0, t) = \frac{1}{A_{\eta_0}} \int_{L_x} \int_{L_y} \tilde{q}(x,y,\eta_0, t) \frac{\partial Z}{\partial \eta} \, dx \, dy,
\]
which is a thickness-weighted average closely related to that of Aiki and Greatbatch (2012) and, more generally, de Szoeeke and Bennett (1993).

\[ \text{b. Mean momentum balance} \]

For the homogeneous equilibrium sea, the only mean divergence of momentum flux in the mean horizontal momentum equations must be the vertical gradient of a vertical flux of horizontal momentum. Denote the effective mean stress divergence, computed by the mass-weighted mean (1)-(4), by \( S_\tau(\eta) \), and the corresponding means of the horizontal velocity and the Cartesian vertical coordinate by \( \bar{U}(\eta) \) and \( \bar{Z}(\eta) \), respectively. An effective mean stress \( \tau(\eta) \) may be defined by integrating \( S_\tau \) with respect to \( \bar{Z} \):

\[
\tau(\eta) = \tau_0 + \int_{\bar{Z}_0}^{\bar{Z}} S_\tau \, d\bar{Z} = \tau_0 + \int_0^{\eta} S_\tau(\eta) \frac{d\bar{Z}}{d\eta} \, d\eta, \tag{8}
\]

where \( \tau_0 \) is the mean wind stress (here taken positive upwards) and \( \bar{Z}_0 = \bar{Z}(\eta = 0) \). Note that \( S_\tau \) will be dominated by the mean vertical stress divergence but may have contributions of order wave-slope squared from horizontal gradient or divergence terms that would vanish in spatial average if \( J \) were constant.

If the symbol \( z \) is now redefined to denote \( \bar{Z}(\eta) - \bar{Z}_0 \) rather than the original Cartesian vertical coordinate, then the corresponding mass-weighted mean horizontal momentum equations have a classical appearance:

\[
\rho_o f \mathbf{k} \times \bar{U}(\eta) = -G_o - \frac{d\tau}{dz}(\eta). \tag{9}
\]

In (9), the constants \( \rho_o \) and \( f \) are the seawater density and the Coriolis parameter, respectively, \( \mathbf{k} \) is the upward unit vector, and \( G_o \) is a constant large-scale pressure gradient. It is assumed that the horizontal velocity is geostrophic far from the interface,

\[
\bar{U} \rightarrow \bar{U}_G = \frac{1}{\rho_o f} \mathbf{k} \times \mathbf{G}_o \quad \text{as} \quad z \rightarrow -\infty. \tag{10}
\]

The deviation of the mean velocity from the geostrophic mean velocity,

\[
\mathbf{U} = \bar{\mathbf{U}} - \bar{\mathbf{U}}_G, \tag{11}
\]
then satisfies
\[
\rho_0 f \mathbf{k} \times \mathbf{U} = -\frac{d\tau}{dz}.
\] (12)

It is clear from (12) that, under the assumed homogeneous, equilibrium conditions, the problem of determining the vertical profile of the mass-weighted mean horizontal velocity – the mean horizontal momentum – is equivalent to the problem of determining the vertical profile of the vertical divergence of effective mean stress, \(d\tau(\eta)/dz\). Note that the redistribution of momentum by waves, including breaking waves (e.g., Pizzo et al. 2019), is explicitly incorporated into \(\tau(\eta)\), as no distinction is made between wave and turbulent fluctuations.

c. Mean drift: linear sinusoidal waves

The main subject of this study is the mean forced velocity response of the near-surface ocean to steady winds. However, it is useful at this point to derive surface-relative averages of two free-wave velocity fields, to provide explicit analytical examples of the method outlined in Section 2.a. Consider a linear deep-water wave with surface displacement \(\zeta\), where
\[
\zeta(x,t) = a \cos(kx - \sigma t),
\] (13)
so that the wave is propagating in the \(x\)-direction with phase speed \(c_p = \sigma/k = (g/k)^{1/2}\) and has the associated horizontal velocity field
\[
u(x,z,t) = \sigma ae^{kz} \cos(kx - \sigma t).
\] (14)

Here \(a, \sigma,\) and \(k\) are wave amplitude, frequency, and wavenumber, respectively, and \(g\) is the acceleration of gravity, while (14) is valid for \(z \leq \zeta\). Assume further that (13)-(14) are accurate to second order in the dimensionless wave slope \(ka\), i.e., the errors in (13)-(14) are of order \((ka)^3\).

The mean (1) may be computed to obtain the mass-weighted mean horizontal surface velocity for the linear sinusoidal wave on surfaces of constant \(\eta\). At time \(t = 0\), or in a frame translating
with the wave, the associated orthogonal curvilinear coordinates are (Phillips 1977)

\[ \xi(x, z) = x - ae^{kz} \sin kx, \quad (15) \]

\[ \eta(x, z) = z - ae^{kz} \cos kx. \quad (16) \]

The transformation (15)-(16) may be inverted to the same accuracy, giving

\[ X(\xi, \eta) = \xi + ae^{\eta} \sin k\xi, \quad (17) \]

\[ Z(\xi, \eta) = \eta + ae^{\eta} \cos k\xi, \quad (18) \]

so that

\[ \hat{u}(\xi, \eta) = u(X, Z) = \sigma a \left( e^{\eta} \cos k\xi - ka_1 e^{2\eta} \cos 2k\xi \right), \quad (19) \]

again to second order in wave slope. To the same order, the mean horizontal velocity on an arbitrary \( \eta \) surface is then

\[ U_s(\eta) = \frac{k}{2\pi} \int_0^{2\pi/k} \hat{u}(\xi; \eta) J(X, Z) \, d\xi = \sigma kao^2 e^{2\eta}, \quad (20) \]

where to first order in wave slope,

\[ J(X, Z) = 1 - 2ka_1 e^{\eta} \cos k\xi, \quad (21) \]

so that \( A_{\eta_0} = 2\pi/k \). The mean depth \( \tilde{Z}(\eta) = \langle Z \rangle(\eta) \) of a given \( \eta \) surface is, similarly,

\[ \tilde{Z}(\eta) = \frac{k}{2\pi} \int_0^{2\pi/k} \tilde{Z}(\xi; \eta) J(X, Z) \, d\xi = \eta + ka^2 \left( e^{2\eta} - 1 \right), \quad (22) \]

so that \( \tilde{Z} = \eta \) to first order.

The mean horizontal velocity profile as a function of mean depth is then

\[ U_s(\tilde{Z}) = \sigma ka^2 e^{2k\tilde{Z}}, \quad (23) \]

again accurate to second order in wave slope. The wave-mean horizontal velocity computed by the mass-weighted Eulerian mean (1) is therefore equal to the classical Lagrangian mean or Stokes
drift (e.g., Phillips 1977; van den Bremer and Breivik 2018). Note that if the Jacobian $J$ in (20) is replaced by $\partial X/\partial \xi$, the resulting mean will be $U_s/2$ (Pollard 1973; Santala and Terray 1992; Smit et al. 2017).

d. Mean drift: Pollard-Gerstner waves

The linear sinusoidal wave was derived for a non-rotating fluid and is a consistent first-order dynamical description of linear ocean surface gravity waves for frequencies $\sigma \gg f$. The second-order mean velocity (23) and the classical Lagrangian Stokes-drift parcel motion, however, are constant in time and thus do not satisfy this frequency constraint. This difficulty was recognized by Ursell (1950), who showed that there could be no such mean drift for inviscid waves on a rotating Earth, and later by Hasselmann (1970). Following on this work, Pollard (1970) presented an exact solution of the Lagrangian equations of motion for a surface gravity wave on a rotating Earth, which for frequencies $\sigma \gg f$ agrees with the non-rotating linear sinusoidal wave (Section 2.c) to first order in the wave slope. In contrast to linear sinusoidal waves, however, and consistent with the arguments of Ursell (1950) and Hasselmann (1970), parcel motions in Pollard waves are exactly periodic and exhibit no mean second-order drift. The Pollard waves are rotating extensions of the non-rotating, exact nonlinear trochoidal-wave solutions of Gerstner (1809), which also have no wave-drift.

The Pollard solution takes the Lagrangian form $x = X^L(x_0, t)$,

\begin{align*}
X_p(x_0, t) &= x_0 - be^{qz_0} \sin(kx_0 - \sigma t), \\
Y_p(x_0, t) &= y_0 + de^{qz_0} \cos(kx_0 - \sigma t), \\
Z_p(x_0, t) &= z_0 + ae^{qz_0} \cos(kx_0 - \sigma t),
\end{align*}

where $x_0 = (x_0, y_0, z_0)$ are the parcel labels and mean parcel positions, the frequency and decay scale are

\begin{equation}
\sigma = \left( gq - f \sigma \frac{q}{k} \right)^{1/2}, \quad q = k \frac{\sigma}{(\sigma^2 - f^2)^{1/2}},
\end{equation}

and the amplitude constants $b$ and $d$ satisfy

\begin{equation}
b = a \frac{q}{k}, \quad d = a \frac{q f}{k \sigma}.
\end{equation}
In (27), \( \dot{f} \) is the horizontal Coriolis parameter. Pressure \( P \) is constant on parcels, \( P(x_0, t) = P(z_0) \), so the free surface may be defined by \( z_0 = 0 \). The free surface deformation is then equivalent to (13).

For \( \sigma \gg f \), the Pollard waves reduce to Gerstner waves, which are (24)-(28) with \( f = \dot{f} = 0 \) and have \( \sigma = (gk)^{1/2} \) and \( q = k \) as for linear sinusoidal waves. The Lagrangian velocity field is then

\[
u_G(x_0, t) = \frac{\partial X_p}{\partial t} = \sigma a e^{kz_0} \cos(kx_0 - \sigma t), \quad v_G(z_0 = 0) = 0. \tag{29}
\]

For small amplitude waves, \( kx_0 \approx kX_p \) and \( kz_0 \approx kZ_p \), and (29) is then likewise equal to (14) to first order in the wave slope. The Gerstner and linear sinusoidal waves are therefore indistinguishable at leading order, for \( \sigma \gg f \) and \( ka \ll 1 \), and their wave coordinates \((\xi, \eta)\) agree with (15)-(16) to first order in wave slope. However, the velocity fields differ at second order. Averaging the Gerstner wave velocity on \( \eta \) surfaces over one wavelength gives

\[
U_G(\eta) = \frac{k}{2\pi} \int_0^{2\pi/k} u_G[x_0(\xi; \eta)] J d\xi = \sigma a \frac{k}{2\pi} \int_0^{2\pi/k} (e^{k\eta} \cos k\xi - 2kae^{2k\eta} \sin^2 k\xi) (1 + 2kae^{k\eta} \cos k\xi) d\xi + O(k^3a^3) = 0 + O(k^3a^3). \tag{30}
\]

Thus, the second-order wave-mean horizontal velocity \( U_G(\eta) \) of the Gerstner wave computed by (1) is zero, as it must be for the exactly periodic parcel motions. Pollard (1970) interpreted this result in terms of the cancellation of the Stokes drift by the second-order mean Eulerian velocity of the Gerstner wave.

