Operational Forecasting and Detection of Mesoscale Gravity Waves

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ABSTRACT
Mesoscale gravity waves display periods of 1–4 h, have wavelengths of 50–500 km, and can have important effects upon the sensible weather. Real-time prediction, detection, and nowcasting of these mesoscale phenomena is shown to be feasible, due to recent major advances in operational observing and modeling systems. The ability to predict the likelihood of a gravity wave event rests upon recognizing the synoptic flow pattern in which such waves are consistently found to occur. The delineation of the most likely region for wave activity can be further refined by computing simple indicators of unbalanced flow and conducting a cursory search for a suitable wave “duct” with meso-Eta Model data. Particular emphasis should be placed on propagating unbalanced fields.

Whenever and wherever a suitable gravity wave environment is found, the Automated Surface Observing System pressure data should be carefully monitored for evidence of gravity wave activity. An automated gravity wave detection system is developed. It is shown that application of a time-to-space conversion adaptation of the Barnes objective analysis scheme to bandpass-filtered 5-min surface observations enables the detection of gravity waves with scales as small as 150 km and their separation from smaller-scale convective phenomena. This scheme requires accurate knowledge of the wave propagation velocity. A method is presented and successfully tested for this purpose, which is based on an adaptation of wave-ducting theory to the mesoscale model forecast fields.

The proposed procedure is demonstrated with a gravity wave event that occurred during STORM-FEST. A solitary wave of depression formed as an upper-level jet streak approached an inflection axis in the diffluent height field downstream of the Rocky Mountains. This wave generation region was diagnosed from mesoscale model forecasts as being unbalanced. A wave duct was diagnosed north of a warm front in both the model forecasts and the STORM-FEST soundings over the region traversed by the observed waves. The analyzed pressure and wind perturbation fields successfully portray the evolution of the gravity wave into a wave train as strong thunderstorms developed with the wave. The mesoscale model produced a gravity wave similar in most respects to that analyzed prior to the development of convection. These results suggest that mesoscale gravity waves can be predicted and analyzed with operationally available data and numerical model guidance.

1. Introduction
Most weather forecasters learn in their undergraduate or military courses in atmospheric dynamics that internal gravity waves result from the action of the restoring force of gravity acting upon air parcels that are displaced vertically in a statically stable atmosphere (e.g., Holton 1992). Forecasters may also have learned of the destabilizing effects of vertical wind shear on internal waves, or studied gravity–inertia waves, in which the Coriolis force further alters wave properties. Another class of gravity waves known as lee waves, which are forced by orography, is fundamentally important to the maintenance of the global momentum balance and to the energetics of the upper atmosphere. Gravity waves are excited by a wide range of phenomena, including convection, density impulses, geostrophic adjustment associated with unbalanced jet streaks and strong frontal systems, topography, and vertical shear instability. Some forecasters are aware that a particular class of gravity waves can exert important controls upon convection and mesoscale precipitation patterns, but in general, the operational community mistakenly perceives gravity waves as being too inconsequential, or occurring too infrequently, or being too difficult to forecast and diagnose, to be worthy of consideration in a daily forecast environment.

Issues that immediately arise in this weather fore-
casting context include the following: 1) What kinds of gravity waves are important to the weather? 2) How frequently do such waves occur? 3) Is there a single synoptic environment in which these kinds of waves are most likely to occur, so that pattern recognition techniques can be used in predicting the occurrence of significant gravity waves? 4) Can operational mesoscale models be used to forecast the likelihood of a gravity wave event or even specific gravity waves? 5) Are the data and analysis tools that are necessary to detect and track these waves available operationally? 6) How do such waves interact with clouds and precipitation? This paper attempts to address these questions. We review the evidence for a class of waves termed “mesoscale gravity waves” that has been shown to produce quite significant effects upon the sensible weather and that gravity waves” that has been shown to produce quite frequent. Recent research using microbarograph data and high-resolution radar imagery from a mesoscale field experiment suggests that mesoscale gravity waves in the wintertime in the central United States may occur quite frequently (34% of the days studied) and that even the strongest wave events with associated precipitation bands occur about 10% of the time (Jin et al. 1996). Furthermore, the strongest events tend to occur in a specific synoptic flow pattern, thereby minimizing the need to be constantly on the lookout for possible gravity waves.

Occasionally, the existence of large-amplitude gravity waves has misled forecasters. In one example, rapid pressure falls associated with a strong gravity wave misled forecasters into believing that secondary coastal cyclogenesis was under way (Bosart and Seimon 1988). Schneider (1990) presented a case in which the minimum pressure associated with one of the wave troughs was lower than that of the synoptic-scale cyclone through which it passed; the isallobaric fall region associated with the wave was noted by forecasters, who lacking knowledge of the wave, mistakenly attempted to correct the model forecasts of the cyclone track.

Unfortunately, we lack a sufficient understanding of the general characteristics and climatology of mesoscale gravity waves, whether there exists a dominant mechanism for exciting and sustaining the waves, and the nature of their interactions with convection and other meteorological phenomena. Mesoscale gravity waves present a real challenge to weather forecasters, because while they are understandably important to the prediction of mesoscale cloud and precipitation patterns, they have been nearly impossible to routinely detect with standard synoptic observations and are not reliably predicted by the current operational models.

The primary reason for the current poor understand-
ing of gravity waves and the acknowledged skepticism in the operational forecast community about their importance is that it is both difficult and tedious to conduct a study of even a single gravity wave event. A traditional case study using synoptic-scale data requires, first of all, that the barographs be digitized and then subjected to digital bandpass filtering and cross-spectral analysis. Bandpass filters are used (e.g., Stobie et al. 1983; Koch and Golus 1988) to extract the mesoscale wave signals from a background of synoptic-scale pressure signals, the semidiurnal pressure oscillation, and higher-frequency phenomena. Finally, the researcher painstakingly attempts to analyze the waves in the absence of detailed spatial information by typically resorting to subjective techniques involving the synthesis of satellite and radar imagery to attempt to track the waves between the stations. Special datasets collected in mesoscale field projects (Koch and Golus 1988; Ferretti et al. 1988; DeMaria et al. 1989) have helped overcome this problem for research studies, but not for operational forecasting.

This dismal picture is rapidly changing because of the National Weather Service (NWS) modernization and restructuring. Therefore, it is of utmost importance for forecasters to understand the basic dynamics of gravity waves and to be able to decipher the existence of gravity waves in model forecasts and observations. The opportunity now presents itself for the research and operational meteorological communities to work together to develop a much improved climatology and basic understanding of mesoscale gravity waves. Because of the recent implementation of the Automated Surface Observing System (ASOS) and the other operational observing systems, as well as digital mesoscale model data [such as the meso-Eta Model from the National Centers for Environmental Prediction (NCEP)], operational prediction, detection, and analysis of these important mesoscale phenomena is now feasible. Operationally available mesoscale data with high temporal resolution include 5-min sampling by ASOS, 6-min wind profiles from the National Oceanic and Atmospheric Administration (NOAA) wind profiler network, and wind profiles obtainable from the WSR-88D velocity–azimuth display. Wind profilers and doppler radars can be used to examine gravity wave vertical structure even in the presence of precipitation, as long as it is not convective (Koch et al. 1993; Ralph et al. 1993; Ramamurthy et al. 1993; Trexler et al. 1996). Since the meso-Eta Model employs a high-resolution grid (29 km), forecasters should be sensitive to the expectation that this model will be capable of producing significant mesoscale gravity waves. Barnes et al. (1996) recently showed that the meso-Eta Model produces coherent gravity waves with wavelengths under 300 km and that these waves can actually dominate the quasigeostrophic fields. Furthermore, strong model coupling of grid-resolvable precipitation and gravity waves should be anticipated in a moist environment.

The objective of this paper is to examine the potential utility of an automated procedure that can be implemented in NWS forecast offices for real-time nowcasting of mesoscale gravity waves using the meso-Eta Model, ASOS, and digital radar (and satellite) data. The discussion of this proposed procedure references the flowchart in Fig. 1. Generally speaking, the first step is to examine the synoptic environment with mesoscale model and rawinsonde data, and apply conceptual models to predict the probability of a gravity wave event. Section 2 discusses this methodology following a review of our present understanding of the environment and initiation–maintenance mechanisms for mesoscale gravity waves. The next step, explained in section 3, involves application of an automated wave detection procedure to ASOS data and synthesis of the results with detailed radar reflectivity composites and satellite imagery. This bandpass filtering–objective analysis technique for tracking mesoscale gravity waves is subjected to tests upon analytical waveforms to examine the expected analysis errors. The entire procedure is demonstrated in a real-data application in section 4, wherein digital surface, radar, and mesoscale model data from the STORMFEST experiment (Cunning and Williams 1993) are utilized.

2. Forecasting of gravity wave occurrence and maintenance

In an attempt to determine the most prevalent excitation mechanism(s) for weather-producing mesoscale gravity waves, Uccellini and Koch (1987, hereafter UK87) performed a climatological survey of 13 published cases. They found that in all the cases, a common synoptic pattern, shown as a conceptual model in Fig. 2, was present. Gravity waves were found to occur in a diffuent region bounded by the 300-hPa ridge axis to the northeast, the 300-hPa inflection axis to the southwest, and a surface frontal boundary to the southeast, though an explanation for the northern boundary of wave activity was not provided. UK87 emphasized that this common synoptic pattern was not sufficient by itself to explain mesoscale gravity wave occurrences, but that it was additionally necessary to have a jet streak propagating away from the upper-level trough and toward the ridge axis. When the jet streak propagates to a position downstream of the geostrophic jet maximum located at the base of the trough, gravity waves maybe triggered near the axis of inflection in the height field, for reasons explained below.

