Time-Dependent Global Energy Modeling

H. A. Dwyer and T. Petersen

*Dept. of Mechanical Engineering, University of California, Davis 95616*

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**ABSTRACT**

A time-dependent model equation of the energy balance of the earth-atmosphere-ocean is derived. A numerical solution to this basic energy equation is formulated. The earth is divided into $10^5$ latitudinal belts with a corresponding sea level temperature for each belt, which is computed yearly. The radiation relationships and the turbulent exchange coefficients are basically the same as used by Sellers.

A number of numerical experiments are performed with the conclusions that: 1) the heat capacity of the oceans play a dominant role in the model's transients; 2) the temperature of the polar latitudinal belts are very sensitive to changes in the temperate and equatorial zone; 3) future internal heat generation by man may cause substantial changes; and 4) with the present method of solution the double solution for the temperature found by Sellers does not exist.

1. **Introduction**

At the present time, there is a serious need to model the energy transport processes in the atmosphere because of the potential for change that has been developed by man. Over the past years, there have been many attempts to achieve this modeling goal, and the approaches have differed considerably. The most basic approach consists of applying the full equations of motion and energy to the atmosphere (averaged over some space and time scales) and then integrating these equations by some numerical technique (Wetherald and Manabe, 1972; Saltzman and Vernekar, 1971; Phillips, 1970). On a global scale, most of the research based on integration of the full equations of motion has not reached a sufficiently accurate state that it can be used for predicting changes in the global temperatures.

Another approach to global energy modeling is to perform an energy balance on the atmosphere and oceans over large latitudinal belts, and develop empirical transport coefficients based on present-day conditions. A model so constructed would be limited to small changes from present conditions, because it assumes no drastic change in atmospheric or ocean structures. The problem of small deviations from present conditions is, however, a significant one, since history has shown that major changes in the environment can be caused by only a few degrees change in sea level temperature. Also, this approach is at the present time much more tractable computationally, and does not involve the excessive amounts of computer time demanded by the more fundamental approach.

Much of the basic research for the energy balance approach has been done by Budyko (1969) and Sellers (1969). They have defined the most important physical processes in the earth-atmosphere system and have carried out steady-state calculations on this system. The paper by Sellers is the more theoretical of the two, but his conclusions are much the same as Budyko's (1970). Sellers performed an energy balance on latitudinal belts, which included radiation, sensible heat transfer in the atmosphere, sensible heat transfer in the ocean, and transport of water vapor. His results agreed with present day temperatures due to changes in atmospheric characteristics such as the albedo and solar constant. The major shortcomings of this work were the lack of transients in the calculations and the use of a crude numerical method which gave double values for a solution. The lack of time-dependent effects in the calculations can be very significant, since transients caused by the thermal heat capacity of the atmosphere and oceans could damp out many of the changes in latitudinal temperatures. Also, the removal of the double solutions would dismiss the speculation concerning their meaning for the stability of atmospheric circulations (Lorenz, 1968; Faegre, 1972).

The model of the global energy balance presented in this paper does include time, and it has been shown that the time-dependent influences do modify all interpretations of the results. Many of the problems that Sellers has posed will yield drastically different interpretations when time is considered as a factor. Also, the use of a better numerical solution technique with the time-dependent equations shows that double solutions do not exist for a properly posed problem.

2. **Basic model**

As mentioned previously, the model employed is based on the energy equation applied to $10^5$ latitudinal
belts in the atmosphere, land and ocean. The belts will be chosen to extend to the top of the atmosphere and to the bottom of the ocean. In deriving the basic equation for the model the energy equation in its time-dependent form was applied to a latitudinal belt (Batchelor, 1967). Instead of using the full energy equation, some simplifications can be accomplished due to approximations which can be justifiably employed. The first approximation is that the molecular heat conduction will be ignored compared to turbulent heat transport. Second, the enthalpy fluxes will be neglected at the bottom of the ocean and the top of the atmosphere. The third approximation is to neglect the local changes in the kinetic and potential energy since they are small compared with the thermal capacity. (Note that the fluxes of kinetic and potential energy between belts has been included implicitly in the transport coefficients.) The fourth approximation is to introduce an average temperature and properties for a latitudinal belt which will make the model axisymmetric around the north-south axis.