A similar, general result can be derived for the mean Pollard wave drift, showing that \( U_P(\eta) = 0 \) for all frequencies \( \sigma \leq f \), as it must. This requires replacing \( e^{kz} \) with \( e^{aq} \) in the averaging coordinates (15)-(16). This illustrates that the mean (1) depends on how the surface-conforming coordinates are chosen.

e. Mean inviscid wave-drifts

For the linear sinusoidal and Pollard-Gerstner wave examples, the Eulerian spatial mean (1) gives the same mean wave-drift velocity for each wave type as the classical Lagrangian mean. These
results are consistent with (5), which shows that the averaging process conserves the total depth-integrated mean momentum. Further, for a random wave field, the mean drift from different waves should sum in the same way as the classical Stokes drift (Kenyon 1969). Thus, it is reasonable to assert that the mean momentum balance (9), interpreted as the result of the mass-weighted, surface-conforming averaging (1)-(4), describes the total mean parcel motions that must result from the total mean forces exerted.

These two examples also illustrate that the mean wave-drift velocity can be different for waves that have identical linear sinusoidal form at leading order. In the case of the Pollard-Gerstner waves, for example, the mean wave-drift vanishes. Although it is not clear that ocean waves generally have the linear sinusoidal form to the required accuracy of second order in wave slope, the classical Stokes drift expression is nonetheless frequently used to estimate surface and near-surface drift velocities from wave states under a variety of conditions and for a variety of purposes (van den Bremer and Breivik 2018). Indeed, in many of these wave states, breaking waves and other evidences of nonlinearity are typical, suggesting that the required linearity to second order may not be satisfied.

The Stokes drift also induces an unbalanced Coriolis force in the linear sinusoidal wave dynamics, leaving the equilibrium theory either inconsistent or incomplete (Ursell 1950). The Pollard (1970) example of rotating Gerstner waves offered one possible resolution of this issue, by showing that exact, weakly nonlinear wave solutions exist that are identical to linear sinusoidal waves at first order but induce no Stokes drift and so also no unbalanced Coriolis force. An alternative resolution, suggested by Hasselmann (1970), is to include the predicted, drift-induced “Stokes-Coriolis” force in a mean momentum balance that is solved independently from the wave momentum balance (e.g., Polton et al. 2005; Rascle et al. 2006; Arduhn et al. 2009). A third approach, which is proposed here, is to assume homogeneous, equilibrium conditions and solve the mean horizontal momentum balance – in a form that by construction captures the total mean parcel motion, including any wave drift – directly.

3. A wave-modified mixing-length model

a. Stress parameterization

The minimal dynamical model of the near-surface, homogeneous-equilibrium wind-drift current proposed here consists of the mean quasi-equilibrium momentum balance (12) supplemented by a
mixing-length parameterization of the effective turbulent stress \( \tau \) (8) that depends on two empirical parameters:

\[
\tau = -\rho_o A_v \frac{dU}{dz},
\]

\[
A_v(z; U_{10N}) = \phi_w(U_{10N}) \kappa u_* [z_{0_o}(U_{10N}) - z], \quad -z_1 < z < 0.
\]

The two parameters, which appear in the eddy viscosity \( A_v \) in (32) along with the von Kármán constant \( \kappa \) and the friction velocity \( u_* \), are an effective ocean roughness length \( z_{0_o} \) and a wave-correction factor \( \phi_w \). Both of these are allowed to depend on \( U_{10N} \). The formulation (32) applies only on the ocean side of the interface \((z < 0)\). The ocean roughness length \( z_{0_o} \) may, and generally will, differ from the atmospheric roughness length \( z_{0_a} \) (see, e.g., Section 4).

The novel wave-correction factor \( \phi_w \) is introduced to account for deviations from classical law-of-the-wall turbulence that can be expected to arise from surface-wave distortions of the interface and near-surface fluid, including breaking waves and the associated momentum transport. In effect, this factor allows deviations of a wave-modified von Kármán constant \( \kappa' = \phi_w \kappa \) from the classical law-of-the-wall value \( \kappa \approx 0.4 \). From the point of view of dimensional analysis, surface waves introduce an additional length scale on which the dimensionless shear may depend, allowing departures of \( \kappa' \) from \( \kappa \). This wave-correction factor is analogous to, but simpler than, the height-dependent stability functions of similarity theory for the atmospheric planetary boundary layer (see, e.g., Sec. 3.3 of Garratt 1992). In the present case, the factor \( \phi_w \) is a constant (with respect to \( z \), for each value of \( U_{10N} \)) rather than a function of stability-scaled height, and from a physical point of view is introduced to represent surface-wave rather than stability effects.

The classical form of (32) without the additional parameter \( \phi_w \) has been previously invoked for the near-surface ocean (see, e.g., Madsen 1977; Jenkins 1989; Lewis and Belcher 2004). The introduction of the novel wave-correction factor \( \phi_w \) is a modest modification to the classical form but allows a much wider variety of near-surface momentum fluxes and velocity profiles, greatly increasing the potential utility of the resulting model, (12) with (31)-(32), as an empirical representation of mean near-surface dynamics. For example, the combination of small \( \phi_w \) and large \( z_{0_o} \) can give an eddy viscosity that is large and approximately constant over a finite depth near the surface, potentially consistent with the structure of turbulence injected at the surface by
breaking waves (e.g., Agrawal et al. 1992; Melville 1996; Sullivan et al. 2004, 2007; Thomson 2012; Thomson et al. 2016). A plausible alternative approach could be to represent the wave effects through a separate, wind-speed-dependent body force (e.g., Kudryavtsev et al. 2008).

b. Near-surface structure, dissipation, and wave-driven mixing

It is important to recognize that, although $A_v$ in (32) has a classical law-of-the-wall mixing-length form, solutions of (12) with (31)-(32) will have a constant-stress, log-layer structure near $z = 0$ only for sufficiently small values of the roughness length $z_{0o}$ (Appendix A). The constant-stress log-layer in these solutions is confined to $z_{0o} - z < 10^{-2} \phi_w \kappa D_o \leq 0.004 D_o$, where $D_o = u_*/f$. For $u_* = 10^{-2} \, \text{m s}^{-1}$ for $U_{10N} \approx 10 \, \text{m s}^{-1}$, $f = 10^{-4} \, \text{s}^{-1}$, and $\phi_w = 1$, this depth is approximately 0.4 m. Below this depth, the stress magnitude and direction and the velocity direction change rapidly. Thus, for $z_{0o} > 1 \, \text{m}$, the logarithmic constant-stress layer may be completely absent from the model profile.

Such large values of $z_{0o}$ are consistent with those inferred under breaking surface waves by Craig and Banner (1994), who used a general, multi-equation, turbulence closure model that is much more complex than (32) but, in the simplest neutral stability limit, reduces to the classical law-of-the-wall eddy viscosity representation near the surface, which is (32) with constant $z_{0o}$ and $\phi_w = 1$. This limiting behavior is shared by a large class of turbulence closure models used in numerical ocean circulation models. While the model constant-stress layer is confined to these shallow depths, the downwind component of model velocity may have an approximately logarithmic profile that extends to depths near $z_{0o} - z < 10^{-1} \phi_w \kappa D_o \leq 0.04 D_o$, which are an order of magnitude deeper than the constant-stress layer but are still no greater than a few meters (Appendix A). For $|z| < z_{0o}$, the near-surface stress and shear will instead both be approximately constant and equal to $u_*/(\phi_w \kappa z_{0o})$.

The shallow confinement of the constant-stress layer may be unexpected but is consistent with understanding of the atmospheric logarithmic velocity layer, which (above a possible wave boundary layer) is often limited by rotation to a height of several tens of meters or less. In Figure 3.2 on p. 46 of Garratt (1992), for example, its extent is estimated similarly as $0.02 D_a$ for $D_a = u_*/f$, which gives a depth of 40 m when $v_* \approx 0.2 \, \text{m s}^{-1}$ for $U_{10N} = 5 \, \text{m s}^{-1}$ (e.g., Fig. 1a). Since $D_o = \alpha D_a$ with $\alpha \approx 0.035$ from (B10), this would suggest that any logarithmic layer on the ocean side would
often be limited to a depth of less than two meters. By this indication, any such constant-stress layer would often be thin relative to the wave-affected layer and difficult to observe even if it were present. Consequently, it is potentially important that (32), for large values of $z_0$, can support solutions without the classical constant-stress, log-layer behavior.

Whether a velocity log layer exists near the ocean surface is uncertain and may depend on wind and wave conditions. Some studies, such as Churchill and Csanady (1983) and Bye (1988) report observations consistent with a near-surface log layer, a conclusion also reached in subsequent analysis of the same observations by Bourassa (2000) and supported by the recent, independent observations of Laxague and Zappa (2020). However, there is no consensus that log-layer behavior is consistently found in existing measurements of near-surface currents.

This question has also been approached indirectly through observations of near-surface kinetic energy dissipation $\epsilon$. For a velocity log layer with $\phi_w = 1$ and small $z_0$ in (31)-(32),

$$\epsilon(z) = A_v \left| \frac{dU}{dz} \right|^2 = u^3_s/[\kappa(z_0 - z)] \approx u^3_s/(\kappa|z|).$$

Some observational analyses (e.g., Anis and Moun 1995; Sutherland et al. 2013; Esters et al. 2018) find support for the scaling (33), while others (e.g., Agrawal et al. 1992; Thomson 2012; Sutherland and Melville 2013, 2015a; Thomson et al. 2016) find disagreement with the $1/|z|$ log-layer structure or with the magnitude of observed dissipation, which especially under breaking waves is often two orders of magnitude larger than this prediction.

In this context, it is important also to recognize that the model energy balance is closed, with the integrated dissipation always equal to the wind work:

$$\tau_w \cdot U(0) = D, \quad D = \int_{-\infty}^{0} \epsilon dz,$$

where $U(0)$ is the mean surface velocity and $\tau_w$ is the mean downward wind stress. To the extent that the model prediction of the mean surface current for a given mean wind stress is correct, the corresponding model prediction of integrated mean-flow dissipation must also be correct.