Although there is some variability among individual events (e.g., it is uncertain how far into the warm sector the waves can appear), mesoscale gravity wave cases that have been published since UK87 (Ferretti et al. 1988; Bosart and Seimon 1988; Lin and Goff 1988; Schneider 1990; Powers and Reed 1993; Ramamurthy et al. 1993) fit the general picture as well. Cyclogenesis is not required for gravity wave occurrence (e.g., see
Fig. 1. Flowchart for proposed operational procedure for predicting and detecting mesoscale gravity waves (MGW). Prediction of MGW consists of determining whether a suitable synoptic environment exists from analysis of surface and upper-air (SFC/UA) data and the meso-Eta Model. Upon satisfying those conditions and determining a region of high MGW potential, the forecaster then activates an automated MGW detection system based on the objective analysis of bandpass-filtered pressure and wind data from ASOS, and examines the relationships of the perturbation fields to mesoscale precipitation patterns to arrive at a precipitation nowcast.

Stobie et al. 1983), nor is it even correlated with wave amplitude. The apparent robustness of this conceptual model leads us to suggest that it can be used by forecasters to make an initial screening check on the likelihood of mesoscale gravity wave occurrence. Whenever such a pattern is found, we suggest that the forecaster should then examine quantitative diagnostic analyses from the meso-Eta Model to determine whether the synoptic environment is dynamically unbalanced and whether it can maintain coherent waves (Fig. 1). The methodology for accomplishing these objectives is explained in the following sections.

a. Diagnosing unbalanced flow from mesoscale model data

The close relationship between mesoscale gravity waves and the upper-tropospheric jet led UK87 to focus on two jet-related mechanisms for generating the waves, namely, shear instability (Einaudi et al. 1978/79) and geostrophic adjustment. The frequent appearance of mesoscale gravity waves in the vicinity of jet exit regions suggested a relatively more important role played by geostrophic adjustment. Van Tuyl and Young (1982) showed that gravity–inertia waves are preferentially generated just downstream of a jet core when the flow is “unbalanced” in the sense that the Lagrangian Rossby number

\[
Ro_L = \frac{|\lambda V(z)|}{f V(z)},
\]

which measures the relative importance of the parcel acceleration to the Coriolis acceleration, is larger than 0.5. Although \(Ro_L\) requires computation of air parcel properties, which is impractical in an operational environment, fortunately an easily calculable surrogate for \(Ro_L\) can be derived from consideration of jet streak dynamics (Koch and Dorian 1988), as
V

This expression is derived by simply replacing the numerator in (1) with the inviscid horizontal equation of motion, \( \mathbf{k} \times \frac{d\mathbf{V}}{dt} = f \mathbf{V}_{ag} \). Thus, instead of being concerned with the parcel acceleration, one merely examines the component of the ageostrophic wind that is directed across height contours if it is in the sense of being “unbalanced” (defined immediately below).

The transverse ageostrophic wind is associated with the deceleration of air parcels in the exit region of a balanced straight jet streak (Uccellini and Johnson 1979). Small \( Ro_t \) is required under the quasigeostrophic system, which is the theoretical basis for understanding jet streak dynamics and the associated ageostrophic winds (typically, the “four cell” pattern of vertical motions). High \( Ro_t \) as defined by a large cross-contour ageostrophic flow directed in a sense that is inconsistent with quasi- or semigeostrophic dynamics—namely, flow from the anticyclonic side of the jet toward lower heights in the exit region of the geostrophic jet—would indicate “imbalance” (Fig. 2). Large \( Ro_t \) and highly unbalanced ageostrophic winds were shown by Koch and Dorian (1988) to have singularly existed in the local generation region for gravity waves that occurred during a mesoscale field experiment.

Large parcel divergence and \( Ro_t > 0.5 \) have been shown to occur as a jet streak approaches a highly diffluent area (as in Fig. 2) in mesoscale model simulations by Kaplan and Paine (1977), Uccellini et al. (1985), and Zack and Kaplan (1987). The latter study is especially pertinent, since these indicators of unbalance occurred in the generation region for gravity waves that were diagnosed by Ferretti et al. (1988). The existence of large parcel divergence violates the nonlinear balance equation (NBE). Zack and Kaplan (1987) found that the highly divergent region displayed a “significantly” nonzero sum \( (\sim 10^{-8} \text{ s}^{-2}) \) of the four terms composing the NBE:

\[
\nabla^2 \phi + 2J(u, v) + f\gamma - \beta u = 0, \quad (3)
\]

where the terms are the Laplacian of geopotential, the Jacobian of the winds, the vorticity term, and the beta-effect term, respectively. Although NBE calculations performed with rawinsonde data may contain considerable uncertainty (Moore and Abeling 1988), the use of mesoscale model data in the operational arena should make it possible to reliably and simply compute the terms of the NBE [e.g., using Automated Weather Information Processing System (N-AWIPS) on the meso-Eta Model data]. This is demonstrated in section 4.

In summary, it is recommended that as digital mesoscale model data is made available operationally, both NBE and \( Ro_t \) be calculated as quantitative indicators of unbalanced flow. Regions diagnosed as being unbalanced mark where significant mesoscale gravity waves may form (they typically propagate downstream of the unbalanced regions). It is important to realize that regions near and downstream of complex orography can often display unbalanced signals and that mesoscale gravity waves have been shown to be produced in such regions (Koch and Dorian 1988; Kaplan and Karyampudi 1992; Jin et al. 1996); the forecaster should be on the lookout for such regions and, especially, for unbalanced fields that propagate. These tests serve as the second level of screening that should be evaluated on a routine basis to check for the likelihood of a significant gravity wave event (Fig. 1). We propose that these fields be computed and examined to refine the most likely region for significant gravity wave activity.

b. Interactions between gravity waves and convection

Convection can generate a broad spectrum of waves, inclusive of everything from short-period waves excited by the development of convective cells along a thunderstorm gust front (Yang and Houze 1995) to largewavelength disturbances resulting from the impulsive release of latent heat in a thunderstorm complex (Lin and Goff 1988; Rottman et al. 1992). The strong wave of depression studied by Bosart and Seimon (1988) formed behind an intense squall line in an area of strong static stability, although this disturbance also shared certain similarities with a wake low often seen with mesoscale convective systems (Johnson and Hamilton 1988). The first attempt to simulate the role of organized convection in the generation of mesoscale gravity waves was by Zhang and Fritsch (1988). The intense gravity
wave event studied by Schneider (1990) was simulated with a mesoscale model by Powers and Reed (1993). Their results indicated that convection was essential for exciting, maintaining, and amplifying the waves. However, Pokrandt et al. (1996) recently cast considerable doubt on this idea, because they showed that the actual wave genesis region was far upstream of the convection and was instead linked to an upper-level jet in a manner more consistent with the Uccellini and Koch (1987) conceptual model. Furthermore, they discuss the present inability of mesoscale models to produce mesoscale waves in the presence of convection for the right reasons due to continued need of models to parameterize sub-grid-scale convection. The interactions between convection and mesoscale gravity waves clearly remains controversial and poorly understood.

According to wave-CISK theory (e.g., Raymond 1984), organized convection is forced by convergence associated with a gravity wave, while latent heat release within the convection provides a source of wave energy. Many mesoscale gravity wave case studies have noted that precipitation bands are preferentially located just ahead of the pressure ridges. A schematic diagram of a ducted mesoscale gravity wave for which the height of the critical level (an altitude where the wave propagation speed $C$ matches the wind component in the direction of wave propagation $\vec{u}$) is equal to one-half the vertical wavelength of the wave (referred to as a “node”) is shown in Fig. 3. The quantity $\vec{u}$ is the wind component in the direction of wave propagation

$$\vec{u} = -u \sin \theta - v \cos \theta,$$

where $\theta$ is along the wave vector (i.e., it is the direction from which the wavefronts are propagating). Note that the vertical motions and resultant isentrope fluctuations associated with a ducted wave are strongest midway through this layer; that is, the wave energy is “ducted” within this layer. In addition, the strongest vertical motions are located halfway between the surface pressure trough and crest (where the perturbation pressure $p' = 0$). These phase relations, which are necessary for the wave to propagate in the direction shown with intrinsic (mean flow-relative) velocity $C^* = C - \vec{U}(z)$ [where $\vec{U}(z)$ is the background wind at a given altitude] imply that the maximum precipitation rate (convective latent heating) occurs just ahead of the wave ridge. According to wave-CISK concepts, convection amplifies the wave trough by compensating subsidence, and it enhances the wave ridge by evaporative cooling of precipitation. Wave-CISK further requires that the strongest convective vertical motions exist at the wave critical level, instead of midway through the stable layer as seen for ducted waves, so that the gravity wave and the convective system move in tandem. These relationships can be diagnosed from mesoscale model output by examining isentropic cross sections that cut perpendicular to

![Fig. 3. Schematic depiction of a ducted mesoscale gravity wave with one-half of a vertical wavelength contained between the ground and a critical level. (a) Vertical cross section in the direction of wave propagation, showing wave-induced horizontal and vertical wind motions (arrows), streamlines or isentropes (solid lines), and the critical level (dashed), for a wave that is propagating with intrinsic (mean flow-relative) phase speed $C^*$ faster than the winds in the duct layer. (b) Vertical profiles of parcel vertical displacement ($\eta$) and adiabatic temperature change ($\Delta T$) applicable for the phase of the wave marked by the heavy arrow in part (c). (c) Wave-induced surface pressure perturbations ($p'$) and wind perturbations in the direction of wave propagation ($\vec{u}'$) drawn for the same wave segment shown in (a). Notice that $\vec{u}'$ and $p'$ are in phase and that they lag behind the phase of the vertical motion within the duct by 90° (after Ralph et al. 1993). Cloud depicts location of rainband axis relative to gravity wave system for a simple nontilted wave structure.](image-url)
the suspected gravity waves. Indeed, the conceptual model in Fig. 3 is extremely useful in the proper interpretation of mesoscale model output, which contains vertical motions caused by quasigeostrophic dynamics, convection, topography, and other sources in addition to gravity waves. Despite the fact that wave-CISK and ducted waves display different vertical motion profiles, convection should not disrupt the phase relationships between the temperature, pressure, and wind perturbations, except perhaps at the surface (Koch et al. 1988; Powers and Reed 1993). It is also possible to find ducted wave-CISK modes, in which the bottom part of the wave displays a local vertical motion maximum without any tilt (as in Fig. 3) and the upper parts of the wave display a second (often stronger) upward velocity center with vertical tilt. Topographically generated disturbances are also easily recognized, since they do not propagate, and hydrostatic mountain waves display upstream tilts.