The only two further modifications of the energy equation required consist first of breaking up the velocity and temperature into their mean and fluctuating components and then time averaging over one year. Then, replacing the differences in \( \theta \) with derivates, the final equation has the form

\[
\left[ \rho \omega \frac{dT}{dt} + \rho \omega' \frac{dT'}{dt'} + \rho \omega \rho A(1 - \omega') \right] a \sin \theta \frac{dT}{dt} = \frac{R_s}{a} \sin \theta + S - \frac{c_p}{a} \frac{\Delta P}{g} \left[ \frac{\Delta T}{\Delta \theta} \sin \theta \right] + \frac{c_p}{a} \frac{\Delta T}{\Delta \theta} \sin \theta \frac{\Delta K}{\Delta \theta} + \frac{c_p}{a} \frac{\Delta T}{\Delta \theta} \sin \theta \frac{\Delta P}{\Delta \theta} \sin \theta \frac{\Delta T}{\Delta \theta} - a \frac{\Delta T}{\Delta \theta} \sin \theta \frac{\Delta P}{\Delta \theta}
\]

In the above equation the first term on the left-hand side represents the heat capacity of the air, land and oceans. In numerical value, this heat capacity is almost completely dominated by the ocean term with the other terms being quite small. The time-dependent change of kinetic and potential energy has not been included in (1), since the model we have developed assumes that there will not be a drastic change in atmospheric circulations. The first two terms on the right-hand side of the equation are the radiation and internal heat generation terms and will be discussed in later sections of the paper. The next two terms in the equations are the advection terms for the energy fluxes of sensible heat and water vapor in the atmosphere. The terms representing the diffusional fluxes of energy can be recognized by the eddy conductivities terms \( K_a \), \( K_o \) and \( K_w \), which are the conductivities for air, ocean and water vapor transport. The last term in the equation accounts for the energy needed to locally change the water vapor content of a latitudinal belt. As written in Eq. (1) we have assumed that all time changes in a belt’s water vapor content are due solely to the temperature of that particular belt and its relative humidity. This assumption is not crucial to our results since the last term in the equation is always less than 1% of the heat capacity term of the oceans. Also, in the derivation of (1), the two following assumptions were made: 1) the sensible heat transport in the ocean is due solely to “turbulent” transport; 2) all transport coefficients are related to the average sea level temperature of the belt (\( T \)).

To proceed further with Eq. (1) data must be found for the transport coefficient, specific humidity, radiation characteristics, and internal heat generation for the various latitudinal belts. In the present paper, extensive use of the data of Sellers (1969) will be employed. In that paper the transport coefficients \( K_a \), \( K_o \) and \( K_w \) were listed as functions of latitude. Also given were the pressure depth (\( \Delta P/g \)), percent ocean (\( \omega' \)), and surface albedo (\( \alpha \)) data. The expression for the net radiation \( R_s \) was

\[
R_s = Q_s (1 - \alpha) - I_s,
\]

where \( Q_s \) is the incident solar radiation and \( I_s \) the infrared emission to space.

The use of a mean meridional velocity (\( v_\theta \)) for the air and water vapor is absolutely necessary to avoid the use of negative transport coefficients. A modified version of Sellers expression is

\[
v_\theta = -A \left( \frac{\partial T}{\partial \theta} + \frac{\partial T}{\partial \theta} \right) \left[ \text{north of } 5N, \right]
\]

\[
v_\theta = -A \left( \frac{\partial T}{\partial \theta} - \frac{\partial T}{\partial \theta} \right) \left[ \text{south of } 5N, \right]
\]

where \( A \) is a function of latitude and \( |\partial T/\partial \theta| \) is the average value of the absolute value of the temperature gradient in all the belts. It should be clearly recognized that \( v_\theta \) is not a physical velocity, but a velocity chosen to give a logical energy transport structure.

The final information needed to complete Eq. (1) is to give the specific humidity for a belt. One solution is to assume that the air is saturated and use the Clausius-Clapeyron equation, and this was employed by Sellers. In the present work a relative humidity of approximately 75% was used throughout the investigation and this is more characteristic of the current conditions.

At this point in the paper it is suitable to make a comment on the eddy transport coefficients by Sellers. The most troublesome thing with these coefficients is
their very large values near the equator, which are not justified by physical reasoning. The reason that the values are so large is that Sellers has used an eddy conductivity model which implies that the energy flux is proportional to the mean annual temperature gradient. Near the equator, the temperature gradient becomes very small, but the energy flux remains finite, and therefore the eddy coefficient must become very large to transport the required energy. Another reason that the coefficients differ from the expected values is because of the fact that they represent averages over extremely large space and time scales. If the model would be changed to shorter time and space scales, the values of the coefficients would no doubt be more realistic; however, the complexity of the calculations would increase by orders of magnitude.1

3. Method of solution

The partial differential equation given by (1) is essentially a time-dependent version of the equation given by Sellers. This equation can be described mathematically as a nonlinear, second-order, parabolic partial differential equation. The nonlinearity is caused by the water vapor terms with their exponential dependence on temperature. In general, equations of this type can be difficult to solve, and one must be careful in choosing a method of solution. In the present paper a finite-difference method will be employed, and it will be shown that this method is superior to the technique used by Sellers.