Because only the mean-flow energy balance and not the wave energy balance is explicitly represented in the model, there is no inconsistency between the large observed wave-breaking dissipation rates, which balance the wind energy input into the wave field, and the much smaller
model dissipation rates that balance the wind energy input into the mean flow. On the other hand, the large dissipation rates observed under breaking waves suggest an alternate interpretation of the roughness term in $A_v$. Let the wave-breaking turbulence be described by a velocity scale $w_\ast \approx 3u_\ast$ and length scale $z_{0w} = z_{0o}/3$, so that $w_\ast^2 \approx 10 u_\ast^2$ and, consistent with observations such as those of Sutherland and Melville (2015a), the wave-breaking dissipation $\epsilon_w \approx w_\ast^2/2z_{0w} \approx 10^2 u_\ast^2/z_{0o} = \epsilon(z_{0o})$. Then (32) may be written as

$$A_v = \phi_w \kappa (w_\ast z_{0w} - u_\ast z), \quad -z_1 < z < 0. \quad (35)$$

From this point of view, the wave-driven mixing sets a minimum value for the near-surface turbulent diffusivity, while the shear-driven mixing is small near the surface but steadily increases and becomes dominant at depths greater than the roughness length $z_{0o}$. This would connect the large effective model roughness length to the energetic wave-breaking turbulence rather than to a geometric roughness of the wavy surface.

A related perspective is taken in the recent observational analysis of the kinetic energy budget of the surface mixed layer by Zippel et al. (2022). In order to focus on the balance analogous to (34), Zippel et al. (2022) separate the mixed-layer into a shallow wave-affected layer, where the large kinetic energy flux into the surface wave field is presumed to be balanced by large wave-breaking dissipation, and the deeper mixed layer, where an integrated balance similar to (34) should hold. In the deeper mixed layer, Zippel et al. (2022) found that the measured dissipation rates roughly balanced the sum of mean-flow wind-work and buoyancy production, where the latter is absent from (34) because of the assumed neutral stability conditions. Like Bourassa (2000) and numerous previous authors, Zippel et al. (2022) allow for a partitioning of the mean wind stress into components that separately drive the mean flow and the wave field. Under the equilibrium assumption made here, however, this partitioning is not necessary, because the wind momentum flux into the wave field, which is in any case typically a relatively small fraction of the total, is balanced by an equal flux from breaking waves into the mean flow.

**c. Piecewise-linear extension**

The solution of (12) with (31)-(32) and $z_1 \to \infty$ can be obtained in terms of a single modified Bessel function (Ellison 1956; Madsen 1977; Lewis and Belcher 2004). This solution (provided in
Appendix A) has an appealing simplicity but also has the unphysical element that the eddy viscosity increases without bound away from the sea surface. A model of nearly equivalent simplicity but with a more physical eddy viscosity profile can be obtained by requiring that the eddy viscosity reach a maximum at the depth \( z = -z_1 \), remain constant for \(-z_2 < z \leq -z_1 \), decrease linearly with depth from its maximum value to a small value over the interval \(-z_3 < z \leq -z_2 \), and then maintain that small value for all depths \( z \leq -z_3 \). In this case, the equations have solutions in terms of modified Bessel functions in the subregions \(-z_1 < z \leq 0\) and \(-z_3 < z \leq -z_2 \) and exponential functions (as for the classical Ekman layer) in the subregions \(-z_2 < z \leq -z_1\) and \( z \leq -z_3 \), with continuity and smoothness conditions imposed on the velocity at \( z = \{-z_1, -z_2, -z_3\} \) (e.g., Samelson et al. 2020). The solution near the surface is not strongly dependent on the definition of these deeper subregions.

This piecewise-linear extension of (32) for \( z < -z_1 \) is adopted here, with \( \{-z_1, z_2, z_3\} \) chosen such that

\[
\phi_w \, k_u^* (z_{0o} + z_1) = A_{v, max} = 0.03 \, u^* \, D_o, \\
z_2 = 2 \, z_1, \\
A_{v, max} - \phi_w \, k_u^* (z_3 - z_2) = A_{v, min} = 0.1 \, A_{v, max},
\]

where the value \( A_{v, max} = 0.03 \, u^* \, D_o \) was suggested by comparison with large-eddy simulations similar to those reported by Skyllingstad et al. (2017). This modification avoids the unphysical unbounded growth of \( A_v \) with depth but has only a minor effect on the model near-surface velocity response, because the depth \( z_1 \approx 0.1 \, D_o \) and so is at least several meters, below the near-surface region of large wind-drift shear (see Sections 3.a, 4.f and Appendix A).

4. Model calibration

a. Motivation

In Section 2, it was argued that under equilibrium conditions, the near-surface current should be determined entirely by the mean momentum balance, rather than, for example, incorporating a separate Stokes-drift component based on an independently specified surface wave field. It was further demonstrated that surface waves with identical first-order linear dynamics can have different second-order wave-drift characteristics, raising doubt regarding the meaning of classical Stokes-
drift estimates for quasi-equilibrium seas. Similar doubts have been raised in ocean (Smith 2006) and laboratory (Monismith et al. 2007) contexts. However, a number of recent studies have argued that model predictions of surface-drift trajectories have been measurably improved by inclusion of a near-surface velocity derived from wave-field estimates by Stokes-drift calculations (van den Bremer and Breivik 2018). Simulations of Langmuir cell development in the turbulent surface boundary layer are also improved when Stokes-drift estimates of near-surface mean shear are included (Skyllingstad and Denbo 1995; McWilliams et al. 1997; Sullivan and McWilliams 2010). These results suggest that, regardless of the underlying dynamics, the Stokes-drift estimates provide at least a useful empirical representation of near-surface ocean drift velocities. Motivated by this recognition, wave-drift velocity profiles are computed in Section 4.d for two quasi-equilibrium wave-state formulations. These profiles are then used in Section 4.e to calibrate the proposed model of near-surface wind drift currents.

Under the quasi-equilibrium hypothesis, a unique wave state should be associated with every wind-speed or wind-stress state. Two complementary approaches to deriving approximate, quasi-equilibrium relations between wave state and wind speed are considered. The first (Section 4.b) is based on a recent analysis and calibration of bulk-flux coefficients for air-sea momentum flux, while the second (Section 4.c) depends more directly on wave observations and a classical assumption of fully developed seas. Both of these are chosen for their simplicity; the differences between them may be taken as a measure of the existing uncertainty in any such approximate relations. A classical estimate of the Stokes-drift associated with each wave state is then obtained in Section 4.d from a monochromatic theory, with a single wave component. A second set of Stokes-drift estimates is obtained from a spectral description of the wave state (Jenkins 1989; Arduhin et al. 2009; Webb and Fox-Kemper 2011; Breivik et al. 2014, 2016).

b. Bulk-flux wave state

The most comprehensive existing analysis of the dependence of air-sea momentum flux on 10-m winds is that of Edson et al. (2013), the results of which are encapsulated in the COARE 3.5 bulk flux algorithm. In the course of that study, the influence of wave age and wave slope on the relation between 10-m winds and surface stress was examined, with the perhaps surprising conclusion that the wave-state variables added no significant information to the wind-speed dependent formulation.
In contrast, for example, significant additional information was found in the measured near-surface (nominal 10-m depth; see Bigorre et al. 2013) ocean current: replacing the 10-m wind with the relative wind – the 10-m wind referenced to the near-surface ocean current – was found to reduce the scatter in the empirical relation between stress and wind. Edson et al. (2013) found a “simple reason” for the lack of independent information in the wave-state variables: statistically, a functional relation existed in the data between the wave state and the wind speed. That is, within the context of existing data sets that constrain the dependence of surface stress on 10-m winds, the wave state was found to be in effective quasi-equilibrium with the winds. Other studies focused specifically on wind and wave conditions have also found evidence for frequent quasi-equilibrium of the wind-wave spectrum (Thomson et al. 2013; Vincent et al. 2019).

An analytical representation of this implied dependence of wave state on 10-m neutral winds can be inferred from the results of Edson et al. (2013). A wave-age dependence

$$\frac{v_s}{c_p} \approx 0.03 \times \left( \frac{U_{10N} - 2}{8} \right) \quad \text{for} \quad U_{10N} > 2, \quad U_{10N} \text{ in units of m s}^{-1},$$

(39)

is suggested by Figure 9 of Edson et al. (2013), where $v_s$ is the air friction velocity. Thus, the wave age has the classical fully-developed-sea value $v_s/c_p \approx 0.03$ for $U_{10N} \approx 10$ m s$^{-1}$, but otherwise differs from this value (Fig. 1d). Edson et al. (2013) find further that the ratio of the rough-flow air roughness length $z_{0a}^\text{rough}$ to the significant wave height $H_s$ is proportional to the square of the wave age,

$$\frac{z_{0a}^\text{rough}}{H_s} \approx D \left( \frac{v_s}{c_p} \right)^2,$$

(40)

where the constant value $D = 0.09$ of the proportionality factor gives good agreement, over all sea states, with the observations on which the COARE 3.5 bulk algorithm is based. Here $z_{0a}^\text{rough}$ is defined in terms of the Charnock relation,

$$z_{0a}^\text{rough} = \alpha_C \frac{v_s^2}{g}, \quad \alpha_C = mU_{10N} + b, \quad (m, b) = (0.0017, -0.005),$$

(41)
Fig. 1. Bulk-stress (thick blue lines) and fully-developed-sea (thin blue lines) wave states vs. 10-m neutral wind $U_{10N}$: (a) air friction velocity $v_*$; (b) air roughness length $z_{0a} \times 10^3$; (c) 10-m neutral drag coefficient $C_{DN10} \times 10^3$; (d) wave age $v_*/c_p$; (e) significant wave height $H_s$; (f) characteristic wavelength $\lambda_s$; (g) wave steepness, $ka$; (h) surface wave-drift from (23); (i) characteristic wavenumber $k$. In (f)-(i), the corresponding values obtained by halving $\lambda$ are indicated (dashed lines). In (h), the classical 3% rule-of-thumb value for surface wind-drift is also indicated (red line).

again for $U_{10N}$ in units of m s$^{-1}$. Note that $z_{0a}^{\text{rough}}$ is related to the COARE 3.5 total roughness length $z_{0a}$ and neutral drag coefficient $C_{DN10}$ by

$$z_{0a} = \gamma \frac{v_a}{v_*} + z_{0a}^{\text{rough}}, \quad C_{DN10} = \left[ \frac{\kappa}{\ln(z_{10}/z_{0a})} \right]^2,$$

where $\gamma = 0.11$, $v_a$ is the kinematic viscosity of air, $\kappa \approx 0.4$ is the von Kármán constant, and $z_{10} = 10$ m.
With $v_s$, $z_{0a}$, and $C_{DN10}$ determined through the COARE 3.5 algorithm for each $U_{10N}$ (Fig. 1a,b,c), and $z_{0a}^{\text{rough}}$ obtained from (42), the significant wave height can be computed from (40) and (39) as

$$H_s(U_{10N}) = \frac{z_{0a}^{\text{rough}}}{D} \left( \frac{8}{0.03 \times (U_{10N} - 2)} \right)^2, \quad U_{10N} > 2 \text{ m s}^{-1}. \quad (43)$$