c. The role of wave-ducting processes in mesoscale gravity wave maintenance

According to the linear theory of Lindzen and Tung (1976), which has been applied in numerous gravity wave case studies, the following atmospheric conditions must be present for an efficient wave duct (Fig. 4): 1) there must be a layer of large static stability of depth \( D_1 \) in the lower troposphere; 2) there must not be a critical level within the lower stable layer, as this would lead to absorption of wave energy; 3) \( D_1 \) must be sufficiently thick as to contain one-quarter of the vertical wavelength that corresponds to the observed phase speed; and 4) there must be a “reflecting layer” of depth \( D_2 \), characterized by conditional instability, in order to prevent the wave from propagating its energy out of the stable duct layer.

Given a suitable stable layer, the intrinsic ducted phase speed \( C_\ast \) is found from

\[
C_\ast = \frac{2N_1D_1}{\pi} = \frac{2}{\pi} \left[ gD_1(\Theta_f - \Theta_b) \right]^{1/2}, \tag{5}
\]

where \( N_1 \) is the Brunt–Väisälä frequency within the stable layer, \( \Theta_f (\Theta_b) \) is the potential temperature at the top (bottom) of the stable duct layer, and \( \Theta \) is the average value within the layer. Thus, the properties of the duct \((N_1, D_1)\) uniquely determine the phase speed of the ducted gravity waves. Criterion 3 above stipulates that the stable layer must have a thickness of at least

\[
D_1 > \frac{\pi C_\ast}{2N_1} = \frac{\pi}{2} \frac{C_\ast}{N_1}. \tag{6}
\]

The “duct depth” is the depth of the statically stable layer, which must be compared to \( D_1 \) from (6) to determine if a suitably thick duct is present. Most recent case studies of gravity waves have examined the ducting characteristics of the regional atmosphere. Typically, a duct is found and the Lindzen and Tung (1976) ducting criteria are met. Computations of the ducted phase speed \((C_d = C_\ast + \bar{U})\) support the notion that the disturbance is a ducted gravity wave when these phase speeds are reasonably close to those observed at the surface (i.e., when \( C \approx C_d \)).

Little is known about the possible relationship between duct structure and wave amplitude. Schneider (1990) found that maximum wave amplitude corresponded to the area where weak midlevel static stability overlaid a strongly stable layer, but the region of ducting was much larger than the region of gravity wave activity. Bosart and Seimon (1988) observed rapid amplification of a wave as it entered a highly stable cold air damming region east of the Appalachians. While these results suggest that wave motion and amplitude might be predictable using ducting theory, rapid changes in duct structure can occur on the 3-h timescale (Ferretti et al. 1988), thus limiting the utility of standard 12-h soundings for this purpose. Reliance on mesoscale model forecast output instead of rawinsonde observations for calculating duct properties remains largely untested, with the single exception of the study by Powers and Reed (1993). Although the duct was determined from the model fields to be highly efficient in the region of...
strongest wave amplitudes, the duct depths were insufficient to support the modeled wave speeds, and an efficient duct was found over a much larger region than the area affected by the gravity waves (in concurrence with Schneider 1990). Another major problem with using ducting theory is that the phase speed calculations are highly sensitive to details within the sounding. Some of the details in rawinsonde profiles can actually be produced by the waves, and few of the details should be expected to be retained by a mesoscale model. Yet, we show below that a simple extraction from ducting theory can be used to obtain a reasonably accurate prediction of the wave propagation velocity and that this is essential for operational wave detection.

3. Gravity wave detection and analysis using surface data

The unbalanced flow diagnostics and ducting analysis serve the dual purposes of refining the predicted area for mesoscale gravity wave activity and establishing a “confidence level” (an increased probability of significant and long-lasting wave activity when all the test results are positive). Having thus been alerted to the likelihood of a gravity wave event, an automated gravity wave detection system should then be activated and monitored over this region by the forecaster (Fig. 1). The remainder of this section describes this proposed ASOS-based detection system.

a. The process of gravity wave detection and analysis using ASOS

It is helpful to first review the “typical” procedure that the researcher follows to isolate and study gravity waves before we consider how this process can be automated. The researcher first digitizes barographs from the region of interest, and then subjects the time series data to a digital bandpass filter to reveal the presence of the waves as pressure perturbations ($p'$). The choice of filter depends upon whether a power spectrum analysis of the pressure data shows a peak in one or more frequency bands (Koch and Golus 1988), or whether one is interested in some preselected wave period (frequency). A cross-spectral analysis should then be performed, using the time delays between all possible pairs of stations in the region to find the propagation velocity of the wave disturbance. This step in the process can be very time consuming when the disturbance is not, as is often the case, a planar wave (meaning that the wave fronts are arcs instead of lines), or local effects like thunderstorm pressure signals are present, or the phase velocity varies temporally or spatially, or waves of similar frequency but differing phase velocities interact. Finally, the researcher attempts to subjectively analyze the spatial wave patterns from the $p'$ fields, together with satellite and radar imagery, to be able to track these mesoscale disturbances between the barograph sites. This is a very tedious and labor-intensive process, and certainly not one amenable to operational adoption.

The availability of digital, high temporal resolution data from ASOS is a very important factor in determining whether operational gravity wave detection is feasible. For one thing, ASOS allows for automated bandpass filtering of digital pressure time series data, since it overcomes the traditional problem of needing to digitize barographs. A prespecified filter whose bandpass frequency interval corresponds to the 1–4-h observed periods of mesoscale gravity waves (UK87) can be chosen. Thus, there is no need to conduct power spectrum analysis. Another advantage realized with ASOS concerns its temporal aspects. The availability of 1–5-min time resolution data can be exploited in a powerful way to open the door to real-time mesoscale analysis. This is true even though the ASOS network (Fig. 5a) does not have a greatly improved spatial resolution over the current surface observing network and is plagued by spatial irregularity caused by having clusters of stations around major metropolitan areas. Adaptation of time-to-space conversion principles to an objective analysis scheme makes it possible to track mesoscale disturbances between the ASOS stations. This capability depends upon having a reasonably accurate estimate of the advection velocity used in the transformation. This vector clearly is the same as the mesoscale gravity wave phase velocity $C$ if one is attempting to track gravity waves. Our proposed method eliminates the need to conduct spectral analyses to estimate $C$, and it makes subjective mesoanalyses of the mesoscale disturbances obsolete and unnecessary.

We begin with the examination of the time-to-space conversion analysis scheme and its sensitivity to errors in the advection velocity vector for a wide range of wave types. We then examine various methods to estimate the wave phase velocity $C$ for use as this vector. Cross-spectral, mesoscale model, and wave-ducting theory approaches are compared for this purpose. Finally, we describe and test a simple method for reliably estimating $C$ from a mesoscale model.

b. An objective analysis technique for analyzing mesoscale gravity waves

Fujita (1963) showed that it is possible to analyze mesoscale pressure systems from synoptic-scale data by using the time series observations at individual stations as surrogates for the sparsely distributed observations. By identifying a steadily propagating disturbance, determining its phase velocity, and using the phase velocity to “advect” off-time observations, one could effectively create “new” observations. During this “time-to-space conversion” (TSC) process, the analyst subjectively determines the value of each off-time observation relative to the current-time data. This typically results in the off-time data being used mainly to
fill in structure between stations. Bosart and Seimon (1988) were the first to apply this subjective TSC technique to the analysis of mesoscale gravity waves. Barnes (1973) showed that TSC could easily be incorporated into an objective analysis scheme. Koch and McCarthy (1982) applied the Barnes TSC technique with mesoscale data, using observed thunderstorm motion to specify the advection velocity vector. They demonstrated that storm features with scales of only twice the average station spacing $\Delta n$ could be reliably resolved. It is not possible to resolve $\lambda = 2\Delta n$ wavelengths with any objective analysis scheme that does not use time series data.

The GEMPAK Barnes objective analysis scheme (Koch et al. 1983) was used to perform the analyses that follow. GEMPAK is widely employed for meteorological analysis at universities, government laboratories, and national and international meteorological centers, and forms a central part of the N-AWIPS software package being employed at NWS offices. We follow the methodology of Barnes (1973), whereby each observation is given a weight that decreases with distance from the grid point being analyzed, as well as the time difference from the analysis time. In so doing, there is no requirement that the gravity wave be in strict “steady state,” though the TSC principle does require that the wave be steadily translating over the period $\tau$ used to convert temporal off-time observations into spatial data (approximately 45 min, as shown in the appendix). The selection of an appropriate value for the parameter $\tau$, which also controls the response characteristics of the objective analysis, is explained in the appendix. In general, larger values for $\tau$ should be used for relatively steady disturbances, and smaller values for rapidly changing ones.