1A complete formulation of the problem would show that the eddy conductivities should be represented in the form of a second-order symmetric tensor, and that the energy flux is not necessarily in the direction of mean temperature gradients.

In order to solve (1), initial conditions on the temperature distribution at some time and boundary must be known. For all of the problems solved, the initial conditions were chosen to be the existing yearly averages, and the boundary conditions were imposed at the poles. The boundary conditions at the poles are that the energy transport into the poles is zero (due to the area going to zero). This condition differs from the zero flux condition at the poles, since zero flux would imply ∂T/∂θ equals zero, and this is not the condition theoretically or experimentally.

The basic method of solution employed was an implicit finite-difference method similar to that used for the unsteady heat conduction equation. With this scheme all derivatives with respect to θ are evaluated at the unknown station i+Δt and the time derivative was evaluated as a backward difference from the conditions at i+Δt. The reason that the implicit scheme was chosen was because of its superior mathematical stability. Previous experience with highly nonlinear parabolic equations has shown that the stability characteristics of this scheme are necessary to get a valid solution.

One interesting aspect of the calculation was the way that the second derivative terms were evaluated. These derivatives were evaluated as the difference in two first-order derivatives because of the uncertainty in the data for the diffusivities. Fig. 1 shows a plot of the eddy diffusivity for sensible heat transport in the air as given by Sellers. As can be seen from Fig. 1, the derivative of ks can be quite large or small, and because of uncertainties in the data it was not thought that accurate derivatives could be obtained from these
data. Therefore, the second derivative terms were evaluated in the following manner:

$$\frac{\partial}{\partial \theta} \left( K_{\theta} \sin \theta \frac{\partial T}{\partial \theta} \right) = \frac{q_{i+1,j+1} - q_{i+1,j-1}}{\Delta \theta},$$

where

$$q_{i+1,j+1} = K_{\theta} j+1 \sin \theta \left( \frac{T_{i+1,j+1} - T_{i+1,j}}{\Delta \theta} \right),$$

and

$$q_{i+1,j-1} = K_{\theta} j-1 \sin \theta \left( \frac{T_{i+1,j-1} - T_{i+1,j}}{\Delta \theta} \right).$$

With this method derivatives of the diffusivities were not required, and accurate solutions were obtained.

In the calculation the nonlinear terms were linearized by evaluating the temperature-dependent coefficients at the previous time step. For the method to be accurate, iteration must be used or small time steps taken. For the calculations carried out in the present paper, it was found that a time step of one year was small enough to obtain converged solutions without iteration. With this scheme the temperatures at the unknown station $t+\Delta t$ had to be solved simultaneously; however, since the matrix of the coefficient of the temperature was tri-diagonal, this was accomplished in a direct fashion.

Another point that should be mentioned in regard to the method of solution is the double roots obtained by Sellers. With the present scheme there are no double roots, either for steady state or the transient conditions. The double roots obtained by Sellers were a direct result of the numerical method he chose to solve for the temperature. Therefore, any speculation concerning the meaning of the alternate solution is without physical basis. In fact, there is not enough physical modeling in Eq. (1) to make any statement concerning the stability of the atmosphere.

4. Results

For the present calculations it was decided that only 18 latitudinal belts should be employed because of the uncertainties in the transport coefficient data. With this number of belts, the calculations agreed exactly with present day conditions for the temperatures and thus checked Sellers' work. One problem that remained before time-dependent calculations could be carried out, was to determine the depth of ocean that would contribute to its heat capacity. To determine this depth, use was made of the fact that annual oscillations of temperature have been recorded as deep as 300 m Tverskoi (1965). Therefore, to be on the conservative side, two depths of ocean for the heat capacity were chosen as 60 and 600 m. (Note that this depth is not directly related to the mixed layer involved in daily transport.) From a study by Carsen and Moses (1963) the depth of land assumed was 3.5 m with the product of the specific heat and the density being 0.5 cal cm$^{-2}$ (°C)$^{-1}$.