Combining (39)-(41) with the deep-water wave dispersion relation $k = g/c_p^2$ gives the corresponding characteristic wavelength $\lambda_s = 2\pi/k$,

$$\lambda_s(U_{10N}) = 2\pi \frac{D}{\alpha_C} H_s(U_{10N}). \quad (44)$$

The resulting significant wave height $H_s$ increases monotonically from zero near $U_{10N} = 3$ m s$^{-1}$ to nearly 7 m for $U_{10N} = 20$ m s$^{-1}$ (Fig. 1e), while the corresponding wavelength $\lambda_s$ has a minimum value of 90 m near $U_{10N} = 7$ m s$^{-1}$ and increases slowly for larger $U_{10N}$ and rapidly for smaller $U_{10N}$ (Fig. 1f,i).

c. Fully developed seas

An alternative empirical relation between the 10-m wind and wave state can be obtained from the results of Resio et al. (1999), who proceed under a fully-developed-sea assumption that is effectively equivalent to assuming a constant value of the wave age,

$$\frac{v_s}{c_p} = \frac{1}{24.18} \approx 0.041, \quad (45)$$

and a constant value of the Charnock coefficient,

$$\alpha_C = 0.015. \quad (46)$$

The result of their considerations is an empirical expression for the significant wave height $H_s$ in terms of a wavelength-referenced wind velocity $U_r$, which is both taken equal to the characteristic wave phase-speed $c_p$ and empirically related to the 10-m wind:

$$H_s = 0.21 \frac{U_r^2}{g}, \quad U_r = 0.516 \times (U_{10N})^{1.244}, \quad (47)$$
again for $U_{10N}$ in units of m s$^{-1}$. With $U_r = c_p$, the deep-water dispersion relation can be used to obtain the corresponding wavelength $\lambda_s$, which is then

$$\lambda_s = \frac{2\pi}{g} U_r^2. \quad (48)$$

The relations (45)-(48) also determine $v_*$, the roughness length $z_{0a} = \alpha_C v_*^2 / g$, and, by the second equation in (42), the neutral drag coefficient $C_{DN10}$ as functions of $U_{10N}$, yielding dependencies of these quantities on $U_{10N}$ that are similar to but weaker than those for the COARE 3.5 algorithm (Fig. 1a,b,c). The assumed constant wave-age contrasts sharply with the variable wave age inferred from COARE 3.5 (Fig. 1d) but the resulting significant wave height dependencies on $U_{10N}$ are nonetheless remarkably similar, differing by more than 1 m only for wind speeds above 15 m s$^{-1}$ (Fig. 1e). In contrast, the wavelength dependencies on $U_{10N}$ differ substantially except near $U_{10N} \approx 11$ m s$^{-1}$, with (48) describing a wavelength that increases quadratically with $U_{10N}$ from near zero at approximately 3 m s$^{-1}$ to over 200 m near $U_{10N} = 16$ m s$^{-1}$ (Fig. 1f).

d. Equilibrium-sea drift profiles

With wave amplitude $a$ taken equal to half of the significant wave (peak-to-trough) height,

$$a(U_{10N}) = \frac{1}{2} H_s(U_{10N}), \quad (49)$$

and $H_s(U_{10N})$ and the corresponding characteristic wavelength $\lambda(U_{10N})$ taken either from (43) and (44) or from (47) and (48), it is straightforward to substitute the wave parameters into (23) to obtain the mean second-order wave drift for linear Stokes waves. The wave drift is second order in the wave steepness $ka$, which depends differently on $U_{10N}$ for the two cases considered: for the bulk-flux wave state (43) and (44), the steepness increases linearly with $U_{10N}$, with $ka = \alpha_C / (2D)$ for $\alpha_C$ in (41), while for the fully-developed wave state (47) and (48), the steepness is constant, with $ka = 0.21/2 = 0.105$ (Fig. 1g). The two steepness estimates are approximately equal for $U_{10N} \approx 14$ m s$^{-1}$.

The resulting monochromatic estimates of the surface wave drift are relatively small, reaching maximum values of less than 10 cm s$^{-1}$ for $U_{10N} \leq 10$ m s$^{-1}$ (Fig. 1h). These values are considerably smaller than typical estimates of near-surface Stokes drift, presumably because of the use of the
monochromatic formulation. They are also smaller than the classical rule of thumb (Keulegan 1951; Van Dorn 1953; Weber 1983; Zelenke et al. 2012; Samelson 2020) according to which the surface drift is 3% of the 10-m wind speed. The present estimates can be made more consistent with the magnitude of typical estimates and with the 3% rule by halving the wavelengths $\lambda$ (Fig. 1f,i) while leaving the amplitudes $a$ fixed. This has the effect of doubling the steepnesses $k a$ (Fig. 1g) and multiplying the wave-drift estimates by $2\sqrt{2}$ (Fig. 1h). The resulting, adjusted, surface drift estimate based on the bulk-flux wave state (43) and (44) remains less than 0.2 m s$^{-1}$ for $U_{10N} < 10$ m s$^{-1}$ but increases rapidly to more than 1 m s$^{-1}$ at $U_{10N} = 20$ m s$^{-1}$ (Fig. 1h). The adjusted surface drift estimate based on the fully-developed-sea wave state (47) and (48), remarkably but presumably accidentally, is nearly indistinguishable from the 3% rule for all wind speeds $U_{10N} \leq 20$ m s$^{-1}$ (Fig. 1h). These adjusted estimates and the associated wave-drift profiles can be regarded as anecdotally verified representations of quasi-equilibrium near-surface ocean wind-drift shear. As such, they are used in Section 3 to provide initial, illustrative calibrations of a simple model of near-surface wind-drift shear.

Corresponding spectral estimates of Stokes drift are obtained from the analytical solution for the Phillips (1985) spectrum given in eq. (11) of Breivik et al. (2014) or Breivik et al. (2016). These spectral estimates provide an alternate prediction for the near-surface shear. For consistency, the surface drift velocities are set equal to those obtained from the monochromatic theory. The peak wavenumber $k_p$, which determines the near-surface decay scale, is computed as $k_p = \omega_p^2 / g$, where the equivalent peak frequency $\omega_p$ is in turn obtained from the relation (Breivik et al. 2016) $U_S = 2\alpha_p g / \omega_p$ with $\alpha_p = 0.0083$.

e. Profile fitting

The generalized mixing-length model considered here consists of (12) with (32) and the piecewise continuous extension of $A_v(z)$ implied by (36)-(38). This model depends primarily on two empirical parameters, the dimensional effective ocean roughness length $z_{0o}$ and the dimensionless wave-effect factor $\phi_w$. Numerical values for these parameters, as functions of $U_{10N}$, could in principle be sought by fitting the model velocity profiles to observations, in a manner analogous to the empirical determination of bulk-stress drag coefficients such as that described by Edson et al. (2013). However, a comparably reliable and comprehensive calibration dataset for near-surface
currents does not exist, largely because of the inherent measurement challenges in the wave-affected near-surface ocean. Instead, the inferred wave-drift profiles obtained in Section 4 are used here to derive preliminary estimates for the two parameters, as functions of $U_{10N}$, and to illustrate the range of near-surface velocity profiles that the model can represent.

Given the large differences between the four sets of wave-drift profiles and the uncertainty inherent in their representations of ocean near-surface wind-drift, the fitting was done manually by subjective comparison of individual profiles, and for only four values of the 10-m neutral wind speed, $U_{10N} = \{5, 10, 15, 20\}$ m s\(^{-1}\), with $f$ evaluated at latitude 40°N. The subjective fits generally emphasized the upper portions of the profiles, where the inferred wave-drift velocity and vertical shear are relatively large. For simplicity, only the magnitude of the model vector velocity was used for the fit in all cases, and the relative wind effect was neglected, with stress computed directly from the 10-m wind $U_{10N}$ by the COARE 3.5 algorithm of Edson et al. (2013). For the bulk-stress wave state (Section 4.b), the inferred wave drifts for $U_{10N} = \{5, 10\}$ m s\(^{-1}\) were only a few centimeters per second, so for those fits the model surface velocity was matched instead to the 3% rule-of-thumb value, $0.03 \times U_{10N}$, with a transition to the wave-drift profile over the upper meter.

In most cases, the fitted model profiles match the inferred monochromatic wave-drift profiles within roughly 10% or, for regions with model velocities below 0.1 m s\(^{-1}\), a few centimeters per second (Fig. 2). A larger difference occurs immediately adjacent to the surface for the bulk-stress wave state with $U_{10N} = \{5, 10\}$ m s\(^{-1}\), because for these cases the model surface velocity was fitted instead to the 3% rule-of-thumb value. For both fitted profiles with $U_{10N} = 5$ m s\(^{-1}\), and for the bulk-flux wave-state fit for $U_{10N} = 10$ m s\(^{-1}\), the velocity shear immediately adjacent to the surface is very large, as the downwind velocity within roughly 0.1 m of the mean sea surface has a classical constant-stress, log-layer structure, analogous to that below 10-m height in the atmospheric planetary boundary layer. For the larger wind-speed fits, with $U_{10N} = \{15, 20\}$ m s\(^{-1}\), the velocity shear is much smaller near the surface and roughly uniform over the upper 10 m, consistent with the anticipated model profile structure for large, order 1-m, roughness lengths. The wave-drift profiles for the two cases with $U_{10N} = 15$ m s\(^{-1}\) are very similar, despite partially compensating differences in $H_s$ and $\lambda$, and consequently it was possible to choose a single set of fitting parameters for both of these cases, giving identical fitted model profiles for these two cases.
Fig. 2. Wave drift (red) and fitted model (blue) velocity magnitude (m s\(^{-1}\)) vs. depth z (m) for the eight calibration profiles, for the bulk-flux (upper panels) and fully-developed-sea (lower panels) wave states and for \(U_{10N}\) values as shown. In each panel, the surface values are indicated (red and blue circles) and the 0.03 \(\times\) \(U_{10N}\) rule-of-thumb wind-drift velocity is also shown (green dot). Profiles were fit independently for the monochromatic (solid) and spectral (dashed) wave-drift profiles.