The TSC analysis technique was tested using both planar and nonplanar (arc shaped) analytic waveforms. The purpose of these sensitivity tests was to evaluate the technique, to attempt to determine suitable analysis values for $\tau$, and to examine the effects of uncertainties in advection (phase) velocity on the ability to objectively analyze the waves. The analysis display domain was based on the STORM-FEST surface data network (Fig. 5b). Stations shown outside this domain (the dashed rectangle) also affected the analyses, in order to avoid extrapolation problems near the domain boundaries. The stations are fairly regularly spaced, with the exception of a high-density network in northeastern Kansas composed of stations that are separated by only 15 km. This kind of irregularity is also characteristic of the full ASOS network (Fig. 5a). The mean station spacing for the network used in our tests, which contains the advantage of the special mesonet stations available during STORM-FEST, is $\Delta n = 56$ km, but if only the ASOS stations are used, $\Delta n$ becomes 80 km. Because the focus of this paper is on future operational wave detection, $\Delta n$ was set to 75 km and the grid mesh $\Delta x$ was set to 35 km in the following tests.

**Fig. 5.** (a) Distribution of ASOS stations when network is fully deployed; (b) surface stations used for the full mesonetwork analytical wave tests (dashed rectangle is the analysis display domain shown in subsequent figures) and the 14 February 1992 gravity wave case study (open triangles depict digital 5-min data, filled triangles represent digitized 15-min resolution barograph sites); (c) approximation of future ASOS network used in “future ASOS” analytical tests.
1) PLANE PERIODIC WAVE

The first set of tests is for a plane periodic wave of amplitude 100 units moving northeastward with a phase velocity of $C = 220^\circ$, $20 \text{ m s}^{-1}$ (Fig. 6). Three different wavelengths were examined (75, 150, and 300 km), corresponding to the $\Delta n$, $2\Delta n$, and $4\Delta n$ waves for this network. This wave motion was chosen because it is similar to that in the STORM-FEST case discussed in section 4, as well as in many observed gravity wave events. The standard Barnes objective analysis is obtained by sampling the analytic wave at the station locations shown in Fig. 5b and objectively analyzing the "observations" to the grid. The TSC Barnes analysis is performed by converting the off-time data at the stations, obtainable by using the well-known function for a traveling plane wave, to spatial data by using the $C$ value to advect the data along the phase velocity direction and then analyzing the observations to the grid. Results in terms of root-mean-square analysis errors (rmse) for the three waves are presented in Fig. 7. The rmse is computed from the difference between the analytic field and the analyzed value at each grid point. The rmse is plotted as a continuous function of $\tau$ for the TSC Barnes analyses, with the various wavelengths indicated by different symbols. Corresponding values for the conventional Barnes analysis without TSC are shown by the isolated points along the ordinate.

Results confirm that better analyses are produced for longer wavelengths since they are better resolved. For example, when using the standard Barnes analysis, the rmse is 30% of the full amplitude of the 300-km wave, whereas an unacceptably large rmse of 65% results for the unresolvable 150-km wave. The TSC analysis with $\tau = 70 \text{ min}$ results in a drastic lowering of the rmse to only 10% for the 150-km wavelength. The analysis error decreases monotonically with increasing $\tau$ for all wavelengths, but most of the benefits are realized by $\tau = 40 \text{ min}$. Assuming that analysis errors of 20% are acceptable, then these results indicate that it is clearly possible to detect and analyze $2\Delta n$ (150 km) plane periodic waves using the TSC Barnes scheme, at least over the complete mesonetwork shown in Fig. 5b.

The analyses of the 150-km wave produced by the original Barnes scheme and the TSC Barnes scheme for three different $\tau$ values are shown in Fig. 8. The Barnes analysis produces a very poor representation of the wave that would be essentially useless for further analysis (rmse = 65, according to Fig. 7). This result is in agreement with accepted sampling theory, which dictates that the $2\Delta n$ wave cannot be represented in an objective analysis. By contrast, the TSC analysis resolves this $2\Delta n$ wave with improved accuracy as $\tau$ increases, at least away from the domain corners. Since the relative added benefit is arguably small beyond $\tau \approx 50 \text{ min}$, we recommend this $\tau$ value for real-time analysis of this kind of wave. This value is nearly identical to that recommended by the theoretical analysis in the appendix.

While these results indicate that the TSC analysis can resolve a 150-km periodic wave using data from a high-density observation network (the one in Fig. 5b), the future operational ASOS network will feature lower resolution. Therefore, the TSC technique was tested on the network shown by the "Future ASOS" symbols in Fig. 5c, which is designed to approximate the fully completed ASOS network in the central United States. To facilitate comparison with the STORM-FEST analyses, we used the same grid size (35 km) and station spacing (75 km), the latter of which determines the Barnes analysis properties (appendix). Resulting rmse's for the 150-km wave are higher, as shown in Fig. 7 by the
Fig. 7. Root-mean-square analysis errors (rmse) averaged over the domain shown in Fig. 6 for analytical plane waves of 75-, 150-, and 300-km wavelengths. The “Barnes” values on the plot show the analysis errors for a customary Barnes objective analysis, whereas the other values show the analysis errors as a function of the TSC conversion interval $\tau$ (min) in the Barnes time-to-space conversion (TSC) scheme. The “Barnes ASOS” and “TSC ASOS” values represent the analysis errors resulting from using only the “Future ASOS” station data (see Fig. 5c).

Fig. 8. Objective analyses of the 150-km wavelength plane periodic wave from (a) a customary Barnes analysis scheme, and the TSC Barnes analysis scheme with (b) $\tau = 25$ min, (c) $\tau = 50$ min, and (d) $\tau = 75$ min. Compare these analyses to the true analytical waveform shown in Fig. 6.
FIG. 9. Analytical wave as in Fig. 6 except for arc-shaped, nonplanar wave.

"Barnes ASOS" symbols. Nevertheless, the TSC analyses "successfully" reproduce the wave (rmse \( \approx 20 \)) for \( \tau > 60 \) min.

These results are important because they indicate that it should be possible to analyze plane periodic gravity waves with wavelengths of \( \sim 150 \) km from the operational ASOS data, insofar as the "ASOS network" shown in Fig. 5a is representative of the actual ASOS network. Of course, there are regions in the eastern United States where major gaps will exist (e.g., over Kentucky and Tennessee), but there are also large regions where higher station density than that used in our tests can be found (e.g., over Indiana and Ohio). Thus, the station network chosen for our analytical wave tests is quite representative of the overall ASOS distribution. Since the vast majority of mesoscale gravity wave events reported in the literature display wavelengths \( \lambda > 150 \) km, these test results are very encouraging for real-time detection of planar mesoscale gravity waves.

2) NONPLANAR PERIODIC WAVES

While some mesoscale gravity waves display a plane wave appearance (Uccellini 1975; Pecnick and Young 1984), it seems that most of them exhibit arc-shaped wave fronts (e.g., Koch and Golus 1988; Ferretti et al. 1988; Ramamurthy et al. 1993). The wave from the STORM-FEST case (section 4) is also highly nonplanar. Therefore, the arc-shaped wave shown in Fig. 9 was used to test the ability of the TSC technique to resolve a nonplanar wave and to determine the number of "advection vectors" needed. The wavelength was set to 150 km, since our focus is on the 2D wave.

The graphs presented in Fig. 10 show that when 60 translation vectors were used (one at each station in the full mesonetwork), the rmse’s were only slightly higher than those of the plane periodic wave (Fig. 7). Most importantly, there is little sensitivity to the number of translation vectors used; in fact, the analysis using only 4 vectors produced equivalent results to the 60-vector analysis, for all practical purposes.\(^1\) Actually, there is not a large degradation of the analysis, even when only one vector is used. In fact, the nonplanar characteristics can be safely ignored by judiciously choosing a \( \tau \) value in the range of 40–60 min as recommended above, since that minimizes the error. These results have important implications for the issue of whether errors in estimating \( C \) severely limit application of the TSC scheme.

3) MULTIPLE PLANE WAVES

Examination of pressure time series from almost any gravity wave case, whether solitary or periodic, reveals variations of wave structure from station to station, even when stations are in close proximity. Some of these variations probably reflect the effects of outside forces such as topography, convection, and variations in atmospheric duct structure. However, some of the variations could also reflect the presence of multiple waves, possibly of different amplitude, wavelength, and frequency, occurring in the same area (Seimon 1994). Here, the ability of the TSC technique to resolve a relatively simple multiple wave event is explored.

The test case consists of the same wave used earlier (150-km wavelength disturbance with amplitude 100 units, moving northeastward at 20 m s\(^{-1}\)), superposed with a 75-km wave with amplitude 50 units, moving eastward at 15 m s\(^{-1}\). The resulting analytic field for the chosen time is shown in Fig. 11a. The presence of the secondary wave deforms the primary wave into a series of isolated, ellipsoidal maxima and minima or-

\(^1\) Variations in phase speed as well as direction would probably also require additional translation vectors, but this was not examined.
ented in varying directions, which are broadly similar to the radar signatures described by Seimon (1994). Since one is generally interested in the wave with the largest amplitude, and also because the primary wave is the only one that would be resolvable by the ASOS network, the TSC analysis was performed using the translation vector of that wave. The analysis using $\tau = 50$ min (Fig. 11b) successfully resolves the structure of the primary wave, though the magnitude is overestimated. Otherwise, the analyzed wave structure is very similar to that shown in Fig. 8c for the single primary wave.

c. Impact of phase velocity errors and a method for their minimization

The tests in the previous section were performed under the assumption that the phase velocity of the wave was known. However, the estimation of phase velocity $C$ may not always be straightforward. Tests were performed to evaluate the potential impact of phase velocity errors on the 150-km periodic wave disturbance. Results reveal significant sensitivities to uncertainties in both wave direction (Fig. 12a) and speed (Fig. 12b). Notice that, in general, the errors first decrease and then increase with increasing $\tau$ values, because at longer periods the effects of inaccurate time-to-space conversion outweigh the benefits of added off-time data. When both direction and speed errors are present (Figs. 12c and 12d), rmse’s are increased, especially for $\tau > 50$ min. The test results indicate that for $\tau = 55$ min analysis errors are only acceptable as long as wave phase speed and direction errors are $\leq 25\%$ (under 5 m s$^{-1}$ and 20°, respectively, for this case).