To show the effect of the depth of ocean on the present model, calculations were carried out with various depths, and are shown in Fig. 2. Fig. 2 shows the time-dependent response of typical equatorial and polar latitudinal belts for a 2% decrease in the solar constant. Two ocean depths, 60 and 600 m, have been employed.

![Fig. 2. Influence of ocean depth on the model's transient response.](image-url)
In the 60 m case a steady state is reached at almost 50 years from the initial decrease in the solar constant. However, when the 600 m ocean is employed, steady state is not nearly obtained after 200 years. Clearly, the heat capacities of the ocean have a dominant effect on changes for the 600 m depth.\footnote{The present calculations have no phase lag between the ocean and atmosphere incorporated in them.}

One very interesting aspect of Fig. 2 is the relatively large changes in sea level temperature, 10–15°C, due to a 2% change in solar constant. These changes have been observed by Sellers, but the important role of the time lags has not been completely comprehended. This long time scale means that the results should be interpreted very cautiously, since changes in atmospheric radiation vary in time. For example, if sinusoidal variation of the solar constant with a period of 100 years is employed with a 2% amplitude, the net effect of the belt temperature will be drastically different. Fig. 3 shows that belt temperature changes are on the order of 1°C from the minimum to maximum location, and this is an order of magnitude reduction. The oceans with their large heat capacity have damped out the effect of solar constant fluctuations.

Another interesting point in Fig. 3 is the phase relationships in time and space. The equatorial belt follows the solar constant fairly rapidly, while in the polar belts a phase lag >90° is observed. The reason for the lag is due to the fact that the polar belts receive most of their energy from the other belts, and not from solar radiation. Therefore, they must wait for changes to occur in the equatorial belts before an effect is felt. The net result is that the polar belts follow the equatorial belts in the temperature fluctuations.

The final calculations which have been completed so far are those with internal heat generation. The source of the heat for the model is that projected to be used by man in the near future. Upper estimates by power specialists indicate that as much as 15 \( \frac{Q}{\text{year}} \) may be used by man in the next 50–100 years (Meyers, 1970). In order to estimate the influence of this energy source, calculations were carried out with an exponential increase in energy consumption to 15 \( \frac{Q}{\text{year}} \). Two different time constants for the increase were chosen, and these were 50 and 100 years. The results of the calculations are seen in Figs. 4 and 5. Fig. 4 shows the initial and final sea level temperature distributions along with the distribution curve for the heat generation in the latitudinal belts. The temperature changes were found to be of the order of 1–2°C with the large changes occurring at the poles. The fact that the largest changes existed near the poles is interesting, since the heat is actually released in the temperate zones. This calculation shows the sensitivity of the poles to changes in the larger and warmer temperate zones.\footnote{The present calculations may be somewhat over-sensitive due to the fact that a physical model of the polar ice sheets has not been included in the present work. Future studies will incorporate models of the polar caps.} Also, the changes in temperature calculated are substantial enough for internal heat generation to be a factor in future climatic changes. Fig. 5 illustrates the time-dependent aspects of the changes for the latitudinal belts 85°S for the two different time constants chosen. For the small constant case, the time phase lag is significant.

One point that should be discussed concerning the present calculations is the potential variation of the transport coefficients with changes in temperature. For example, it may be argued that the transport coeffici-
ents given by Sellers will change drastically with small changes in atmospheric temperatures. The counter-argument for this is that temperature must first change before the transport coefficient, and therefore, the question is really how far can the present model be used away from present day conditions. The answer to this question will only come with more experimental data or with future success of the more fundamental models discussed previously.

5. Conclusions

The major results and conclusions which have been obtained from the above research are:

1) A time-dependent model equation of the energy balance of the earth has been derived. The nature of this equation has been analyzed and an efficient method of solution has been developed.

2) The heat capacity of the ocean plays a dominant
role in transient processes in the environment. The neglect of the ocean heat capacity will lead to serious misinterpretation of results.

3) The temperatures of polar latitudinal belts are very sensitive to changes in the temperate and equatorial zones. Substantial time lags may, however, exist before the changes are felt.

4) Internal heat generation by man could cause substantial changes in global temperatures. Also, time lags do exist before the changes may be felt.

5) No double solutions for temperature exist for the present method of solution, and the existence of double solutions seems highly unlikely on both physical and mathematical grounds.

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APPENDIX

List of Symbols

\[ \theta \] latitude angle
\[ V \] volume
\[ v \] velocity

Subscripts
\[ a \] air
\[ o \] ocean
\[ l \] land
\[ w \] water vapor

REFERENCES