All of the spectral wave-drift profiles have larger near-surface shear than the corresponding monochromatic profiles (Fig. 2). This reflects the relatively large contributions of short waves, which have relatively small vertical scales, to the total Stokes drift. The model fits to these profiles generally have similar accuracy to those for the monochromatic profiles. An exception is the bulk-flux wave state for \(U_{10N} = 20\) m s\(^{-1}\), for which the spectral profile has a rapid transition near 0.5 m depth between a large near-surface shear and a small deeper shear that was difficult to fit with the model.

f. Fitted parameters

The fitted parameters \(z_{00}(U_{10N})\) and \(\phi_w(U_{10N})\) show consistent dependencies on \(U_{10N}\), with \(z_{00}\) increasing and \(\phi_w\) decreasing for increasing \(U_{10N}\) (Fig. 3ab). The inferred \(z_{00}\) values vary by more
than 5 orders of magnitude, from $5 \times 10^{-5}$ m for both bulk-flux wave-state profiles with $U_{10N} = 5$ m s$^{-1}$ to 35 m for the monochromatic fully-developed-sea profile with $U_{10N} = 20$ m s$^{-1}$. The inferred $\phi_w$ values decrease from a maximum of 1 for both $U_{10N} = 5$ m s$^{-1}$ bulk-flux profiles to values near 0.1 for both $U_{10N} = 20$ m s$^{-1}$ monochromatic profiles. The differences between the two wave-state parameter fits are relatively large at each of the two lower wind speeds and relatively small at each of the two higher wind speeds. The spectral-profile $z_{0o}$ values are generally smaller, and the $\phi_w$ values larger, than the corresponding monochromatic-profile values, reflecting the larger near-surface shear in the spectral wave-drift profiles.

The surface value of the eddy viscosity, $A_v(0) = \phi_w k u_* z_{0o}$, for the fitted parameters increases monotonically by 4 orders of magnitude from $U_{10N} = 5$ m s$^{-1}$ to $U_{10N} = 20$ m s$^{-1}$ (Fig. 3c). For both bulk-flux wave-state profiles with $U_{10N} = 5$ m s$^{-1}$, the fitted value of $A_v(0)$ is less than the kinematic viscosity of seawater, suggesting that a laminar sublayer would be present (Appendix A). The spectral-profile $A_v(0)$ values are generally smaller than the corresponding monochromatic-profile values, again consistent with the larger near-surface shear in the spectral wave-drift profiles.

The general dependence of the fitted parameters $z_{0o}$ and $\phi_w$ on $U_{10N}$ can be characterized by analytical expressions that nominally interpolate between the fitted values at the four $U_{10N}$ values
(Fig. 3). This gives, for the monochromatic estimates:

\[
\log_{10} z_{0o}(U_{10N}) = -5 + 7 \left(1 - e^{-U_{10N}/10}\right),
\]

\[
\phi_w(U_{10N}) = e^{-U_{10N}/10},
\]

and for the wave-spectrum estimates:

\[
\log_{10} z_{0o}(U_{10N}) = -6 + 7.5 \left(1 - e^{-U_{10N}/10}\right),
\]

\[
\phi_w(U_{10N}) = e^{-U_{10N}/15},
\]

for \(z_{0o}\) and \(U_{10N}\) in units of m and m s\(^{-1}\), respectively.

5. Results

a. Calibrated model solutions

For either of the two nominal parameterizations, (50)-(51) or (52)-(53) model solutions can be computed for all wind speeds \(U_{10N} \leq 20\) m s\(^{-1}\). The resulting solutions can be illustrated by the magnitude and direction relative to surface stress of the velocity at the surface and a set of fixed depths as a function of wind speed (4). The model solutions for the first parameterization, (50)-(51), have surface wind-drift velocity magnitude that is within a few cm s\(^{-1}\) of the 3% rule for \(U_{10N} < 15\) m s\(^{-1}\) and exceeds that estimate by roughly 10% for larger wind speeds (Fig. 4a). For wind speeds \(U_{10N} < 10\) m s\(^{-1}\), the direction of this surface velocity is also within roughly ±5° of the 15-degree directional component (“15 degrees to the right of the wind [stress]”) of the 3% rule (Fig. 4b). The shear – in direction (Fig. 4b) as well as magnitude (Fig. 4a) – is concentrated in the upper 1 m or less for \(U_{10N} < 10\) m s\(^{-1}\) and becomes progressively more uniform over the upper 5-10 m for \(U_{10N} > 10\) m s\(^{-1}\). The model solutions for the second parameterization, (52)-(53), are generally similar but retain a larger near-surface shear and downwind velocity for wind speeds greater than 10 m s\(^{-1}\) (Fig. 4c,d).

For both parameterizations, the surface velocity shows systematic departures in speed and direction from the log-layer estimate, especially for wind speeds greater than 15 m s\(^{-1}\) (Fig. 4). For wind speeds less than 3 m s\(^{-1}\), for which \(z_{0o}\) is small and the log-layer approximation would otherwise
Fig. 4. Model velocity vs. $U_{10N}$ for the nominal parameterizations (50) and (51), for $f$ evaluated at latitude 40°N. Wind-drift velocity (a,c) magnitude and (b,d) angle relative to surface stress, at the surface (thick blue line) and at depths of 1 m (thin), 2 m (dashed), 5 m (dashed-dotted) and 10 m (dotted), with the 3%, 15° rule-of-thumb estimate of surface drift (red). In (b,d), the 45° angle of the classical Ekman-layer surface velocity is shown for reference (dotted black). The log-layer estimate (A8) of surface velocity magnitude and direction is also shown (green dashed).

apply, this departure arises from the laminar sublayer correction (Appendix A). For wind speeds greater than 10 m s$^{-1}$, this difference arises from the large roughness lengths, which approach and then exceed the cut-off depth for the log layer (see Section 3.b and Appendix A).

The change in the near-surface shear regime as the roughness length $z_{0o}$ increases with wind speed is particularly evident in the solutions with the first parameterization (Fig. 5a,b). The region of constant near-surface shear for $|z| < z_{0o}$ becomes progressively more apparent as $U_{10N}$ increases and is particularly evident when the velocity magnitude is viewed vs. the logarithm of depth (Fig. 5b). The transition to a uniform near-surface shear regime with approximately uniform dissipation $\varepsilon \approx u_{*}^3/(\phi_{w} \kappa z_{0o})$ is also evident in the model dissipation profiles (Fig. 5c,d). The depth $0.3 z_{0o}$ is roughly comparable to $0.3 H_s$ (Fig. 5c,d), the estimated depth of the wave-breaking layer (Sutherland and Melville 2015a; Thomson 2012), consistent with $z_{0o}$ representing wave-breaking
Fig. 5. Profiles vs. $U_{10N}$ and (a,c) depth $z$ or (b,d) base-10 logarithm of depth $\log_{10} z$ for model solutions as in Fig. 4a,b. (a,b) Magnitude of model vector velocity (m s$^{-1}$; shading, white contours at 0.1 m s$^{-1}$ intervals). (c,d) 10-base logarithm of model dissipation $\epsilon$ from (33), for $\epsilon$ in units of m$^2$ s$^{-3}$ (shading, white contours at unit intervals). The depths $z_{0o}$ (solid white line), $0.3z_{0o}$ (dashed white), and $0.3H_s$ for $H_s$ from Fig. 1e (red lines) are also shown.

effects (Section 3.b). For wind speeds less than 3 m s$^{-1}$, the uniform near-surface dissipation arises from the laminar sublayer.

b. Comparison with near-surface current observations

The model solutions described in Section 5.a illustrate that the calibrated wave-modified mixing length model, despite depending on only two wind-speed dependent parameters, is able to represent a broad range of near-surface velocity and shear profiles. The nominal calibrations (50)-(51) and (52)-(53), were obtained indirectly, by fitting the model velocity profiles to a set of equivalent wave-drift velocity profiles that were taken as representative of near-surface wind drift, as described in Section 3. This indirect approach was taken because a comprehensive dataset of measured mean near-surface currents that would be adequate to provide a direct calibration, analogous to the eddy-covariance flux dataset on which bulk stress algorithms such as that of Edson et al. (2013) are based, is not presently available. It is possible, however, to compare the model predictions with various sets of measurements of near-surface currents that do exist. Four examples of such measurements are considered here, described respectively by Churchill and Csanady (1983); Kudryavtsev et al.
(2008), Morey et al. (2018), and Ardhuin et al. (2009). Because these measurements were not used in the calibration, the comparisons provide an independent test of the model. The monochromatic calibration (50)-(51) is used for these comparisons; results obtained using (52)-(53) were not substantially different.

Churchill and Csanady (1983) measured near-surface water velocities in Lake Huron and Cape Cod Bay using drifters and drogues whose positions were determined from acoustic travel time and compass-sighting methods. Relative velocity magnitude and direction between 1.2 cm or 2.5 cm depths, from surface drifters with 2.4 cm or 5 cm thicknesses, and 1.8 m, from a drifter with drogue 0.3 m in vertical extent centered at 1.8 m depth, were reported as a function of surface wind speed at 3 m height. A total of approximately 24 such measurements were reported (Churchill and Csanady 1983, Table 1). The relative velocity magnitudes were mostly between 0.06 and 0.1 m s\(^{-1}\) but showed little systematic dependence on wind speed. A difficulty with this comparison, which also arises for the Kudryavtsev et al. (2008) data, is that the observations do not clearly define a systematic empirical relation between wind speed and near-surface currents. Presumably, this is a sampling problem: either the conditions were not close enough to the ideal homogeneous-equilibrium state or the number of measurements were not adequate to reduce the noise from random fluctuations.

The corresponding model-predicted shear is comparable to the Churchill and Csanady (1983) observations for wind speeds below roughly 7 m s\(^{-1}\) but larger than the observations for larger wind speeds (Fig. 6a). For this comparison, observed wind speeds at 3 m height were corrected to nominal 10-m height using a logarithmic profile with constant atmospheric roughness length \(z_{0a} = 0.152 \times 10^{-3}\) m; these corrections were of order 10% in observed wind speed and do not affect the basic comparison. Although the model surface current closely follows the 3%-of-wind-speed rule, the near-surface shear is so large that the model-predicted shears between 1.2 cm or 2.5 cm and 1.8 m depths are significantly below 3% of the wind speed, consistent with the measurements. The model near-surface shear is sufficiently large that the predicted currents at 1.2 cm and 2.5 cm differ by a factor or two or more depending on whether they are computed as point values in the vertical or as averages over the 2.4-cm and 5-cm vertical extents of the drifters (Fig. 6a). Larger shears relative to 1.8 m are obtained for the vertical averages, because these contain contributions
Fig. 6. Comparisons of model predictions with drifter observations from (a) Churchill and Csanady (1983), (b) Kudryavtsev et al. (2008), and (c) Morey et al. (2018). (a) Observations of 2.5 cm (red squares) and 1.2 cm (blue squares) to 1.8 m relative velocity magnitude vs. 10-m wind speed, with corresponding model predictions (red and blue lines, respectively) using vertical averages (solid) or point values (dashed). (b) Observations of 0.5 m to 5 m relative velocity magnitude (squares) vs. 23-m wind speed, with corresponding model predictions using vertical averages (blue solid line) or point values (red dashed). The additional lines are model fits from Kudryavtsev et al. (2008). Reprinted with permission from Kudryavtsev et al. (2008), with added overlays. (c) Observations of 10 cm and 1 m (red solid disk and open circle, respectively, showing approximate extents of the reported elliptical 95%-confidence intervals about the mean) to 5 cm normalized velocity difference, with corresponding model predictions (blue solid and dashed lines, respectively). Constant relative velocity magnitude ratios of 1 (equal magnitudes), 0.75, and 0.5 are indicated (dotted circles centered on the origin, i.e., \( u_r^2 + v_r^2 = \text{const} \)). Departures of the \( u_r \) value (abscissa) from zero indicate directional differences in the relative velocity. Adapted from Morey et al. (2018), with added overlays.

from the fast, 3%-of-wind surface currents. The point-value estimates show a weaker dependence on wind speed, more consistent with the measurements than the vertical averages.