These test results provide useful guidance for the required accuracy of any phase velocity estimation technique, at least for the simplest of mesoscale gravity wave types. While subjective techniques are certainly not plausible approaches for estimating $C$ in an operational environment, there are a number of other candidates to consider. Spectral methods, operational mesoscale models, and wave-ducting theory are each evaluated for this purpose below. It must always be remembered that microbarographs record fluctuations in pressure that can arise from a wide variety of causes besides gravity waves, including fronts, thunderstorms, turbulent pressure fluctuations, and other unknown sources. Coherent signals that allow for the determination of a “propagation velocity,” even if they appear wavelike and have a dominant frequency, are not unambiguous indicators of gravity waves, though they will eliminate turbulent pressure fluctuations as the source. The clearest indicator of the presence of waves is a high perturbation pressure–wind correlation ($p \ n$), since this is consistent with the wave impedance relationship and is a direct measure of the wave signal-to-noise ratio (Gossard and
Sweezy 1974; Gossard and Hooke 1975), as is also evident in the wave schematic in Fig. 3. Since the wind perturbation in the direction of wave propagation ($\vec{u}^\prime$) is determined by the phase velocity $C$, this emphasizes the need to obtain a good estimate of the phase velocity, apart from the requirements of the TSC objective analysis.

1) SPECTRAL METHODS FOR DETERMINING THE PHASE VELOCITY

If a cross-spectral analysis is performed between all possible station pairs, the lags that maximize the cross-spectral amplitude can be used to determine $C$ by a least squares procedure (e.g., Einaudi et al. 1989; Karyampudi et al. 1995). This technique was tested over part of the high-density surface network used during STORM-FEST (Fig. 5b). One experiment involved seven of the mesonet stations in the dense network of portable automated mesonet (PAM) stations in northeastern Kansas, which are separated by only 15 km. A second set of tests was performed with the ASOS station MHK replacing one of the PAM stations; this increased the maximum between-station distance to $\Delta n_{\text{max}} = 120$ km, which is more representative of the future ASOS distribution (Fig. 5a). Separate tests were performed on sinusoidal plane periodic and solitary waves of varying wavelengths. Results (Table 1) reveal that the phase velocity for the 150-km wavelength plane periodic wave was nearly perfect for the small PAM array but failed...
miserably for the network with MHK included. This failure occurred because $\Delta n_{\text{max}}$ exceeded one-half the wavelength, which led to phase ambiguities in the cross-spectral analysis (Koch and Golus 1988). Results for the solitary wave were much better. The phase velocity estimates for a 300-km periodic wave were essentially error free for both station arrays and wave types.

Spectral techniques implicitly assume that there is one and only one propagation velocity and that it is constant over the network. Standard power spectrum techniques work best on a time series containing multiple monochromatic waves, but more typically, the signal is quite complex and characterized by a broad frequency band. In fact, when a power spectrum displays two peaks at different frequencies, it is not straightforward to decide whether the two components occurred at the same time or one after the other. In addition, it is necessary to determine the time window for the spectral analysis, in order to attempt to provide several wave cycles in the spectral analysis. This represents a major limitation for real-time gravity wave analysis. Short-lasting wave events (“wave bursts”) are particularly difficult to detect in power spectral analyses.

2) MESOSCALE MODEL PREDICTION OF WAVE PHASE VELOCITY

Since mesoscale models (operational and research) have the ability to produce a wide range of mesoscale phenomena including gravity waves, this naturally raises the question whether these models can be used to obtain an accurate estimate for the true advection vector (wave propagation vector) is 220°, 20 m s$^{-1}$.

Fig. 12. RMSEs as in Fig. 7 except showing the effects of uncertainties in the advection vector speed and direction on the analysis of the 150-km wave using the TSC Barnes scheme: (a) effects of 20° and 40° directional errors, compared to the control case with no directional errors (“Correct”); (b) effects of 5 m s$^{-1}$ and 10 m s$^{-1}$ speed errors; (c) effects of 20°, 5 m s$^{-1}$ and 20°, 10 m s$^{-1}$ vector errors; and (d) effects of 40°, 5 m s$^{-1}$ and 40°, 10 m s$^{-1}$ vector errors. The true advection vector (wave propagation vector) is 220°, 20 m s$^{-1}$.

---

2 A less restrictive approach is complex principal component analysis, which was applied by DeMaria et al. (1989) to a gravity wave case.

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to predict the value of $C$ accurately. Considerable research is occurring along similar lines; for example, Kaplan and Karyampudi (1992) simulated ~20-km-scale waves along a dryline similar to those observed by Koch (1979). The STORM-FEST case presented in section 4 demonstrates a more successful prediction of the phase velocity for an observed gravity wave. Additional research is clearly needed before mesoscale models can be recommended for the purposes of predicting wave phase velocity from the simulated gravity waves in the model.

3) Predicting the Phase Velocity from Gravity Wave Ducting Theory

Next we attempt to determine whether a relationship can be found between the observed wave propagation velocity and the characteristics of the environment, in particular the wave-ducting properties. We recall that the conditions required for an efficient wave duct from linear theory are that there exists a stable lower atmosphere with a conditionally unstable layer above, a critical level within the upper layer, and that the stable layer is sufficiently thick to accomodate the observed intrinsic phase speed. However, there are several problems with this approach. First, real-time detection of critical levels $Z_c$ is difficult, if not impossible, with the operational rawinsonde network, since these features can change very quickly (Ferretti et al. 1988), and as noted above, mesoscale models have not proven to be sufficiently reliable predictors of $C$ (hence, $Z_c$). Second, as discussed earlier, phase speed calculations from ducting theory are highly sensitive to details within the sounding, particularly the properties of the duct layer. Third, it is not the intrinsic phase speed that is needed for use as the TSC advection velocity, but rather the ground-relative value of $C$; unfortunately, this requires knowledge of the duct depth in order to know which layer-mean wind to add to the intrinsic phase speed ($C_{r} = C_{i} + \bar{U}$).

Nevertheless, ducting theory can still be of considerable value in predicting the ground-relative value of $C$. Since ducting theory requires a critical level to be present somewhere within the conditionally unstable layer above the duct layer, and since most of the problematic details in soundings (in particular, inversions and vertically varying wind direction) occur in the lower part of the troposphere, we argue that calculations of the vector mean wind in the conditionally unstable layer can be used as a surrogate to obtain an “accurate” estimate of $C$ ($\pm 5$ m s$^{-1}$ and 20$^\circ$). We will now show that even 12-h rawinsonde data can be used to estimate $C$ with sufficient accuracy in most cases.

The dataset used for the development of this method is based on publications of mesoscale gravity wave events over approximately the last 15 yr, when the most reliable and detailed studies have been performed. The wave event from Bosart and Seimon (1988) was excluded from the sample, since the “gravity wave” displayed many of the characteristics of a “wake depression” to the rear of a squall line. We altered the stated wave propagation vectors in two of the papers. The two wave events that were diagnosed by Ferretti et al. (1988) were reinterpreted as a single highly arc-shaped wave, since the pressure perturbation maps actually display such a character; when the “wave axis-tracking method” (Koch and Golus 1988) was applied to the combined wave instead of the “wave center-tracking method,” a wave propagation direction 20$^\circ$–50$^\circ$ to the left of the $C$ values quoted by the authors resulted. Second, since the 14 January 1989 wave event reported by Ramamurthy et al. (1993) displayed a highly variable wave propagation direction over the large region affected by the gravity wave, we used the orientation of the rainband associated with the wave as seen in the local radar display from Champaign, Illinois, to obtain a representative $C$ value near the Champaign rawinsonde site.

Table 2 shows the resulting nine case studies and 11 wave events used in our sample. A single sounding was chosen as most representative of each of the wave events whenever the authors relied on only one sounding. In other cases, our reason for selecting one among multiple soundings varied from whether the sounding occurred in the wave generation region, to whether the sounding contained a critical level with small Richardson number [this is a necessary condition for the gravity wave to be able to extract energy from the mean flow (e.g., see Gossard and Hooke 1975)]. We also noted when deep convection was present during, before, or after wave
Table 2. Observed wave propagation velocities and predicted velocities, using the simplified mean wind ducting theory method described in the text, for selected studies of gravity waves published in the literature. Critical levels are above ground level (AGL).

<table>
<thead>
<tr>
<th>Reference</th>
<th>Wave event</th>
<th>Representative sounding*</th>
<th>Deep convection present?</th>
<th>Nonplanar wave fronts?</th>
<th>Critical level altitude</th>
<th>Wave velocity</th>
<th>Mean wind</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>mb km</td>
<td>(m s⁻¹)</td>
<td>(°)</td>
</tr>
<tr>
<td>Bosart and Sanders (1986)</td>
<td>Waves B + C</td>
<td>GS0/1200/11 Feb 83</td>
<td>Later</td>
<td>No</td>
<td>457 6.3</td>
<td>235 16.0</td>
<td>241 22.2</td>
</tr>
<tr>
<td>Koch and Dorian (1988)</td>
<td>Wave episode I</td>
<td>KNO/1009/12 Jul 81</td>
<td>Yes</td>
<td>Yes</td>
<td>500 5.6</td>
<td>243 19.0</td>
<td>239 24.9</td>
</tr>
<tr>
<td>Lin and Goff (1988)</td>
<td>Wave episode I</td>
<td>DAY/1200/06 Mar 69</td>
<td>Prior</td>
<td>No</td>
<td>315 8.9</td>
<td>240 55.0</td>
<td>253 39.5</td>
</tr>
<tr>
<td>Penczak and Young (1984)</td>
<td>Composite</td>
<td>CCOM/0211/11 Apr 79</td>
<td>Yes</td>
<td>No</td>
<td>269 9.9</td>
<td>240 32.0</td>
<td>210 27.4</td>
</tr>
<tr>
<td>Powers and Reed (1993)</td>
<td>Average</td>
<td>SLO/0000/15 Dec 87</td>
<td>Later</td>
<td>Yes</td>
<td>605 4.1</td>
<td>202 28.7</td>
<td>227 33.0</td>
</tr>
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<td>Ramamurthy et al. (1993)</td>
<td>5 Jan 1989</td>
<td>CMI/2100/5 Jan 89</td>
<td>Yes</td>
<td>No</td>
<td>492 5.7</td>
<td>260 25.0</td>
<td>234 25.2</td>
</tr>
<tr>
<td>Ramamurthy et al. (1993)</td>
<td>14 Jan 1989</td>
<td>CMI/1230/14 Jan 89</td>
<td>Yes</td>
<td>No</td>
<td>356 8.0</td>
<td>220 29.0</td>
<td>220 25.7</td>
</tr>
<tr>
<td>Schneider (1990)</td>
<td>Average</td>
<td>UMN/0000/15 Dec 87</td>
<td>Later</td>
<td>Yes</td>
<td>645 3.6</td>
<td>207 32.3</td>
<td>217 37.8</td>
</tr>
<tr>
<td>Stobie et al. (1983)</td>
<td>Wave A</td>
<td>LBF/1400/9 May 79</td>
<td>Yes</td>
<td>No</td>
<td>600 3.3</td>
<td>217 29.4</td>
<td>207 24.2</td>
</tr>
<tr>
<td>Stobie et al. (1983)</td>
<td>Wave B</td>
<td>LBF/1400/9 May 79</td>
<td>Yes</td>
<td>No</td>
<td>614 3.1</td>
<td>215 20.0</td>
<td>207 24.2</td>
</tr>
<tr>
<td>AVERAGE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>467 6.2</td>
<td>±141 ±2.5</td>
<td>±229 ±27.8</td>
</tr>
</tbody>
</table>

*Sounding format used: station/time (UTC)/date.

*Use was made of the ‘axis-tracking’ method by combining the two separate diagnosed waves into one highly arc-shaped wave front.

**Highly arc-shaped waves occurred after the development of strong thunderstorms in this case.

The wave propagation direction was highly irregular in this case. Local radar imagery was used to improve upon the stated overall wave motion.

Fig. 13. Scattergram plots of predicted vs observed (a) gravity wave propagation directions (°), and (b) gravity wave propagation speeds (m s⁻¹). Predicted values are derived from application of wave ducting theory, as discussed in the text. Heavy and light grey regions represent predicted regions of acceptable phase velocity predictions (±20° ±5 m s⁻¹).
independent estimates of the vector mean wind by the two authors of this paper agreed within 6° and 1 m s\(^{-1}\). Other tests showed that vertical interpolation of the sounding data to equal height increments was unnecessary. Thus, it appears that use of the mean wind vector in the conditionally unstable layer above stably stratified duct layers is a useful predictor of actual wave propagation vectors, with some qualification. When strong convection is present, it might be expected that wave fronts may deviate to the right of those that would otherwise be predicted. Koch et al. (1988) suggested that forcing of new convective cells along the thunderstorm gust front on the right side of the convective system can explain such behavior. A possible approach to take when strong convection is present is to use a value that is perhaps 30° rightward of the predicted direction as the TSC advection vector. Highly arc-shaped wave fronts occur in many of the reported cases, and unless this factor is taken into account, significant errors may result. It is shown below how special soundings or, by implication, mesoscale model fields can be used to predict spatially and/or temporally variable phase velocities.

4. Application to the STORM-FEST gravity wave case of 14 February 1992

a. The wave environment

The concepts and techniques described in the preceding sections are now tested on a gravity wave event that occurred on 14–15 February 1992 during STORM-FEST. The environment within which the wave formed is depicted in Fig. 14a at 1500 UTC 14 February. The analyses depicted here were produced from the 27-h forecasts made by the 60-km version of the MM4 mesoscale model that was run in real time in support of field operations (Cunning and Williams 1993). We note that this model dynamically assimilated many of the special observations that were taken during STORM-FEST, including the 3-h NWS and Cross-chain Linked Atmospheric Sounding System (CLASS) soundings, hourly wind profiler data, ARINC Communications Addressing and Reporting System aircraft observations, and surface data. Of course, the special sounding data and some of the surface data would not be available to the operational meso-Eta Model. Diagnostic analyses from the 20-km nested grid version of the model are shown in Fig. 14b.

The analyses produced from the MM4 model forecasts show that the local environment of the wave was consistent with the UK87 conceptual model (Fig. 2). First, a strong 300-hPa jet streak over southwestern Oklahoma located to the east of the upper-level trough (over eastern Colorado and New Mexico) was approaching the inflection axis in the height field, whereas the geostrophic wind maximum was over eastern New Mexico. Second, the gravity wave propagated along and north of a warm front ahead of a surface cyclone (Fig. 15). The wave first appeared as a singular wave of depression in southwestern Kansas at approximately the same time in the model and in the observations (1500 UTC). No precipitation was observed or simulated near the wave genesis region. In fact, the wave formed within a large-scale dry intrusion overspreading the cyclone from the southwest. This is somewhat atypical; in most cases, mesoscale gravity waves occur within or near the back edge of a precipitation shield. The wave eventually propagated farther ahead of the surface low and strong convection developed along the wave crest after 2000 UTC in the observations, but not in the model.

Having thus found a favorable environment for the wave, and employing the conceptual model in Fig. 2, the forecaster would, according to the precepts of Fig. 1,

1) issue a “wave alert” standby for a region extending from the Oklahoma panhandle downstream throughout most of Kansas, northern Oklahoma, Missouri, and possibly surrounding regions;
2) determine whether there are any indications that the flow is actually unbalanced to increase confidence in the wave alert; and
3) conduct a wave-ducting analysis to refine further the wave outlook region for the purposes of real-time monitoring with the ASOS detection system.

There are several indications from the model that the flow was unbalanced. The Texas panhandle existed in the exit region of the geostrophic wind maximum and the entrance region of the actual jet streak (Fig. 14a). Unbalanced ageostrophic winds (leftward-directed ageostrophic flow in the exit region of the geostrophic wind maximum) resulted in the eastern Texas panhandle (Fig. 14b). Furthermore, the sum of the terms in the nonlinear balance equation (Fig. 14b) also indicates unbalanced conditions in the same region. The NBE results consistently showed this small region of unbalance, which slowly propagated northeastward from 1300 to 1700 UTC (other maxima in the NBE field in eastern Kansas were transitory features related to convection).

Since this diagnosed imbalance area in the Texas panhandle occurred in the absence of any model precipitation, we can safely associate it with jet streak dynamics. Given the persistence of this feature and the fact that the gravity wave in the model (and in the observations) was triggered precisely in this region, it is suggested that the NBE field has short-term predictive capability with high precision. In summary, from an operational forecasting perspective, there are strong indications that a gravity wave should develop in the northern Texas panhandle and propagate northeastward into southwestern Kansas.

It might be wondered at this point whether this demonstration of the value of using unbalanced flow diagnostic fields for gravity wave prediction is a single exception. Another example of the value of computing the NBE terms using mesoscale model fields is demonstrated by the fact that Zack and Kaplan (1987) found a
Fig. 14. Environment of the STORM-FEST gravity wave event at 300 hPa as forecast by the MM4 model verifying at 1500 UTC 14 February 1992: (a) geopotential heights (solid, m), wind vectors and isotachs \( \pm 50 \text{ m s}^{-1} \) (thick solid, contour interval = 5 m s\(^{-1}\)), and isotachs of geostrophic wind \( \pm 58 \text{ m s}^{-1} \) (dashed, contour interval = 5 m s\(^{-1}\)) from the 60-km model grid; (b) ageostrophic streamlines and the sum of the terms in the nonlinear balance equation (shaded, contour interval = \(3 \times 10^{-3} \text{ s}^{-2}\)) from the 20-km model grid over the region depicted by the rectangle in (a).
larger NBE region to occur uniquely in the gravity wave generation region diagnosed from observations by Ferretti et al. (1988). This event occurred during a severe thunderstorm outbreak. Koch and Dorian (1988) also demonstrate the value of unbalanced flow fields to deduce gravity wave generation in a case of widespread severe convection.

A wave ducting analysis was conducted upon the special STORM-FEST soundings, as well as the model fields. The classic wave duct sounding (Fig. 4) was evident in every single sounding examined across Kansas, Missouri, Nebraska, Iowa, and Illinois, but not in Oklahoma or west Texas. A critical level was also present in the conditionally unstable layer above the duct ev-
erywhere and all the time (1200 UTC 14 Feb–0300 UTC 15 Feb) over this region. However, the depth of the stable duct layer was sufficient to support the observed gravity wave speed [as required by (6)] only over central and eastern Kansas, Missouri, and Illinois.

In order to most easily define the efficiency of the duct to a forecaster, we suggest that the following “duct factor” be computed with sounding and mesoscale model data:

$$DF = \Theta(800) - \Theta(950) + \Theta_e(800) - \Theta_e(400).$$  \hspace{1cm} (7)

The potential temperature $\Theta$ and equivalent potential temperature $\Theta_e$ at the noted pressure levels determine the duct factor. The reasoning behind this easily calculable parameter is that an efficient duct, according to the linear theory of Lindzen and Tung (1976) is one in which there exists a conditionally unstable layer (800–400 mb) above a very stable surface-based layer (950–800 mb). Although this parameter ignores the additional requirements of a sufficiently thick duct and the existence of a critical level, our experience is that the duct factor provides sufficient information for operational decision making to justify its use. Results with the duct factor computed from the STORM-FEST soundings (Fig. 16) show that the strongest wave duct was centered over central Kansas at 1200 UTC and over north-central Missouri by 0000 UTC 15 February. It is shown below that this behavior agrees very well with the observed track of the gravity wave. Quite similar results to those found in Fig. 16, though with greater detail, are derived from an analysis of the MM4 model forecast fields (Fig. 17). Since operational forecasters will not have access to special sounding datasets like those collected in STORM-FEST, but they will be able to readily obtain the sounding data, the duct factor provides a useful tool for operational decision making.