Kudryavtsev et al. (2008) measured near-surface velocities in the Black Sea using drifters with drogues 0.32 m in vertical extent centered at depths between 0.5 m and 5 m, whose positions were determined by laser range-finding. Relative velocity magnitude and direction between drifters at various depths were reported as a function of wind speed measured at 23 m height. The data were not reported numerically, so for this comparison, the model predictions for shear between the 0.5-m and 5-m drifters have been overlaid on a figure from Kudryavtsev et al. (2008), with the model wind speeds at 10 m height corrected to nominal 23-m height, again using a logarithmic profile.
with constant atmospheric roughness length $z_{0a} = 0.152 \times 10^{-3}$ m. These corrections were again of order 10% in observed wind speed and do not affect the basic comparison.

The model predictions show a basic consistency with the Kudryavtsev et al. (2008) observations (Fig. 6b). The observed relative velocity magnitudes were between 0.03 and 0.12 m s$^{-1}$ and showed little systematic dependence on wind speed. The model predictions are in this same range for wind speeds below 8 m s$^{-1}$ but are two to three times larger for wind speeds above 12 m s$^{-1}$. With the uppermost profile point at 0.5 m, there is no significant difference between the model predictions obtained from point values and from vertical averages over the drogue extent.

Morey et al. (2018) measured near-surface water velocities in the Gulf of Mexico using GPS-tracked surface drifters with 5-cm and 10-cm vertical extent and with a drifter with drogue 0.9 m in vertical extent centered at 1 m depth. The observed velocities of the 10-cm and drogued drifters relative to the 5-cm drifter were reported in terms of distributions of a normalized vector velocity difference, for which exact equality of a pair of velocities corresponds to the point (0,1) in the normalized velocity difference plane (their Figure 6, showing normalized velocity differences defined in their Equation 1). Dependence of the relative velocities on wind speed was not discussed, so the data are compared here to equivalent normalized vector velocity differences for the full range of model relative velocities predicted for all values of $U_{10N}$ less than 12 m s$^{-1}$, the maximum wind speed reported by Morey et al. (2018) from buoy observations during the observation period (their Figure 5). The observations were conducted in regions of energetic geostrophic motion but it seems likely that these motions did not strongly affect the shear above 1-m depth.

Although the wind speed dependence is not addressed in this comparison, the range of predictions from the model have a basic quantitative similarity to the observations (Fig. 6c). For the relative velocity between 10-cm and 5-cm depths, the model predictions over the full wind-speed range are all within the elliptical 95%-confidence interval about the mean that is reported by Morey et al. (2018). For the relative velocity between the 1-m and 5-cm depths, the model predictions show a normalized relative velocity difference that is roughly twice as large as observed, with a monotonic wind-speed dependence that gives better agreement with the observations at the lowest wind speeds, for which the model values are again within the elliptical 95%-confidence-interval about the mean. With the uppermost profile point at 5 cm, there is again no significant difference
between the model predictions obtained from point values and from vertical averages over the drogue extent.

Ardhuin et al. (2009) analyzed two years of coastal radar surface-current and buoy wind observations approximately 80–100 km offshore in water of approximately 120 m depth. The coastal radar measurements of surface currents in the approximate effective depth range 0–1.6 m, reported by Ardhuin et al. (2009) as velocity $U_R$, were binned vs. 10-m wind speed for wind speeds up to 19 m s$^{-1}$. The binned mean surface current magnitude was a nearly constant percentage of the wind speed, increasing slightly from 1.2% at low wind speeds to 1.6% at high wind speeds (Fig. 7a,b). The binned mean wind-relative surface current direction decreased from -40° to -15° over the same range (Fig. 7c). Most of the observations were for wind speeds below 10 m s$^{-1}$. Ardhuin et al. (2009) used a wave model to separate the velocity $U_R$ into Stokes drift and quasi-Eulerian components but such a decomposition is not necessary for this comparison because the model predicts the total mean parcel motion. For these observations, the assumption of wind-wave equilibrium is questionable, as significant independence was found between measured $H_s$ and 10-m wind.

Model predictions for 0–1.6 m average and 0.8 m depth current magnitude and direction show a rough agreement with the coastal radar observations for wind speeds between 3 and 10 m s$^{-1}$, with model currents increasing with wind speed from 0.7% to 2% of $U_{10N}$ and wind-relative direction decreasing from -45° to -30° over the same range (Fig. 7). For larger wind speeds, the model predicts stronger currents than the observations, at greater relative direction to the wind.

c. Relative wind and drag coefficient

As noted above, the Edson et al. (2013) bulk-stress algorithm should use the relative wind rather than the absolute 10-m wind $U_{10N}$, where in the calibration with measurements, the ocean surface current is generally taken from observations near 10-m depth. For $U_{10N} > 15$ m s$^{-1}$, the model wind drift velocity at 10-m depth can reach 0.2-0.4 m s$^{-1}$, comparable to or greater than typical open-ocean mesoscale geostrophic currents. This model wind-drift contribution to the velocity at 10-m depth has been neglected in the model calculations of surface stress from $U_{10N}$, although in principle the equivalent velocity would be included in observational estimates using a relative wind referenced to observed currents at 10-m depth. From this point of view, the drag coefficient
Fig. 7. Comparisons of model predictions with coastal radar observations from Arduhin et al. (2009). (a) Observed (black line, ×) and model 0–1.6 m mean (blue) and 0.8 m (red dashed) velocity magnitude vs. observed 10-m wind and model $U_{10N}$, respectively. (b) Velocity magnitudes in (a) as percentage of corresponding wind speed. (c) Angles of velocities in (a) relative to wind direction. Observational data digitized from Figure 6 of Arduhin et al. (2009).

used in the model is, effectively, up to 1% larger than the COARE 3.5 coefficient for $U_{10N} > 10$ m s$^{-1}$ (Fig. 8).

At the surface, the model wind-drift surface velocity is approximately 3% of $U_{10N}$ (Fig. 4a). The effective drag coefficient computed for the 10-m wind, using stress computed from the relative wind referenced to the model surface velocity, is up to 2% smaller than the effective 10-m wind drag coefficient computed for wind relative to the model current at 10-m depth (Fig. 8). If this model surface drift proves to be consistent with observations, the drag coefficient from the Edson
et al. (2013) COARE 3.5 algorithm or any similar formulation would have a small systematic bias if used to compute stress from the wind referenced to the current at the ocean surface rather than at 10-m depth. This effect could be incorporated into the bulk-flux iteration to remove the bias. The effect is smaller than the 6% that might be inferred as the quadratic effect of a 3% reduction in relative wind because of the dependence of the COARE 3.5 drag coefficient on $U_{10N}$, i.e., on the relative wind.

d. Coupled air-sea boundary layer

The close agreement of the model surface current magnitude with the 3% rule (Fig. 4a) motivates comparison of the coupled ocean-atmosphere boundary-layer profiles with the structure assumed by Samelson (2020), who derived the exact result (B11) for the interface drift velocity in the case in which there is only a single dimensional length scale in each boundary layer. In that case, universality considerations dictate that the dimensionless boundary-layer deviation profiles will be antisymmetric relative to one another, which determines the surface velocity as the weighted mean (B11) of the geostrophic current and wind outside the boundary layer, lending theoretical
support to the empirical 3% rule. For this comparison, a form equivalent to (32) was used for the atmospheric eddy viscosity, with an equal dimensionless atmospheric value of $z_1$ in (36), but with $\phi_w = 1$ as in classical log-layer theory. For simplicity, the eddy viscosities were held constant at the respective values of $A_{v_{\text{max}}}$ for $|z| > z_1$ in both fluids, i.e., the limits $z_2, z_3 \to \infty$ were taken in (37)-(38).

The dimensionless model problem for the coupled boundary layers then consists of (B6) with (32) and (B7), plus their equivalents for the atmosphere, with the continuity conditions for both stress (B8) and velocity (B7) at the mean air-sea interface. The COARE 3.5 drag coefficient (Edson et al. 2013) was used to obtain the surface stress from the 10-m wind, again neglecting – for simplicity and consistency with the previous results – any ocean current contribution to the relative wind. The values of $z_{0o}$ and $\phi_w$ for the ocean profile are taken from the nominal parameterization (50)-(51). Solutions are found for zero geostrophic ocean current and for a geostrophic atmospheric wind that is directed along the $x$-axis, with magnitude such that the 10-m wind speed takes a specified value. Note that the 10-wind is then directed roughly 10° to the left of the geostrophic wind rather than along the $x$-axis, because of the rotation of velocity with height in the boundary layer.

The coupled boundary layer solutions for the two 10-m wind values $U_{10N} = \{5, 15\}$ m s$^{-1}$ illustrate the differing character of the response at low and at high wind speeds. For $U_{10N} = 5$ m s$^{-1}$, the dimensionless ocean velocity is approximately an antisymmetric reflection of the atmospheric profile (Fig. 9a). The primary asymmetry arises from the truncation of the profile at the ocean roughness length $z_{0o} \approx 5 \times 10^{-3}$ m, which, from a dimensionless perspective, is substantially larger than the atmospheric roughness, i.e., $z_{0o} \gg \alpha z_{0a}$ for the COARE 3.5 $z_{0a}$ from (42). This leads to a small clockwise rotation of the surface velocity past the geostrophic wind direction (Fig. 9b). The dimensional stress profiles for this coupled boundary layer solution are likewise nearly symmetric (Fig. 9c). For these small wind speeds and small values of $z_{0o}$, the model response thus approximately exhibits the universality invoked by Samelson (2020).