Fig. 16. Duct factor analysis (°C) derived from (7) using STORM-FEST soundings for (a) 1200 UTC 14 February and (b) 0000 UTC 15 February. Boxes are NWS sounding sites, triangles are CLASS sounding sites.

Fig. 17. Duct factor analysis from MM4 model forecasts valid at the same times shown in Fig. 16.
mesoscale model datasets from the meso-Eta Model, these intercomparisons are very important in demonstrating the potential for operational wave ducting analysis. In fact, as gravity waves are begun to be examined in an operational context, it will be interesting to compare computed duct factors with wave amplitudes from many cases, instead of just a single case as shown here.

b. Nature of the gravity wave in the surface data and mesoscale model

At this point in the flowchart (Fig. 1), the forecaster would have delineated a region extending from southwestern Kansas to western Illinois north of the warm front to be the area of greatest potential for gravity wave activity on this day. The forecaster would then activate the automated ASOS gravity wave detection system and monitor the perturbation pressure fields over this region, and relate these fields to the evolving precipitation patterns in the WSR-88D composite reflectivity and Geostationary Operational Environmental Satellite imagery.

The surface time series data reveal that the disturbance began as a solitary wave of depression ($T_1$) with an amplitude of approximately 1.5 hPa at DDC (Fig. 18). At this time, a mesoscale cloud band was developing in association with the gravity wave, but no precipitation was yet observed. As wave trough $T_1$ moved across Kansas it amplified, with the pressure drop exceeding 2.5 hPa between MHK and the dense mesonet in northeastern Kansas (Fig. 19). A weak pressure ridge ($R_1$) began to form just ahead of the trough in association with the development of precipitation within the cloud band. Intense convection developed as the gravity wave neared the Missouri border. During this period, the original wave of depression ($T_1$) began to weaken while the preceding pressure ridge ($R_1$) amplified (note station traces P16, P17, P18 in Fig. 18). In addition, a second ridge ($R_2$), also associated with convection, began to amplify in the wake of the first disturbance. Immediately thereafter, the wave disturbance became characterized by a double wave of elevation ($R_1$ and $R_2$), and a new trough ($T_2$) formed just ahead of the first ridge ($R_1$). This new trough took on a sharp appearance as a thunderstorm mesohigh developed within the cold outflow associated with $R_1$.

c. Objective phase velocity estimates

The isochrones of the original wave of depression ($T_1$) shown in Fig. 19 were obtained from painstakingly prepared subjective analyses of the pressure perturbation fields. The disturbance took the form of a pronounced arc that appeared to emanate from the Oklahoma panhandle. The vectors show subjectively estimated phase velocities for the initial wave of depression ranging in speed from 10 to 22 m s$^{-1}$ and in direction from 180° to 270°. These multiple wave propagation vectors, one at each station, were used in the TSC objective analyses that follow, since the range of values exceeds limits that resulted from the TSC tests (Fig.
12), even though most of the outliers were from the edges of the wave corridor, where the waves were weak and hard to track. While such an array of $C$ vectors is impressive and leads to the best set of wave analyses, it is impractical to think that such a labor-intensive estimation of wave vectors can be accomplished under operational constraints. Hence, we compared this range of vectors to the results from cross-spectral methods and to the results from the ducting theory prediction scheme described earlier.

The cross-spectral methods were tested on three selected station networks (marked with dashed lines on Fig. 19). Notice that one of these networks shares similar spatial distributions to the “superclusters” of ASOS stations near large metropolitan areas like Kansas City–Topeka. The station arrays over northeastern Kansas, eastern Kansas, and western Illinois feature average station spacings of 15, 90, and 116 km, respectively. The results (not shown) revealed that cross-spectral techniques produced phase velocities that were within 20 and 4.6 m s$^{-1}$ of the subjective value in the vicinity of all three networks. Most likely the success of the techniques reflects the presence of at most two high-amplitude waves, which reduces the chance for phase ambiguities. Cases featuring periodic waves of several cycles would be more sensitive to the station spacing. Nevertheless, these objective results certainly confirm our subjective estimates for the highly variable wave propagation vector.

The next question concerns whether the ducting theory prediction for $C$ would produce sufficiently accurate results for real-time application. We applied this method to all STORM-FEST soundings that were released at least 3 h, but no more than 9 h, before wave passage (to minimize the wave effects on the atmosphere while maximizing the predictive value). In addition, soundings selected must have exhibited a duct structure (Fig. 4). Table 3 shows the comparisons between the predicted and observed (from Fig. 19) phase velocities. Individual differences are generally no larger than 20 and 4.6 m s$^{-1}$, and mean differences are negligible. Clearly, this method holds much promise for predicting gravity wave phase velocities, just as was suggested by the earlier results from the independent dataset (Fig. 13 and Table 2).

d. Objective analysis of wave fields and relation to evolving precipitation patterns

The pressure and wind data from the surface stations shown in Fig. 19 were bandpass filtered in a way that did not significantly affect wave amplitudes in the range of 30-min to 3-h periodicity. Objective analyses of $p'$ and $u'$ were produced using both the conventional Barnes and TSC Barnes techniques. Since most phase speeds in the area of maximum wave amplitude were 15–20 m s$^{-1}$ (Table 3), and the spectral analysis revealed the wave period to be approximately 150 min, the wavelength of the primary wave trough $T_1$ is approximately 130–180 km. This wavelength is about equal to twice
the average station spacing over most of the network; thus, it could be anticipated that, with accurate knowledge of $C$, the TSC technique should be able to resolve the evolving structure of this gravity wave event with success, while the conventional Barnes scheme should fail. Selected results from the conventional Barnes and TSC Barnes analyses are presented in Fig. 20, and corresponding radar imagery with overlaid $p'$ isobars are shown in Fig. 21.

Because the wave trough and ridge were located primarily between stations in west-central Kansas at 1700 UTC, the wave is essentially absent in the conventional Barnes analysis. By contrast, the TSC Barnes analysis produces a distinct wave that exhibits one complete phase (two troughs and a ridge). Light rain was just beginning to occur with the singular wave at this time.

A well-defined rainband with a hailstorm at its southern end had developed along the leading ridge $R_1$ just ahead of the wave of depression $T_1$ by 2100 UTC (Fig. 21b). Notice that the conventional Barnes analysis produces a “bull’s-eye” pattern that only vaguely resembles the actual waves (Fig. 20). The TSC analysis suggests that the wave was approaching its maximum amplitude over northeastern Kansas and displayed a marked arc shape. Comparison of this analysis to the radar imagery reveals excellent correspondence between the location and shape of the rainband and that of the analyzed gravity wave. It is important to appreciate the highly nonlinear wave shape, meaning that the pressure fall following ridge $R_1$ and preceding trough $T_1$ was much steeper than the rise in pressure preceding $R_1$ (see station MHK in the meteorograms in Fig. 18). This non-sinusoidal pressure tendency pattern produces the appearance of a rainband leading the wave trough in the objectively analyzed pressure perturbation fields (Fig. 21b), whereas in actuality it is along the wave ridge that shortly precedes the trough axis. This relationship is consistent with the wave schematic (Fig. 3), particularly if there is a vertical tilt to the wave structure, which causes the rainband to be located precisely at the ridge axis instead of ahead of it (Koch and Golus 1988).

Strong convection continued developing within the rainband through 0000 UTC (Fig. 21c), as the singular wave evolved into a wave train. The TSC analysis shows that the original trough $T_1$ weakened over northwestern Missouri while the downstream ridge $R_1$ and trough $T_3$ amplified, as is consistent with the raw data (Fig. 18). The modest improvement of the conventional Barnes analysis at 0000 UTC compared to the earlier analyses (Fig. 20) is the result of the fact that the wave system lies within the densest part of the station network.

Finally, by 0300 UTC the original wave of depression can no longer be identified as the wave system and associated mesoscale rainband complex had progressed to Iowa, extreme northeastern Missouri, and western Illinois (Fig. 21d). The dominant feature in the TSC analysis is the arcing pressure ridge $R_1$. Although the conventional Barnes analysis reproduces the structure of the larger-scale trough–ridge couplet fairly well, the wavelength is considerably overestimated, the amplitude is greatly underestimated, and the presence of a double-ridge structure is not resolved.

Similar objective analyses were performed for the $u'$ fields (not shown). The patterns and overall evolution closely mirrored that of the $p'$ analyses. The pressure–wind correlation $\overline{p' u'}$ was calculated. The areas where values of $\overline{p' u'} > 0.80$ are shaded in Fig. 19. We draw the following conclusions from this analysis.

1. A high pressure–wind correlation within the area of maximum wave amplitude over northeastern Kansas supports the characterization of the initial pressure trough as a gravity wave.
2. The values of $\overline{p' u'}$ decreased as the wave crossed into Missouri, simultaneously with the onset of strong convection just ahead of the wave of depression. Such a behavior is consistent with that found in another case by Koch et al. (1988), who attributed the reduction of gravity wave signal to local feedback effects of strong convection.
3. A subsequent increase in $\overline{p' u'}$ occurred as the wave train traveled into northeastern Missouri and western Illinois, as the convection weakened in intensity. Since the convection did not disappear altogether,
these results suggest that the convection and waves may have evolved into some kind of equilibrium state (perhaps wave-CISK in nature).

Comparison of the MM4 forecasts (Fig. 15) with the objectively analyzed perturbation pressure fields (Fig. 20) reveals that the model accurately forecast the occurrence of the original wave of depression with an amplitude of ~1 hPa at 1500 UTC and 3 hPa 5 h later. This changing wave amplitude is in excellent agreement with the time series observations of the wave of depression (Fig. 18). The forecast wave location and shape agree well with the isochrone analysis of the observed wave (Fig. 20), though the forecast position was ~100 km too fast at 1500 UTC. However, the model forecasts departed significantly from the analyzed gravity wave after convection developed along the wave (not shown). Gravity wave–convection feedback interactions remain poorly understood and currently have poor demonstrated predictability.

5. Discussion and recommendations

The ability to operationally predict, detect, and analyze weather producing mesoscale gravity waves has
been discussed in the light of recent research findings and the results of sensitivity tests shown in this paper. Tests of easily implementable wave analysis techniques were conducted upon idealized waves, and the results were then applied to actual data collected during the 14 February 1992 gravity wave event in the STORM-FEST project. The stepwise process for prediction and real-time analysis of gravity waves has been discussed in terms of a flowchart that uses operationally available observational data and mesoscale models.

Reliable prediction of significant gravity wave occurrences hinges on the identification of a characteristic synoptic flow pattern in which such waves are systematically observed. This pattern was identified by Uccellini and Koch (1987) as one in which a jet streak is seen to propagate away from the upper-level trough axis toward the inflection axis in the height field. Mesoscale gravity waves appear to form near the inflection axis and north of a warm or stationary frontal boundary. More recent research has indicated that this kind of pattern is conducive to being dynamically unbalanced. Simple diagnostic indicators of such imbalance have been described herein, namely the nonlinear balance equation and a simplified form of the Lagrangian Rossby number. The ability to make real-time computations of these fields is only now becoming possible, due to the introduction of digital mesoscale model data (the NCEP meso-Eta Model) and complementary meteorological processing software (AWIPS) in NWS forecast offices.

The ability to operationally detect gravity waves hinges on the ability of NWS offices to acquire and process 5-min digital ASOS data. Processing consists of the application of a prespecified bandpass filter, an accurate method for predicting the wave propagation vector \( \mathbf{C} \), and the incorporation of time-to-space conversion to the Barnes objective analysis scheme (used in N-AWIPS), in which the vector \( \mathbf{C} \) is used for the TSC advection velocity. Subjective and cross-spectral methods for estimating \( \mathbf{C} \) were shown to be either impractical for real-time use, or subject to large uncertainties. Cross-spectral methods can only provide reliable estimates for relatively simple gravity wave systems. A real-time method for prediction of \( \mathbf{C} \) has been developed herein. The technique is based on a simplification of wave-ducting theory and entails the calculation of the vector mean wind in the conditionally unstable layer above a statically stable duct layer. Using data acquired from published accounts of mesoscale gravity waves, it was shown that sufficiently accurate estimates of \( \mathbf{C} \) (±20°, 5 m s\(^{-1}\)) are possible, but care must be taken when applied in the presence of strong convection. In addition, surface pressure fluctuations revealed by these filtering and objective analysis techniques must be properly interpreted to avoid confusion about their source. Only when the perturbation pressure–wind correlation is large can one be confident that such fluctuations are due to gravity waves, rather than convection, fronts, or some other source.

Application of these concepts and techniques to the STORM-FEST case met with success. The observed wave formed in a region consistent with the synoptic-scale conceptual model. The diagnostic indicators of unbalance calculated from mesoscale model fields clearly suggested the potential for a gravity wave event, and the ducting analyses conducted upon the STORM-FEST soundings and the model data further indicated that the wave, once formed, would be maintained by ducting processes. A highly arc-shaped wave of depression developed into a train of wave troughs and ridges. This nature to the wave fronts was captured only when spa-
tially variable advection vectors were used in the TSC Barnes scheme, whereas a conventional Barnes scheme failed in many respects, primarily because of the short 150-km wavelength (which is typical of mesoscale gravity waves). The high pressure–wind correlation clearly revealed that the disturbance was a gravity wave. The correlation weakened temporarily as strong convection developed within a pressure ridge ahead of the wave of depression. It appears that the initial gravity wave was temporarily disrupted by strong convection but eventually reached an equilibrium with the convection. The analyses also revealed that the solitary wave of depression evolved into a wave train and that the convective rainband intensified as the downstream ridge– trough couplet amplified while the original depression wave weakened. The final wave system structure featured a strong mesoscale pressure ridge under the precipitation band and a strong trough farther downstream. The MM4 model forecasts accurately depicted the wave genesis and the general characteristics of the wave but departed from reality once convection erupted along the gravity wave. The details of this transition will be discussed in a future paper involving the MM4 model analyses.

The techniques discussed in this paper hold the promise for operational application in the majority of mesoscale gravity wave events and, furthermore, allow the analyst to easily extract the wave signals from a background of other pressure signals, in particular the higher-frequency, convectively forced pressure systems. Real-time detection of mesoscale gravity waves is needed to improve our understanding of the wave environment, the effects of gravity waves on precipitation, and the practical utility of mesoscale models. Hence, the opportunity now exists for the weather forecast community to provide valuable information to the research community about these characteristics.

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APPENDIX

Mathematics of the Time-to-Space Conversion

Barnes Scheme

The Barnes (1973) scheme is a multiple-pass, successive corrections technique. There are four selectable parameters that determine the analysis: 1) the weight function $\kappa$ resulting from the data distribution, 2) a numerical convergence parameter $\gamma$ that effectively determines the weight function on successive correction passes, 3) the cutoff radius $r_c$ beyond which the weight function is set to zero, and 4) the number of correction passes applied. Barnes (1994b,c) has shown that the analysis error for derivative fields (such as divergence) can be minimized by using three or four passes. Nevertheless, the two-pass scheme used in GEMPAK (Koch et al. 1983) and AWIPS produces sufficiently accurate results of nonderivative fields like pressure perturbation being examined in this paper. The value of $r_c$ is chosen to be a very large number and is not really a factor here. The weight assigned to a datum on the first pass is governed by the distance between the station and the grid point being analyzed $(r)$, and the parameter $\kappa$ according to

$$w_1 = \exp(-r^2/\kappa), \quad (A1)$$

where in the GEMPAK Barnes scheme, $\kappa$ depends entirely on the average station spacing $\Delta n$ as follows (Koch et al. 1983):

$$\kappa = 5.052(2\Delta n/\pi)^2. \quad (A2)$$

The response function after this first pass through the data is given as

$$R_1 = \exp[-(\kappa(\pi/\lambda)^2] \quad (A3)$$

where $\lambda$ is the wavelength. Two passes are made through the data to achieve the full ability to resolve the finest scales, with the final response being controlled by the value of $\gamma$ ($0 < \gamma < 1$). A final response $R_2 = 0.37$ results for the $2\Delta n$ wave when $\gamma = 0.2$ and (A2) are used (the highest response allowed in the GEMPAK Barnes scheme). Thus, it is not possible to resolve disturbances with horizontal wavelengths equal to or smaller than twice the average station spacing. This limitation is not unique to the Barnes objective analysis technique, and in fact, in many cases a more conservative response is desired, particularly when attempting to calculate derivative fields from the data. Additional problems are encountered in choosing the “optimum” value of $\kappa$ when the stations are highly irregularly distributed; the GEMPAK Barnes scheme employs a “random data spacing” calculation for $\Delta n$, whereas a more complicated approach has been offered by Barnes (1994a).

Incorporation of off-time observations makes it possible to analyze the $2\Delta n$ waves in circumstances wherein a steadily propagating and well-behaved disturbance exists. The modified weight function, incorporating time-to-space conversion, is given by
\[ \tilde{u}_1 = \exp(-r^2/\tilde{k} - \Delta t^2/\tau). \]  \hspace{1cm} (A4)

Here, \( \Delta t \) is the difference between the analysis time and the data time, and \( \tau \) is a parameter that controls the time response. Since the goal of TSC is to increase the resolution of the analysis, \( \tilde{k} \) represents a modified value of \( k \) that allows for nearly full resolution of the \( 2\Delta n \) wave. If

\[ \tilde{k} = 0.693(2\Delta n/\pi)^2 \]  \hspace{1cm} (A5)

and \( \gamma = 0.2 \), a second-pass response of 0.94 results for the \( 2\Delta n \) wave. These values were used in all the analytical wave experiments shown herein.

The following objective method can be employed for determining \( \tau \) for operational purposes. Assume a rectangular array of stations each separated by distance \( \Delta n \), an analysis grid of resolution \( \Delta x = \Delta n/2 \), and a plane periodic wave of wavelength \( \lambda = 2\Delta n \) moving due east at speed \( C \). In this situation, recovery of the maximum wave amplitude between stations requires that the time series data extend to a distance of at least \( D = \Delta x = \Delta n/2 \) from each station. Thus, we require

\[ D = C \tau \geq \Delta n/2, \text{ or } \tau \geq \Delta n/(2C) = \lambda/(4C). \]  \hspace{1cm} (A6)

For \( C = 20 \text{ m s}^{-1} \) and \( \lambda = 150 \text{ km} \), this yields \( \tau = 31 \text{ min} \). However, as noted by Barnes (1973), there should be some spatial overlap of the time series data due to observing errors and wave evolution. For this example, using \( \tau = 62 \text{ min} \) would produce 100% overlap, while \( \tau = 46 \text{ min} \) would produce about 50% overlap. In a dataset with irregular data spacing, it would probably be wise to allow for at least 50% overlap. The tests described in section 3b support this recommendation.

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