For $U_{10N} = 15$ m s$^{-1}$, the dimensionless ocean velocity departs substantially from the antisymmetric reflection of the atmospheric profile, with a much larger deflection of the surface velocity past the geostrophic wind direction (Fig. 9d,e). The main source of the asymmetry is the removal of most of the constant-stress log layer in the ocean through the truncation of the profile at the ocean roughness length $z_{0o} \approx 2.7$ m, a value that is several orders of magnitude larger than the atmospheric...
Fig. 9. Hodographs of ocean (blue) and atmosphere (green) model profiles of (a,d) dimensionless and (b,e) dimensional velocity, and (c,f) dimensionless downward stress, for the coupled boundary layer solutions with (a,b,c) $U_{10N} = 5 \, \text{m s}^{-1}$ and (d,e,f) $U_{10N} = 15 \, \text{m s}^{-1}$. The interface values (○; solid or open; blue or green), the 10-m height in the atmosphere (×) and the levels $z_1^{(o,a)}$ (+) are marked. In (a), the antisymmetric reflection of the atmospheric profile is indicated (green dotted line).

roughness length and is larger than the thickness of the ocean log layer that would obtain if $z_{0o}$ were comparable to $z_{0a}$. The dimensional stress profiles for this coupled boundary layer solution remain approximately symmetric in hodograph but are distorted in relative dimensionless height (Fig. 9c,f). Thus, while the model surface velocity magnitude for $U_{10N} = 15 \, \text{m s}^{-1}$ remains close to the 3% rule, the corresponding profiles depart significantly from the antisymmetry that would be implied by the turbulent universality argument of Samelson (2020). These departures arise as asymmetries induced by the the nominal parameterization (50)-(51) for the ocean roughness length.
and wave correction factor, which result in a dimensionless ocean eddy viscosity profile that differs significantly from its atmospheric counterpart.

6. Summary

A nominal model has been proposed here for the wind-drift current in the upper few meters of the ocean under homogeneous, equilibrium conditions. The model consists of the mean momentum balance (12) with the mixing-length eddy viscosity (31)-(38) and the empirical parameterizations (50)-(51) or (52)-(53) for the effective roughness length $z_{0o}$ and wave correction factor $\phi_w$ as a function of 10-m neutral wind speed. This model is envisioned as a continuation into the near-surface ocean of the model atmospheric boundary-layer wind profile that is implicit in bulk-stress algorithms such as that of Edson et al. (2013).

The inferred $z_{0o}$ increases with $U_{10N}$ from a minimum value of $10^{-5}$ m for calm conditions to roughly 10 m for $U_{10N} = 20$ m s$^{-1}$, while $\phi_w$ decreases from 1 to roughly 0.1 over the same range of wind speed. The largest values of $z_{0o}$ are comparable to the value inferred by Craig and Banner (1994) in a closely related study that used a more complex turbulence closure formulation with an independently predicted turbulent kinetic energy variable. The model profiles show a log-layer or laminar sublayer shear structure at the smaller wind speeds and a nearly uniform near-surface shear at the larger wind speeds. The associated near-surface dissipation profiles are inversely proportional to depth for moderate wind speeds with a transition to a uniform near-surface regime for larger wind speeds. The nominal model ocean surface velocity magnitude closely follows the 3%-of-wind-speed, 15°-rotation rule but, for 10-m winds greater than 10 m s$^{-1}$, shows a greater rotation away from the wind direction than the 15° suggested by the rule.

The model predictions of near-surface wind-drift show a basic consistency with the observations described by Churchill and Csanady (1983), Kudryavtsev et al. (2008), Morey et al. (2018) and Arduhin et al. (2009) but the comparison is not definitive. A small correction to the observed bulk aerodynamic drag coefficient is implied by this nominal model, when the relative wind is computed with respect to the ocean current at the sea surface rather than at 10-m depth. Because the wind-drift model parameters are determined by the 10-m wind, the correction could be incorporated directly into the drag coefficient. For observations, where measurements of currents above 10-m depth are difficult and remain rare, this correction may in general not be required. For numerical
models with sufficiently fine vertical resolution near the ocean surface, however, it may be more generally appropriate (e.g., Samelson et al. 2021).

In many theoretical modeling studies of near-surface shear, the wave and locally wind-driven motions are treated separately. In this “wave-kinematic” approach, a mean drift motion is inferred from the wave state using the theory of Stokes (1847) and its modern extensions (e.g., Phillips 1977; Bye 1988; Jenkins 1989; McWilliams et al. 1997; Ardhuin et al. 2009; Breivik et al. 2016; Suzuki and Fox-Kemper 2016). The wave state may be imposed arbitrarily or it may be derived from an independent dynamical model of the wave field. The approach taken here relies instead on the assumption of a homogeneous, equilibrium sea state. In this homogeneous-equilibrium setting, the surface-wave dynamics that may drive any Stokes-drift motions are indistinguishable in the mean momentum balance from the rest of the wave-turbulent shear flow dynamics that control the mean momentum fluxes. Consequently, the equilibrium surface and near-surface velocity response can and should be characterized generally as a wind drift, because all the motions, including those that might invite interpretation as waves or wave-related, are driven and maintained by the wind. There is therefore neither need nor advantage in this setting to enforce a separation of wave and other motions. All that matters is the mean effective vertical flux of horizontal momentum.

An intriguing question concerns the relation of the modeled wind-drift to roll instabilities such as Langmuir circulations and other similar features of ocean surface boundary layer turbulence (Leibovich 1983; Skyllingstad and Denbo 1995; Sullivan et al. 2004, 2007; Skyllingstad et al. 2017). These features are generally recognized to be driven by a mean near-surface shear that is often interpreted as a wave-driven Stokes drift. The model is intended to incorporate all forms of turbulent momentum transport and the effect of such structures is therefore implicitly included in the parameterizations (50)-(51) or 52)-(53). On the other hand, the model near-surface shear magnitude has been calibrated to fit Stokes-drift profiles. This raises the possibility that some of the model-predicted profiles of mean wind-driven shear could support roll instabilities, which might suggest alternate interpretations of instability mechanisms.

The nominal model calibrations (50)-(51) and (52)-(53) have been obtained here in an indirect manner. Comparisons with existing observations are not definitive and the model will remain preliminary until a reliable calibration dataset for near-surface ocean currents as a function of wind speed, similar to the eddy covariance datasets for air-sea momentum exchange (e.g., Edson et al.
2013), is available. The time and space scales for achieving the assumed equilibrium have not been addressed but presumably depend on rotational (Lewis and Belcher 2004) and wind-wave processes. Recent, renewed attention to measurement of near-surface currents and an associated new generation of remote sensing and in-situ instruments (Ardhuin et al. 2017; Callies et al. 2017; Lund et al. 2018; Morey et al. 2018; Rodriguez et al. 2018, 2019; Hughes et al. 2020; Laxague and Zappa 2020; Grare et al. 2021; Lenain and Pizzo 2021), supplemented by novel analysis approaches, promise a rich set of new observations relevant to this problem in the near future. It is hoped that the present model will provide a useful framework for comparison and analysis of these new observations.
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Data availability statement. No datasets were generated and only previously published datasets were analyzed during the current study.

APPENDIX A

Simplified analytical solution

As noted in Section 2.c, the solution of (12) with (31)-(32) can be obtained in terms of a single modified Bessel function when $z_1 \to \infty$. This simplified solution for the vector velocity $(U, V)$ may be written in complex form as

$$W(z) = U(z) + iV(z) = AK_0(\xi),$$  \hspace{1cm} (A1)

and also closely approximates the finite-$z_1$ solution near the surface, where $|z| \ll z_1$. Here

$$A = \left( \frac{u^3}{\phi w f z_{0o}} \right)^{1/2} e^{-i\pi/4} K_1(\xi_0);$$ \hspace{1cm} (A2)

$K_0$ and $K_1$ are the modified Bessel functions of order 0 and 1, respectively;

$$\xi(z) = (1 + i) \left[ \frac{2f}{\phi w \kappa u_s} (z_{0o} - z)^{1/2} \right], \quad \xi_0 = \xi(0);$$ \hspace{1cm} (A3)

and the $x$-axis has been aligned with the direction of the wind stress (taken positive downward), so that

$$\tau_w = \rho_0 u^2_s.$$ \hspace{1cm} (A4)

This approximate formulation could be used, for example, to provide a prediction of near-surface current shear within a finite-thickness surface grid-cell or layer of a numerical ocean model, with values of $z_{0o}$ and $\phi w$ computed from the 10-m neutral wind velocity corresponding to the imposed wind stress $\tau_w$. More generally, ocean turbulence mixing-length submodels could be formulated.
Fig. A1. (a) $K_0(\xi)$ and (b) dimensionless stress $2(\zeta_0 - \zeta) dK_0(\xi) / d\zeta$ vs. $\zeta_0 - \zeta$. In (a,b), the real parts (thick blue lines) and imaginary parts (thin blue) correspond to the downwind and cross-wind directions. In (c), the relative directions are shown as angles relative to the downwind and cross-wind directions for $K_0(\xi)$ (thick blue) and stress (thin blue). In (a), the approximation $K_0(\xi) \approx -\ln \xi$ (real part: red dotted; imaginary part: green dotted) is shown. In (b), the unit surface stress value (red dotted) is shown. The reference values (a) $\zeta_0 - \zeta = 10^{-1}$ and (b,c) $\zeta_0 - \zeta = 10^{-2}$ are also indicated (dashed black).

to match (31)-(32) in the neutrally stratified limit rather than the classical law-of-the-wall with $\phi_w = 1$.

For sufficiently small $|\xi|$ and $\xi_0$, $K_0(\xi) \approx -\ln \xi$ and $K_1(\xi_0) \approx 1/\xi_0$, so that $A = A_r + i A_i \approx A_r$ and the downwind and crosswind components of $W(z)$ correspond to the real and imaginary parts of $K_0(\xi)$. The downwind velocity $U$ is then approximately logarithmic for $\zeta_0 - \zeta$ of order $10^{-1}$ or less, where

$$\zeta = \frac{f}{\phi_w k u_*} z, \quad \zeta_0 = \frac{f}{\phi_w k u_*} z_{0o}. \quad (A5)$$

However, the direction of the vector velocity begins to depart appreciably from its surface value for $\zeta_0 - \zeta$ of order $10^{-2}$ (Fig. A1a,c). For $\zeta_0 - \zeta$ of order $10^{-2}$, the downward dimensionless stress $2(\zeta_0 - \zeta) dK_0(\xi) / d\zeta$ also begins to depart appreciably in both magnitude and direction from the constant-stress, log-layer value $\tau_w / (\rho_0 u_*^2) = 1$ (Fig. A1b,c).

For finite values of $\zeta_0$, the solution $K_0(\xi)$ truncates at the surface value $K_0(\xi_0)$, where $\zeta = 0$. Viewed as a function of dimensionless depth $|\zeta|$ as $|\zeta| \to 0$, however, the solution is approximately linear, $K_0(\xi) \approx K_0(\xi_0) + \zeta / (2\xi_0)$, for $|\zeta| < |\xi_0|$ (Fig. A2). For the velocity $W(z)$, this corresponds to an approximately uniform downwind shear $dU / dz \approx u_*/(\phi_w k z_{0o})$ for $|z| < z_{0o}$.
Fig. A2. $K_0(\xi)$ as in Figure A1a but vs. dimensionless depth $|\xi|$ for $\zeta_0 = 0.005$ (solid blue lines). As a function of $\zeta$, $K_0(\xi) \approx K_0(\xi_0) + \zeta/(2\zeta_0)$ for $|\xi| < \zeta_0$ (red line). As a function of $\zeta_0 - \zeta$, as in Figure A1a, $K_0(\xi)$ truncates (real part: dashed blue) at $K_0(\xi_0)$, where $\zeta = 0$ and $\zeta_0 - \zeta = 0.005$ (dashed black lines); for smaller $\zeta_0$, the real part of $K_0(\xi)$ would extend as in Figure A1a (dotted blue). The imaginary part of $K_0(\xi)$ asymptotes to a constant as $\zeta_0 - \zeta \to 0$ (Fig. A1a) and is essentially the same (thin blue line) in both cases for $|\xi| \geq \zeta_0 = 0.005$.

If the surface roughness $z_{0_o}$ is sufficiently small that

$$z_{0_o} < \frac{\nu}{\phi_w K u_*},$$

so that the surface value $A_{r0}$ of the eddy diffusivity (32) is less than the molecular viscosity $\nu \approx 1.3 \times 10^{-6}$ m$^2$ s$^{-1}$ of sea water, then a laminar viscous sublayer should be inserted at the surface. For such a small $z_{0_o}$, the solution (A1) will be accurately approximated near $z = 0$ by a log layer,

$$W(z) = W_0 + \frac{u_*}{\phi_w K} \ln \left( \frac{z_{0_o} - z}{z_{0_o}} \right),$$

where

$$W_0 = -\frac{u_*}{\phi_w K} \left[ \frac{\pi}{2} i + 2 \ln 2 + \ln \left( \frac{z_{0_o} f}{\phi_w K u_*} \right) \right],$$

while the laminar sublayer will have the form

$$W^{lam}(z) = W_0^{lam} + \frac{u_*^2}{\nu} z.$$ 

Both of these are constant-stress layers, with kinematic stress equal to $u_*^2$. Consequently, the laminar sublayer may be inserted by replacing all of the velocity values $W(z)$ by the laminar
sublayer values $W_{Iam}(z)$ in the region where the sublayer correction $|W_{Iam}(z) - W_{Iam}^0|$ is less than the log-layer correction $|W(z) - W_0|$: 

$$W(z) = W(z_{Iam}) + \frac{u^2}{\nu}(z - z_{Iam}) \quad \text{for} \quad z_{Iam} < z < 0, \quad (A10)$$

where

$$z_{Iam} = -\frac{\nu}{\phi_0 k u_s} \ln \left( \frac{\nu}{\phi_0 k u_s z_{0o}} \right) \quad (A11)$$

is the depth at which, to leading order, the two corrections are equal.

**APPENDIX B**

**Boundary conditions and dimensionless formulation**

If the stress $\tau$ is taken to depend on the mean flow $U$ by relations such as (31)-(32), then (12) can be regarded as a differential equation, the solution of which is the desired velocity profile. In the classical theory of Ekman (1905), for example, the stress is taken proportional to the mean shear and the solution can be obtained in terms of complex exponentials. In general, to solve the resulting differential equation, boundary conditions are needed. The assumption (10) implies that the velocity deviation $U$ vanishes far from the interface:

$$U \to 0 \quad \text{as} \quad z \to -\infty, \quad (B1)$$

which provides one of the needed boundary conditions.

The averaging with respect to surfaces of constant $\eta$ in each fluid is assumed to preserve the standard conditions of continuity of velocity and stress at the interface of two viscous fluids (Batchelor 1967). These conditions are therefore taken to hold for the mean velocities and stress at the mean air-sea interface $z = 0$:

$$\bar{U} = \bar{V} \quad \text{at} \quad z = 0, \quad (B2)$$

or, equivalently,

$$U(z = 0) - V(z = 0) = -(\bar{U}_G - \bar{V}_G) \quad \text{at} \quad z = 0, \quad (B3)$$
\[ \tau(z \to 0^+) = \tau(z \to 0^-) = \tau_0. \]  \hspace{1cm} (B4)

In (B4), \( \tau_0 \) is the mean stress at the interface, with \( z \to 0^+ \) and \( z \to 0^- \) denoting the limits as the interface \( z = 0 \) is approached from above and below, respectively. In the continuity condition (B2) or (B3) for velocity, \( \mathbf{V}, \mathbf{V}_G \) and \( \mathbf{V} \) are the atmospheric mean, geostrophic, and boundary-layer deviation (ageostrophic) velocities, analogous to \( \mathbf{U}, \mathbf{U}_G \) and \( \mathbf{U} \). The continuity at the interface holds also for the mean pressure.

The averaged stress \( \tau_0 \) incorporates both the viscous stress at the interface and the momentum flux from correlations of pressure and interface slope fluctuations, such as those associated with wind-driven wave fields. As no distinction is made between wave and turbulent fluctuations, no assumption need be made regarding the relative contributions of the pressure and viscous components to the total momentum flux \( \tau_0 \) across the interface. The vertical integral of (12) gives the classical Ekman transport relation (Ekman 1905),

\[ \mathbf{T}_E = \int_{-\infty}^{0} \mathbf{U} d\mathbf{z} = \frac{1}{\rho_0 f} \mathbf{k} \times \tau_0 = -\frac{1}{\rho_0 f} \mathbf{k} \times \tau^w, \]  \hspace{1cm} (B5)

which shows that the depth-integrated horizontal boundary-layer transport \( \mathbf{T}_E \) is directed to the right of the downward wind stress \( \tau^w = -\tau_0 \) for \( f > 0 \), and is independent of any assumption regarding the relation between the mean stress and mean shear.

When the mean stress is taken proportional to the mean shear, the resulting vector differential equation (12) is second order in each component, regardless of whether the proportionality factor is constant or depends on \( z \), and only two vector boundary conditions are in general required. Typically, the two boundary conditions (B4) and (B1) are imposed, and the third condition (B2) is treated as a prediction of the surface wind drift in response to the imposed stress (e.g., Ekman 1905; Madsen 1977; Weber 1983). This approach is taken for most of the ocean boundary layer model solutions considered here (Sections 4 and 5.b).

A similar approach to that outlined in Section 2.b can be taken to obtain mean momentum equations for the atmospheric flow above the sea surface, under similar homogeneous, equilibrium conditions. It is natural then to solve the resulting differential equations simultaneously in both boundary layers and to impose the continuity conditions for both stress (B4) and velocity (B3) at
the interface (e.g., Samelson 2020). This approach is taken for the illustrative coupled boundary layer model solutions in Section 5.d.

The fundamental velocity and time scales, \( u_* = |\tau_0/\rho_a|^{1/2} \) and \( f^{-1} \) may be used to express (12) in dimensionless form:

\[
\mathbf{k} \times \mathbf{U} = -\frac{d\tau}{dz}, \quad z < 0. \tag{B6}
\]

In (B6), the dimensional velocity, stress, and depth have been scaled by \( u_*, \rho_0 u_*^2, \) and \( D_o = u_*/f, \) respectively. The corresponding dimensionless boundary conditions are:

\[
\begin{align*}
U & \to 0 \quad \text{as} \quad z \to -\infty, \quad \tag{B7}
U - V/\alpha &= -(U_G - V_G/\alpha) \quad \text{at} \quad z = 0, \quad \tag{B8}
\tau &= 1 \quad \text{at} \quad z = 0, \quad \tag{B9}
\end{align*}
\]

where

\[
\alpha = \frac{u_*}{v_*} = \left(\frac{\rho_a}{\rho_o}\right)^{1/2} \approx 0.035 \tag{B10}
\]

is the ratio of \( u_* \) to the atmospheric friction velocity \( v_* = |\tau_0/\rho_a|^{1/2} \) and the ocean and atmospheric velocities have been scaled by \( u_* \) and \( v_* \) respectively.

The dimensionless problem for the coupled boundary layers, consisting of (B6) and (B7)-(B10) supplemented by the atmospheric equivalents of (B6) and (B7), would have a universal structure if there were no additional relevant length scales. Samelson (2020) has shown that for coupled boundary layers with this universality, the drift velocity \( \mathbf{U}_0 \) at the interface is the \( \alpha \)-weighted average of the free-stream velocities, which for the rotating system is

\[
\mathbf{U}_0 = \frac{U_G + \alpha V_G}{1 + \alpha}. \tag{B11}
\]

For the ocean-atmosphere case, this prediction would be consistent with the classical rule-of-thumb for surface drift, according to which \( \mathbf{U}_0 \) is directed 15° to the right of the wind (in the Northern Hemisphere) at 3% of the wind speed. However, there is in this case at least one another relevant length scale, associated with the presence of gravity waves at the air-sea interface. For example, Charnock length scales \( l_{Co^*} = u_*^2/g \) for the ocean and \( l_{Ca^*} = v_*^2/g \) for the atmosphere can be defined, where \( g \) is the acceleration of gravity. The corresponding dimensionless length scales \( l_{Co} = u_* f/g \)
and \( l_{Cn} = v_* f / g \) differ by a factor of \( \alpha \), suggesting the possibility of departures from the universal, antisymmetric structure.

Consistent with general experience, the estimate (B11) indicates that under most conditions the dimensional \( |U_0| \ll V_{10} \), where \( V_{10} \) is the 10-m wind speed, so that to first order the stress \( \tau_0 \) is independent of the surface drift. This motivates the traditional neglect of the third boundary condition (B3) or (B8), the continuity of the velocity at the air-sea interface, in the specification of the ocean boundary layer problem. Note, however, that ocean surface velocities can be detected in observational estimates of the dependence of air-sea momentum flux on the 10-m wind, as the scatter in the observed statistical relationship is measurably reduced when the relative wind – the difference between the 10-m wind and the ocean surface velocity – is used in place of the 10-m wind (Edson et al. 2013). In the observational estimates, this ocean surface velocity is generally taken as the velocity near 10-m depth (Bigorre et al. 2013), the shallowest depth at which reliable ocean current measurements are typically available. It thus may differ significantly from the interface velocity \( U_0 \), suggesting that a correction to the observed drag coefficient would be necessary if the relative wind were instead computed with respect to the interface velocity, as would be most appropriate theoretically (Section 5.c).
References


